LAB 5 – VELOCITY CONTROL OF A DC BRUSHED MOTOR

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***VELOCITY CONTROL OF A BRUSHED DC MOTOR***

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# Cover Letter

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Dr. Ebadi,

The following document contains an analysis of control systems for a DC brushed motor. DC motors are generally controlled via pulse width modulation and a microprocessor, however for the purpose of demonstration, we controlled DC motors with power op-amp driven proportion control, integral control, and proportional-integral control.

These control systems were compared against each other in terms of functionality- the motor parameters have been determined through experimentation.

The body of this report comprises of the results of inputs to the system and recorded system response.

Best Regards,

Jesse Feng

Simon Popecki

Reilly Webb

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# Objectives

The objective of this experiment was to compare different methods of control using a power operational amplifier. Proportional, Integral, and Proportional-Integral control systems controlled a motor under load to be measured by a second motor. The parameters of the motor were calculated, and the system response determined from experimental data. The motor back EMF, open-loop, and closed-loop response were analyzed in particular. Quantitative analysis was performed and interpreted. All relevant values are tabulated/listed. Gain values were confirmed by root locus analysis.

# Executive Summary

In this experiment three types of power op-amp control systems were used to control the speed of a DC brushed motor. Proportional control, Integral control, and Proportional-Integral control systems were used. PID control was not used in this experiment. Motors were controlled by voltage, rather than the conventional method of pulse width modulation. The objective of the control systems was to maintain motor speed regardless of the load placed on the motor. Motor speed was measured by a tachometer.

This experiment assumes that the electrical time constant was much less than the mechanical time constant – so that it may be neglected in the analysis.

The DC motor’s back EMF constant was found to be #. This value was determined by back driving another motor, referred to in this report as the MUT (motor under test). The torque constant Kt was calculated to be # - it was assumed that Kt = Ke. The resulting motor torque value was calculated to be # oz\*in/A. This value matches the specifications provided by the motor manufacturer.

The motor’s open-loop response for a disturbance torque and input voltage step was recorded and analyzed. The time constant and system gain were calculated – a disturbance load was implemented with a switch to measure steady state error – the steady state error is the difference between the motor’s no-load speed and speed under load. The motor time constant was found to be #, and the motor gain value was found to be #. The stall torque of the motor was calculated to be #. The moment of inertia and damping constant of the system were calculated to be # and # respectively.

Closed-loop motor response to a voltage step input and disturbance torque was recorded with a tachometer in the same way as the open-loop response. The power op-amps were controlled by a 741 op-amp – proportional, integral, and proportional-integral control were implemented and analyzed. SHOW SPEEDS OF EACH CONTROL SYSTEM. In a purpose-built closed-loop system the magnitude of each control element (P,I,D) is modified such that it has a larger or smaller role to fit the individual needs of the system. Generic control parameters were used for this experiment (non-optimized).

# Theory and Experimental Methods

The experimental setup for the open-loop response is shown in Figure 1 below.

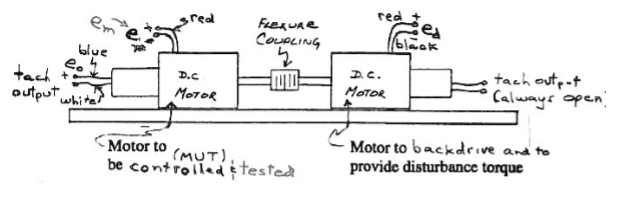


Figure 1

The motor left most is the MUT – the motor under test, and the motor on the right is the motor which is back driven and used to provide a load for the MUT. The two motors are coupled with a flex-coupling to allow for minor shaft misalignment. The MUT was supplied with power from a power op-amp. The MUT was connected to a tachometer, which outputted a voltage proportional to the rotational velocity.

Figure 2 below shows the schematic describing the power op-amp setup.

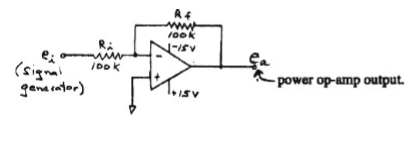


Figure 2

The gain of the power op-amp is -1, which means the motor spins the opposite direction (not relevant to the results). Voltages of +15 and -15 volts. The power op-amp was supplied by BK Precision, it provides the power for the motor. Both resistors are 100 kΩ.

