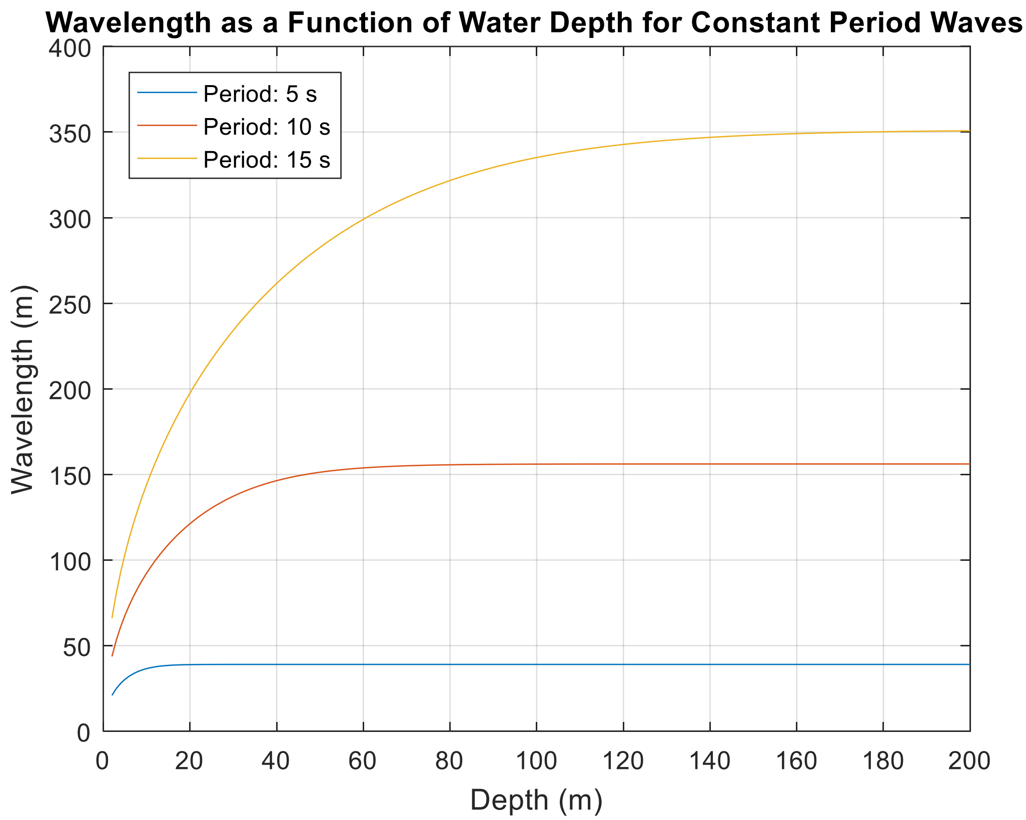
Simon Popecki

OE 757

13 September 2017

Homework 1

1. Sensitivity of wavelength and celerity to water depth and wave period

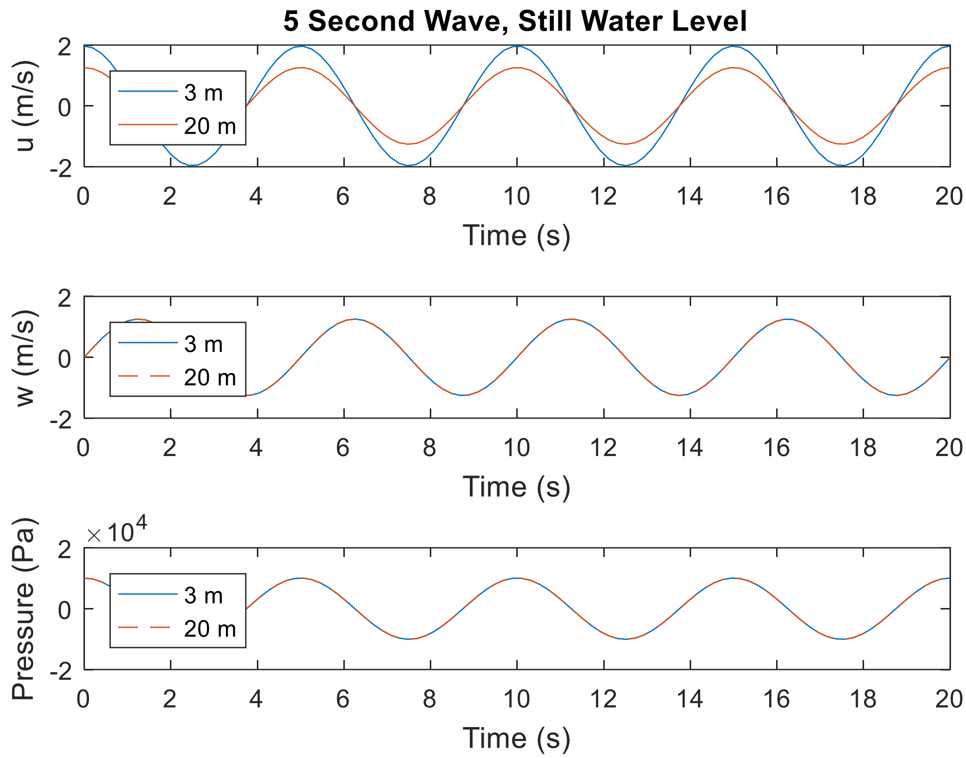


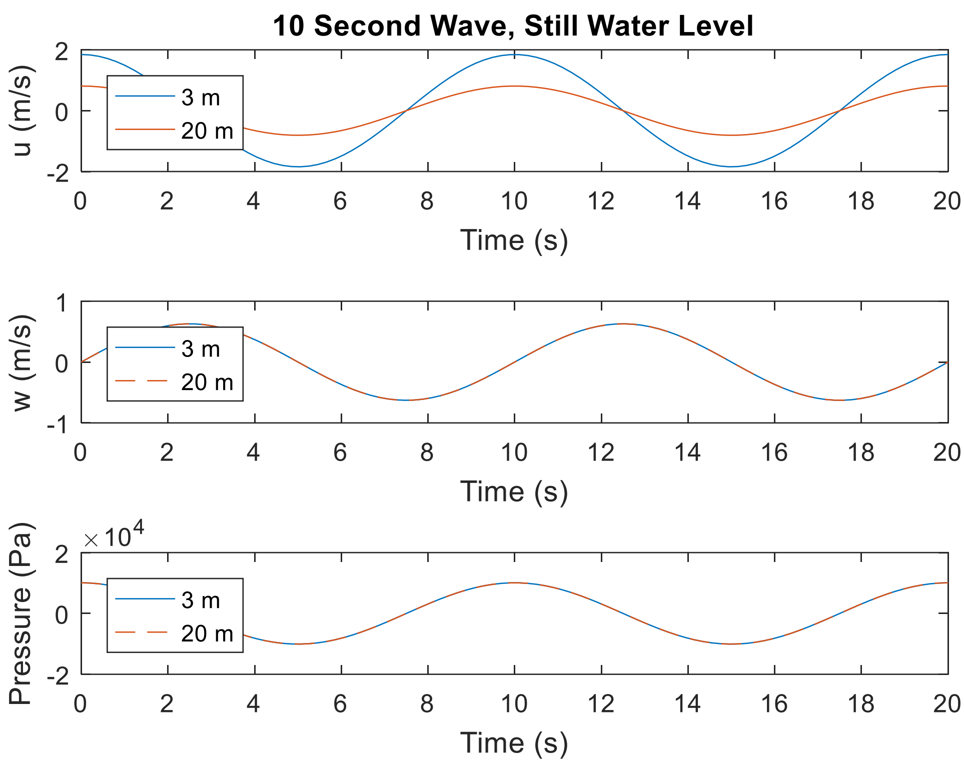
Wavelength and celerity share a linear relationship – that is to say, a plot of celerity vs water depth would look identical with the exception of a different scale on the Y-axis. What can be considered deep or shallow is dependent on the wavelength of the wave. Waves with shorter periods reach their 'deep water depth' in shallower water when compared to waves with longer wavelengths. For example – at twenty meters depth, a wave of five second period is in deep water. At the same depth of twenty meters, a fifteen second wave is confidently in the shallow water range. Waves can be considered 'deep' when depth divided by wavelength is greater than one half. A wave can be considered to be in shallow water when depth divided by wavelength is less than one twentieth.

