University of New Hampshire

College of Engineering and Physical Sciences
Department of Mechanical Engineering
Kingsbury Hall, W101
33 Academic Way
Durham, NH 03824

Course Number and Name: ME 747	
Semester and Year: Fall 2017	Name of Lab Instructor: Alireza Ebadi
Lab Section and Meeting Time: Section 2B, Tuesday 2-5 pm	Report Type: Internal Group Report
Title of Experiment: Time and Frequency Response of RC and RLC Circuits	
Date Experiment Performed: 9/19/17	Date Report Submitted: 10/3/17
Names of Group Members: Zhangxi Feng Simon Popecki Reilly Webb	Grader's Comments: Grade:

Table of Contents

List of Figures

List of Tables

Objectives

A first order (RC) circuit and a second order (RLC) circuit were subjected to step and sinusoidal inputs and their outputs were measured to determine their responses. The system parameters of the circuits were derived and calculated from the outputs. The unknown inductance and capacitances of the circuits were calculated from the system parameters.

Executive Summary

In part 1 of this lab, a first order (RC) circuit was investigated to determine its responses for two step inputs at different DC offsets and amplitudes, and a sinusoidal input with no DC offset. The differential equation and transfer function were found for the circuit, as well as the time constant, $\tau = 6.424 * 10^{-4}\ seconds$. The resistor value was known and the capacitance was calculated from the results, $C = 58.77\ nF$. The amplitude and phase shift of the frequency response- the bode plot, was created from measurements. The results were then compared to Matlab simulations. From the bode plot, τ and C were calculated again to be $6.366 * 10 - 4\ seconds$ and $57.35\ nF$ respectively.

In part 2 of this lab, a second order (RLC) circuit was investigated to determine its responses for the same step as well as sinusoidal inputs. The damping ratio (ζ) and natural frequencies (ω_n) were determined from the step responses, 0.0620 and 313.1 Hz, and the bode plot, 0.0275 and 1959.7 Hz. The resistor value of the circuit was known, and the inductance and capacitance were calculated for the step response, 1.27 H and $20.7\ \mu F$, and the frequency response, 0.568 H and $4.585\ \mu F$. The results were compared between the responses and Matlab simulations.

In part 3, the same two circuits were connected to Labview via a Data Acquisition (DAQ) device to collect data for creating the bode plots. The system parameters and component values of each circuit were calculated from the bode plots and compared to the results from previous two parts. The frequency response of the circuit was tested via an input of simulated white noise. For the RC circuit, τ was found to be $6.124 * 10^{-4}\ seconds$ and $C = 5.517 * 10^{-8}\ F$. The RLC circuit returned a damping ratio of 0.0284 and an undamped natural frequency of 360 Hz. From these, L was found to be 0.5083 H and C was $3.85 * 10^{-7} F$.

Theory and Experimental Methods

Theory

1. *Step Response of a First Order System*

Two methods were used to experimentally determine the time constant from the response data. The initial slope method locates τ by extrapolating the initial slope of the data. This slope is extended to the intersection with the steady-state voltage, and this point gives the time constant in seconds.

The second method is defined as follows

$$at\ t = \tau, \qquad V_c(\tau) = V_{ss}(1 - e^{-1}) = 0.632 V_{ss} \tag{1}$$

In short τ is the point where the output reaches 63.2% of the final steady state value. Kirchhoff's Voltage Loop law can be applied to the RC circuit, yielding the following equations

$$e_i(t) - Ri(t) - \frac{1}{C}\int i(t)dt = 0 \tag{2}$$

$$e_o(t) = \frac{1}{C}\int i(t)dt \tag{3}$$

Solving for an equation relating $e_o(t)$ and $e_i(t)$ through $i(t)$

$$Ce_o(t) = \int i(t)dt \tag{4}$$

$$\frac{d\big(Ce_o(t)\big)}{dt} = i(t) \tag{5}$$

$$e_i(t) - RC\frac{de_o(t)}{dt} - \frac{1}{C}\int \frac{d\big(Ce_o(t)\big)}{dt}dt = 0 \tag{6}$$

$$e_i(t) = RC\frac{de_o(t)}{dt} + e_o(t) \tag{7}$$

To solve for capacitance C

$$C = \frac{\tau}{R} \tag{8}$$

The measured resistance for the RC circuit was $R_1 = 11.1\ k\Omega$

Multiple data points and methods were used to find τ, the average calculated capacitance will be taken.

1. *Frequency Response of a First Order System*

To find the transfer function of the system, first take the Laplace transform of the differential equation.

$$\mathcal{L}(e_i(t)) = \mathcal{L}\left(RC\frac{de_o(t)}{dt}\right) + \mathcal{L}(e_o(t)) \tag{9}$$

$$E_i(s) = RCsE_o(s) + E_o(s) \tag{10}$$

Then rearrange to find $\frac{E_o(s)}{E_i(s)}$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{(RCs + 1)} \tag{11}$$

The time constant is related to the breakpoint frequency as follows

$$\tau = RC = \frac{1}{2\pi f_c} \tag{12}$$

Experimentally the break frequency was found to be 250 Hz. With a measured resistance $R_1 = 11.1\ k\Omega$, the calculated capacitance is $C = 5.73 * 10^{-8}\ F$

2. *Step Response of a Second Order System*

The RLC circuit can be modeled using a 2nd order differential equation. Using Kirchhoff's Voltage Loop law,

$$Loop\ 1:\ \ e_i - L\frac{di_1}{dt} - \frac{1}{C}\int(i_1 - i_2)dt = 0 \tag{13}$$

$$Loop\ 2:\ \ \frac{1}{C}\int(i_2 - i_1)dt + i_2R = 0 \tag{14}$$

With the equivalent voltage output relating the equations

$$e_o = Ri_2 => i_2 = \frac{e_o}{R} \tag{15}$$

$$e_i - L\frac{di_1}{dt} - \frac{1}{C}\int(i_1 - \frac{e_o}{R}dt) = 0 \tag{16}$$

$$\frac{1}{C}\int(\frac{e_o}{R} - i_1)dt + \frac{e_o}{R}R = 0 => -Ce_o = \int(\frac{e_o}{R} - i_1)dt => -C\frac{de_o}{dt} = \frac{e_o}{R} - i_1 \tag{17}$$

$$C\frac{de_o}{dt} + \frac{e_o}{R} = i_1 => e_i - L\frac{d(C\frac{de_o}{dt} + \frac{e_o}{R})}{dt} - \frac{1}{C}\int(C\frac{de_o}{dt} + \frac{e_o}{R} - \frac{e_o}{R})dt = 0 \tag{18}$$

$$e_i - L\frac{d(C\frac{de_o}{dt} + \frac{e_o}{R})}{dt} - \frac{1}{C}\int(C\frac{de_o}{dt} + \frac{e_o}{R} - \frac{e_o}{R})dt = 0 \tag{19}$$

$$e_i - LC\frac{d^2e_o}{dt} - \frac{L}{R}\frac{de_0}{dt} - e_o = 0 \tag{20}$$

$$e_i = LC\frac{d^2e_o}{dt} + \frac{L}{R}\frac{de_0}{dt} + e_o \tag{21}$$

The damping ratio can then be found from the raw data using the log decrement method on any two successive peaks

$$\zeta = \frac{1}{\sqrt{1 + (\frac{2\pi}{\delta})^2}} \quad where \quad \delta = \frac{1}{n}\ln\left(\frac{x(t)}{x(t+nT)}\right) \tag{22}$$

where $x(t)$ is the amplitude at time t, T is the period, and n is the number of positive successive peaks. The undamped natural frequency can then be found as follows

$$\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}} \tag{23}$$

L and C can then be found with the following equations derived from the general form of the system's differential equation

$$L = \frac{2R\zeta}{\omega_n} \tag{24}$$

$$C = \frac{1}{2R\zeta\omega_n} \tag{25}$$

3. Frequency Response of a Second Order System

The transfer function of the system can be derived by taking the Laplace transform of the differential equation

$$\mathcal{L}(e_i) = \mathcal{L}\left(LC\frac{d^2e_o}{dt}\right) + \mathcal{L}\left(\frac{L}{R}\frac{de_0}{dt}\right) + \mathcal{L}(e_o) \tag{26}$$

$$\frac{E_o}{E_i} = \frac{1}{LCs^2 + \frac{L}{R}s + 1} \tag{27}$$

The damping ratio and undamped natural frequency were then determined from a bode plot of the transfer function, where max peak is in dB and ω_n correlates to this peak location.

$$\max peak = \frac{1}{2\zeta} \tag{28}$$

L and C can be found using the same equations (24) and (25)

3. Frequency Response Using LabView

The time constant and capacitance was found from the 1st order LabView results using a bode plot and the following relationships

$$\omega_b = \frac{1}{\tau} \quad and \quad C = \frac{\tau}{R} \tag{28}$$

The damping ratio for the 2nd order system was found using the same max peak equation, equation (28). The natural frequency also correlates to the location of this peak.

L and C can be found using the same equations (24) and (25)

Experimental Methods

1. First Order System

A first order RC circuit was set up as shown in Figure 1. A T-junction BNC connector was used to connect a function generator to both a digital scope and the circuit input, $e_i(t)$.

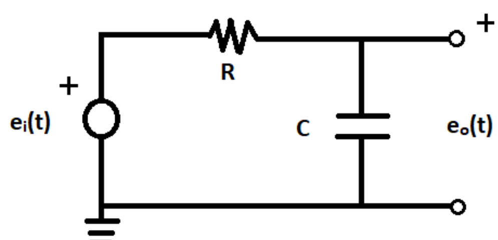


FIGURE 1: RC CIRCUIT

The resistance, R (14.1 Ω), was measured using a digital multimeter. A square wave with an amplitude of 2 volts was sent through the circuit and the input and output curves were recorded using a digital oscilloscope (Figure 2).

Next a 4 volt DC offset was applied so that the square wave amplitude was from -3.0 V to +5.0 V, and the step response was recorded and compared.

The function generator (Figure 4) was then set to generate a sine wave with a 2 volt amplitude and no dc offset. The output amplitude and phase delay was recorded using the scope cursor for a range of input frequencies. These input frequencies ranged from 10-10000 Hz, with a higher density of points near the break frequency (250 Hz).

2. Second Order System

A second order RLC circuit was set up as shown in Figure 3. A T-junction BNC connector was used to connect a function generator to both a digital scope and the circuit input, $e_i(t)$.

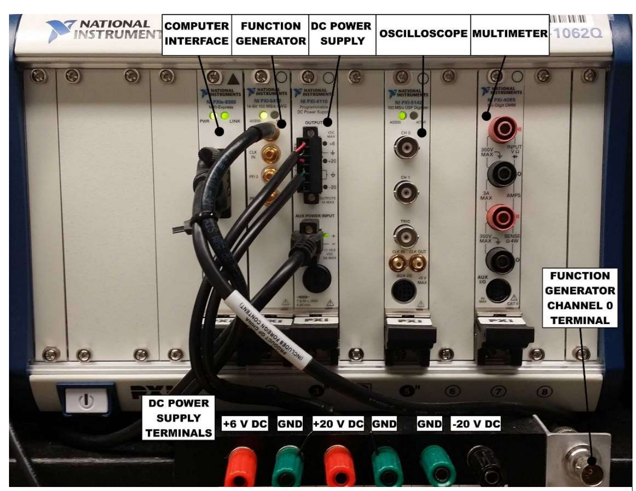


FIGURE 2: LAB INSTRUMENTATION PANEL

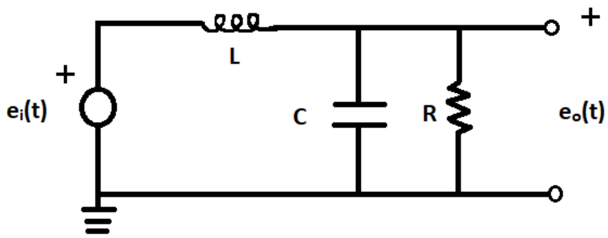


FIGURE 3: RLC CIRCUIT

The resistance, R (20 kΩ), was measured using a digital multimeter. A square wave with an amplitude of 2 volts was sent through the circuit input, and the input and output curves were recorded for a complete step response.

A 4 volt DC offset was applied so that the square wave amplitude is from -3.0 V to +5.0 V, and once again the input and output curves for a full step response were recorded using the scope.

The function generator (Figure 4) was then set to sinusoidal input with an amplitude of 2 V and no DC offset. The break frequency (317 Hz) was estimated by sweeping the frequency of the sin wave from 10 Hz to 10 kHz. The output amplitude and phase shift was recorded for input frequencies of 1/20, 1/4, 1/2, 3/2, 2 and 4 times the break frequency.

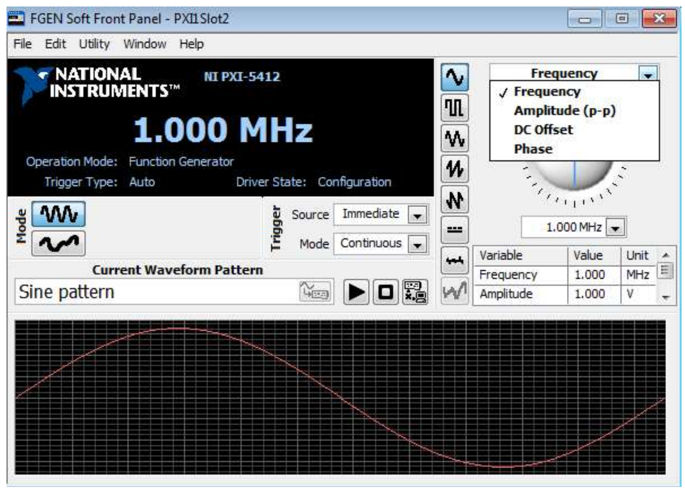


FIGURE 4: FUNCTION GENERATOR

3. Frequency Response Using LabView

LabView was used as an alternative method for determining the frequency response for both systems. Broadband noise was sent through the input of the circuit to test the whole spectrum of frequency inputs.

The following procedure was performed on both the RC and RLC circuits.

The filter input was connected to the NI DAQ (Figure 5) output terminals. The RC circuit output was connected to scope Ch.1 and the input to scope Ch. 0.

Select the "Data View" tab, then press the "Run" button. Once the curve has converged, hit the "Stop" button. Export the bode plot and use the zoom tools in LV-SE to estimate the break frequencies.

Reference "Using LabView for Frequency Responses" by M.H. deLeon [1] for a more in depth procedure.

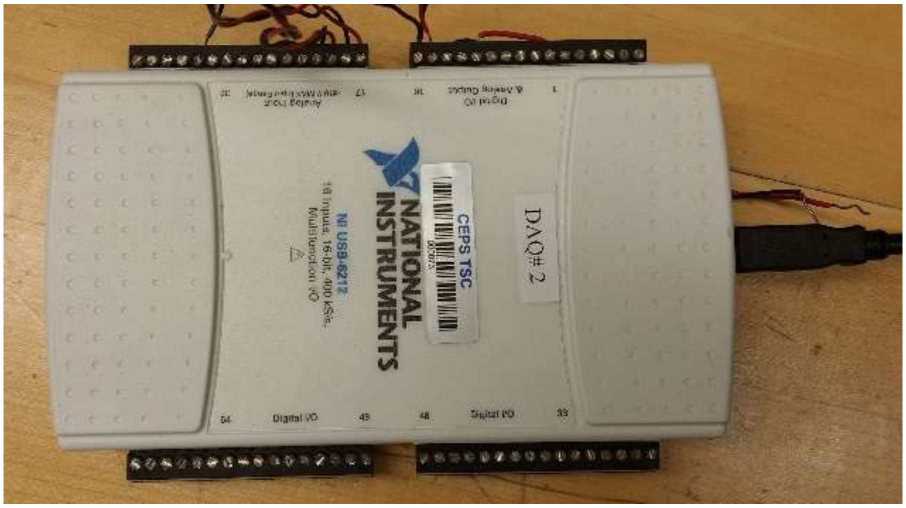


FIGURE 5: DAQ DEVICE

Results and Discussion

1.1

Two methods were used for calculating the time constants for both the unadjusted square wave input and the 1 V DC offset square wave input. It is clear from Table 1 that each method resulted in similar time constants, with only a 5% difference between the furthest values. It is also apparent that adding a DC offset to the input did not affect the time constant of the system.

TABLE 1: CALCULATING TAU AND CAPACITANCE FROM STEP RESPONSE

Method	Tau Adjusted (s)*	Calculated Capacitance (nF)
No Offset Initial Slope	6.68155e-4	60.19
No Offset 63.2%	6.54000e-4	58.92
DC Offset Initial Slope	6.35428e-4	57.25
DC Offset 63.2%	6.52000e-4	58.74
Average	6.52396e-4	58.77

NOTE: Adjusted Tau is the value found taking into account that the data did not start at time = 0.

From Table 1 it can be seen that the calculated capacitance (averaged between all of the methods) is

$$C = 5.87 * 10^{-8}\ F$$

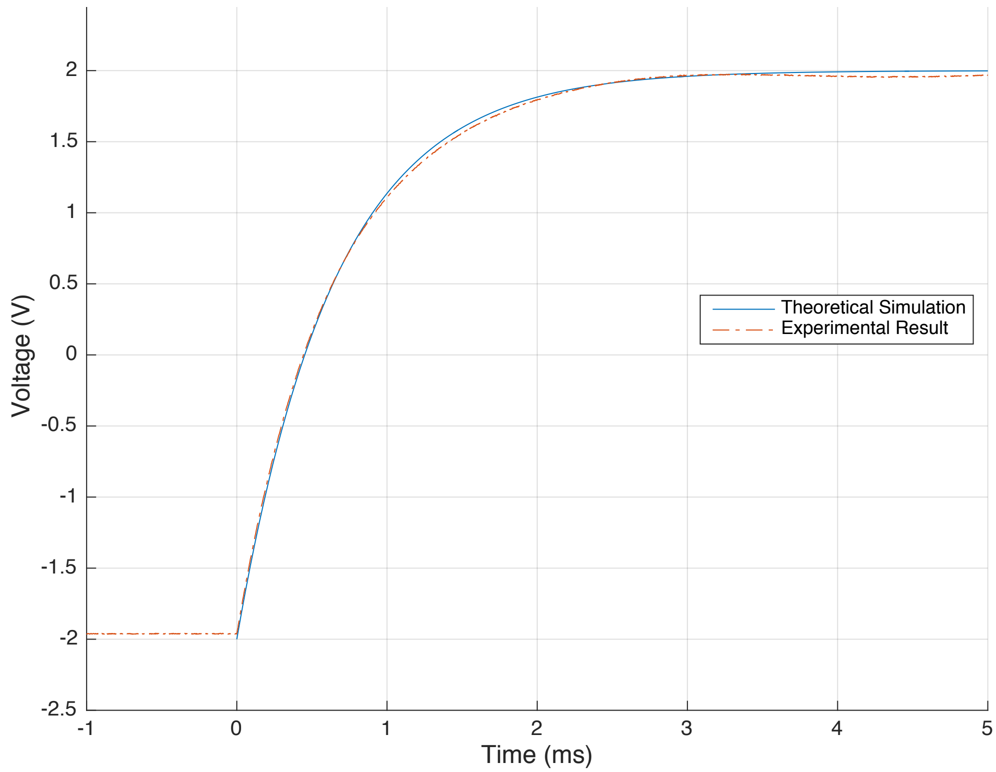


FIGURE 6: COMPARING THEORETICAL AND EXPERIMENTAL STEP RESPONSE

The theoretical simulation curve and the experimental result curve match closely (see Figure 6). Small deviations are caused by assumptions made to simplify theoretical calculations, such as ignoring resistance in the wire and temperature sensitive circuit elements.

The only noticeable difference between the initial step response and the offset step response is the actual voltage offset. The curve shape was unaffected. The time constant will not change from a DC offset.

1.2

The Bode plot of the first order system would be unaffected by a dc offset. The Bode a plot displays response behavior, and remains the same for a circuit regardless of input.

TABLE 2: 1ST ORDER FREQUENCY RESPONSE DATA

ω_i [Hz]	V	ϕ	dB
10	3.93	-2.2	-0.08795
30	3.9	-6.8	-0.15451

100	3.64	-21.7	-0.75378
200	3.05	-38.5	-2.28981
230	2.85	-42.4	-2.87891
240	2.78	-43.8	-3.09491
250	2.71	-44.9	-3.31642
260	2.64	-46.1	-3.54373
270	2.57	-47.2	-3.77714
500	1.75	-63.2	-7.11504
1000	0.956	-75.6	-12.3666
5000	0.197	-86.4	-26.0864
10000	0.099	-87.5	-32.0631

Table 2 displays selected data over a range of 3 decades of frequencies. The transfer function for the RC circuit was found to be

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{(RCs + 1)}$$

This transfer function was used to build the bode plot in Figure 7, which also has an overlay of an experimental bode plot that was calculated form the experiment data. Visually, both bode plots look very similar, especially the points close to the breakpoint frequency.

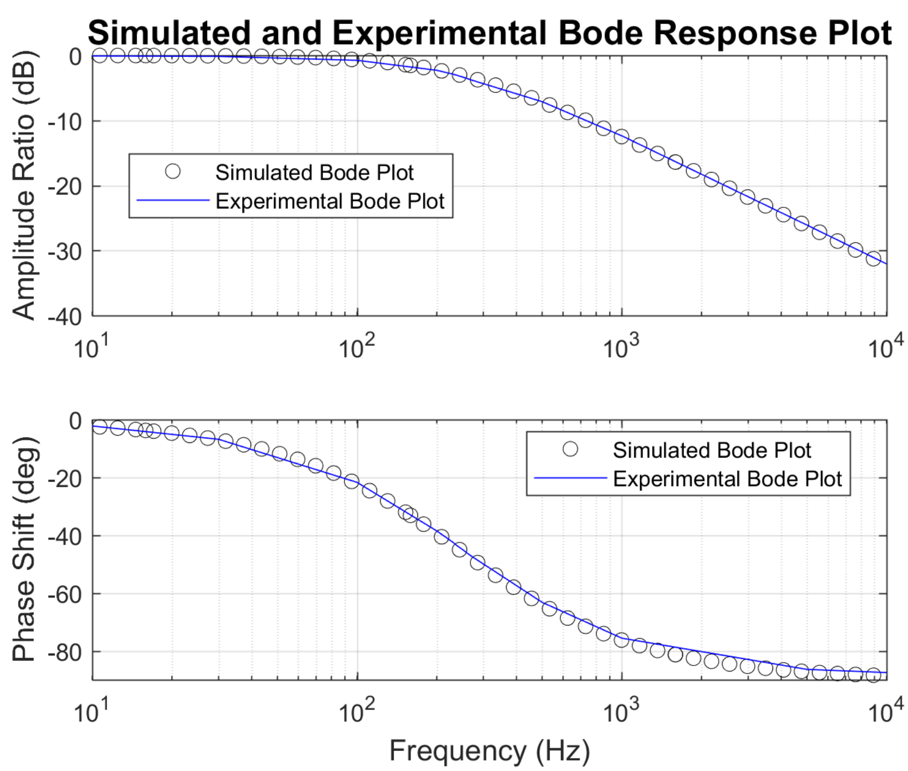


FIGURE 7: 1ST ORDER FREQUENCY RESPONSE BODE PLOT

The experimental time constant was calculated to be 6.366e-04 seconds, which resulted in a capacitor value of 5.735e-08 F. This is only a 2.3% difference from the capacitor value found from the step response analysis (5.87 e-08 F).

The decibel drop in amplitude ratio at the breakpoint frequency of 250 Hz was found to be -3.118 dB.

2.1

For the unaltered square wave input, the damping ratio was found to be 0.0620, with an undamped natural frequency of 313.1 Hz. The DC offset square wave resulted in a damping ratio of 0.0581, and a natural frequency of 301.7 Hz. Theoretically the offset should not cause a difference in these values. In reality, the inductor, capacitor, or resistor could be dependent on voltage level.

The inductance of the system (without the offset), L, was calculated to be 1.27 H. The capacitance of the system was found to be 2.07 e-07 F

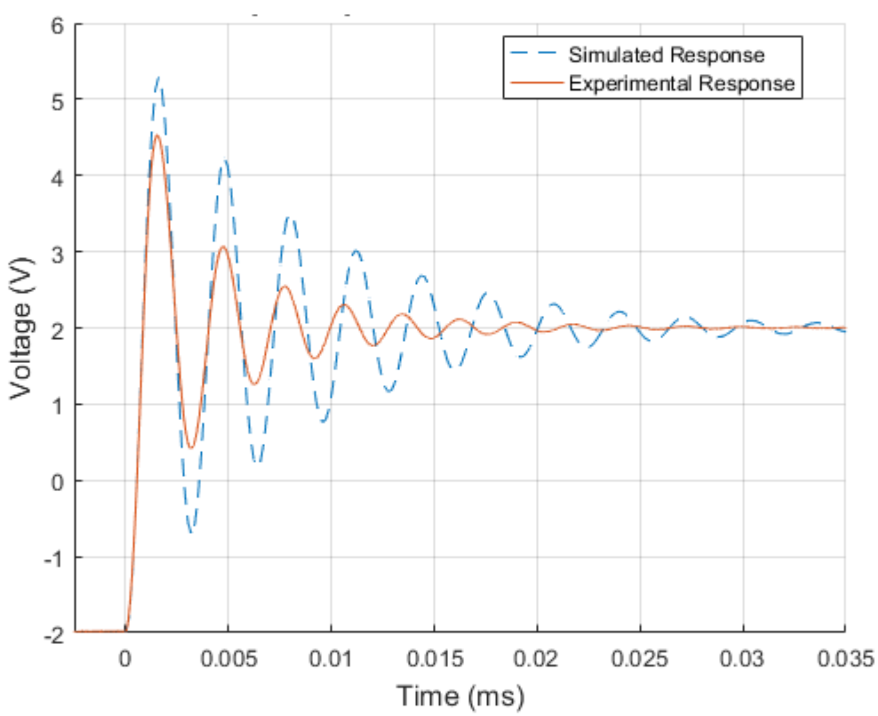


FIGURE 8: SIMULATED STEP RESPONSE OF THE RLC CIRCUIT

Figure 8 further demonstrates the inaccuracy of the experimental damping ratio. It is approximately twice what the simulated damping ratio is. However, the same general shape of the curves is consistent.