The open-loop block diagram of the motor is shown below in Figure 3.

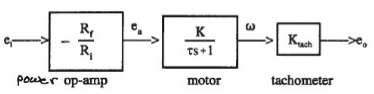


Figure 3

The time constant of the electrical system is much shorter than the time constant of the mechanical system, it is therefore neglected. In this case, the mechanical pole is dominant.

1.1 DC Motor Systems Modeling



Figure 4: DC Brush Motor Electro-mechanical model

The electro-mechanical model of a DC brush motor is shown in **figure 4**. The electrical circuit takes an input voltage and generates a voltage drop across the rotor. The electrical energy from the voltage drop can be related to the resulting angular velocity, , through the motor velocity constant, :

(Equation 1)

The current of the circuit, , can also be coupled to the resulting torque in the motor,, through the back-EMF constant, :

(Equation 2)

Performing electrical and mechanical analysis on the system using equations 1 and 2 leads to the following transfer function relating the output angular velocity to the input voltage:

(Equation 3)

Equation 3 is typically written in the following format for ease of identifying the system parameters, system gain, , and time constant, :

Therefore, the equations for the system gain and time constant of a motor are:

(Equation 5)

(Equation 4)

The detailed derivation steps for the motor transfer function are attached in Appendix A1.1 in this document.

1.2 Experimental Setup and Methods

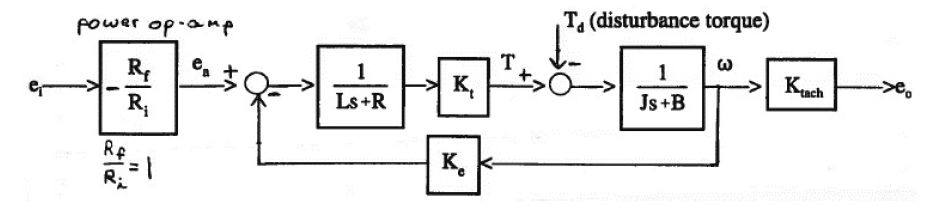


Figure 5: Experimental Setup Block Diagram

The full experimental setup is shown in the block diagram in **figure 5**. The generated input voltage is passed through a power op-amp with a gain of to produce the motor input voltage . The motor’s resulting angular velocity is fed back to complete a closed-loop feedback system through the motor velocity constant. The difference (error) between the input voltage and the output angular velocity controls the motor. The term represents the electrical system’s resulting voltage drop at the rotor, which converts the electrical energy into torque through the back-EMF constant. Potential disturbance loading is included in the schematic. The angular velocity is measured by a tachometer with a gain of and outputs a voltage measurable with a scope.

It is worth noting that when the disturbance load is equal to the generated torque from a given voltage, , the input into the mechanical model would be zero, resulting in zero angular velocity. But the motor is still generating a torque . This torque is the stall torque at the given voltage .

A back-drive motor is connected to the motor under test (MUT). In the first part, the back-drive motor delivers torque to the MUT, converting the motor into a generator, and the generated voltage and the tachometer output for the back-drive motor are measured. The data is used to determine the back-EMF constant, , of the MUT by linearly fitting the angular velocity with the MUT output, assuming the loss between the back-drive and MUT connection is negligible.

In the second part, after is determined, the MUT is directly connected to determine the motor’s time constant, gain, and response to disturbance load. The steady-state error of the disturbance response is the difference between the output before and after engaging the disturbance.