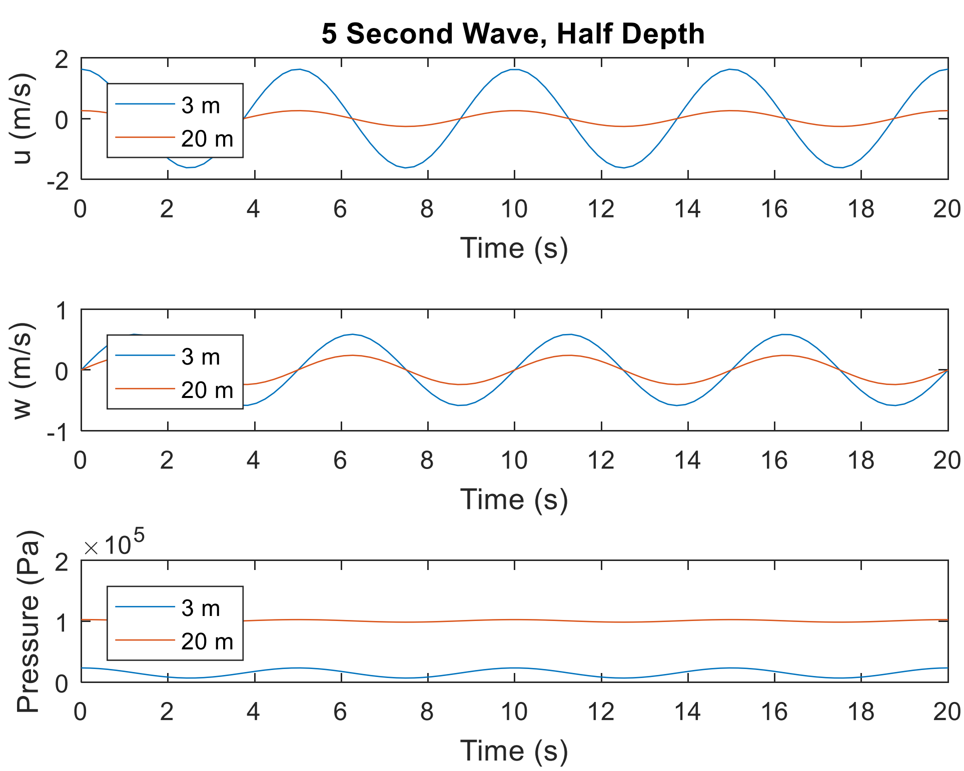
2. For horizontal water particle velocity: The hyperbolic term varies between 0.87 to 1.19 for a 5 second wave (z between the trough and peak of the wave respectively), and 0.97 - 1.05 for a 10 second wave (same z range). The gT/L term is 1.97 for the 5 second wave and 1.85 for the 10 second wave. The cosine theta term varies between zero and one depending upon the phase of the wave. Wave amplitude (H/2) depends on the wave height – 2 meters was used for this problem, making wave amplitude 1 meter. Wave amplitude is limited by depth i.e. wave amplitude cannot exceed depth, and will not come close in deep water. Overall horizontal water particle velocity was calculated (at still water level with phase equal to zero) to be 1.97 meters per second for the 5 second wave and 1.85 for the 10 second wave. The gT/L term has the most impact in the final answer, followed by the hyperbolic term (1 at still water level), wave amplitude and the cosine term were both equal to 1.