2.2

TABLE 3: FREQUENCY RESPONSE DATA OF THE 2ND ORDER SYSTEM

ω_i [Hz]	V	ϕ	dB
10	3.97	-0.2	0
16	3.99	-0.3	0.04364
80	4.18	-1.5	0.44771
158	5.12	-5.9	2.20958
317	11.1	-89	8.93064
476	2.95	-162	-2.57934
500	2.57	-164	-3.77714
634	1.39	-169	-9.11551
1000	0.512	-174	-17.7904
1268	0.311	-175	-22.1206
5000	0.019	-178	-46.4007
7000	0.0087	-178	-53.1854

Sample measurements for amplitude ratio and phase angle can be found in Table 3.

The transfer function for the 2nd order system was determined using the differential equation of the system.

$$\frac{E_o}{E_i} = \frac{1}{LCs^2 + \frac{L}{R}s + 1}$$

From this, a bode plot can be constructed. (Figure 9)

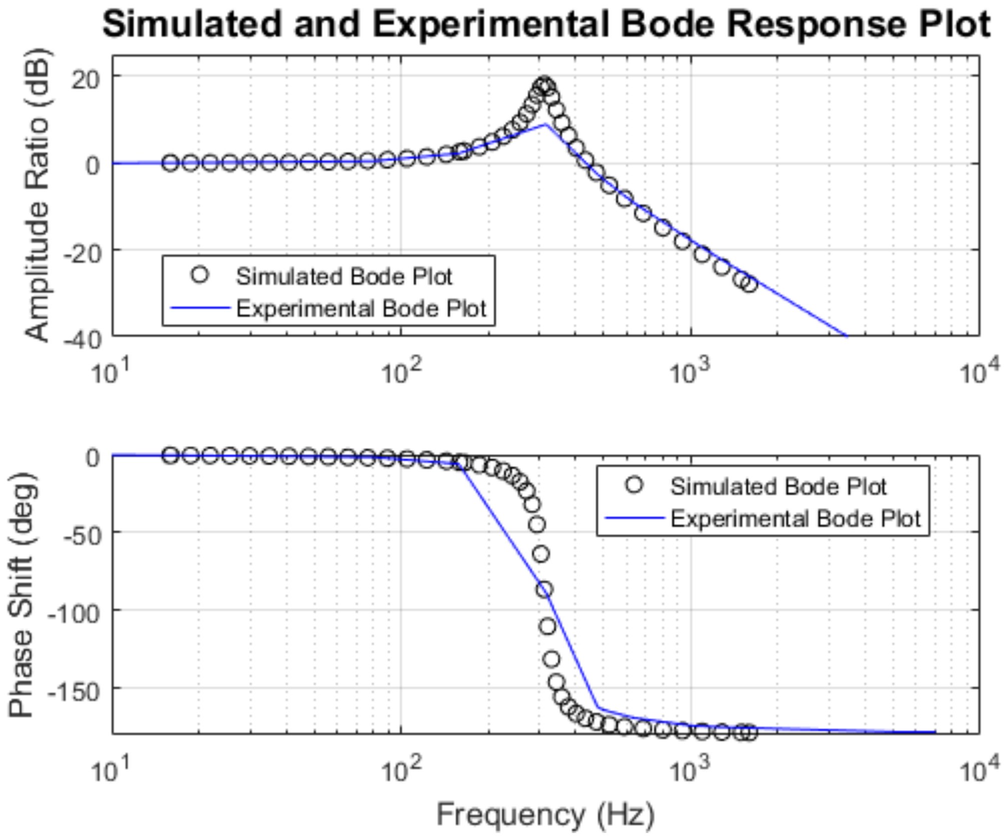


FIGURE 9: 2ND ORDER SYSTEM FREQUENCY RESPONSE BODE PLOT

Experimental data points are plotted to the same bode plot in Figure 9. It is clear that the simulated damping ratio is much higher than experimentally determined damping ratio.

From the bode plot, the damping ratio is 0.0275 with an undamped natural frequency of 1959.7 Hz. These values are much different compared to the damping ratio of 0.0620 and ω_n of 313.1 Hz that was found in the step response. This discrepancy could be the result of not picking enough data points that were close to the break frequency to get an accurate cusp for analysis.

Using these measurements from the bode plot, the inductance was found to be 0.5679 H, and the capacitance was found to be 4.585 e-07 F. These differ greatly from the L and C that resulted from the step response analysis There is a 55.2% difference in inductance (1.27 H) and a 54.9% difference in capacitance (2.07 e-07F) of both methods. Again, this could be error that propagated from not having enough data points to get an accurate estimate of the system characteristics.

3.1

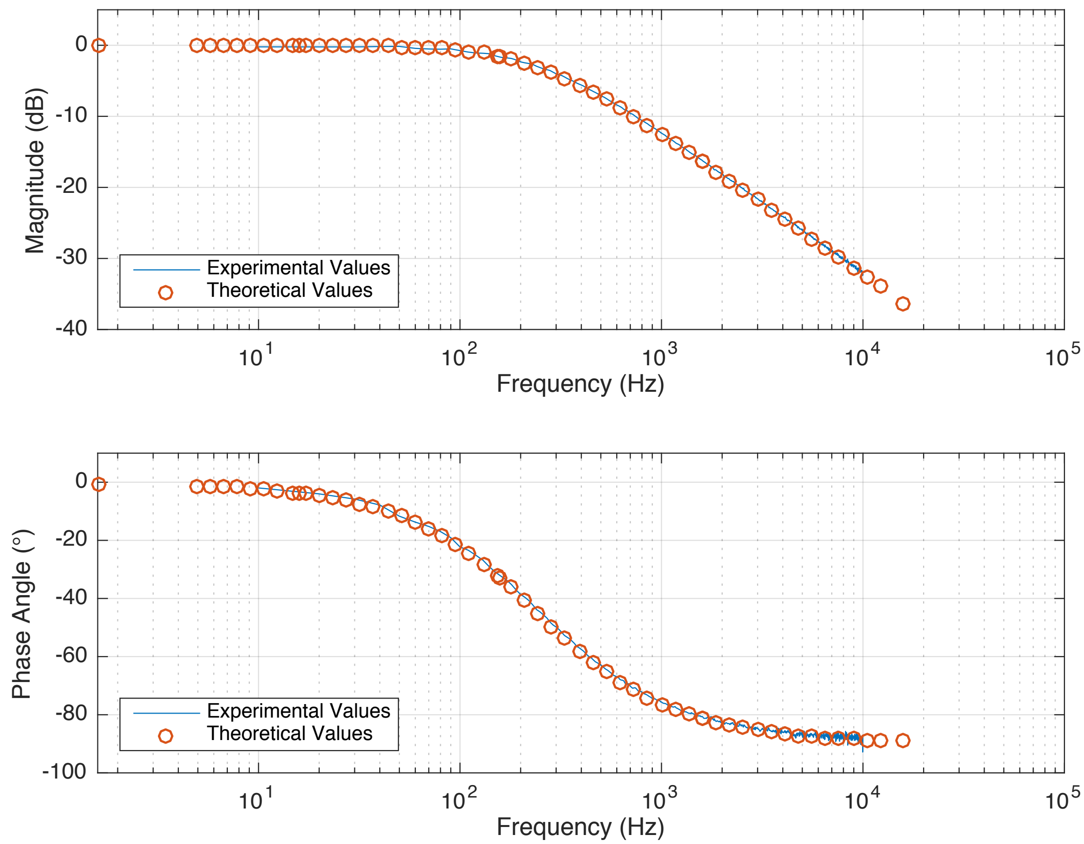


FIGURE 10: EXPERIMENTAL AND THEORETICAL LABVIEW RESULTS FOR THE 1ST ORDER SYSTEM

The experimental values obtained from LabView for the first order system match the theoretical values closely (Figure 10).