Finally, the system is then connected to proportional, integral, and proportional-integral controllers to form a closed-loop system. The three controllers’ circuit diagram is shown in **figure 6** (next page). The P, I, and PI controllers’ transfer functions are shown in equations 6 to 8. The P controller’s gain is 1.5:

(Equation 6)

(Equation 8)

(Equation 7)

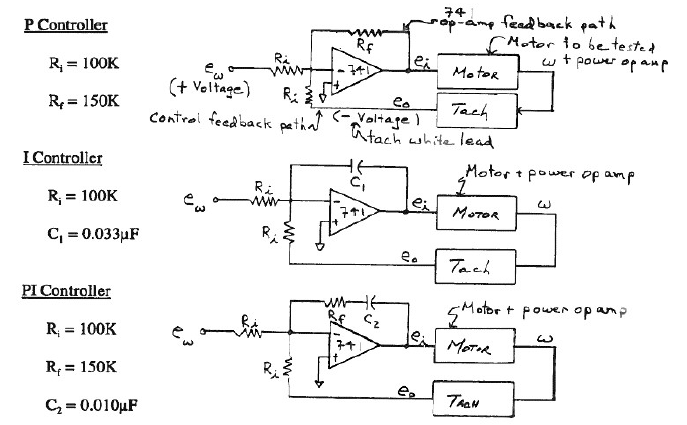


Figure 6: P, I, and PI Controller circuit diagrams showing connection to the rest of the system

# Results and Discussion

The back-EMF constant was found by driving the MUT with varying voltages, and measuring the resulting RPM. The resulting curve is a linear response, which is used to find the motor’s torque constant, Kt. The curve is shown in Figure 7 below.



Figure 7 – MUT output voltage vs rotational speed

The motor voltage constant was found using the slope of the line in Figure 7. This was used to find the motor torque constant. the motor voltage constant was found to be 4.9817 volts per thousand RPM, which is consistent with the motor specifications. the motor torque constantwas found to be 6.7273 Oz-in/A which is also consistent with what the manufacturer specified in their publication. See appendix for calculations. The motor voltage constant was specified to the range of 4.39 to 5.37 volts per thousand RPM, the value calculated by this group lands about midway through that estimate. The motor torque constant was specified to be between 10 percent plus or minus of 6.6 oz\*in/A. The calculated value falls within 2% of this range.

A step response of square wave profile with amplitude of 4 V was inputted Into the system using closed-loop control. This allowed for the calculation of first order system parameters. The motor stall torque, T was calculated to be 9.62 oz\*in. The system time constant, τ was calculated to be .009 s and the system gain, K was found to be .1912.

There were issues with calculating the damping constant – the calculated value was 3.19, however a value of .2 was used for calculations. This was discussed with professor Fussell who attributed the high damping constant to the motor’s age and friction, and approved our use of .2. Since the B value was compromised, two different motor inertia values were calculated for. A motor inertia, J of 4.125x10-4 was found using the calculated B value, and a value of 7.21x10-4 was found using .2. Units for both values are oz\*in\*s^2. Equations for the calculations of all these values can be found in the appendix.

The step response of the square wave is shown below in Figure 8 (recall op-amp gain of -1).



Figure 8 – Tachometer output voltage with respect to time.

Figure 9 shows just the step response, zoomed it to show the point of the first time constant.



Figure 9

The time constant was calculated again using the disturbance load as a metric. The steady state value is equivalent to the disturbance torque from the engagement of the 5 Ohm resistor.

The expected time constant from the spec sheet was 8.4 ms, the calculated time constant using the method of disturbance torque was 6.7 ms (20% error). The expected damping constant was .863 oz\*in/KRPM, the calculated result was

# Conclusions

# Appendix

A1.1 DC Motor Electromechanical Model Derivation

Given the following system for a DC motor:

From the circuit, perform Kirchhoff’s loop law:

The voltage drop is proportional to the resulting angular velocity of the mechanical system, and the current is proportional to the resulting torque by:

Using and , the mechanical system equation is:

Since the inductance is three magnitudes less than the resistance, the inductance term is neglected. Laplace transform and combine the equations:

The transfer function relating output angular velocity to input voltage is then:

A1.2 Controller transfer function derivations

Given the P controller setup shown in **figure 3**, assuming ideal op-amp, performing nodal analysis on the negative lead node gives:

Ideal op-amp means , and is grounded to zero. Therefore:

Rearranging:

Similarly, for I controller:

The PI controller is slightly more complicated. The resistor-capacitor in series need to be combined into one element:

The repeat the same steps: