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ME 646

LAB 3 – STRAIN GAUGES AND VIBRATION

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Ink Drawings
BEAM AND STRAIN GAUGE RESPONSE

**Diagram of the Physical Setup:**

**Determination of Bridge Sensitivity Using Shunt Resistors:**

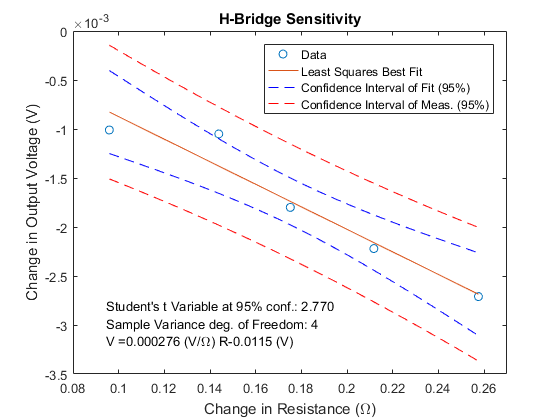


Figure 1 – Experimental response of the Wheatstone bridge. The large confidence interval was caused by an anomalous point, data for subsequent portions of this experiment will be borrowed from Jesse Feng.

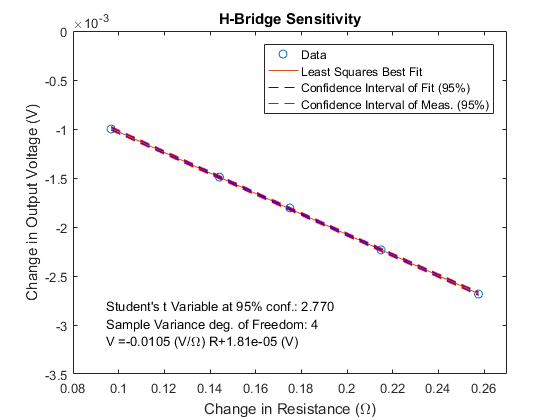


Figure 2 – The data of Figure 1 were replaced with cleaner data.

**V = .0115 (V/Ω)+2.76e-4 (V)**



**Comparison of Measured Bridge Sensitivity with Calculated Bridge Sensitivity**

The measurements of bridge sensitivity were taken by adding a shunt resistor to one of the legs of the Wheatstone bridge. The calculations of the bridge sensitivity are made using the quarter Wheatstone bridge model. The voltage supplied to the Wheatstone bridge was 5 V – this value is represented by Ei in the equations below. Eo represents the output voltage across the Wheatstone bridge (see equation 1).

(1)

Since only one resistor is being changed in value, the following is true:

(2)

(3)

With the introduction of changes in voltage and resistance the equation for output voltage of the Wheatstone bridge is shown as:

(4)

The input voltage is approximated as:

(5)

If the output voltage is assumed to be zero at the point where there is no change in resistance. Strain is represented by , and gauge factor is represented by GF.

(6)

Solving for the bridge sensitivity by rearranging equation 6:

(7)

The experimental bridge sensitivity had an error less than 1% using the data of Jesse Feng. Using the original data, the error percentage was 11%.

**Solving for Strain Given Gauge Factor and Amplifier Output**

The experimental Wheatstone bridge was a half bridge – there were two strain gauges and two static 120 Ω resistors.

(8)

Strains three and four are equal to zero (since there are only two strain gauges – one on top of the beam and one below the beam). Strains one and two are equal to the bending strain plus the thermal strain (which is comparatively small) strain one is roughly equal in magnitude to strain two if they are on opposing sides of the beam. Equation 8 reduces to:

(9)

Rearranging equation 9, and accounting for gain allows for the isolation of strain, which can now be solved for given a change in output voltage.

(10)

**Determining Strain vs. Load and Strain vs. Displacement**

The strain in a beam can be calculated by equation 11.

(11)

Where ‘E’ is the Young’s modulus, ‘Thickness’ is the thickness of the beam (smallest dimension measured), and ‘Width’ is the width of the beam (second smallest dimension measured). Length is the distance between the hook and strain gauges. Force can be determined either through equation 12 (mass) or through equation 13 (displacement).

