I)

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + g|x|\right)\psi(x) = E\psi(x)$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - g|x|\right)\right]\psi(x) = 0$$

$$\left[\frac{mE}{\hbar^2}\right] = L^{-2} \quad , \left[\frac{mg}{\hbar^2}\right] = L^{-3}$$

$$\left[\left(\frac{mE}{\hbar^2}\right)^3\right] = \left[\left(\frac{mg}{\hbar^2}\right)^2\right]$$

$$[E] = \left[\left(\frac{\hbar^2 g^2}{\cancel{2}m}\right)^{1/3}\right]$$

$$\Rightarrow \quad E_n = \left(\frac{\hbar^2 g^2}{m}\right)^{1/3} f(n)$$

$$\psi_{a,c}(x) = c\,\Theta(x+a)\Theta(a-x)\left(1 - \frac{|x|}{a}\right)$$

$$\hookrightarrow 1 \overset{!}{=} \int \psi^*(x)\,\psi(x)\,dx$$

$$= |c^2| \int \Theta(x+a)\Theta(a-x)\left(1 - \frac{|x|}{a}\right)^2 dx$$

$$= |c|^2 \int_{-a}^{a} \left(1 - \frac{|x|}{a}\right)^2 dx \quad = \frac{2a}{3}|c|^2$$

$$\Rightarrow |c|^2 = \frac{3}{2a}$$

$$\bar{H} = \int \psi^* H \psi dx$$

$$= \underbrace{-\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \psi^*(x) \frac{d^2}{dx^2} \psi(x) dx}_{(I)} + \underbrace{g \int_{-\infty}^{\infty} \psi^*(x) \, |x| \, \psi(x) \, dx}_{(II)}$$

$$(II) = |c|^2 \left(-\int_{-a}^{0} x \left(1 - \frac{x}{a}\right)^2 dx + \int_{0}^{a} x \left(1 - \frac{x}{a}\right)^2 dx \right)$$

$$= \frac{a^2 |c|^2}{6} = \frac{a}{4}$$

$$\frac{d}{dx} \psi(x) = c \, \delta(x+a) \, \theta(a-x) \left(1 - \frac{|x|}{a}\right)$$

$$- c \theta(x+a) \, \delta(x-a) \left(1 - \frac{|x|}{a}\right)$$

$$+ c \, \theta(x+a) \, \theta(x-a) \left(- \frac{|x|}{x} \frac{1}{a}\right)$$

$$(I) = \int_{-\infty}^{\infty} \psi^*(x) \frac{d^2}{dx^2} \psi(x)$$

$$= \psi^*(x) \frac{d}{dx} \psi(x) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx$$

$$= -|c|^2 \int_{-a}^{a} \left(- \frac{|x|}{xa}\right)^2 dx = - \frac{2|c|^2}{a} = - \frac{3}{a^2}$$

$$\Rightarrow \quad \bar{H} = \frac{3\hbar^2}{2ma} + \frac{ag}{4}$$

$$\psi_{a,c}(x) = c\,\Theta(x+a)\,\Theta(a-x)\left(1-\frac{|x|}{a}\right)$$

$$\frac{\delta \bar{H}}{\delta a} = -\frac{3\hbar^2}{ma^3} + \frac{g}{4} \overset{!}{=} 0 \quad \Rightarrow \quad a = \left(\frac{12\hbar^2}{gm}\right)^{1/3}$$

$$\bar{H} = \frac{3\hbar^2}{2m}\left(\frac{gm}{12\hbar^2}\right)^{2/3} + \frac{g}{4}\left(\frac{12\hbar^2}{gm}\right)^{1/3} = \frac{3}{4}\left(\frac{3\hbar^2 g^2}{2m}\right)^{1/3}$$

II) Zentralpot.: $f = 2\hbar \quad , s = \hbar$

$$H_{so} = a\hat{L}\hat{S} \qquad \mathcal{B} = \{H, J^2, J_z, L^2, S^2\}$$

$$\phi_{jm_jls} \equiv \phi : \quad J^2\phi_{jm_jls} = \hbar^2 j(j+1)\phi$$

$$L^2\phi_{jm_jls} = \hbar^2 l(l+1)\phi$$

$$S^2\phi = \hbar^2 s(s+1)\phi$$

$$J_z\phi = \hbar m_j\phi$$

$$J = L + S$$

$$\Rightarrow H_{so} = aLS = \frac{1}{2}a(J^2 - L^2 - S^2)$$

$$\Rightarrow \quad E_{SO} = \frac{\hbar^2}{2} a \left[\bar{j}(\bar{j}+1) - \ell(\ell+1) - s(s+1) \right]$$

$$= \begin{cases} 2a\hbar^2 & , \bar{j} = 3 \\ -a\hbar^2 & , \bar{j} = 2 \\ -3a\hbar^2 & , \bar{j} = 1 \end{cases}$$

Entartung:

$$d = 2\bar{j} + 1 = \begin{cases} 7 & , \bar{j} = 3 \\ 5 & , \bar{j} = 2 \\ 3 & , \bar{j} = 1 \end{cases}$$