Using LabView, the time constant was found to be 6.1244e-04 seconds. This is a 6.125% difference from the time constant calculated with the step response analysis (6.52396e-4 sec) and only a 3.8% difference for the frequency response analysis (6.366e-04 sec).

The capacitor C value was calculated to be 5.5175e-08 F in LabView. This is a 3.94% difference from the frequency response calculated capacitor value (5.735e-08 F), and is a 6.0% difference from the capacitor value found from the step response analysis (5.87 e-08 F).

3.2

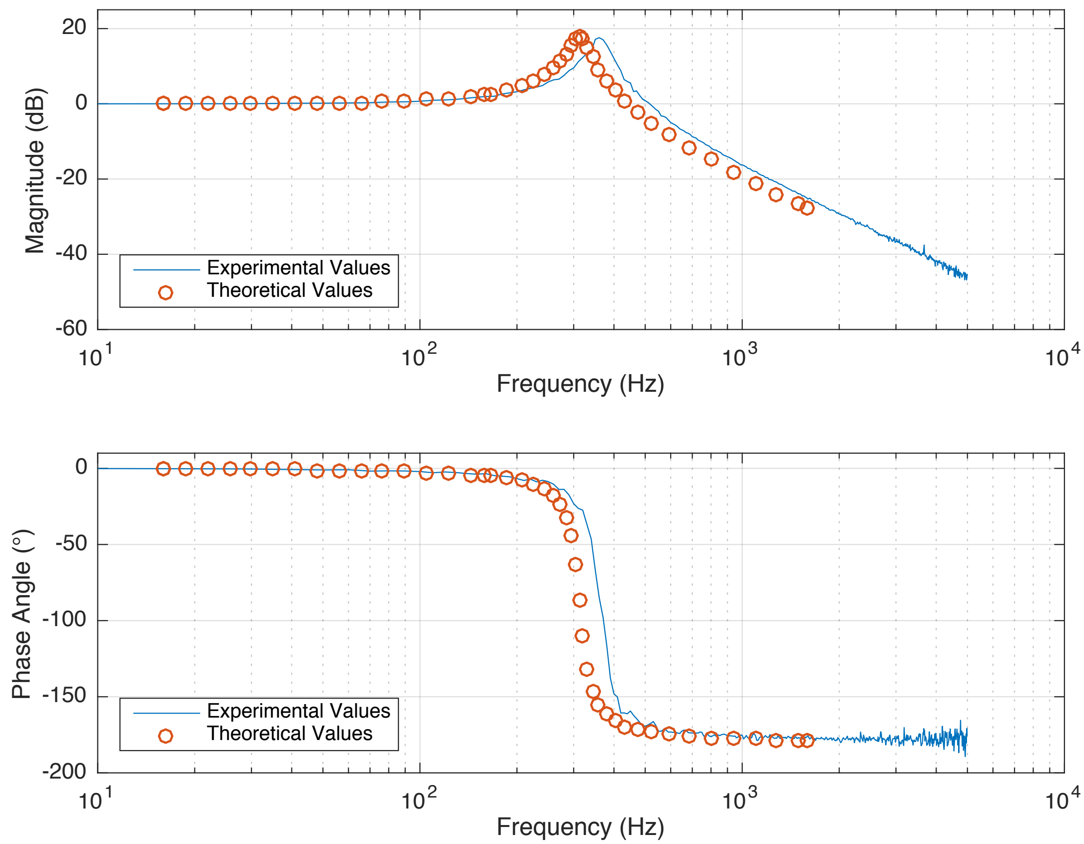


FIGURE 11: EXPERIMENTAL AND THEORETICAL LABVIEW RESULTS FOR THE 2ND ORDER SYSTEM

The curves in Figure 11 are the same general shape, however there is a 30 Hz offset between the experimental and the theoretical values. This could be from each component of the 2^{nd} order circuit not being "ideal". For example, the capacitor and inductor are assumed to have no resistance, when realistically every element of the circuit has a resistance. Wires comprising the circuit may also have an impact – as they each have a resistive and inductive identity.

The experimental damping ratio was found in LabView to be 0.0284 with a natural frequency of 360 Hz. The results from the square wave response shows a 54.19% difference in damping ratio (0.0620), with a 13.0% difference in undamped natural frequency (313.1 Hz). The frequency response analysis yielded a damping ratio with 3.17% difference (0.0275) and a difference in undamped natural frequency of 81.6% (1959.7 Hz).

The characteristics determined by LabView resulted in an inductance of 0.5083 H. This is 10.5% different from the inductance calculated from the frequency response (0.5679 H), and is 59.98% different from the step response inductance (1.27 H)

The capacitance C was calculated to be 3.8491e-07 F using LabView. This is 16.1% different from the frequency response capacitance calculation (4.585 e-07 F), and is 46.2% different from the step response analysis.

Conclusions

Zhangxi Feng:

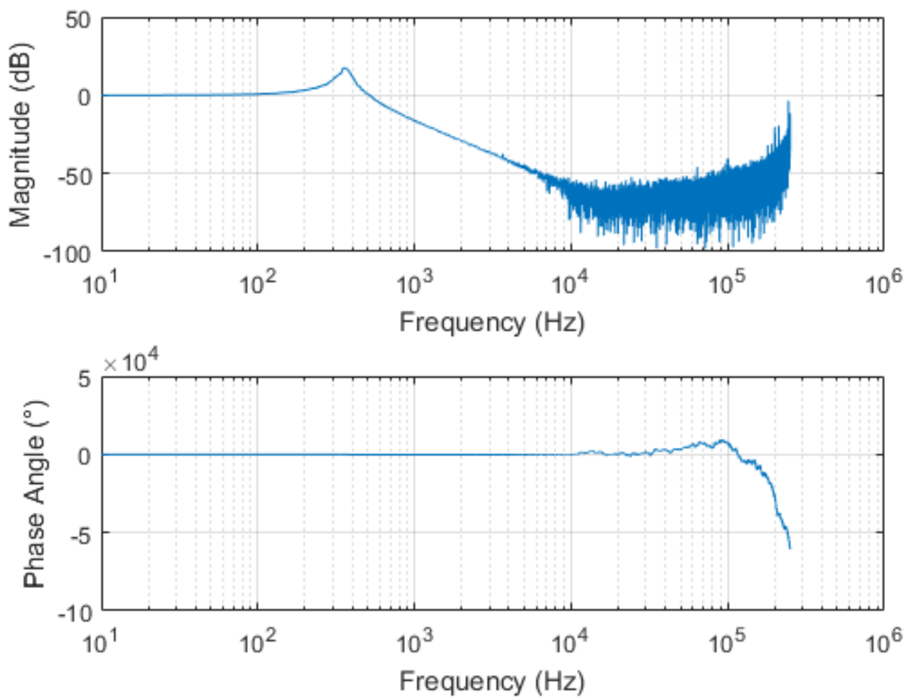
The capacitor in the first order circuit was found to have a capacitance of 58.7 nF. The capacitor in the second order circuit was found to have a capacitance of 0.207 μF. The inductor in the second order circuit was found to have an inductance of 1.27 H. The time constant of the first order circuit was found to be 0.65 ms

Simon Popecki:

The experiments performed with these first and second order circuits show a very low deviance between theoretical and expected values. It is worth noting that with mechanical systems, it takes a considerably higher degree of effort to yield the same results. This is largely in part due to the consistency and predictability of electronics, which have become far more standardized than mechanical systems. For the first order system, theoretical and actual values regarding step response were within a few percent difference – so close in fact, that measurement resolution/sampling rate becomes a question with experimental results.

Differences in experimental and theoretical results of the second order response to a step input clearly show that there were not enough data (Figure 11), it is especially evident in the phase angle plot that data are missing, as discrete points which lie in areas predicted by calculation form lines cutting through the phase change. As far as the individual points are concerned, there is no prominent difference between the theoretical results and experimental results.

However, when determining the broadband frequency response to noise, the predicted values were clearly different than the experimental values recorded in LabView. Error in this operation was considerably high – up to 84%. This could have been caused by a multitude of factors, if the issue is of the hardware variety likely the input was the primary problem. Also likely, is poor data acquisition – an extreme amount of noise was collected on the LabView samples, possibly contributing to error.



Reilly Webb:

For the most part, the gap between theoretical and experimental results remained low throughout this lab. One outlier was the huge difference in experimental and theoretical data for the 2nd order LabView frequency response plots, but other than that our calculated values were agreeing with each other.