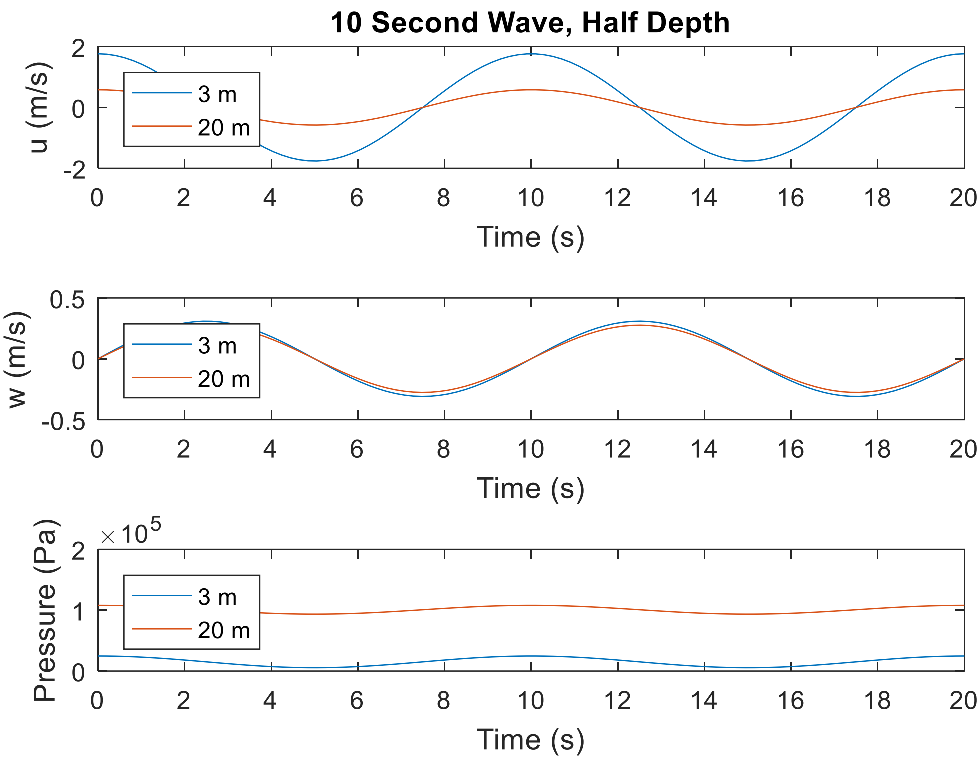
   Vertical water particle velocity shared all terms except the sinh/cosh term (note that the wave phase was changed to pi/2 to solve for maximum values). The sinh/cosh terms were 0.64 for the 5 second wave and 0.34 for the 10 second wave at still water level – both values lower than the respective horizontal velocities at maximized phase. Overall vertical particle velocity was calculated to be 1.26 meters per second for the 5 second wave, and 0.63 meters per second for the 10 second wave.

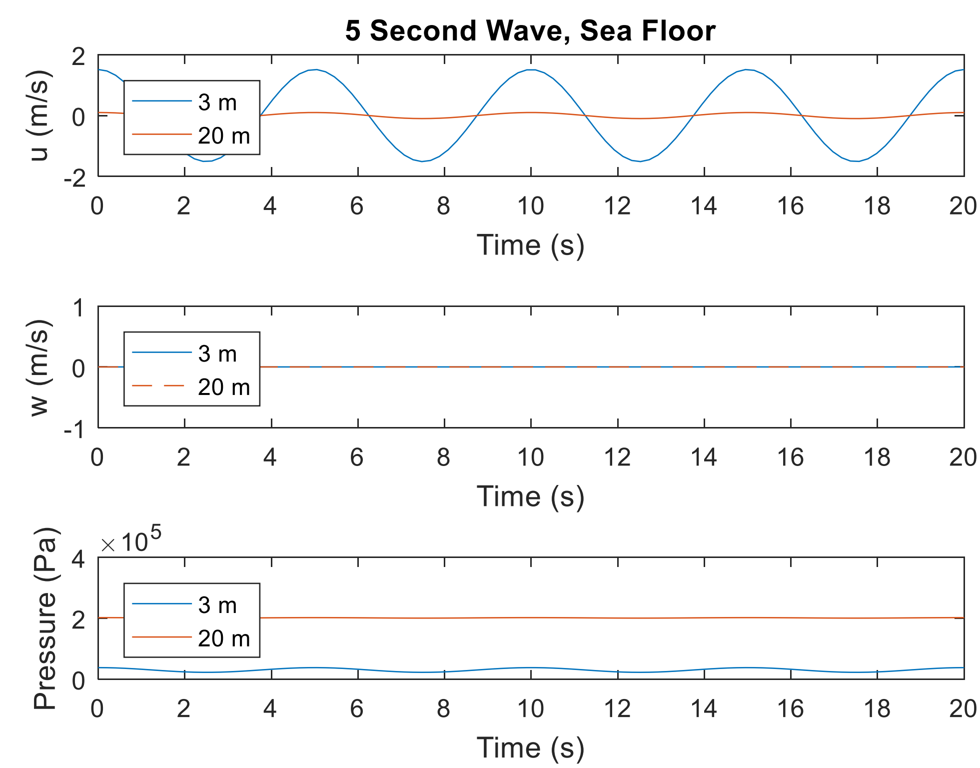
3.