(12)

(13)

Where m is the mass suspended by the beam, g is the gravitational acceleration constant on Earth, and deflection is the amount the beam was displaced by the micrometer. Substituting equation 13 into equation 11 causes the moment of inertia and Young’s modulus to cancel, yielding:

(14)

The comparison between the values recorded by the strain gauges and the predictions derived from Mass (Figure 3) and from displacement (Figure 4) is shown below. Figure 5 shows the linear least squares 95% confidence intervals of measurement and fit.

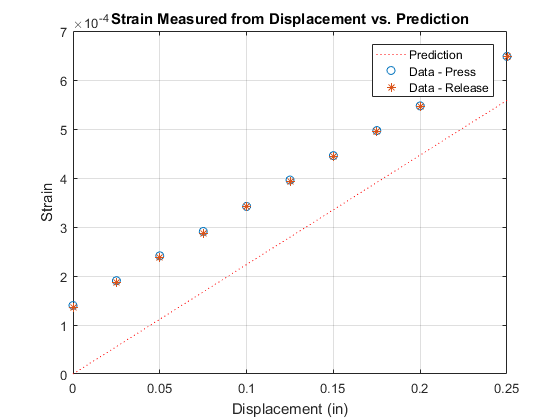


Figure 3 – Strain calculated from displacement compared to data recorded by the strain gauges

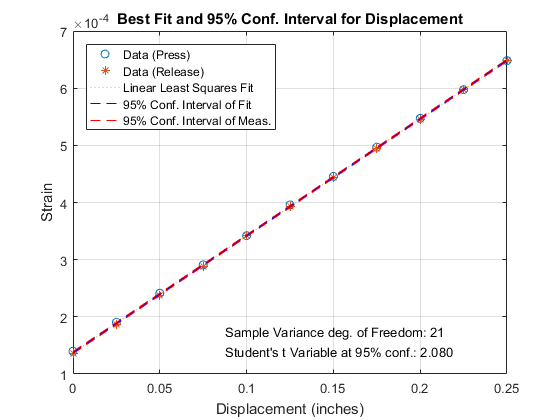


Figure 5 – The data above show a relatively narrow 95% confidence interval.

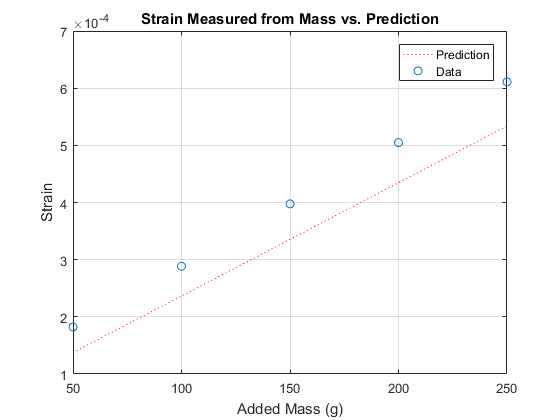


Figure 4 – Strain measured by adding mass to the end of the beam vs. strain gauge measurement

**Determination of Hysteresis for the Displacement Measurements**

Hysteresis is the difference between the y values f(x) for any function of x. In this lab the hysteresis will be the largest value measured from the strain during beam depression (downwards) by the micrometer, minus the strain at the corresponding displacement from the beam’s return to its starting position. Table 1 shows the difference between each *point* corresponding to a specific displacement, as well as the total hysteresis.

|  |  |  |
| --- | --- | --- |
| Total Hysteresis (ε): | 5.122 x10-4 | |
| Displacement (inches): | Press – Release (εx10-4): | Hysteresis (εx10-4): |
| 0 | 1.398 – 6.482 | -5.084 |
| .025 | 1.904 – 5.978 | -4.074 |
| .050 | 2.412 – 5.454 | -3.042 |
| .075 | 2.908 – 4.942 | -2.034 |
| .100 | 3.420 – 4.434 | -1.014 |
| .125 | 3.958 – 3.930 | .0280 |
| .150 | 4.456 – 3.416 | 1.040 |
| .175 | 4.968 – 2.872 | 2.096 |
| .200 | 5.472 – 2.370 | 3.102 |
| .225 | 5.974 – 1.868 | 4.106 |
| .250 | 6.482 – 1.360 | 5.122 |

*Table 1*

CANTILEVER BEAM VIBRATION

**Harmonics of the Beam:**

The beam was pulled back and let go, the subsequent oscillations were recorded, trimmed such to start at a time of 0 seconds, and normalized such that the strain gauge outputs zero volts at the rest position of the beam. The waveform can be seen in Figure 6.