If this experiment were to be repeated, I would measure the true inductance and capacitance of each of the circuits. These values would be useful in checking our analysis techniques and calculations without making the analysis less challenging.

It would be interesting to analyze a different form of an RLC circuit, such as a series RLC circuit, and see how the frequency response compares. An RL circuit could also make for a good analysis.

References

[1] DeLeon, M H. "Using LabView for Frequency Responses." 3 Sept. 2014.

Appendices

A. Data Tables

1st order Frequency Response Table

ω_i [Hz]	V	ϕ	dB
10	3.93	-2.2	-0.08795
30	3.9	-6.8	-0.15451
100	3.64	-21.7	-0.75378
200	3.05	-38.5	-2.28981
230	2.85	-42.4	-2.87891
240	2.78	-43.8	-3.09491
250	2.71	-44.9	-3.31642
260	2.64	-46.1	-3.54373
270	2.57	-47.2	-3.77714
500	1.75	-63.2	-7.11504
1000	0.956	-75.6	-12.3666
5000	0.197	-86.4	-26.0864
10000	0.099	-87.5	-32.0631

2nd Order Frequency Response Table

ω_i [Hz]	V	ϕ	dB
10	3.97	-0.2	0
16	3.99	-0.3	0.04364
80	4.18	-1.5	0.44771
158	5.12	-5.9	2.20958
317	11.1	-89	8.93064
476	2.95	-162	-2.57934
500	2.57	-164	-3.77714
634	1.39	-169	-9.11551

1000	0.512	-174	-17.7904
1268	0.311	-175	-22.1206
5000	0.019	-178	-46.4007
7000	0.0087	-178	-53.1854

B. Sample Calculations

Matlab

One.m

```
clear all;
close all;

%% Reading lvm data for first output
filename1 = '1.1.5 Data.lvm';
output1 = importdata(filename1,'\t',33);
responsedata1 = output1.data(:,3:4);

%% a)
% Output 1 from 1.1.5
% Observed from data, t = -3.13e-7 is the threshold
% Takes first 5 values to find the slope.
% Second method

threshold1 = -3.13e-7; % this is the first observable starting point of the
response curve
responseFinalValue1 = mean(responsedata1(4950:end,2));
responseInitialValue1 = mean(responsedata1(1:350,2));

[onetauX1,onetauX2,onetauY1,onetauY2,onetauPer1,onetauPer2,p1] = ...

FindTaus(responsedata1,threshold1,responseFinalValue1,responseInitialValue1);

onetauTime1 = onetauX1 - threshold1;
onetauTime2 = onetauX2 - threshold1;

% The initial slope line
time1 = linspace(0,onetauTime1,100);
out1 = time1*p1(1) + p1(2);

% Plot the results
figure(1);
hold on;
grid on;
xlim([-1 5]);
ylim([-2.5 2.5]);
% plot the original data with initial time offset adjusted
plot((responsedata1(:,1)-threshold1)*1e3,responsedata1(:,2),'-.');
% plot the line
plot(time1*1e3,out1);
% plot the tau location on the curve from the initial slope method
plot(onetauTime1*1e3,onetauY1,'O');
```

```
% plot the tau location on the curve from the second method
plot(onetauTime2*1e3,onetauY2,'X','markers',12);
% plot a drop down from the tip of the line to the tau location
plot([onetauTime1*1e3 onetauTime1*1e3],[onetauY1 out1(end)],'--');
legend('Location','best','Original response data','Initial slope line',...
    'Tau point from initial slope method','Tau point from 63.2% method',...
    'Drop down indicator line for initial slope method');
title('First Order Step Response 0 Offset Analysis','FontSize',14);
xlabel('Time (ms)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);

%% Reading lvm data for second output
filename2 = '1.1.6 Data.lvm';
output2 = importdata(filename2,'\t',33);
responsedata2 = output2.data(:,3:4);

%% a)
% Output 1 from 1.1.6
% observed from data, t = 2.717e-7 is the threshold
% Takes first 5 values to find the slope.

threshold2 = 2.717e-7; % this is the first value higher than the initial
noise
responseFinalValue2 = mean(responsedata2(4950:end,2));
responseInitialValue2 = mean(responsedata2(1:350,2));

[twotauX1,twotauX2,twotauY1,twotauY2,twotauPer1,twotauPer2,p2] = ...

FindTaus(responsedata2,threshold2,responseFinalValue2,responseInitialValue2);

twotauTime1 = twotauX1 - threshold2;
twotauTime2 = twotauX2 - threshold2;

% The initial slope line
time2 = linspace(0,twotauX1,100);
out2 = time2*p2(1) + p2(2);

% Plot the results
figure(2);
hold on;
grid on;
xlim([-1 5]);
ylim([-3.5 5.5]);
% plot the original data with initial time offset adjusted
plot((responsedata2(:,1)-threshold2)*1e3,responsedata2(:,2),'-.');
% plot the line
plot(time2*1e3,out2);
% plot the tau location on the curve from the initial slope method
plot(twotauTime1*1e3,twotauY1,'O');
% plot the tau location on the curve from the second method
plot(twotauTime2*1e3,twotauY2,'X','markers',12);
% plot a drop down from the tip of the line to the tau location
plot([twotauTime1*1e3 twotauTime1*1e3],[twotauY1 out2(end)],'--');
legend('Location','best','Original response data','Initial slope line',...
    'Tau point from initial slope method','Tau point from 63.2% method',...
    'Drop down indicator line for initial slope method');
title('First Order Step Response 1V Offset Analysis','FontSize',14);
```

```
xlabel('Time (ms)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);

%% e) Theoretical Simulation
R = 11.1e3;
C = 58.77e-9;
sys = tf(1,[R*C 1]); % transfer function

% creates the step offset conditions
opt1 = stepDataOptions('InputOffset',-2,'StepAmplitude',4);

% step response
[V1,t1] = step(sys,opt1);

% Find tau
for i = 1:length(V1)
    if (V1(i) >= 0.632*(V1(end)-V1(1))+V1(1))
        tauX = t1(i);
        tauY = V1(i);
        break;
    end
end

figure(3);
hold on;
grid on;
xlim([-1 5]);
ylim([-2.5 2.5]);
plot(t1*1e3,V1);
plot(tauX*1e3,tauY,'*','markers',14);
plot([-1 5],[tauY tauY],'k--');
title('Simulated Response of the First Order Circuit','FontSize',14);
xlabel('Time (ms)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);
text(2,-0.5,['Tau = ',num2str(tauX*1e3),' ms']);
text(2,-0.8,['Tau Voltage = ',num2str(tauY),' V']);

figure(4);
hold on;
grid on;
xlim([-1 5]);
ylim([-2.5 2.5]);
plot(t1*1e3,V1);
plot((responsedata1(:,1)-threshold1)*1e3,responsedata1(:,2),'-.');
legend('Location','best','Theoretical Simulation','Experimental Result');
title('Comparing Theoretical and Experimental Step Response','FontSize',14);
xlabel('Time (ms)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);

%% 1.2 b) Bode Plots
% source data
omega_i = [10,30,100,200,230,240,250,260,270,500,1000,5000,10000];
dB = [-0.087959128,-0.15451799,-0.753782462,-2.289813348,-2.878912935,...
    -3.094914217,-3.316424318,-3.543731598,-3.777147669,-7.115049162,...
    -12.36665229,-26.08648561,-32.06310624];
phi = [-2.2,-6.8,-21.7,-38.5,-42.4,-43.8,-44.9,-46.1,-47.2,-63.2,-75.6,-
86.4,-87.5];
```

```
% retrieve bode plot data from bode(sys)
[mag,phase,wout] = bode(sys);
phase = squeeze(phase); % reduce the 3D matrix into 2D
mag = squeeze(mag); % reduce the 3D matrix into 2D
wout = wout/(2*pi); % convert from rad/s to Hz

% magr = (dB/max(dB)).^2; % Power of the ratio
% dB3 = interp1(magr,[omega_i' phi' dB'],0.5,'spline'); % interpret for Wb

% convert mag1 to decibels
magdb = 20*log10(mag);

figure(5);
subplot(2,1,1);
semilogx(wout,magdb,'kO');
hold on;
grid on;
semilogx(omega_i,dB,'b');
xlim([10 10000]);
ylabel('Amplitude Ratio (dB)','FontSize',12);
title('Simulated and Experimental Bode Response Plot','FontSize',14);
legend('Location','best','Simulated Bode Plot','Experimental Bode Plot');

subplot(2,1,2);
semilogx(wout,phase,'kO');
hold on;
grid on;
semilogx(omega_i,phi,'b');
xlim([10 10000]);
ylim([-90 0]);
xlabel('Frequency (Hz)','FontSize',12);
ylabel('Phase Shift (deg)','FontSize',12);
legend('Location','best','Simulated Bode Plot','Experimental Bode Plot');
%% 1.2 c)
breakfrequency=250; %Hz
tauC=1/(2*pi*breakfrequency);
CapC=tauC/R;
%% 1.2 e)
[magE,phaseE,woutE]=bode(sys,250*2*pi);
magdbE = 20*log10(magE);
```

Two.m

```
clear all;
close all;

%% 2.1 b) Finding Experimental damping ratio and undamped natural frequency
filename1 = '2.1.4 Data.lvm';
output1 = importdata(filename1,'\t',33);
responsedata1 = output1.data(:,3:4);

% find peaks
[maxtab,mintab] = peakdet(responsedata1(:,2),0.05);

% check peaks
```

```
figure(1);
grid on;
hold on;
plot(responsedata1(:,1),responsedata1(:,2));
plot(responsedata1(maxtab(:,1),1),maxtab(:,2),'O');
title('Experimental 0V Offset Step Response Data','FontSize',14);
xlabel('Time (s)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);
legend('Location','best','Data','Peaks');

% Log decrement method
n = 2; % next peak is 2 periods away
sigma = (1/(n-1))*log(maxtab(1,2)/maxtab(2,2));
zeta = sigma/sqrt(4*pi^2+(sigma)^2); % the damping ratio
T = diff(responsedata1(maxtab(:,1),1));
T = T(1); % The first damped period
wd = 2*pi/T; % The damped frequency in rad/s -> 2pi rad/cyc div s/cyc
wn = wd/(sqrt(1-zeta^2)); % Natural frequency in rad/s
wnHz = wn/(2*pi); % Natural frequency in Hz -> rad/s div 2pi rad/cyc

%% c)
% From derived differential equation, LC = 1/wn^2, L/R = 2*zeta/wn
R = 20200; % R = 20.2k ohms measured in lab
L = R*2*zeta/wn; % result = 1.3 mH
C = 1/wn^2/L; % result = 179.21 muF

sys = tf(1,[L*C,L/R,1]);

%% d) Step Responses
% creates the step offset conditions
opt1 = stepDataOptions('InputOffset',-2,'StepAmplitude',4);

% step response
[V1,t1] = step(sys,opt1);

figure(2);
hold on;
grid on;
xlim([-0.0025 0.035]);
ylim([-2 6]);
plot(t1,V1,'--');
plot(responsedata1(:,1),responsedata1(:,2));
title('Simulated 0V Offset Step Response of the Second Order
Circuit','FontSize',14);
xlabel('Time (ms)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);
legend('Location','best','Simulated Response','Experimental Response');

%% d) second offset
filename2 = '2.1.5 Data.lvm';
output2 = importdata(filename2,'\t',33);
responsedata2 = output2.data(:,3:4);
% creates the step offset conditions
opt2 = stepDataOptions('InputOffset',-3,'StepAmplitude',8);

% step response
[V2,t2] = step(sys,opt2);
```

```
figure(3);
hold on;
grid on;
xlim([-0.0025 0.035]);
ylim([-4 12]);
plot(t2,V2,'--');
plot(responsedata2(:,1),responsedata2(:,2));
title('Simulated 1V Offset Step Response of the Second Order
Circuit','FontSize',14);
xlabel('Time (ms)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);
legend('Location','best','Simulated Response','Experimental Response');

% find peaks
[maxtab2,mintab2] = peakdet(responsedata2(:,2),0.05);

% check peaks
figure(4);
grid on;
hold on;
plot(responsedata2(:,1),responsedata2(:,2));
plot(responsedata2(maxtab2(:,1),1),maxtab2(:,2),'O');
title('Experimental 1V Offset Step Response Data','FontSize',14);
xlabel('Time (s)','FontSize',12);
ylabel('Voltage (V)','FontSize',12);
legend('Location','best','Data','Peaks');

% Log decrement method
n = 2; % next peak is 2 periods away
sigma2 = (1/(n-1))*log(maxtab2(1,2)/maxtab2(2,2));
zeta2 = sigma2/sqrt(4*pi^2+(sigma2)^2); % the damping ratio
T2 = diff(responsedata2(maxtab2(:,1),1));
T2 = T2(1); % The first damped period
wd2 = 2*pi/T2; % The damped frequency in rad/s -> 2pi rad/cyc div s/cyc
wn2 = wd2/(sqrt(1-zeta2^2)); % Natural frequency in rad/s
wnHz2 = wn2/(2*pi); % Natural frequency in Hz -> rad/s div 2pi rad/cyc

%% 2.2 a) - done in excel sheet

%% b)
omega_i = [10,16,80,158,317,476,500,634,1000,1268,5000,7000]; % Hz
dB = [0,0.043647778,0.4477155,2.209589084,8.93064944,-2.579369816,... % dB
    -3.777147669,-9.11551413,-17.79041092,-22.12060235,-46.40073812,-
53.18542508];
phi = [-0.2,-0.3,-1.5,-5.9,-89,-162,-164,-169,-174,-175,-178,-178]; % deg

% retrieve bode plot data from bode(sys)
[mag,phase,wout] = bode(sys);
phase = squeeze(phase); % reduce the 3D matrix into 2D
mag = squeeze(mag); % reduce the 3D matrix into 2D
wout = wout/(2*pi); % convert from rad/s to Hz

% convert mag1 to decibels
magdb = 20*log10(mag);
```

```
figure(5);
subplot(2,1,1);
semilogx(wout,magdb,'kO');
hold on;
grid on;
semilogx(omega_i,dB,'b');
xlim([10 10000]);
ylim([-40 25]);
ylabel('Amplitude Ratio (dB)','FontSize',12);
title('Simulated and Experimental Bode Response Plot','FontSize',14);
legend('Location','best','Simulated Bode Plot','Experimental Bode Plot');

subplot(2,1,2);
semilogx(wout,phase,'kO');
hold on;
grid on;
semilogx(omega_i,phi,'b');
xlim([10 10000]);
ylim([-180 0]);
xlabel('Frequency (Hz)','FontSize',12);
ylabel('Phase Shift (deg)','FontSize',12);
legend('Location','best','Simulated Bode Plot','Experimental Bode Plot');

%% c)
[maxPeak,maxInd] = max(magdb); % maximum point on the measured bode plot
wnFromBode = wout(maxInd)*2*pi; % frequency where maxPeak happened
zetaExp = 1/(2*maxPeak); % Approximate height = 1/2zeta
%% d)
% From derived differential equation, LC = 1/wn^2, L/R = 2*zeta/wn
LFromBode = R*2*zetaExp/wnFromBode; %
CFromBode = 1/wnFromBode^2/LFromBode; %
```

