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| Semester and Year:  Fall 2017 | Name of Lab Instructor:  Alireza Ebadi |
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| Grade: |

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# Objectives

A first order (RC) circuit and a second order (RLC) circuit were subjected to step and sinusoidal inputs and their outputs were measured to determine their responses. The system parameters of the circuits were derived and calculated from the outputs. The unknown inductance and capacitances of the circuits were calculated from the system parameters.

# Executive Summary

***(need to add in numbers from results)***

In part 1 of this lab, a first order (RC) circuit was investigated to determine its responses for two step inputs at different DC offsets and amplitudes, and a sinusoidal input with no DC offset. The differential equation and transfer function were found for the circuit, as well as the time constant, . The resistor value was known and the capacitance was calculated from the results, . The amplitude and phase shift of the frequency response- the bode plot, was created from measurements. The results were then compared to Matlab simulations. From the bode plot, and were calculated again to be and respectively.

In part 2 of this lab, a second order (RLC) circuit was investigated to determine its responses for the same step as well as sinusoidal inputs. The damping ratio () and natural frequencies () were determined from the step responses, 0.0620 and 313.1 Hz, and the bode plot, 0.0275 and 1959.7 Hz. The resistor value of the circuit was known, and the inductance and capacitance were calculated for the step response, 1.27 H and , and the frequency response, 0.568 H and . The results were compared between the responses and Matlab simulations.

In part 3, the same two circuits were connected to Labview via a Data Acquisition (DAQ) device to collect data for creating the bode plots. The system parameters and component values of each circuit were calculated from the bode plots and compared to the results from previous two parts. The frequency response of the circuit was tested via an input of simulated white noise. For the RC circuit, τ was found to be and . The RLC circuit returned a damping ratio of 0.0284 and an undamped natural frequency of 360 Hz. From these, L was found to be 0.5083 H and C was

# Theory and Experimental Methods

**Theory**

1. Step Response of a First Order System

Two methods were used to experimentally determine the time constant from the response data. The initial slope method locates τ by extrapolating the initial slope of the data. This slope is extended to the intersection with the steady-state voltage, and this point gives the time constant in seconds.

The second method is defined as follows

In short τ is the point where the output reaches 63.2% of the final steady state value.

Kirchhoff’s Voltage Loop law can be applied to the RC circuit, yielding the following equations

Solving for an equation relating and through

To solve for capacitance

The measured resistance for the RC circuit was

Multiple data points and methods were used to find τ, the average calculated capacitance will be taken.

1. Frequency Response of a First Order System

To find the transfer function of the system, first take the Laplace transform of the differential equation.

Then rearrange to find

The time constant is related to the breakpoint frequency as follows

Experimentally the break frequency was found to be 250 Hz. With a measured resistance , the calculated capacitance is

1. Step Response of a Second Order System

The RLC circuit can be modeled using a 2nd order differential equation. Using Kirchhoff’s Voltage Loop law,

With the equivalent voltage output relating the equations

The damping ratio can then be found from the raw data using the log decrement method on any two successive peaks

where is the amplitude at time , is the period, and is the number of positive successive peaks. The undamped natural frequency can then be found as follows

and can then be found with the following equations derived from the general form of the system’s differential equation

1. Frequency Response of a Second Order System

The transfer function of the system can be derived by taking the Laplace transform of the differential equation

The damping ratio and undamped natural frequency were then determined from a bode plot of the transfer function, where max peak is in dB and correlates to this peak location.

and can be found using the same equations as before, **EquationNumber**

3. Frequency Response Using LabView

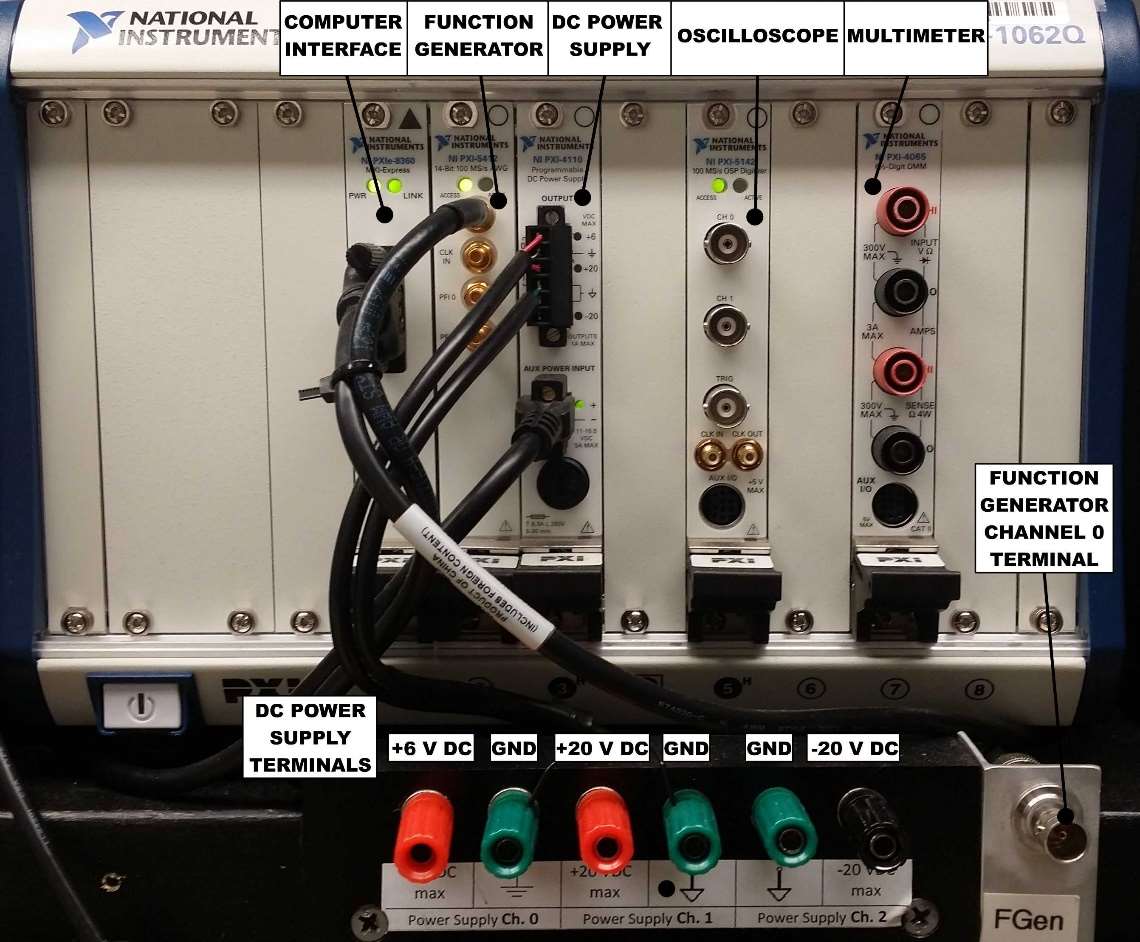
The time constant and capacitance was found from the 1st order LabView results using a bode plot and the following relationships

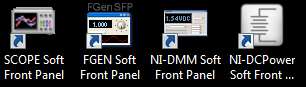
The damping ratio for the 2nd order system was found using the same max peak equation, **EquationNumber**. The natural frequency also correlates to the location of this peak.

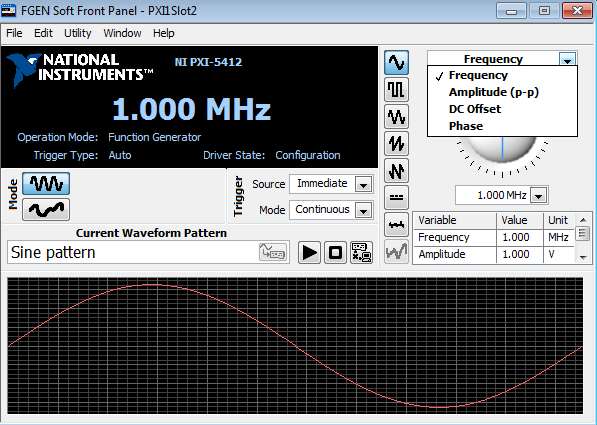
and can be found using the same equations as before, **EquationNumber**

