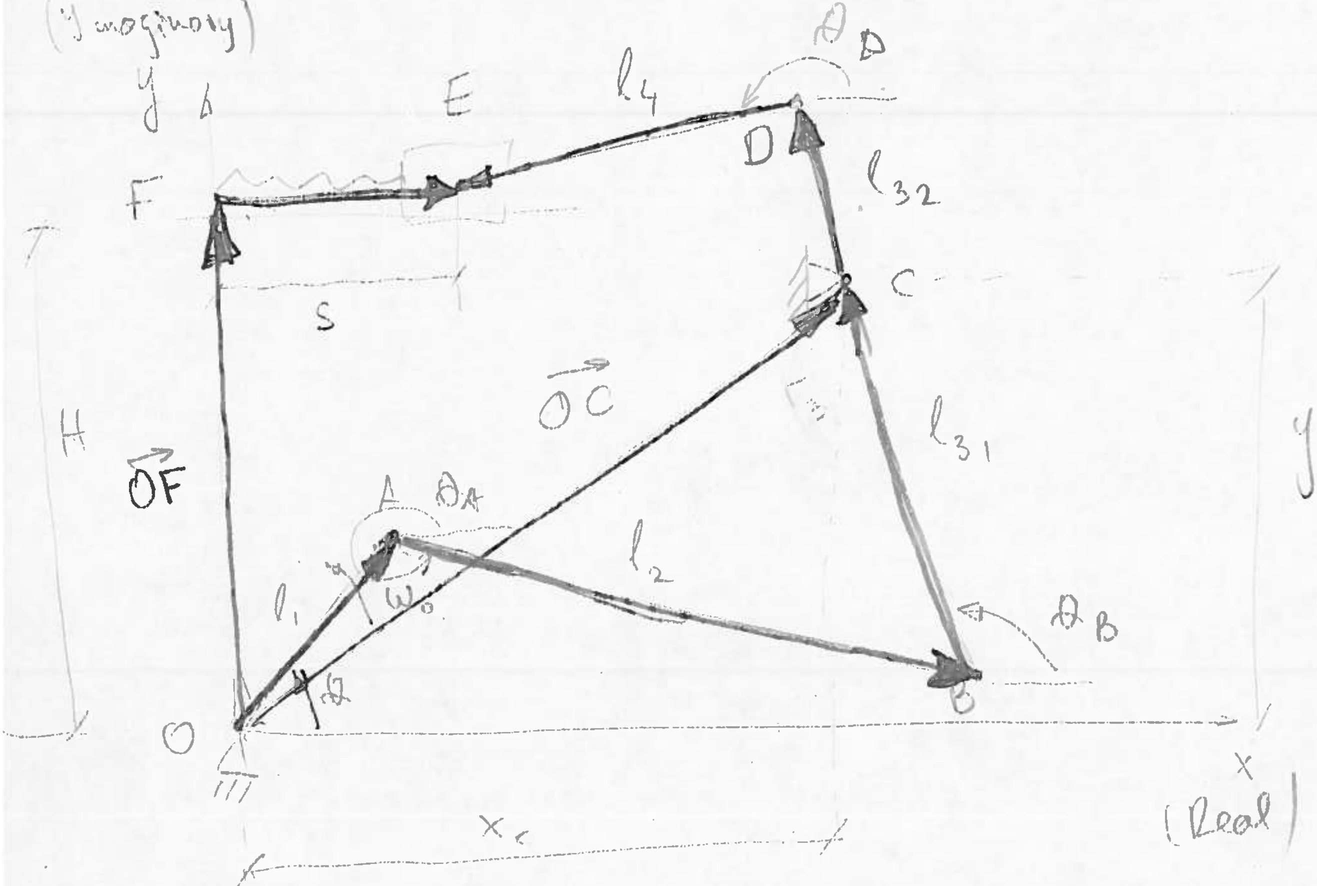


$l_3 = l_{31} + l_{32}$

$x_c = 5\,in$ $\qquad l_1 = 0.5\,in$

$y_c = 2\,in$ $\qquad k = 1\ \frac{lbf}{in}$

$H = 3\,in$

Position

$$\vec{OC} + \vec{r_{32}} + \vec{r_4} - \vec{s} - \vec{OF} = 0$$

x: $\quad x_c + l_{32}\cos\theta_B + l_4\cos\theta_D - s = 0 \qquad (1)$

y: $\quad y_c + l_{32}\sin\theta_B + l_4\sin\theta_D - H = 0 \qquad (2)$

$$\vec{r_1} + \vec{r_2} + \vec{r_{31}} - \vec{OC} = 0$$

x: $\quad l_1\cos\theta + l_2\cos\theta_A + l_{31}\cos\theta_B - x_c = 0 \qquad (3)$

y: $\quad l_1\sin\theta + l_2\sin\theta_A + l_{31}\sin\theta_B - y_c = 0 \qquad (4)$

unknowns: $\theta_B,\ \theta_D,\ s,\ \theta_A$

Velocity

$$\vec{OC} + l_{32} e^{i\theta_B} + l_4 e^{i\theta_D} - s\cancel{e^{i0}}^{1} - \vec{OF} = 0 \quad \Big/ \frac{d}{dt}$$

$$l_{32} e^{i(\theta_B + \frac{\pi}{2})} \dot{\theta}_B + l_4 e^{i(\theta_D + \frac{\pi}{2})} \dot{\theta}_D - \dot{s} = 0$$

$$x:\ l_{32} \dot{\theta}_B \cos\left(\theta_B + \frac{\pi}{2}\right) + l_4 \dot{\theta}_D \cos\left(\theta_D + \frac{\pi}{2}\right) - \dot{s} = 0 \quad (1)$$

$$y:\ l_{32} \dot{\theta}_B \sin\left(\theta_B + \frac{\pi}{2}\right) + l_4 \dot{\theta}_D \sin\left(\theta_D + \frac{\pi}{2}\right) = 0 \quad (2)$$

$$l_1 e^{i\theta} + l_2 e^{i\theta_A} + l_{31} e^{i\theta_B} - \vec{OC} = 0 \quad \Big/ \frac{d}{dt}$$

$$l_1 \dot{\theta} e^{i(\theta + \frac{\pi}{2})} + l_2 \dot{\theta}_A e^{i(\theta_A + \frac{\pi}{2})} + l_{31} \dot{\theta}_B e^{i(\theta_B + \frac{\pi}{2})} = 0$$

$$x:\ l_1 \cos\left(\theta + \frac{\pi}{2}\right) + l_2 \dot{\theta}_A \cos\left(\theta_A + \frac{\pi}{2}\right) + l_{31} \dot{\theta}_B \cos\left(\theta_B + \frac{\pi}{2}\right) = 0 \quad (3)$$

$$y:\ l_1 \sin\left(\theta + \frac{\pi}{2}\right) + l_2 \dot{\theta}_A \sin\left(\theta_A + \frac{\pi}{2}\right) + l_{31} \dot{\theta}_B \sin\left(\theta_B + \frac{\pi}{2}\right) = 0 \quad (4)$$

unknowns: $\dot{\theta}_B$, $\dot{\theta}_D$, $\dot{s}$, $\dot{\theta}_A$

Acceleration

$$l_{32} e^{i(\theta_B + \pi)} \dot{\theta}_B^2 + l_{32} e^{i(\theta_B + \frac{\pi}{2})} \ddot{\theta}_B + l_4 e^{i(\theta_D + \pi)} \dot{\theta}_D^2 +$$

$$+ l_4 e^{i(\theta_D + \frac{\pi}{2})} \ddot{\theta}_D - \ddot{s} = 0$$

$$x:\ \dot{\theta}_B^2 l_{32} \cos(\theta_B + \pi) + \ddot{\theta}_B l_{32} \cos\left(\theta_B + \frac{\pi}{2}\right) + l_4 \dot{\theta}_D^2 \cos(\theta_D + \pi) +$$

$$+ \ddot{\theta}_D l_4 \cos\left(\theta_D + \frac{\pi}{2}\right) - \ddot{s} = 0 \quad (1)$$

$$y:\ \dot{\theta}_B^2 l_{32} \sin(\theta_B + \pi) + \ddot{\theta}_B l_{32} \sin\left(\theta_B + \frac{\pi}{2}\right) + l_4 \dot{\theta}_D^2 \sin(\theta_D + \pi) +$$

$$+ \ddot{\theta}_D l_4 \sin\left(\theta_D + \frac{\pi}{2}\right) = 0 \quad (2)$$

$$l_1 e^{i(\theta+\pi)}\dot{\theta}^2 + l_2\dot{\theta}_A^2 e^{i(\theta_A+\pi)} + l_2\ddot{\theta}_A e^{i(\theta_A+\frac{\pi}{2})} + l_{31}\dot{\theta}_B^2 e^{i(\theta_B+\pi)} +$$

$$+ l_{31}\ddot{\theta}_B e^{i(\theta_B+\frac{\pi}{2})} = 0$$

$$: l_1\dot{\theta}^2\cos(\theta+\pi) + l_2\dot{\theta}_A^2\cos(\theta_A+\pi) + l_2\ddot{\theta}_A\cos(\theta_A+\frac{\pi}{2}) + l_{31}\dot{\theta}_B^2\cos(\theta_B+\pi) + l_{31}\ddot{\theta}_B\cos(\theta_B+\frac{\pi}{2}) =$$

$$: l_1\dot{\theta}^2\sin(\theta+\pi) + l_2\dot{\theta}_A^2\cos(\theta_A+\pi) + l_2\ddot{\theta}_A\sin(\theta_A+\frac{\pi}{2}) + l_{31}\dot{\theta}_B^2\sin(\theta_B+\pi) + l_{31}\ddot{\theta}_B\sin(\theta_B+\frac{\pi}{2}) = 0$$

unknowns: $\ddot{\theta}_B$, $\ddot{\theta}_D$, $\ddot{s}$, $\ddot{\theta}_A$

$$\vec{r}_A = l_1 e^{i\theta} \quad / \frac{d}{dt}$$

$$\vec{V}_A = l_1\dot{\theta} e^{i(\theta+\frac{\pi}{2})}$$

$$x: \quad V_{Ax} = l_1\dot{\theta}\cdot\cos(\theta+\frac{\pi}{2})$$

$$y: \quad V_{Ay} = l_1\dot{\theta}\cdot\sin(\theta+\frac{\pi}{2})$$

$$\vec{a}_A = l_1\dot{\theta}^2 e^{i(\theta+\pi)}$$

$$x: \quad a_{Ax} = l_1\dot{\theta}^2\cos(\theta+\pi)$$

$$y: \quad a_{Ay} = l_1\dot{\theta}^2\sin(\theta+\pi)$$

Element 1

$$\vec{r}_{cm,1} = \frac{l_1}{2} e^{i\theta}$$

$$\vec{v}_{cm,1} = \frac{l_1}{2} \dot{\theta} e^{i(\theta+\frac{\pi}{2})}$$

$$\vec{a}_{cm,1} = \frac{l_1}{2} \dot{\theta}^2 e^{i(\theta+\pi)}$$

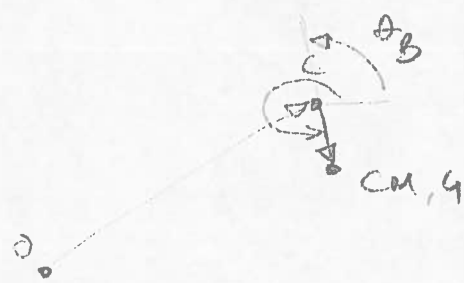
element 2

$$\vec{r}_{cm,2} = \vec{r}_A + \frac{l_2}{2} e^{i\theta_A}$$

$$\vec{v}_{cm,2} = \vec{v}_A + \frac{l_2}{2} \dot{\theta}_A e^{i(\theta_A+\frac{\pi}{2})}$$

$$\vec{a}_{cm,2} = \vec{a}_A + \frac{l_2}{2} \ddot{\theta}_A e^{i(\theta_A+\frac{\pi}{2})} + \frac{l_2}{2} \dot{\theta}_A^2 e^{i(\theta_A+\pi)}$$

element 3



$$\vec{r}_{cm,3} = \vec{OC} + \frac{l_{31}+l_{32}}{2} e^{i(\theta_B+\pi)}$$

$$\vec{v}_{cm,3} = \frac{l_{31}+l_{32}}{2} \dot{\theta}_B e^{i(\theta_B+\frac{3\pi}{2})}$$

$$\vec{a}_{cm,3} = \frac{l_{31}+l_{32}}{2} \ddot{\theta}_B e^{i(\theta_B+\frac{3\pi}{2})} + \frac{l_{31}+l_{32}}{2} \dot{\theta}_B^2 e^{i(\theta_B+2\pi)}$$

element 4

$$\vec{r}_{cm,4} = \vec{OC} + l_{32} e^{i\theta_B} + \frac{l_4}{2} e^{i\theta_D}$$

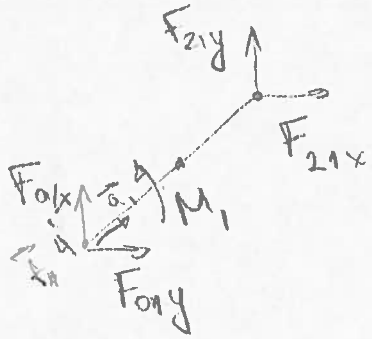
$$\vec{v}_{cm,4} = l_{32} \theta_B e^{i(\theta_B+\frac{\pi}{2})} + \frac{l_4}{2} e^{i(\theta_D+\frac{\pi}{2})} \dot{\theta}_D$$

$$\vec{a}_{cm,4} = l_{32} \ddot{\theta}_B e^{i(\theta_B+\frac{\pi}{2})} + l_{32} \dot{\theta}_B^2 e^{i(\theta_B+\pi)} +$$

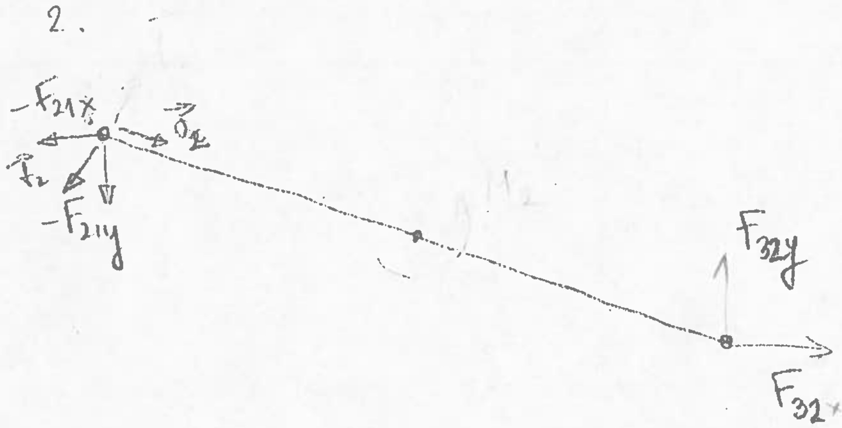
$$+ \frac{l_4}{2} \ddot{\theta}_D e^{i(\theta_D+\frac{\pi}{2})} + \frac{l_4}{2} \dot{\theta}_D^2 e^{i(\theta_D+\pi)}$$

Loads

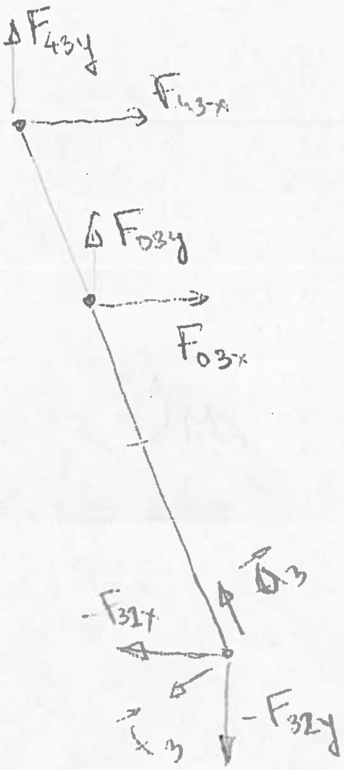
1.