Three.m

```
%This script is responsible for Part 3 of the analysis
%Author: Simon Popecki
%Date: 30 September 2017
%Class: ME 747

%Part 3 concerns the response of the first and second order systems to a
%random noise input. The experimental values are compared to the
%theoretical predictions.

%% Data Importation
clear all;
close all;

load Part3Data.mat
%DATA FORMAT:
%For amplitude response plots, the X-value from the raw data is frequency
(Hz)
%The Y-value is the corresponding magnitude (dB)
%For phase response plots, the X-vlue from the raw data is frequency (Hz)
%The Y-value is the corresponding phase angle (degrees)

%Signal Conditioning
```

```
%The raw bode plots from LabView have a lot of noise past 10 kHz (1st
%order) and 1.4 kHz (2nd order)

%Trimming the 1st order bode plot at ~10 kHz
cutoff1 = 1000; %frequency index cutoff number, first order
frequencyAR1 = frequencyAR1(1:cutoff1);
magnitudeAR1 = magnitudeAR1(1:cutoff1);
frequencyPS1 = frequencyPS1(1:cutoff1);
phaseAnglePS1 = phaseAnglePS1(1:cutoff1);

%Trimming the 2nd order bode plot at 1.4 kHz
cutoff2 = 500; %frequency index cutoff number, second order
frequencyAR2 = frequencyAR2(1:cutoff2);
magnitudeAR2 = magnitudeAR2(1:cutoff2);
frequencyPS2 = frequencyPS2(1:cutoff2);
phaseAnglePS2 = phaseAnglePS2(1:cutoff2);


%% Freqeuncy Response of a First Order System Using LabView
%First order known values (R and C get overwritten)
R = 11.1e3; %Ohms, the resistance of the resistor
C = 58.77e-9; %Farrads, the capacitance of the capacitor
sys1 = tf(1,[R*C 1]); %The transfer function of the 1st order system

[mag,theoreticalPhaseAngle1,theoreticalFrequency1] = bode(sys1); %Extracting
data points from system object, sys1
theoreticalPhaseAngle1 = squeeze(theoreticalPhaseAngle1); %force array
mag1 = squeeze(mag); %force array
TheoreticalMagnitudeAR1 = 20*log10(mag1); %converting from absolute magnitude
to decibels
theoreticalFrequency1 = theoreticalFrequency1/(2*pi); %convert from rad/s to
Hz

%1/Tau (the time constant) occurs at the point where phase is 45 degrees
%omega is ~250 Hz
locationMap = find(phaseAnglePS1<=-45); %finding the indices of phase angles
that meet the condition <= -45 degrees
freqInd = locationMap(1); %finding the index of the first value meeting the
<= -45 degrees condition
omega1 = frequencyPS1(freqInd); %Hz, 1/tau - the natural frequency of the
first order system
tau1 = 1/(omega1*6.28); %seconds, the time constant of the first order system
(multiplied by 6.28 to convert to radians per second
disp('Circuit 1 time constant (seconds):')
disp(tau1)
C1 = 1/(R*omega1*6.28); %Farrads, the calculated capacitance of the first
order system using data from LabView
disp('Circuit 1 capacitance (Farrads):')
disp(C1)

figure(1)
subplot(2,1,1)
semilogx(frequencyAR1,magnitudeAR1,theoreticalFrequency1,TheoreticalMagnitude
AR1,'o')
% title('Theoretical and Experimental Frequency Response')
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
```

```
legend('Experimental Values','Theoretical Values','location','southwest')
grid on
xmin = 0;
xmax = 100000;
ymin = -40;
ymax = 5;
axis ([xmin xmax ymin ymax])
subplot(2,1,2)
semilogx(frequencyPS1,phaseAnglePS1,theoreticalFrequency1,theoreticalPhaseAng
le1,'o')
xlabel('Frequency (Hz)')
ylabel('Phase Angle (∞)')
legend('Experimental Values','Theoretical Values','location','southwest')
grid on
xmin = 0;
xmax = 100000;
ymin = -100;
ymax = 10;
axis ([xmin xmax ymin ymax])


%% Frequency Response of a Second Order System Using LabView
%Second order known values
L = 1.272690763057150; %H, inductance
R = 20200; %Ohms, the resistance of the resistor
C = 2.030234051171926e-07; %Farrads, the capacitance of the capacitor
sys2 = tf(1,[L*C L/R 1]); %The transfer function of the 1st order system

[mag,theoreticalPhaseAngle2,theoreticalFrequency2] = bode(sys2); %Extracting
data points from system object, sys1
theoreticalPhaseAngle2 = squeeze(theoreticalPhaseAngle2); %force array
mag2 = squeeze(mag); %force array
TheoreticalMagnitudeAR2 = 20*log10(mag2); %converting from absolute magnitude
to decibels
theoreticalFrequency2 = theoreticalFrequency2/(2*pi); %convert from rad/s to
Hz

%Finding Experimental Damping Ratio
peakdB = max(magnitudeAR2); %finding the magnitude of the peak
zeta = 1/(peakdB*2); %calculating the damping ratio given the peak magnitude
disp('Circuit 2 damping ratio:')
disp(zeta)

%Finding Experimental Natural Frequency
[~,indexMaxMag] = max(magnitudeAR2); %finding the index of the max amplitude
natFrequency = frequencyAR2(indexMaxMag); %Hz, finding the frequency at the
break point
disp('Circuit 2 natural frequency (Hz):')
disp(natFrequency)

%Calculating L and C values from LabView experimental results
LC = 1/((natFrequency*6.28)^2);
L = R*(2*zeta/(natFrequency*6.28)); %H, calculating inductance
C = LC/L;
disp('Circuit 2 Inductance (H):')
disp(L)
disp('Circuit 2 Resistance (Ohms):')
```

```
disp(R)
disp('Circuit 2 Capacitance (F):')
disp(C)

figure(2)
subplot(2,1,1)
semilogx(frequencyAR2,magnitudeAR2,theoreticalFrequency2,TheoreticalMagnitude
AR2,'o')
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
legend('Experimental Values','Theoretical Values','location','southwest')
grid on
xmin = 0;
xmax = 10000;
ymin = -60;
ymax = 25;
axis ([xmin xmax ymin ymax])
subplot(2,1,2)
semilogx(frequencyPS2,phaseAnglePS2,theoreticalFrequency2,theoreticalPhaseAng
le2,'o')
xlabel('Frequency (Hz)')
ylabel('Phase Angle (∞)')
legend('Experimental Values','Theoretical Values','location','southwest')
grid on
xmin = 0;
xmax = 10000;
ymin = -200;
ymax = 10;
axis ([xmin xmax ymin ymax])
```

peakdet.m

```
function [maxtab, mintab]=peakdet(v, delta, x)
%PEAKDET Detect peaks in a vector
%        [MAXTAB, MINTAB] = PEAKDET(V, DELTA) finds the local
%        maxima and minima ("peaks") in the vector V.
%        MAXTAB and MINTAB consists of two columns. Column 1
%        contains indices in V, and column 2 the found values.
%
%        With [MAXTAB, MINTAB] = PEAKDET(V, DELTA, X) the indices
%        in MAXTAB and MINTAB are replaced with the corresponding
%        X-values.
%
%        A point is considered a maximum peak if it has the maximal
%        value, and was preceded (to the left) by a value lower by
%        DELTA.

% Eli Billauer, 3.4.05 (Explicitly not copyrighted).
% This function is released to the public domain; Any use is allowed.

maxtab = [];
mintab = [];

v = v(:); % Just in case this wasn't a proper vector

if nargin < 3
  x = (1:length(v))';
```

```
else
  x = x(:);
  if length(v)~= length(x)
    error('Input vectors v and x must have same length');
  end
end

if (length(delta(:)))>1
  error('Input argument DELTA must be a scalar');
end

if delta <= 0
  error('Input argument DELTA must be positive');
end

mn = Inf; mx = -Inf;
mnpos = NaN; mxpos = NaN;

lookformax = 1;

for i=1:length(v)
  this = v(i);
  if this > mx, mx = this; mxpos = x(i); end
  if this < mn, mn = this; mnpos = x(i); end

  if lookformax
    if this < mx-delta
      maxtab = [maxtab ; mxpos mx];
      mn = this; mnpos = x(i);
      lookformax = 0;
    end
  else
    if this > mn+delta
      mintab = [mintab ; mnpos mn];
      mx = this; mxpos = x(i);
      lookformax = 1;
    end
  end
end
```

findtaus.m

```
function [tauX1,tauX2,tauY1,tauY2,tauPer1,tauPer2,p] = ...
    FindTaus(responsedata,threshold,responseFinalValue,responseInitialValue)
    %% Calculate tau using initial slope method

    % initialize variables
    indSt = 1;

    % cut off the initial flat data
    for i = 1:size(responsedata,1)
        % This condition will be met after the initial index, therefore if
this
        % condition is met, the loop will exit
        if (responsedata(i,1) > threshold)
            indSt = i;
```

```
            break;
        end
    end
    indEnd = indSt + 4;

    % polyfit the initial slope segment data to find the equation of a line
    % then use the equation values and final value to find the intersection
    p = polyfit(responsedata(indSt:indEnd,1),responsedata(indSt:indEnd,2),1);
    tauX1 = (responseFinalValue - p(2))/p(1); % x = (y - y(0))/m
    for i = indEnd:size(responsedata,1) % we know the tau must be beyond
indEnd
        if (responsedata(i,1) >= tauX1)
            tauY1 = responsedata(i,2);
            break;
        end
    end

    % check for the tau percentage
    tauPer1 = (tauY1-responseInitialValue)/(responseFinalValue-
responseInitialValue);

    %% Calculate tau using the 63.2% method
    % Finds the 63.2% of final value point, then scan the data until that
    % point is reached.
    tauY2 = (responseFinalValue-responseInitialValue)*0.632 +
responseInitialValue;
    for i = indEnd:size(responsedata,1)
        if (responsedata(i,2) >= tauY2)
            tauX2 = responsedata(i,1);
            break;
        end
    end

    % check for the tau percentage
    tauPer2 = (tauY2-responseInitialValue)/(responseFinalValue-
responseInitialValue);

end % end of function
```

Statement of Shared Effort - Lab 1 ME 747

Monday, October 2, 2017 8:04 PM

All parties contributed an equal amount and the grade should be split evenly.

Simon Popecki

SP

Jesse Feng

[illegible] F

Reilly Webb

RW