Vertically averaged transport in a lake

Project Numerical Methods for Stochastic Differential Equations (TW3750TU)

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Consider a 2D Rectangular lake $(-1 \le x \le 1,\ -1 \le y \le 1)$ with water depth:

$$H(x,y) = 15 + 5x$$

The dispersion coefficients in $x-$ and $y-$ directions are respectively:

$$D_x(x,y) = 1 + \cos(\pi x)$$
$$D_y(x,y) = 1 + \cos(\pi y)$$

The flow of the water in $x-$ and $y-$ directions are respectively

$$u(x,y) = -y\left(1-x^2\right)/H(x,y)$$
$$v(x,y) = x\left(1-y^2\right)/H(x,y)$$

The initial release of a pollutant is at $x = \frac{1}{2},\ y = \frac{1}{2}$

a) Derive a particle model for the simulation of the spreading of pollutants based on the 2D vertically averaged advection diffusion equation.

b) Explain that a particle can never cross a boundary of the lake.

c) What is the stationary particle density for $t \to \infty$ (use your physical intuition here).

d) Implement the Euler and the Milstein scheme. Use these schemes for the simulation of the particles. Study by means of simulations the convergence behavior of the schemes for $\Delta t \to 0$ in the strong sense. Discuss the results.

e) Verify experimentally the weak order of convergence. Discuss the results.

f) Because of numerical errors, particles can leave the domain of the problem. Study the performance of the various numerical schemes with respect to this numerical artefact.