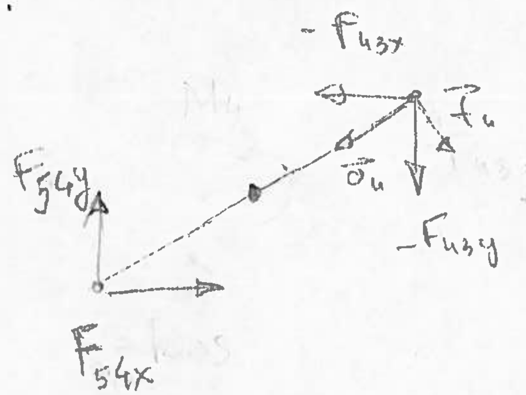
2.



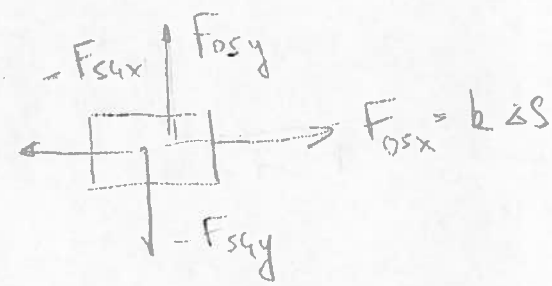
3.



4.



5.



element 1

$$\Sigma X = 0 \qquad F_{01x} + F_{21x} = m_1 a_{cm,1x} \qquad (1)$$

$$\Sigma Y = 0 \qquad F_{01y} + F_{21y} = m_1 a_{cm,1y} \qquad (2)$$

$$\Sigma M_{cm1} = 0 \qquad M_1 + R_{01x} F_{01y} - R_{01y} F_{01x} + R_{21x} F_{21y} - R_{21y} F_{21x} = 0 \qquad (3)$$

element 2

$$\sum X = 0 \quad -F_{21x} + F_{32x} = m_2 a_{cm,2x} \qquad (4)$$

$$\sum Y = 0 \quad -F_{21y} + F_{32y} = m_2 a_{cm,2y} \qquad (5)$$

$$\sum M = 0 \quad -R_{12x} F_{21y} + R_{12y} F_{21y} + R_{32x} F_{32y} - R_{32y} F_{32x} = I_2 \varepsilon_2 \qquad (6)$$

element 3

$$\sum X = 0 \quad -F_{32x} + F_{43x} + F_{03x} = 0 \qquad (7)$$

$$\sum Y = 0 \quad -F_{32y} + F_{43y} + F_{03y} = 0 \qquad (8)$$

$$\sum M = 0 \quad -R_{23x} F_{32y} + R_{23y} F_{32x} + R_{43x} F_{43y} - R_{43y} F_{43x} + R_{03x} F_{03y} - R_{03y} F_{03x} = I_3 \varepsilon_3 \qquad (9)$$

element 4

$$\sum X = 0 \quad F_{54x} - F_{43x} = m_4 a_{cm4,x} \qquad (10)$$

$$\sum Y = 0 \quad F_{54y} - F_{43y} = m_4 a_{cm4,y} \qquad (11)$$

$$\sum M = 0 \quad -R_{34x} F_{43y} + R_{34y} F_{43x} + R_{54x} F_{54y} - R_{54y} F_{54x} = I_4 \varepsilon_4 \qquad (12)$$

element 5

$$\sum X \quad -F_{54x} + F_{05x} = m_5 a_{5,x} \Rightarrow F_{54x} = F_{05x} - m_5 a_{5,x}$$

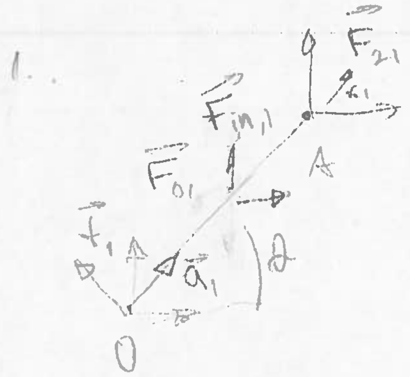
$$\sum Y \quad -F_{54y} + F_{05y} = 0 \Rightarrow F_{05y} = F_{54y}$$

12 equations

unknowns: F_{01x}, F_{01y}, F_{21x}, F_{21y}, M_1, F_{32x}, F_{32y}, F_{43x}, F_{43y}, F_{54y}, F_{03x}, F_{03y}

Shear force, axial force and bending moment diagrams

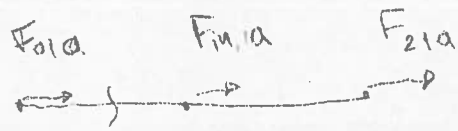
1.



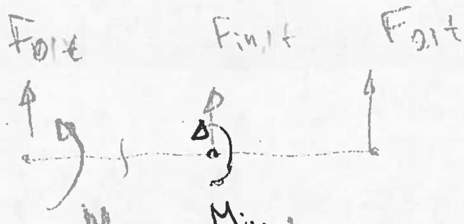
$$\vec{a}_1 = \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} \quad ; \quad \vec{t}_1 = \begin{Bmatrix} \cos(\theta + \frac{\pi}{2}) \\ \sin(\theta + \frac{\pi}{2}) \end{Bmatrix}$$

$$F_{01a} = \vec{F}_{01} \cdot \vec{a}_1 \quad ; \quad F_{01t} = \vec{F}_{01} \cdot \vec{t}_1$$

$$\vec{F}_{in,1} = \begin{Bmatrix} -m_1 a_{cw,1x} \\ -m_1 a_{cw,1y} \end{Bmatrix} \quad ; \quad M_{in,1} = -J_1 \varepsilon_1 = 0$$



$$A_1 = \begin{cases} -F_{01a} & x_1 \le \frac{l_1}{2} \\ F_{21a} & x_1 > \frac{l_1}{2} \end{cases}$$

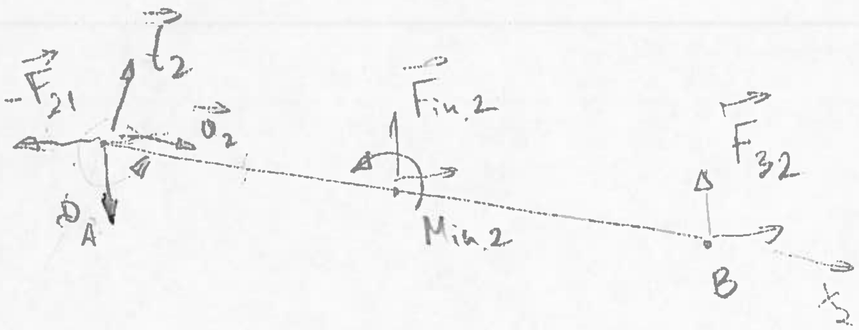


$$V_1 = \begin{cases} F_{01t} & x_1 < \frac{l_1}{2} \\ -F_{21} & x_1 > \frac{l_1}{2} \end{cases}$$

$$M_1 = \begin{cases} F_{01t} \cdot x_1 + M_1 & x_1 < \frac{l_1}{2} \\ F_{21t} \cdot (l_1 - x_1) & x_1 > \frac{l_1}{2} \end{cases}$$

2.