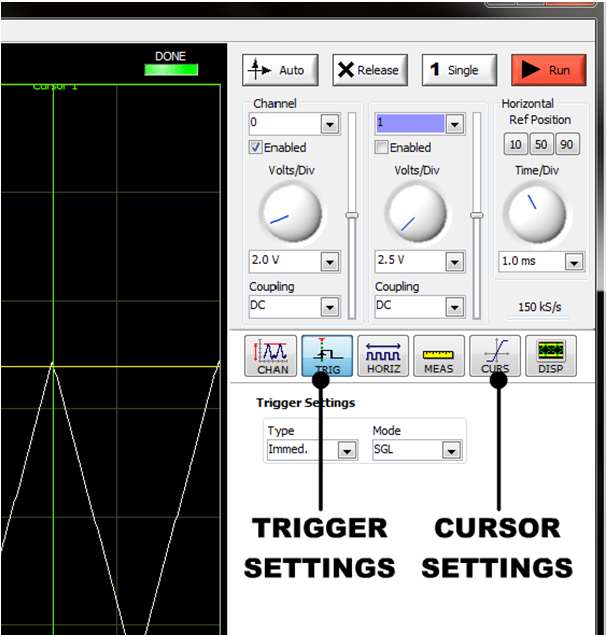
**Experimental Methods**

Useful figures taken from Lab 0









1. First Order System

**A first order RC circuit was set up as shown in FigureNumber. A T-junction BNC connector was used to connect a function generator to both a digital scope and the circuit input, ei(t).**



**The resistance, R (14.1 Ω), was measured using a digital multimeter. A square wave with an amplitude of 2 volts was sent through the circuit and the input and output curves were recorded using a digital oscilloscope.**

**Next a 4 volt DC offset was applied so that the square wave amplitude was from -3.0 V to +5.0 V, and the step response was recorded and compared.**

**The function generator was then set to generate a sine wave with a 2 volt amplitude and no dc offset. The output amplitude and phase delay was recorded using the scope cursor for a range of input frequencies. These input frequencies ranged from 10-10000 Hz, with a higher density of points near the break frequency (250 Hz).**

2. Second Order System

**A second order RLC circuit was set up as shown in FigureNumber. A T-junction BNC connector was used to connect a function generator to both a digital scope and the circuit input, ei(t).**



**The resistance, R (20 kΩ), was measured using a digital multimeter. A square wave with an amplitude of 2 volts was sent through the circuit input, and the input and output curves were recorded for a complete step response.**

**A 4 volt DC offset was applied so that the square wave amplitude is from -3.0 V to +5.0 V, and once again the input and output curves for a full step response were recorded using the scope.**

**The function generator was then set to sinusoidal input with an amplitude of 2 V and no DC offset. The break frequency (317 Hz) was estimated by sweeping the frequency of the sin wave from 10 Hz to 10 kHz. The output amplitude and phase shift was recorded for input frequencies of 1/20, 1/4, 1/2, 3/2, 2 and 4 times the break frequency.**

3. Frequency Response Using LabView

**LabView was used as an alternative method for determining the frequency response for both systems. ADD REFERENCE TO EXPLANATION ON BROADBAND NOISE OR CROSS POWER SPECTRAL DENSITY.**

**ADD SCREENSHOTS OF LABVIEW? Maybe just reference the procedure by M.H. deLeon instead of regurgitating these instructions.**

**The following procedure was performed on both the RC and RLC circuits.**

**The filter input was connected to the NI DAQ output terminals. The RC circuit output was connected to scope Ch.1 and the input to scope Ch. 0.**

**Select the “Data View” tab, then press the “Run” button. Once the curve has converged, hit the “Stop” button. Export the bode plot and use the zoom tools in LV-SE to estimate the break frequencies.**



# Results and Discussion

Summarize your results in a topic sentence. Relate results to Objective. This is the place for graphs, tables and figures. Explain the results of the experiment. Comment on the shapes of the curves, compare obtained results with expected results, give possible reasons for discrepancies. Answer all questions and solve any problems presented in the instructions. Tell why things happened, not only that they did happen. Experimental error should be discussed here. Calculations and formulas are not presented in this section. Avoid space consuming zeros.

