

$$H|\varphi\rangle = E_n |\varphi\rangle$$

$$H|n \ell m_\ell m_s\rangle = E_n |n \ell m_\ell m_s\rangle$$

$$H = H_0 + H_1 \quad , \quad H_1 = \frac{\mu_B}{\hbar} (L + 2S) \vec{B} = \frac{\mu_B}{\hbar} B (L_z + 2S_z)$$

$$H|n \ell m_\ell m_s\rangle = E |n \ell m_\ell m_s\rangle$$

$$E = E_n + \mu_B B (m_\ell + 2m_s)$$

$$\Rightarrow n - |m_\ell|$$

12.2.2

$$\vec{\nabla}^2 = -\frac{1}{\hbar^2} (\hat{p}_\varphi^2 + \hat{p}_z^2 + \frac{1}{\rho^2} \hat{L}_z^2) \quad , \quad \hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + v(\hat{g})$$

$$[H, \hat{L}_z] = [H, \hat{p}_z] = [\hat{L}_z, \hat{p}_z] = 0$$

$$\Rightarrow H|\varphi\rangle = E|\varphi\rangle$$

$$\hat{L}_z|\varphi\rangle = \hbar m|\varphi\rangle$$

$$\hat{p}_z|\varphi\rangle = \hbar k|\varphi\rangle$$

Welche Struktur hat $|\varphi\rangle$?

$$|\varphi\rangle = \phi(g, \varphi, z) \equiv f(g) \phi(\varphi) z(z)$$

$$\hat{p}_z \phi(g, \varphi, z) = -i\hbar \frac{\partial}{\partial z} \phi(g, \varphi, z) = \hbar k \phi(g, \varphi, z)$$

$$\hat{L}_z \phi(g, \varphi, z) = -i\hbar \frac{\partial}{\partial \varphi} \phi(g, \varphi, z) = \hbar m \phi(g, \varphi, z)$$

$$\Rightarrow z(z) \sim e^{ikz}$$

$$\phi(\varphi) \sim e^{im\varphi}$$

$$\Rightarrow \phi(g, \varphi, z) = f(g) e^{ikz} e^{im\varphi}$$

k : reell

m : ganzzahlig

12.2.3

$$\begin{aligned}
 H\phi &= E\phi \Rightarrow \left(-\frac{\hbar^2}{2\mu} \nabla^2 + V(s)\right) \phi(s, \varphi, z) = E\phi(s, \varphi, z) \\
 &\Rightarrow \left(-\frac{\hbar^2}{2\mu} \left(\frac{\partial^2}{\partial s^2} + \frac{1}{s} \frac{\partial}{\partial s} + \frac{1}{s^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}\right) + V(s)\right) \phi = E\phi \\
 &\Rightarrow -\frac{\hbar^2}{2\mu} \left(f''e + \frac{1}{s} f'e - \frac{1}{s^2} m^2 f e - k^2 f e\right) + V(s) f e = E f e \\
 &\Rightarrow -\frac{\hbar^2}{2\mu} \left(f''e + \frac{1}{s} f'e - \frac{1}{s^2} m^2 f e - k^2 f e\right) = (E - V(s)) f e \\
 &\Rightarrow -\frac{\hbar^2}{2\mu} \frac{\partial^2 f}{\partial s^2} - \frac{\hbar^2}{2\mu s} \frac{\partial f}{\partial s} + f \left(V + \frac{m^2 \hbar^2}{2\mu s^2} + \frac{k^2 \hbar^2}{2\mu} - E\right) = 0
 \end{aligned}$$

12.2.4

$$\hat{Z}_Y \begin{pmatrix} s \\ \phi \\ z \end{pmatrix} \rightarrow \begin{pmatrix} s \\ -\phi \\ z \end{pmatrix}$$

$$[\hat{Z}_Y, p_s^2] = 0 = [\hat{Z}_Y, p_z^2]$$

$$[\hat{Z}_Y, L_z] = ?$$

$$\begin{aligned}
 Z_Y L_z \phi(\varphi) &= Z_Y \left(-i\hbar \frac{\partial \phi(\varphi)}{\partial \varphi}\right) = i\hbar \frac{\partial \phi(-\varphi)}{\partial \varphi} \\
 &= -L_z \phi(-\varphi) = -L_z Z_Y \phi(\varphi)
 \end{aligned}$$

$$Z_Y L_z^2 = -L_z Z_Y L_z = L_z^2 Z_Y \Rightarrow [Z_Y, L_z^2] = 0$$

$$\begin{aligned}
L_z(Z_Y \phi(\rho, \varphi, z)) &= L_z Z_Y f(\rho) e^{im\varphi} e^{ikz} \\
&= L_z f(\rho) e^{-im\varphi} e^{ikz} \\
&= -i\hbar \frac{\partial}{\partial \varphi} f(\rho) e^{-im\varphi} e^{ikz} \\
&= -f(\rho) e^{ikz} i\hbar (-im) e^{-im\varphi} \\
&= -\hbar m \phi(\rho, -\varphi, z) \\
&= -\hbar m Z_Y \phi(\rho, \varphi, z)
\end{aligned}$$

12.3.1

$$H = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega r^2, \quad \frac{i}{\hbar} [H, r \cdot p] = ?$$

$$\frac{i}{\hbar} [H, r \cdot p] = \frac{i}{\hbar} [H, r] p + \frac{i}{\hbar} r [H, p]$$

$$\begin{aligned}
\text{mit } [H, r] &= \frac{1}{2\mu} [p^2, r] = \frac{1}{2\mu} p [p, r] + \frac{1}{2\mu} [p, r] p \\
&= \frac{\hbar}{i\mu} p
\end{aligned}$$

$$\begin{aligned}
[H, p] &= \frac{1}{2}\mu\omega [r^2, p] = \frac{1}{2}\mu\omega r [r, p] + \frac{1}{2}\mu\omega [r, p] r \\
&= -\frac{\hbar}{i} \nabla V(r)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \frac{i}{\hbar} [H, r \cdot p] &= \frac{p^2}{\mu} - (r \nabla) V(r) \\
&= \frac{p^2}{\mu} - \left(r \frac{\partial}{\partial r}\right) \left(\frac{1}{2}\mu\omega r^2\right)
\end{aligned}$$

$$= \frac{p^2}{m} - r^2 \mu \omega = 2T - 2V$$

12.3.2

$$\begin{aligned} \langle \psi | [H, r p] | \psi \rangle &= \langle \psi | H r p | \psi \rangle - \langle \psi | r p H | \psi \rangle \\ &= E (\langle \psi | r p | \psi \rangle - \langle \psi | r p | \psi \rangle) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \langle \psi | [H, r p] | \psi \rangle &= 0 \\ &= \langle \psi | 2T - 2V | \psi \rangle \\ &= \langle \psi | T | \psi \rangle - \langle \psi | V | \psi \rangle \\ &\Rightarrow \langle T \rangle = \langle V \rangle \end{aligned}$$