$$\vec{a}_2 = \begin{Bmatrix} \cos(\theta_A) \\ \sin(\theta_A) \end{Bmatrix}, \quad \vec{t}_2 = \begin{Bmatrix} \cos(\theta_A + \frac{\pi}{2}) \\ \sin(\theta_A + \frac{\pi}{2}) \end{Bmatrix}$$



$$F_{12a} = -\vec{F}_{21} \cdot \vec{a}_2$$

$$F_{12t} = -\vec{F}_{21} \cdot \vec{t}_2$$

$$F_{32a} = \vec{F}_{32} \cdot \vec{a}_2$$

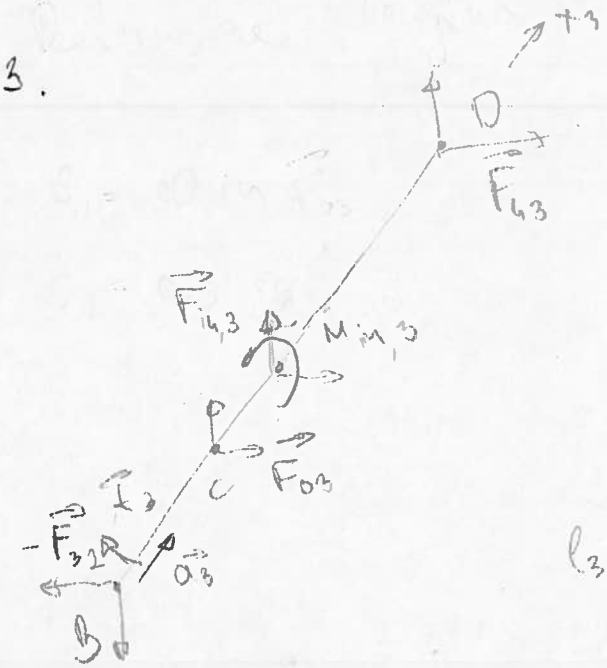
$$F_{32t} = \vec{F}_{32} \cdot \vec{t}_2$$

$$A_2 = \begin{cases} -F_{12a} & x_2 < \frac{l_2}{2} \\ F_{32a} & x_2 > \frac{l_2}{2} \end{cases}$$

$$V_2 = \begin{cases} F_{12t} & x_2 < \frac{l_2}{2} \\ -F_{32t} & x_2 > \frac{l_2}{2} \end{cases}$$

$$M_2 = \begin{cases} F_{12t} \cdot x_2 & x_2 < \frac{l_2}{2} \\ F_{32t} \cdot (l_2 - x_2) & x_2 > \frac{l_2}{2} \end{cases}$$

3.



$$A_3 = \begin{cases} -F_{32a} & x_3 < l_{31} \\ -F_{32a} - F_{03a} & l_{31} < x_3 < \frac{l_{31}+l_{32}}{2} \\ F_{43a} & x_3 > \frac{l_{31}+l_{32}}{2} \end{cases}$$

$l_{31} < l_{32}$

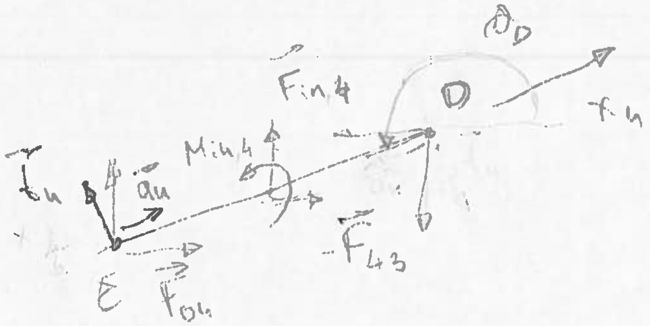
$$F_{32a} = -\vec{F}_{32} \cdot \vec{a}_3$$
$$F_{32t} = -\vec{F}_{32} \cdot \vec{t}_3$$

Lengths l_{31} and l_{32} affect these graphs.

$$V_3 = \begin{cases} F_{32t} & x_3 < l_{31} \\ F_{32t} + F_{03t} & l_{31} < x_3 < \frac{l_{31}+l_{32}}{2} \\ -F_{43t} & x_3 > \frac{l_{31}+l_{32}}{2} \end{cases}$$

$$M_3 = \begin{cases} F_{32t} \cdot x_3 & x_3 < l_{31} \\ F_{32t} \cdot x_3 + F_{03t} \cdot (x_3 - l_{31}) & l_{31} < x_3 < \frac{l_{31}+l_{32}}{2} \\ F_{43t} \cdot (l_{31} + l_{32} - x_3) & x_3 > \frac{l_{31}+l_{32}}{2} \end{cases}$$

4.



$$\vec{a}_4 = \begin{Bmatrix} -\cos\theta_D \\ -\sin\theta_D \end{Bmatrix} ; \vec{t}_4 = \begin{Bmatrix} -\cos(\theta_D+\frac{\pi}{2}) \\ -\sin(\theta_D+\frac{\pi}{2}) \end{Bmatrix}$$

$$F_{34a} = -\vec{F}_{43} \cdot \vec{a}_4 \quad ; F_{54a} = \vec{F}_{54} \cdot \vec{a}_4$$

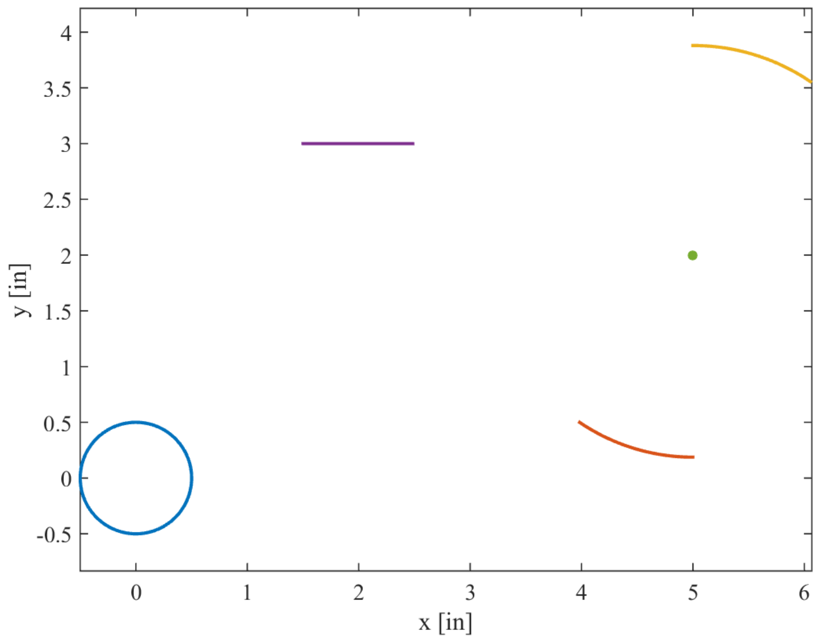
$$F_{34t} = -\vec{F}_{43} \cdot \vec{t}_4 \quad ; F_{54t} = \vec{F}_{54} \cdot \vec{t}_4$$

$$A_3 = \begin{cases} -F_{54a} & x_4 < \frac{l_4}{2} \\ F_{34a} & x_4 > \frac{l_4}{2} \end{cases}$$

$$V_3 = \begin{cases} F_{54t} & x_4 < \frac{l_4}{2} \\ -F_{34t} & x_4 > \frac{l_4}{2} \end{cases}$$

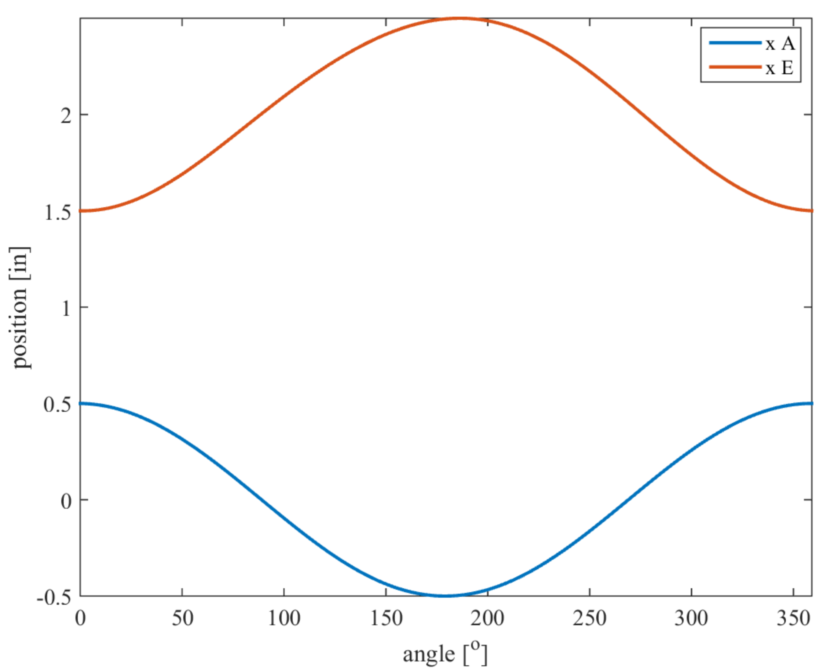
$$M_4 = \begin{cases} F_{54t} \cdot x_4 & x_4 < \frac{l_4}{2} \\ F_{34t}(l_4 - x_4) & x_4 > \frac{l_4}{2} \end{cases}$$

b)