1.1

a.)

Two methods were used for calculating the time constants for both the unadjusted square wave input and the 1 V DC offset square wave input. It is clear from **TableNumber** that each method resulted in similar time constants, with only a 5% difference between the furthest values. It is also apparent that adding a DC offset to the input did not affect the time constant of the system.

|  |  |  |
| --- | --- | --- |
| Method | Tau Adjusted (s)\* | Calculated Capacitance (nF) |
| No Offset Initial Slope | 6.68155e-4 | 60.19 |
| No Offset 63.2% | 6.54000e-4 | 58.92 |
| DC Offset Initial Slope | 6.35428e-4 | 57.25 |
| DC Offset 63.2% | 6.52000e-4 | 58.74 |
| Average | 6.52396e-4 | 58.77 |

\*Adjusted Tau is the value found taking into account that the data did not start at time = 0.

From **TableNumber** it can be seen that the calculated capacitance (averaged between all of the methods) is



The theoretical simulation curve and the experimental result curve match closely (see **FigureNumber**). Small deviations are caused by assumptions made to simplify theoretical calculations, such as ignoring resistance in the wire and temperature sensitive circuit elements.

The only noticeable difference between the initial step response and the offset step response is the actual voltage offset. The curve shape was unaffected. The time constant will not change from a DC offset.

1.2

The Bode plot of the first order system would be unaffected by a dc offset. The Bode a plot displays response behavior, and remains the same for a circuit regardless of input.

|  |  |  |  |
| --- | --- | --- | --- |
| [Hz] | V | φ | dB |
| 10 | 3.93 | -2.2 | -0.08795 |
| 30 | 3.9 | -6.8 | -0.15451 |
| 100 | 3.64 | -21.7 | -0.75378 |
| 200 | 3.05 | -38.5 | -2.28981 |
| 230 | 2.85 | -42.4 | -2.87891 |
| 240 | 2.78 | -43.8 | -3.09491 |
| 250 | 2.71 | -44.9 | -3.31642 |
| 260 | 2.64 | -46.1 | -3.54373 |
| 270 | 2.57 | -47.2 | -3.77714 |
| 500 | 1.75 | -63.2 | -7.11504 |
| 1000 | 0.956 | -75.6 | -12.3666 |
| 5000 | 0.197 | -86.4 | -26.0864 |
| 10000 | 0.099 | -87.5 | -32.0631 |

**TableNumber** displays selected data over a range of 3 decades of frequencies. The transfer function for the RC circuit was found to be

This transfer function was used to build the bode plot in **FigureNumber**, which also has an overlay of an experimental bode plot that was calculated form the experiment data. Visually, both bode plots look very similar, especially the points close to the breakpoint frequency.



The experimental time constant was calculated to be 6.366e-04 seconds, which resulted in a capacitor value of 5.735e-08 F. This is only a 2.3% difference from the capacitor value found from the step response analysis (5.87 e-08 F).

The decibel drop in amplitude ratio at the breakpoint frequency of 250 Hz was found to be -3.118 dB.

2.1

For the unaltered square wave input, the damping ratio was found to be 0.0620, with an undamped natural frequency of 313.1 Hz. The DC offset square wave resulted in a damping ratio of 0.0581, and a natural frequency of 301.7 Hz. Theoretically the offset should not cause a difference in these values. In reality, the inductor, capacitor, or resistor could be dependent on voltage level. **THESE NUMBERS SEEM WRONG**

The inductance of the system (without the offset), L, was calculated to be 1.27 H. The capacitance of the system was found to be 2.07 e-07 F



**FigureNumber**  further demonstrates the inaccuracy of the experimental damping ratio. It is approximately twice what the simulated damping ratio is. However, the same general shape of the curves are consistent.