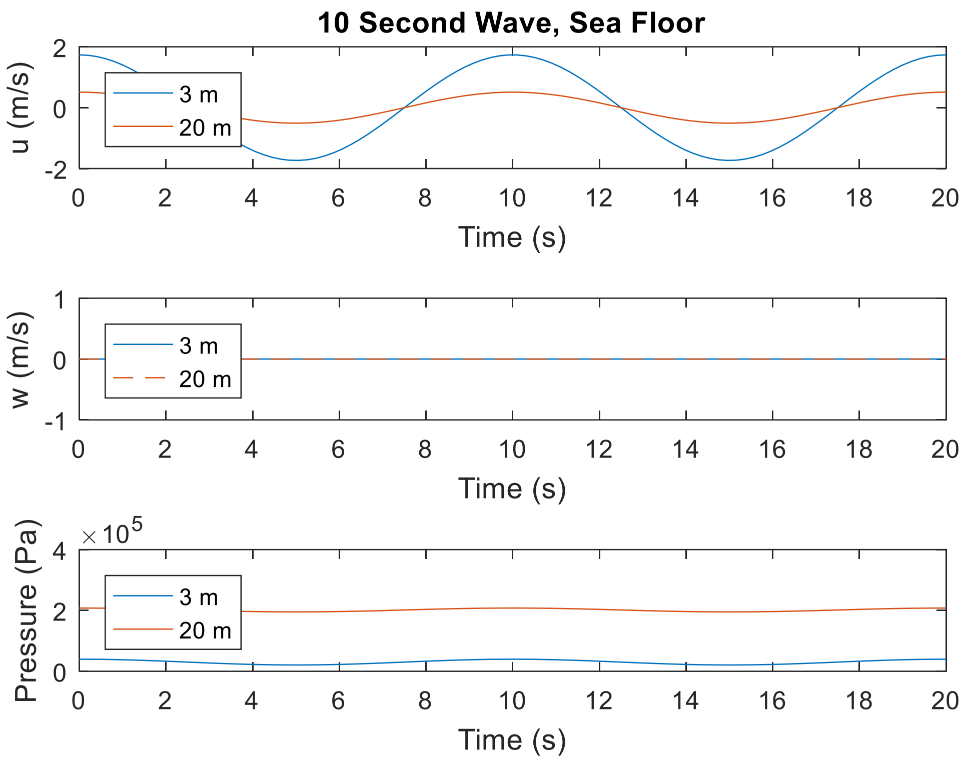

5 Second Wave, Still Water Level
u (m/s)
3 m
20 m
Time (s)
w (m/s)
3 m
20 m
Time (s)
Pressure (Pa)
$\times 10^4$
3 m
20 m
Time (s)


10 Second Wave, Still Water Level
u (m/s)
w (m/s)
Pressure (Pa)
×10^4
Time (s)
3 m
20 m


5 Second Wave, Half Depth
u (m/s)
w (m/s)
Pressure (Pa)
×10^5
Time (s)
3 m
20 m


10 Second Wave, Half Depth
u (m/s)
w (m/s)
Pressure (Pa)
×10^5
3 m
20 m
Time (s)


5 Second Wave, Sea Floor
u (m/s)
w (m/s)
Pressure (Pa)
×10^5
3 m
20 m
Time (s)



Horizontal water particle velocity decays with relation to wave period and depth – a 10 second wave will still maintain much of its velocity at the seafloor while a 5 second wave under the same conditions will die out. At the seafloor, all vertical velocity is zero, because the seafloor is considered impermeable. This applies to all wave periods. At the surface, the vertical velocity for a wave of constant period is the same regardless of depth, because the surface of the water moves up and down as a function of wave height. Waves with longer period tend to lose their particle velocity more quickly with increasing depth. Pressure at the surface is the same for waves of any period, because the water-air interface is a free surface. Depth can be qualitatively found by looking at the pressure and waveform – pressure is lower is shallower water, and the waveform is flatter in deeper water. Pressure goes up by roughly 1 atmosphere every 10 meters.

4. Total wave energy does not depend on the water depth or wave period – only fluid density and wave height. Since a height of 2 m was assumed for all waves, the total

wave energy was calculated to be 5,037 joules. Wave power is shown in the table below:

	5 Second Wave	10 Second Wave
3 meter Water Depth	21,360 W	25,717 W
20 meter Water Depth	19,998 W	46,719 W

Longer period waves generate more power than shorter period waves, and waves in deeper water generate more power than waves in comparatively shallow water.