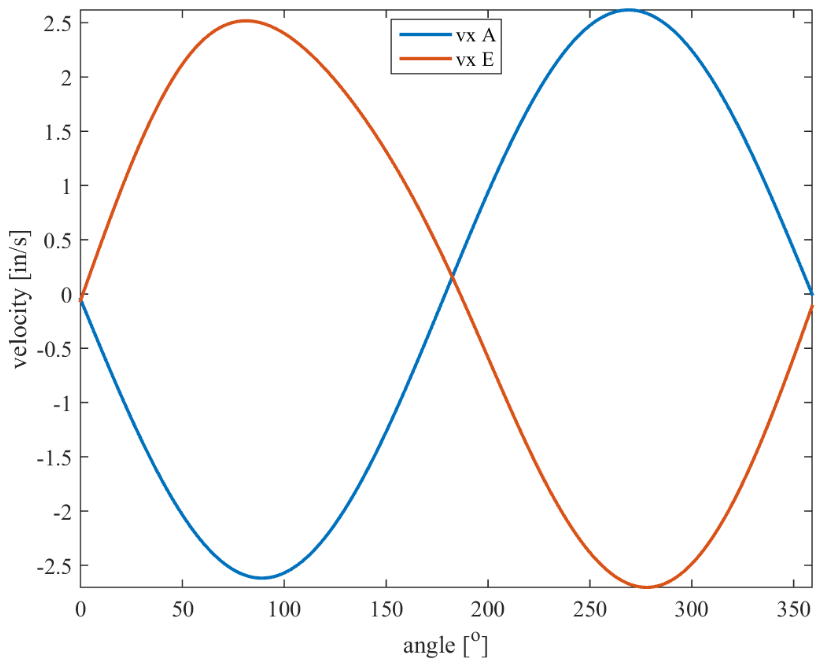
y [in]
x [in]
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
6

c)

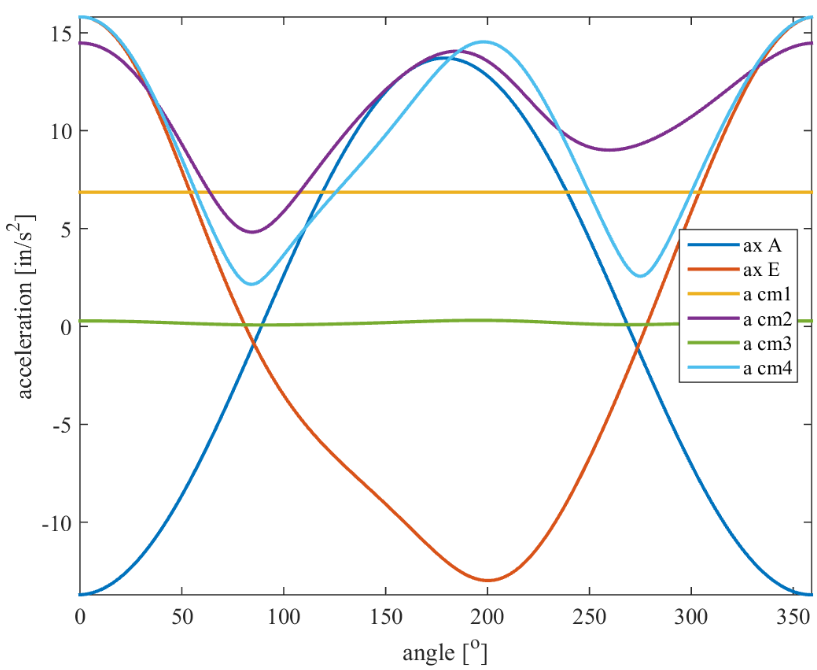


x A
x E
position [in]
angle [^{o}]
2
1.5
1
0.5
0
-0.5
0
50
100
150
200
250
300
350

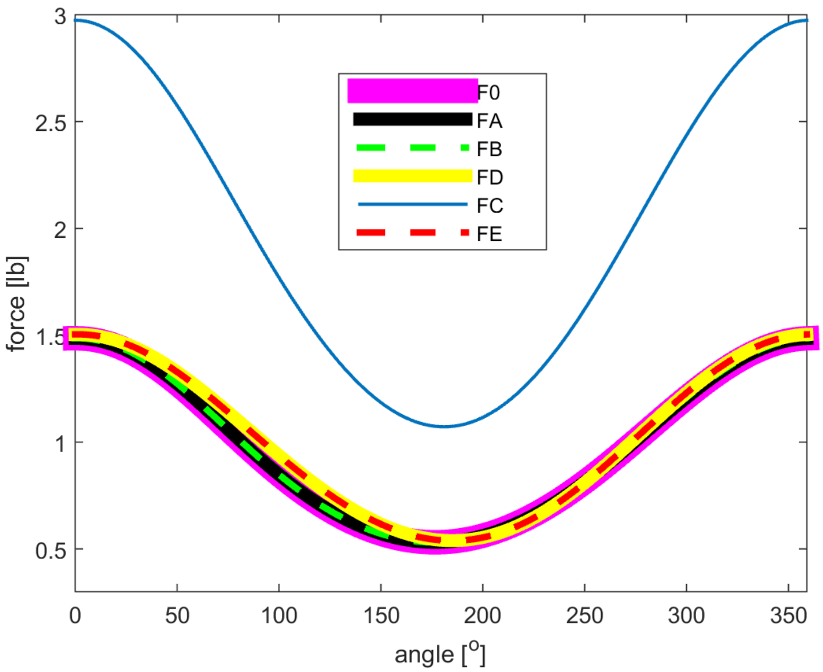
d)


vx A
vx E
velocity [in/s]
angle [°]
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
50
100
150
200
250
300
350

e)

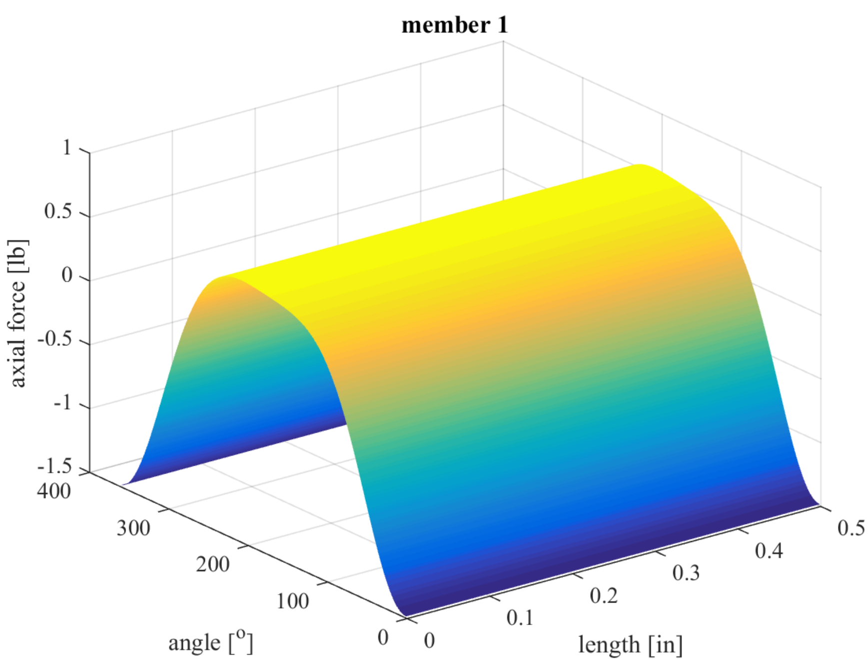

ax A
ax E
a cm1
a cm2
a cm3
a cm4
acceleration [in/s^2]
angle [°]
15
10
5
0
-5
-10
0
50
100
150
200
250
300
350

f)

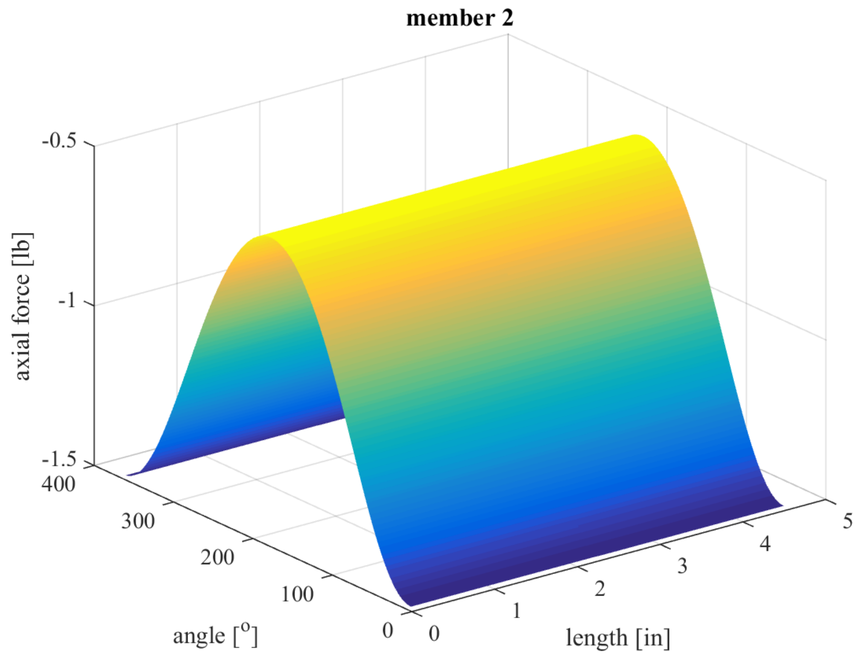


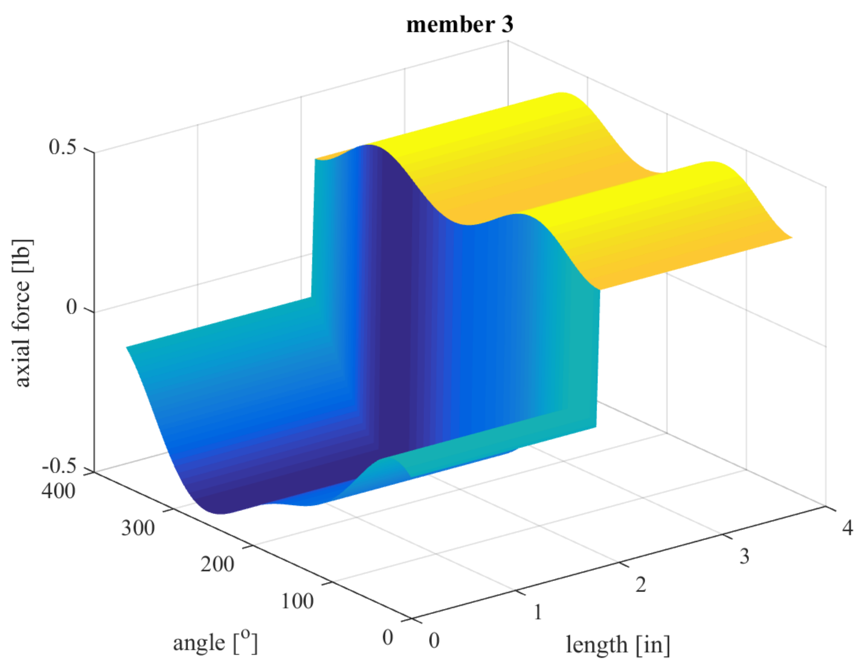
3
2.5
2
1.5
1
0.5
0
50
100
150
200
250
300
350
force [lb]
angle [^{o}]
F0
FA
FB
FD
FC
FE

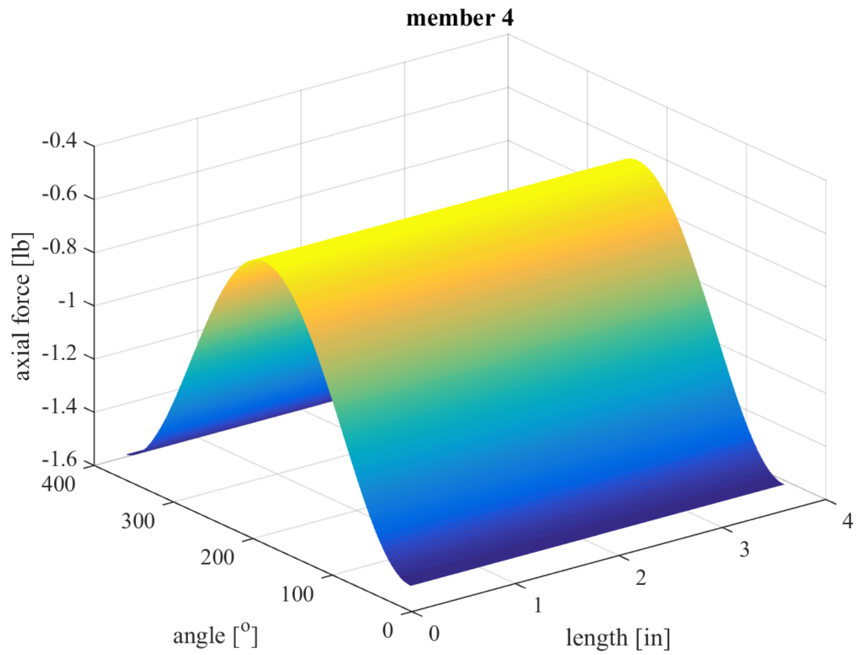
h)



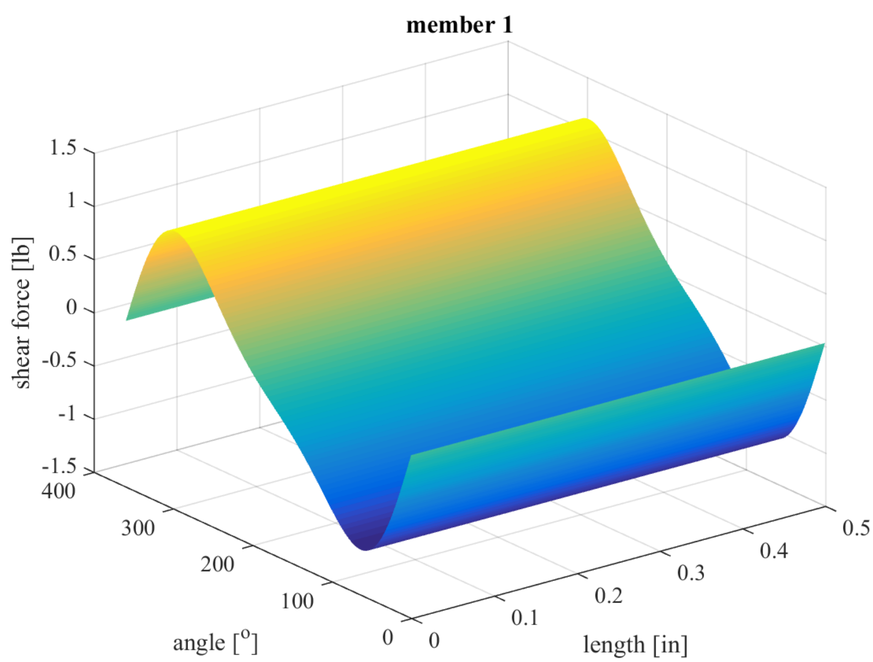
member 1
1
0.5
0
-0.5
-1
-1.5
axial force [lb]
400
300
200
100
0
angle [^{o}]
0
0.1
0.2
0.3
0.4
0.5
length [in]