2.2

|  |  |  |  |
| --- | --- | --- | --- |
| [Hz] | V | φ | dB |
| 10 | 3.97 | -0.2 | 0 |
| 16 | 3.99 | -0.3 | 0.04364 |
| 80 | 4.18 | -1.5 | 0.44771 |
| 158 | 5.12 | -5.9 | 2.20958 |
| 317 | 11.1 | -89 | 8.93064 |
| 476 | 2.95 | -162 | -2.57934 |
| 500 | 2.57 | -164 | -3.77714 |
| 634 | 1.39 | -169 | -9.11551 |
| 1000 | 0.512 | -174 | -17.7904 |
| 1268 | 0.311 | -175 | -22.1206 |
| 5000 | 0.019 | -178 | -46.4007 |
| 7000 | 0.0087 | -178 | -53.1854 |

Sample measurements for amplitude ratio and phase angle can be found in **TableNumber**.

The transfer function for the 2nd order system was determined using the differential equation of the system.

From this, a bode plot can be constructed **FigureNumber**



Experimental data points are plotted to the same bode plot in **FigureNumber**. It is clear that the simulated damping ratio is much higher than experimentally determined damping ratio.

From the bode plot, the damping ratio is 0.0275 with an undamped natural frequency of 1959.7 Hz. These values are much different compared to the damping ratio of 0.0620 and of 313.1 Hz that was found in the step response. This discrepancy could be the result of not picking enough data points that were close to the break frequency to get an accurate cusp for analysis.

Using these measurements from the bode plot, the inductance was found to be 0.5679 H, and the capacitance was found to be 4.585 e-07 F. These differ greatly from the L and C that resulted from the step response analysis There is a 55.2% difference in inductance (1.27 H) and a 54.9% difference in capacitance (2.07 e-07F) of both methods. Again, this could be error that propagated from not having enough data points to get an accurate estimate of the system characteristics.

3.1



The experimental values obtained from LabView for the first order system match the theoretical values closely (**FigureNumber**). **Maybe calculate deviation in the code?**

Using LabView, the time constant was found to be 6.1244e-04 seconds. This is a 6.125% difference from the time constant calculated with the step response analysis (6.52396e-4 sec) and only a 3.8% difference for the frequency response analysis (6.366e-04 sec).

The capacitor C value was calculated to be 5.5175e-08 F in LabView. This is a 3.94% difference from the frequency response calculated capacitor value (5.735e-08 F), and is a 6.0% difference from the capacitor value found from the step response analysis (5.87 e-08 F).

3.2



The curves in **FigureNumber** are the same general shape, however there is a 30 Hz offset between the experimental and the theoretical values. This could be from each component of the 2nd order circuit not being “ideal”. For example, the capacitor and inductor are assumed to have no resistance, when realistically every element of the circuit has a resistance. Wires comprising the circuit may also have an impact – as they each have a resistive and inductive identity.

The experimental damping ratio was found in LabView to be 0.0284 with a natural frequency of 360 Hz. The results from the square wave response shows a 54.19% difference in damping ratio (0.0620), with a 13.0% difference in undamped natural frequency (313.1 Hz). The frequency response analysis yielded a damping ratio with 3.17% difference (0.0275) and a difference in undamped natural frequency of 81.6% (1959.7 Hz).

The characteristics determined by LabView resulted in an inductance of 0.5083 H. This is 10.5% different from the inductance calculated from the frequency response (0.5679 H), and is 59.98% different from the step response inductance (1.27 H)

The capacitance C was calculated to be 3.8491e-07 F using LabView. This is 16.1% different from the frequency response capacitance calculation (4.585 e-07 F), and is 46.2% different from the step response analysis.

# Conclusions

Zhangxi Feng:

The capacitor in the first order circuit was found to have a capacitance of 58.7 nF. The capacitor in the second order circuit was found to have a capacitance of 0.207 µF. The inductor in the second order circuit was found to have an inductance of 1.27 H. The time constant of the first order circuit was found to be 0.65 ms

# References

# Appendices

1. **Data Tables**
2. **Sample Calculations**
3. **Equipment List**
4. **Raw Data Sheets**
5. **Lab Instructions**