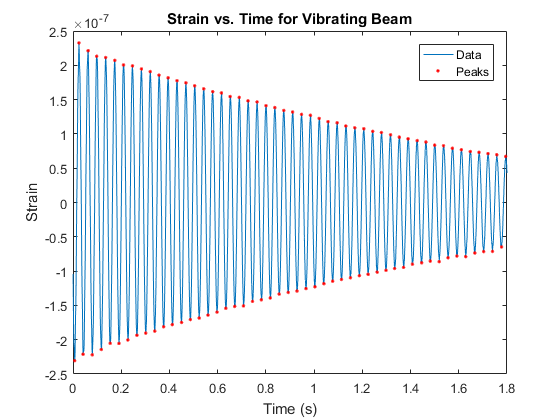


Figure 6 – The waveform of the beam vibration

The damping ratio of the beam was calculated using two different methods. The first method used least squares to determine a single damping ratio value ξ, which was found to be .0041.

(15)

The second method used equation 16 and produced an array of values.

(16)

The second method’s results are shown below in Table 2. The damped natural frequency was found to be 165.3470 Hz. The undamped natural frequency was found to be 165.3483 Hz. These values are very close – the only damping present was the viscosity of air and the internal friction of the steel bar.

|  |  |
| --- | --- |
| **Average ξ:** .0040 | |
| **Standard Deviation of ξ:** 9.49x10-4 | |
| **ξValues** (Peaks in ascending order from top left to bottom right – e.g. first peak is 0 last peak is .0041…)**:** | |
| 0 | 0.00377858 |
| 0.00792578 | 0.00369839 |
| 0.00694509 | 0.00377963 |
| 0.00507497 | 0.00386316 |
| 0.00460476 | 0.00380683 |
| 0.00472383 | 0.0039008 |
| 0.00411706 | 0.0038961 |
| 0.00404618 | 0.00383685 |
| 0.00393465 | 0.00389408 |
| 0.00398214 | 0.00385965 |
| 0.00392523 | 0.0038971 |
| 0.00390827 | 0.00394379 |
| 0.00381773 | 0.00394874 |
| 0.00383253 | 0.00396374 |
| 0.003854 | 0.00396093 |
| 0.00387075 | 0.00407046 |
| 0.00373783 | 0.00401506 |
| 0.00383384 | 0.00409199 |
| 0.00369097 | 0.00409366 |
| 0.00377688 | 0.0041298 |
| 0.00367551 | 0.00409081 |
| 0.00378715 | 0.00409292 |
| 0.0037558 | 0.00408467 |
| 0.00380678 | 0.00411787 |
| 0.00376281 |  |

*Table 2*

**Comparing Calculated and Experimental Beam Stiffness**

The stiffness of the beam was calculated knowing stiffness is equal to force divided by displacement. The equation for beam stiffness (equation 19) was derived from equations 17 and 18 and the previously stated relationship. I is the moment of inertia of the rectangular beam, l is the length of the beam, E is the Young’s modulus of steel, and Y is the distance from the center of the beam to the top of the beam.

(17)

(18)

(19)

The calculated stiffness was found to be 435.35 N/m using equation 19. The stiffness was also calculated experimentally – the voltage outputs of the force (mass) driven measurements were compared to the voltage outputs of the displacement driven measurements. Linearly interpolated arrays were used to increase the resolution of the force driven voltage measurements. Using the relationship between force and voltage, and voltage and displacement, a point of interest could be solved for giving the force applied for a given displacement. The stiffness figure from this method was 535.65 N/m, a figure within 19% error of the calculated value.

**Comparison Between Stiffness, Effective Mass, and the Natural Frequency**

The mass of the steel beam was estimated by multiplying the volume by the density of steel. The mass of the beam was calculated to be 20.4 grams. The mass of the hook was given as 7.4 grams. The effective mass is one quarter of the total mass. The effective mass and calculated/experimental stiffnesses are substituted into equation 20.

(20)

The natural frequency calculated using the experimental value for K (535.65 N/m) was found to be 139.8 Hz. The natural frequency using the calculated value for K (435.35) was found to be 126.1 Hz. The expected natural frequency was 165.3 Hz. The error percentages for the experimental and calculated stiffness were 15% and 24% respectively. The damping of the system was likely caused by air resistance or the internal friction of the beam.

**Fast Fourier Transformations Using Function Generator**

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