$$H|\varphi\rangle = E_n|\varphi\rangle$$

$$H|n\,l\,m_l\,m_s\rangle = E_n|n\,l\,m_l\,m_s\rangle$$

$$H = H_0 + H_1 \quad , \quad H_1 = \frac{\mu_B}{\hbar}(L + 2S)\vec{B} = \frac{\mu_B}{\hbar}B(L_z + 2S_z)$$

$$H|n\,l\,m_l\,m_s\rangle = E|n\,l\,m_l\,m_s\rangle$$

$$E = E_n + \mu_B B(m_l + 2m_s)$$

$$\Rightarrow n - |m_l|$$

12.2.2

$$\hat{\nabla}^2 = -\frac{1}{\hbar^2}\left(\hat{p}_\rho^2 + \hat{p}_z^2 + \frac{1}{\rho^2}\hat{L}_z^2\right) \quad , \hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{\rho})$$

$$[H, L_z] = [H, \hat{P}_z] = [L_z, P_z] = 0$$

$$\Rightarrow \quad H|\varphi\rangle = E|\varphi\rangle$$
$$L_z|\varphi\rangle = \hbar m|\varphi\rangle$$
$$P_z|\varphi\rangle = \hbar k|\varphi\rangle$$

Welche Struktur hat $|\varphi\rangle$?

$$|\varphi\rangle = \phi(\rho, \varphi, z) \equiv f(\rho)\,\phi(\varphi)\,Z(z)$$

$$P_z\,\phi(\rho,\varphi,z) = -i\hbar\frac{\partial}{\partial z}\phi(\rho,\varphi,z) = \hbar k\,\phi(\rho,\varphi,z)$$

$$L_z\,\phi(\rho,\varphi,z) = -i\hbar\frac{\partial}{\partial\varphi}\phi(\rho,\varphi,z) = \hbar m\,\phi(\rho,\varphi,z)$$

$$\Rightarrow \quad Z(z) \sim e^{ikz}$$
$$\phi(\varphi) \sim e^{im\varphi}$$

$$\Rightarrow \quad \phi(\rho,\varphi,z) = f(\rho)\,e^{ikz}\,e^{im\varphi}$$

k: reell

m: ganzzahlig

12.2.3

$$H\phi = E\phi \Rightarrow \left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(\rho)\right)\phi(\rho,\varphi,z) = E\phi(\rho,\varphi,z)$$

$$\Rightarrow \left(-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}\right) + V(\rho)\right)\phi = E\phi$$

$$\Rightarrow -\frac{\hbar^2}{2\mu}\left(f''e + \frac{1}{\rho}f'e - \frac{1}{\rho^2}m^2fe - k^2fe\right) + V(\rho)fe = Efe$$

$$\Rightarrow -\frac{\hbar^2}{2\mu}\left(f''e + \frac{1}{\rho}f'e - \frac{1}{\rho^2}m^2fe - k^2fe\right) = (E - V(\rho))fe$$

$$\Rightarrow -\frac{\hbar^2}{2\mu}\frac{\partial^2 f}{\partial\rho^2} - \frac{\hbar^2}{2\mu\rho}\frac{\partial f}{\partial\rho} + f\left(V + \frac{m^2\hbar^2}{2\mu\rho^2} + \frac{k^2\hbar^2}{2\mu} - E\right) = 0$$

12.2.4

$$\hat{\Sigma}_y\begin{pmatrix}\rho\\ \phi\\ z\end{pmatrix} \to \begin{pmatrix}\rho\\ -\phi\\ z\end{pmatrix}$$

$$[\hat{\Sigma}_y, P_\rho^2] = 0 = [\hat{\Sigma}_y, P_z^2]$$

$$[\hat{\Sigma}_y, L_z] = ?$$

$$\Sigma_y L_z\phi(\varphi) = \Sigma_y\left(-i\hbar\frac{\partial\phi(\varphi)}{\partial\varphi}\right) = i\hbar\frac{\partial\phi(-\varphi)}{\partial\varphi}$$

$$= -L_z\phi(-\varphi) = -L_z\Sigma_y\phi(\varphi)$$

$$\Sigma_y L_z^2 = -L_z\Sigma_y L_z = L_z^2\Sigma_y \Rightarrow [\Sigma_y, L_z^2] = 0$$

$$L_z(Z_y \phi(\rho,\varphi,z)) = L_z\ Z_y f(\rho) e^{im\varphi} e^{ikz}$$
$$= L_z f(\rho) e^{-im\varphi} e^{ikz}$$
$$= -i\hbar \frac{\partial}{\partial\varphi} f(\rho) e^{-im\varphi} e^{ikz}$$
$$= - f(\rho) e^{ikz} i\hbar(-im) e^{-im\varphi}$$
$$= -\hbar m\ \phi(\rho, -\varphi, z)$$
$$= -\hbar m Z_y \phi(\rho, \varphi, z)$$

12.3.1

$$H = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega r^2 \quad , \quad \frac{i}{\hbar}[H, r\cdot p] = ?$$

$$\frac{i}{\hbar}[H, r\cdot p] = \frac{i}{\hbar}[H,r]p + \frac{i}{\hbar} r[H,p]$$

$$\text{mit } [H,r] = \frac{1}{2\mu}[p^2, r] = \frac{1}{2\mu} p[p,r] + \frac{1}{2\mu}[p,r]p$$
$$= \frac{\hbar}{i\mu} p$$

$$[H,p] = \frac{1}{2}\mu\omega[r^2,p] = \frac{1}{2}\mu\omega r[r,p] + \frac{1}{2}\mu\omega[r,p]r$$
$$= -\frac{\hbar}{i} \nabla V(r)$$

$$\Rightarrow \frac{i}{\hbar}[H, r\cdot p] = \frac{p^2}{\mu} - (r\nabla) V(r)$$
$$= \frac{p^2}{\mu} - \left(r\frac{\partial}{\partial r}\right)\left(\frac{1}{2}\mu\omega r^2\right)$$

$$= \frac{p^2}{\mu} - r^2\mu\omega = 2T - 2V$$

12.3.2

$$\langle \varphi | [H, rp] | \varphi \rangle = \langle \varphi | H rp | \varphi \rangle - \langle \varphi | rpH | \varphi \rangle$$

$$= E(\langle \varphi | rp | \varphi \rangle - \langle \varphi | rp | \varphi \rangle$$

$$= 0$$

$$\langle \varphi | [H, rp] | \varphi \rangle = 0$$

$$= \langle \varphi | 2T - 2V | \varphi \rangle$$

$$= \langle \varphi | T | \varphi \rangle - \langle \varphi | V | \varphi \rangle$$

$$\Rightarrow \langle T \rangle = \langle V \rangle$$