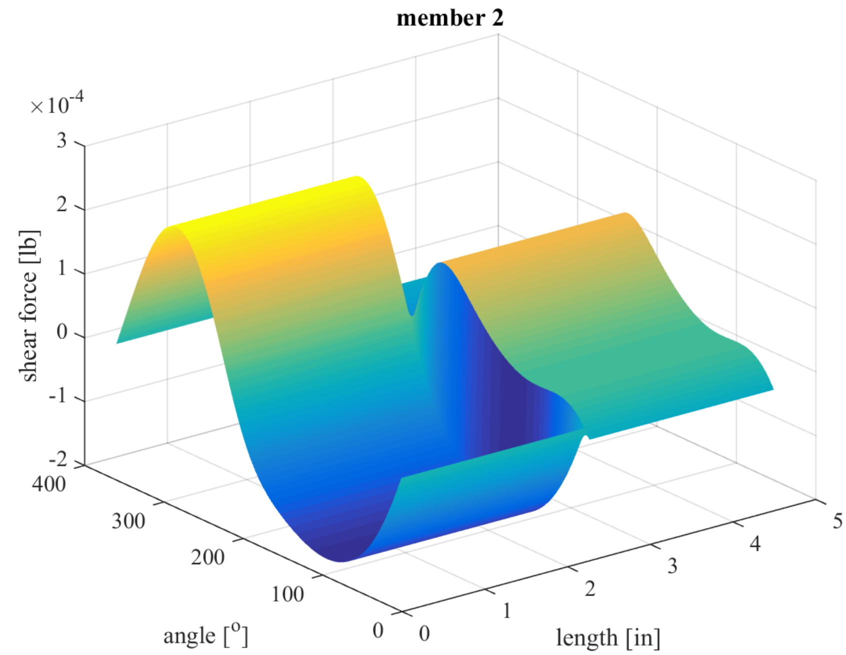

member 2
axial force [lb]
-0.5
-1
-1.5
400
300
200
100
0
angle [o]
0
1
2
3
4
5
length [in]


member 3
axial force [lb]
0.5
0
-0.5
400
300
200
100
0
angle [o]
0
1
2
3
4
length [in]

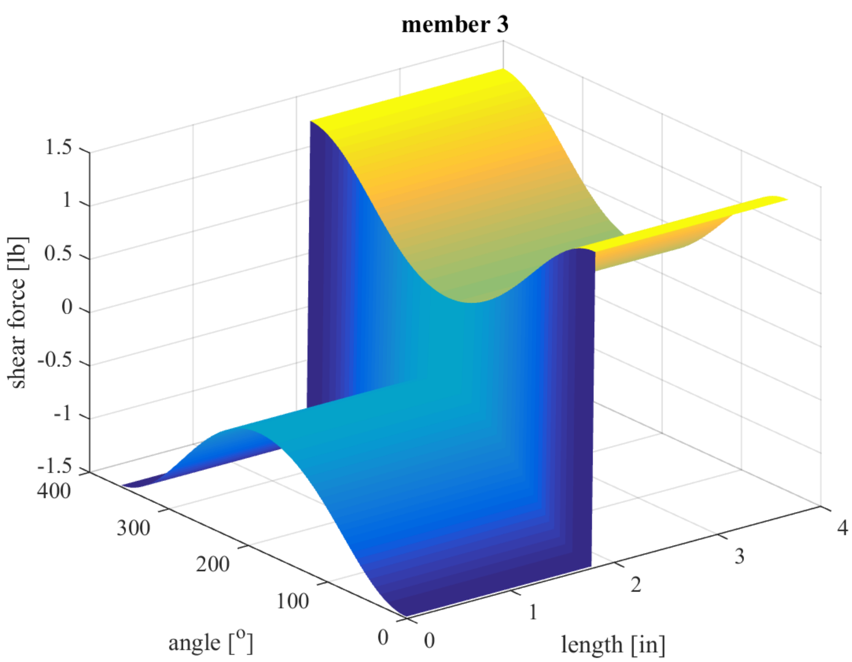

member 4
axial force [lb]
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
400
300
200
100
0
angle [°]
0
1
2
3
4
length [in]

i)

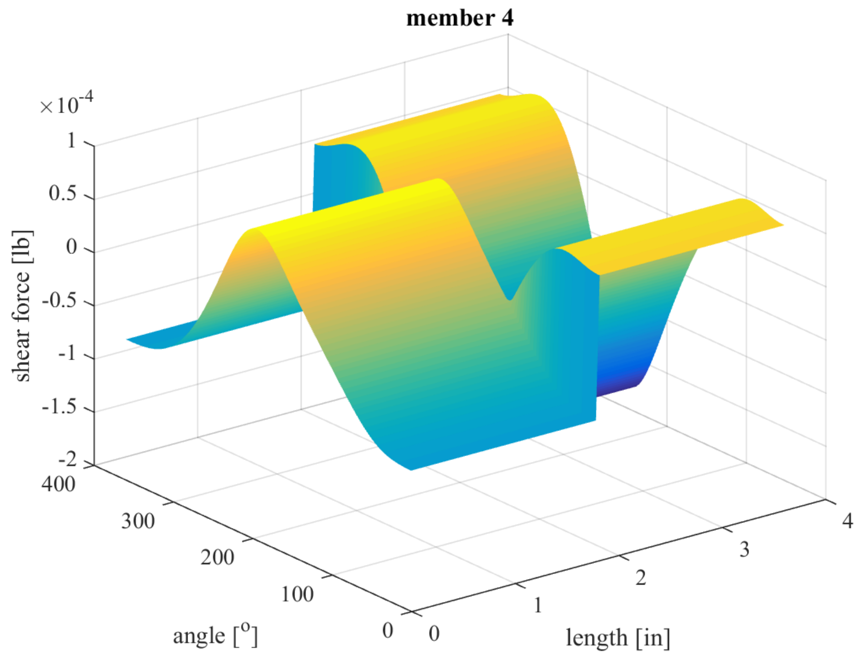

member 1
shear force [lb]
1.5
1
0.5
0
-0.5
-1
-1.5
400
300
200
100
0
angle [°]
0
0.1
0.2
0.3
0.4
0.5
length [in]



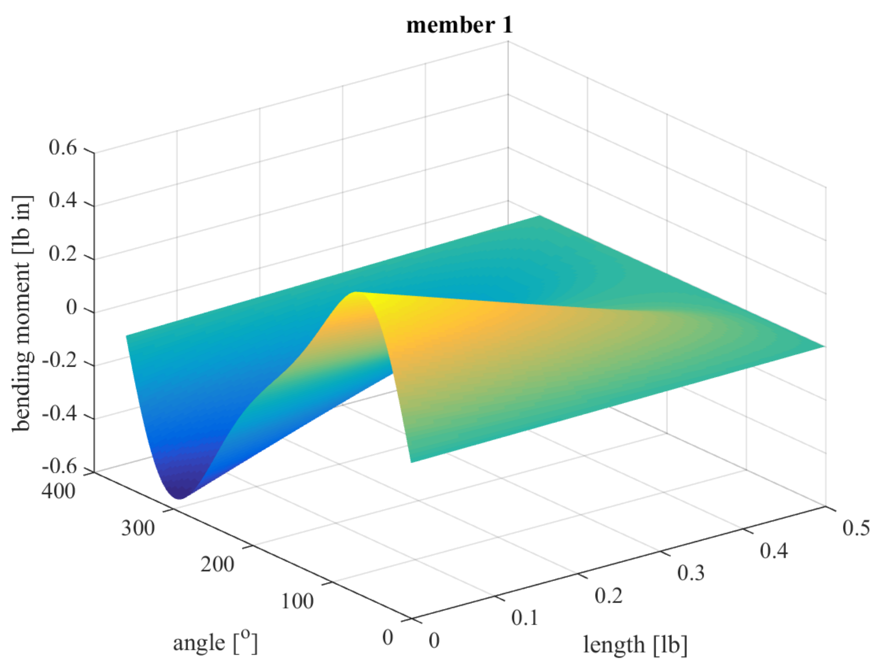
member 2
×10^-4
3
2
1
0
-1
-2
shear force [lb]
400
300
200
100
0
angle [°]
0
1
2
3
4
5
length [in]

