Homework #2 – 2017

1. What is the uncertainty in measuring the tensile strength for nominally 12 mm diameter specimens if the diameter measurement uncertainty is 0.01 mm and the force measurement uncertainty is 50 N and the typical breaking load is 8 kN? Provide an algebraic answer before evaluating the uncertainty.

$$\sigma_{ut} = \frac{F}{\frac{\pi d^2}{4}}$$

$$u_{\sigma_{ut}} = \sqrt{\left(\frac{\partial \sigma_{ut}}{\partial F} u_F\right)^2 + \left(\frac{\partial \sigma_{ut}}{\partial d} u_d\right)^2}$$

$$u_{\sigma_{ut}} = \sqrt{\left(\frac{1}{\frac{\pi d^2}{4}} u_F\right)^2 + \left(\frac{-2F}{\frac{\pi d^3}{4}} u_d\right)^2}$$

$$u_{\sigma_{ut}} = \sigma_{ut}\sqrt{\left(\frac{u_F}{F}\right)^2 + \left(\frac{-2u_d}{d}\right)^2} = \frac{8000}{\frac{\pi(0.012)^2}{4}}\sqrt{\left(\frac{50}{8000}\right)^2 + \left(\frac{0.01}{12}\right)^2} = 4.46x10^5\ Pa$$

$$\frac{u_{\sigma_{ut}}}{\sigma_{ut}} = 0.0063$$

2. The elastic modulus of a composite was determined by measuring the strain using an extensometer, a load cell, and measurements of the width, w, thickness, t, and gage length, l.

   The expression for elastic modulus is given by $E = \frac{\frac{F}{wt}}{\frac{\Delta l}{l}} = \frac{Fl}{wt\Delta l}$. The uncertainty in F is 0.1%, the uncertainty in the dimensional measurements, $w, t, and\ l$ is 0.025 mm, and the uncertainty in the change in gage length, Δl is 0.1 μm (10^{-4} mm).

   a. (5 points) Provide an algebraic expression with all partial derivatives performed for the uncertainty of the modulus based on the individual uncertainties, $u_l\ , u_w, u_t, u_{\Delta l}, \text{ and} u_F$.

$$E = \frac{Fl}{wt\Delta l}$$

$$u_E = \sqrt{\left(\frac{\partial E}{\partial F} u_F\right)^2 + \left(\frac{\partial E}{\partial l} u_l\right)^2 + \left(\frac{\partial E}{\partial w} u_w\right)^2 + \left(\frac{\partial E}{\partial t} u_t\right)^2 + \left(\frac{\partial E}{\partial \Delta l} u_{\Delta l}\right)^2}$$

$$u_E = E\sqrt{\left(\frac{u_F}{F}\right)^2 + \left(\frac{u_l}{l}\right)^2 + \left(\frac{u_w}{W}\right)^2 + \left(\frac{u_t}{t}\right)^2 + \left(\frac{u_l}{\Delta l}\right)^2}$$

b. (5 points) Evaluate the fractional uncertainty (u_E / E) for w = 25 mm, t = 1 mm, $l =$ 75 mm, and $\Delta l =$ 0.075 mm.

$$u_E = E\sqrt{\left(\frac{u_F}{F}\right)^2 + \left(\frac{u_l}{l}\right)^2 + \left(\frac{u_w}{W}\right)^2 + \left(\frac{u_t}{t}\right)^2 + \left(\frac{u_l}{\Delta l}\right)^2}$$

$$\frac{u_E}{E} = \sqrt{(0.001)^2 + \left(\frac{0.025}{75}\right)^2 + \left(\frac{0.025}{25}\right)^2 + \left(\frac{0.025}{1}\right)^2 + \left(\frac{0.0001}{0.075}\right)^2} = 0.025$$

$$= \sqrt{1x10^{-6} + 1.11x10^{-7} + 1x10^{-6} + 6.25x10^{-4} + 1.78x10^{-6}}$$

c. (5 points) Which component is the strongest contributor to the uncertainty?

Thickness uncertainty.

3. Estimate the uncertainty in a 22 m/sec air velocity measurement using a Pitot tube. The pressure for each tap is measured with separate manometers that have a resolution of 0.5 mm. The manometers use water as the fluid and each are oriented vertically. The absolute pressure (which gives you the density of air) is known with an uncertainty of 0.1 kPa. The temperature is known with an uncertainty of 1°C. Assume the stagnation pressure, p_t, is indicated by a static height of 7 mm.