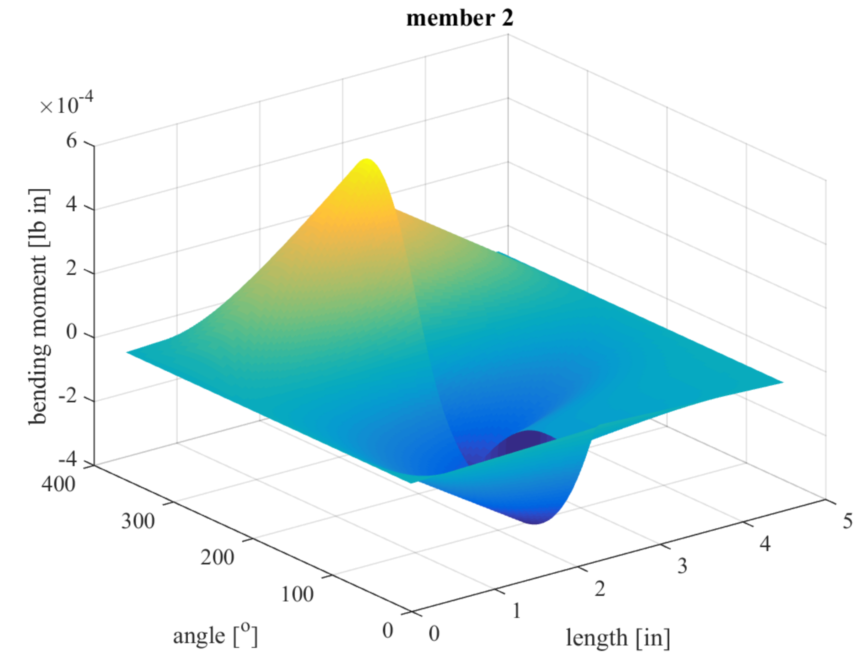


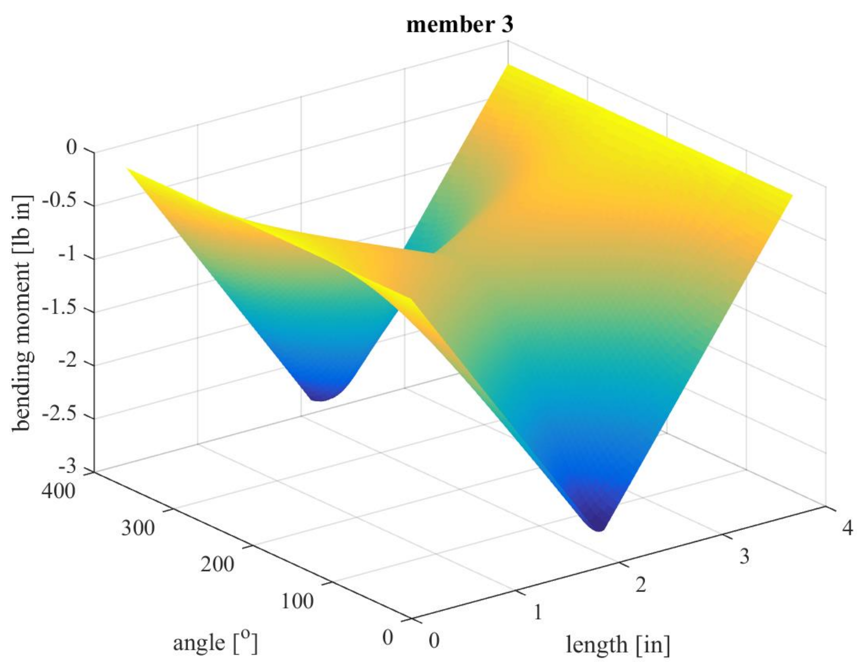
member 3
1.5
1
0.5
0
-0.5
-1
-1.5
shear force [lb]
400
300
200
100
0
angle [°]
0
1
2
3
4
length [in]

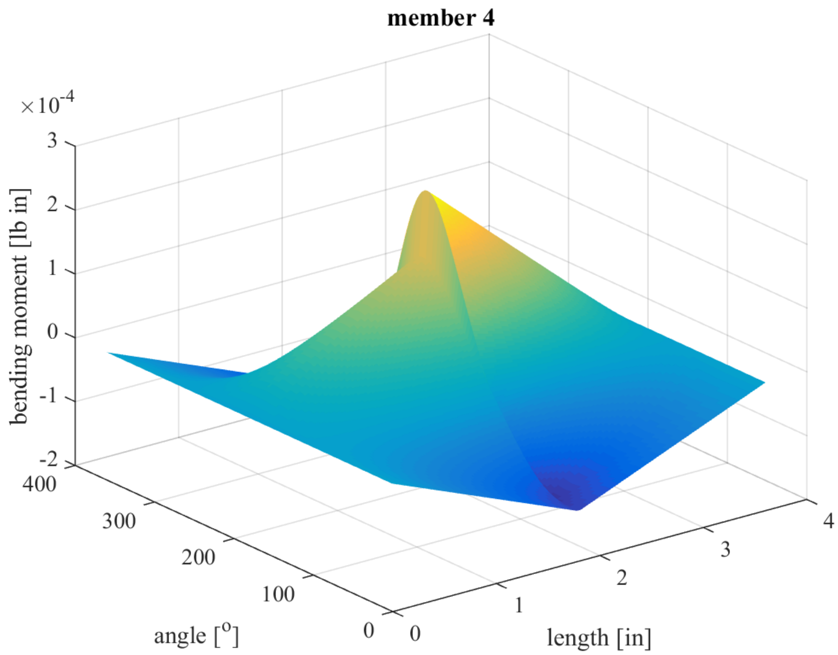

member 4
$\times 10^{-4}$
1
0.5
0
-0.5
-1
-1.5
-2
shear force [lb]
400
300
200
100
0
angle [^{o}]
0
1
2
3
4
length [in]

j)


member 1
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
bending moment [lb in]
400
300
200
100
0
angle [^{o}]
0
0.1
0.2
0.3
0.4
0.5
length [lb]


member 2
×10^-4
6
4
2
0
-2
-4
bending moment [lb in]
400
300
200
100
0
angle [°]
0
1
2
3
4
5
length [in]


member 3
0
-0.5
-1
-1.5
-2
-2.5
-3
bending moment [lb in]
400
300
200
100
0
angle [°]
0
1
2
3
4
length [in]



member 4
×10^-4
3
2
1
0
-1
-2
bending moment [lb in]
400
300
200
100
0
angle [°]
0
1
2
3
4
length [in]

Proposed lengths of elements:

Link	AB	BC	CD	DE
Length (in)	4.50	1.81	1.88	3.61

Range of bending moments (and corresponding axial forces):

Link	M_{min} (lbf in)	M_{max} (lbf in)	P_{ax}@M_{min} (lbf)	P_{ax}@M_{max} (lbf)
AB	0	0	-1.49	-0.53
BD	-2.67	-0.72	0.37 (-0.06)	0.37 (-0.36)
DE	0	0	-1.50	-0.54

Maximum transverse force in BD (at point C) is equal to 2.94 lbf and is acting at the center of BD (approximately). This is used to estimate deflections in BD.

Maximum compressive force in each link (to analyze buckling):

Link	P_c (lbf)
AB	1.49
BD (BC region)	0.46
DE	1.50

Maximum force acting in each pin:

Pin	P_{max} (lbf)
A	1.49
B	1.49
D	1.50
E	1.50

Proposed lengths of elements:

Link	AB	BC	CD	DE
Length (m)	0.114	0.046	0.048	0.092

Range of bending moments (and corresponding axial forces):

Link	M_{min} (N m)	M_{max} (N m)	$P_{ax}@M_{min}$ (N)	$P_{ax}@M_{max}$ (N)
AB	0	0	-6.628	-2.358
BD	-0.302	-0.081	1.646 (-0.267)	1.646 (-1.601)
DE	0	0	-6.672	-2.402

Maximum transverse force in BD (at point C) is equal to 13.078 N and is acting at the center of BD (approximately). This is used to estimate deflections in BD.

Maximum compressive force in each link (to analyze buckling):

Link	P_c (N)
AB	6.628
BD (BC region)	2.046
DE	6.672

Maximum force acting in each pin:

Pin	P_{max} (N)
A	6.628
B	6.628
D	6.672
E	6.672