$$p_{stag} - p_{atm} = -\rho_{H_2O} g h_{stag}$$
$$p_{static} - p_{atm} = -\rho_{H_2O} g h_{static}$$
$$U_x = \sqrt{\frac{2\left(p_{stag} - p_{static}\right)}{\rho}} = \sqrt{\frac{2\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{\frac{pM}{RT}}} = \sqrt{\frac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}$$
$$u_{U_x} = \sqrt{\left(\frac{\partial U_x}{\partial h} u_{h_{stag}}\right)^2 + \left(\frac{\partial U_x}{\partial h} u_{h_{static}}\right)^2 + \left(\frac{\partial U_x}{\partial p} u_p\right)^2 + \left(\frac{\partial U_x}{\partial T} u_T\right)^2}$$
$$u_{U_x} = \sqrt{\begin{array}{l}\left(\dfrac{-\dfrac{2RT\rho_{H_2O} g}{pM}}{2\sqrt{\dfrac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}} u_h\right)^2 + \left(\dfrac{\dfrac{2RT\rho_{H_2O} g}{pM}}{2\sqrt{\dfrac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}} u_h\right)^2 \\ .....+\left(\dfrac{-\dfrac{2R\rho_{H_2O} g\left(h_t - h_x\right)}{pM}}{2\sqrt{\dfrac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}} u_T\right)^2 + \left(\dfrac{-\dfrac{2RT\rho_{H_2O} g\left(h_t - h_x\right)}{M}}{2\sqrt{\dfrac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}} \dfrac{-1}{p^2} u_p\right)^2\end{array}}$$
$$= \frac{\frac{2RT\rho_{H_2O} g}{pM}}{2\sqrt{\frac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}} \sqrt{2\left(u_h\right)^2 + \left(h_t - h_x\right)^2\left(\left(\frac{u_T}{T}\right)^2 + \left(\frac{-u_p}{p}\right)^2\right)}$$
$$= \frac{\frac{2RT\rho_{H_2O} g\left(h_{stat} - h_{stag}\right)}{pM}}{2\sqrt{\frac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}} \sqrt{2\left(\frac{u_h}{\left(h_t - h_x\right)}\right)^2 + \left(\left(\frac{u_T}{T}\right)^2 + \left(\frac{-u_p}{p}\right)^2\right)}$$
$$u_{U_x} = \frac{1}{2}\sqrt{\frac{2RT\rho_{H_2O} g\left(h_{static} - h_{stag}\right)}{pM}}\sqrt{2\left(\frac{u_h}{\left(h_t - h_x\right)}\right)^2 + \left(\frac{u_T}{T}\right)^2 + \left(\frac{-u_p}{p}\right)^2}$$
$$u_{U_x} = \frac{U_x}{2}\sqrt{2\left(\frac{u_h}{\left(h_t - h_x\right)}\right)^2 + \left(\frac{u_T}{T}\right)^2 + \left(\frac{-u_p}{p}\right)^2}$$
$$h_{static} = h_{stag} - \frac{U_x{}^2 pM}{2RT\rho_{H_2O} g} = 0.007m - \frac{\left(22m/s\right)^2\left(100x10^3 Pa\right)\left(0.029kg/mole\right)}{2\left(8.314J/mol\cdot K\right)\left(393K\right)\left(1000kg/m^3\right)\left(9.8m/sec^2\right)} = 0.205m$$

$$u_{U_x} = \frac{U_x}{2}\sqrt{2\left(\frac{u_h}{(h_t - h_x)}\right)^2 + \left(\frac{u_T}{T}\right)^2 + \left(\frac{-u_p}{p}\right)^2}$$

$$= \frac{22m/s}{2}\sqrt{2\left(\frac{0.5mm}{219mm - 7mm}\right)^2 + \left(\frac{1^0C}{393K}\right)^2 + \left(\frac{0.1kPa}{100kPa}\right)^2} = 0.0475m/s$$

4. Estimate the uncertainty for measuring the coefficient of drag of 0.1 on an object with a planform area A = 0.5 m^2 as a function of velocity for velocities ranging from 1 m/sec to 100 m/sec ($C_D = \frac{D}{\frac{1}{2}\rho V^2 A}$) using a force balance that has a resolution of 1 N and a range of 1000N. The area is known with an uncertainty of 0.15%, and the velocity is known with an uncertainty of 0.1 m/s. The fluid density is inferred from the ideal gas law and where the temperature is known with an uncertainty of 1°C and the pressure is known with a uncertainty of 0.2 kPa. Assume room temperature is 20°C and the pressure is atmospheric pressure.

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \frac{D}{\frac{1}{2}\left(\frac{pM}{RT}\right)V^2 A} = \frac{2DRT}{pMV^2 A} \qquad M = molecular\ \ weight$$

$$u_{C_D} = \sqrt{\left(\frac{\partial C}{\partial D}u_D\right)^2 + \left(\frac{\partial C}{\partial T}u_T\right)^2 + \left(\frac{\partial C}{\partial p}u_p\right)^2 + \left(\frac{\partial C}{\partial V}u_V\right)^2 + \left(\frac{\partial C}{\partial A}u_A\right)^2}$$

$$u_{C_D} = \sqrt{\left(\frac{2RT}{pMV^2A}u_D\right)^2 + \left(\frac{2DR}{pMV^2A}u_T\right)^2 + \left(\frac{-2DRT}{p^2MV^2A}u_p\right)^2 + \left(\frac{-4DRT}{pMV^3A}u_V\right)^2 + \left(\frac{-2DRT}{pMV^2A^2}u_A\right)^2}$$

$$\frac{u_{C_D}}{C_D} = \sqrt{\left(\frac{u_D}{D}\right)^2 + \left(\frac{u_T}{T}\right)^2 + \left(\frac{-u_p}{p}\right)^2 + \left(\frac{-2u_V}{V}\right)^2 + \left(\frac{u_A}{A}\right)^2}$$

To solve this, you need to calculate the drag, D, and the uncertainty as a function of V. Use p = 100 kPa, T = 293.15 K, M = 0.029 kg/mole, R = 8.314 J/mole·K.

V(m/sec)	D(N)	uC_D/C_D
1	0.029635	3.375833
2	0.118541	0.84507
5	0.740883	0.13646
10	2.963533	0.035242
20	11.85413	0.009976
50	74.08833	0.003031
100	296.3533	0.002117

5. Problem 5.33 in the book.

Assume R and M are known with certainty.

$$\rho = \frac{pM}{RT}$$

$$u_\rho = \sqrt{\left(\frac{\partial \rho}{\partial p} u_p\right)^2 + \left(\frac{\partial \rho}{\partial T} u_T\right)^2}$$

$$= \rho\sqrt{\left(\frac{u_p}{p}\right)^2 + \left(\frac{u_T}{T}\right)^2} = \rho\sqrt{(0.01)^2 + \left(\frac{2}{298}\right)^2} = 0.012\rho$$

6. Problem 5.38 in the book

$$\Gamma = e^{-t/\tau}$$

$$\tau = -t / \ln\Gamma$$

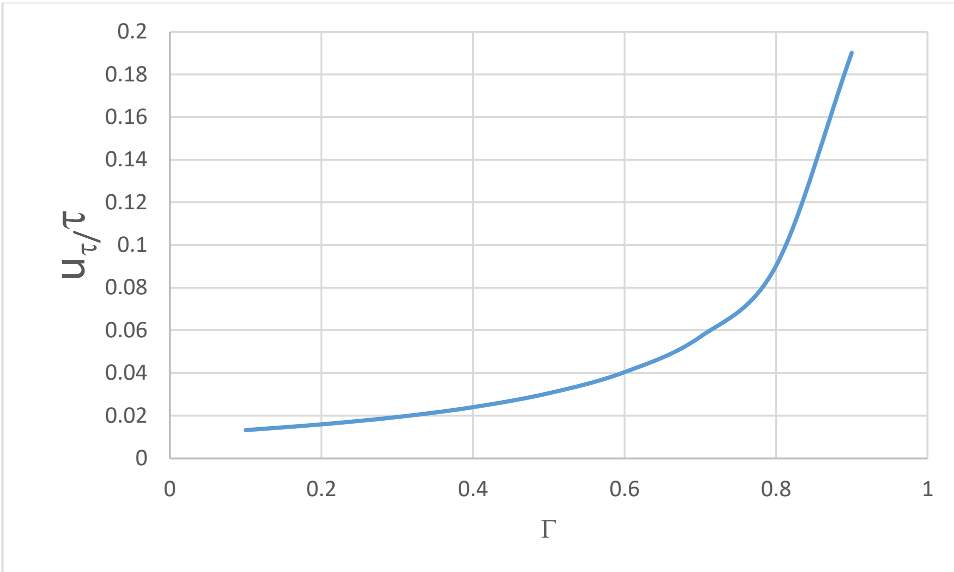
$$u_\tau = \sqrt{\left(\frac{\partial \tau}{\partial t} u_t\right)^2 + \left(\frac{\partial \tau}{\partial \Gamma} u_\Gamma\right)^2}$$

$$= \sqrt{\left(\frac{-1}{\ln\Gamma} u_t\right)^2 + \left(\frac{-t}{\Gamma(\ln\Gamma)^2} u_\Gamma\right)^2}$$

$$= \sqrt{\left(\frac{-0.01t}{\ln\Gamma}\right)^2 + \left(\frac{-t(0.02\Gamma)}{\Gamma(\ln\Gamma)^2}\right)^2}$$

$$= \tau\sqrt{(0.01)^2 + \left(\frac{0.02}{\ln\Gamma}\right)^2}$$

Γ	u_τ/τ
0.1	0.013246
0.2	0.015951
0.3	0.019389
0.4	0.024009
0.5	0.030538
0.6	0.040409
0.7	0.056958
0.8	0.090185
0.9	0.190088


0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
u_τ/τ
0
0.2
0.4
0.6
0.8
1
Γ