# Chapitre 1

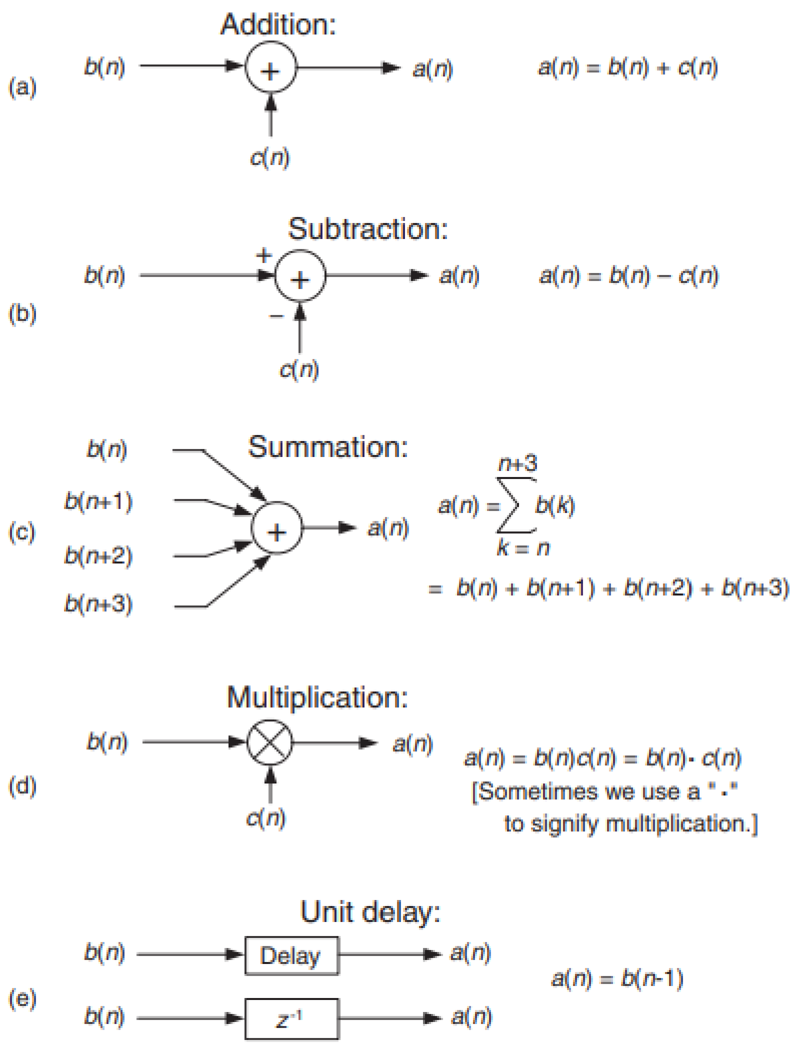
## 1.1

> *discrete-time signal* : a signal whose independent time variable is quantized so that we know only the value of the signal at discrete instants in time.

**Note**

If a continuous sinewave represents a physical voltage, we could sample it once every $t_s$ seconds and represent the sinewave as a sequence of discrete values.

## 1.3




# 1.4

> *LTI* : linear time-invariant

# 1.5

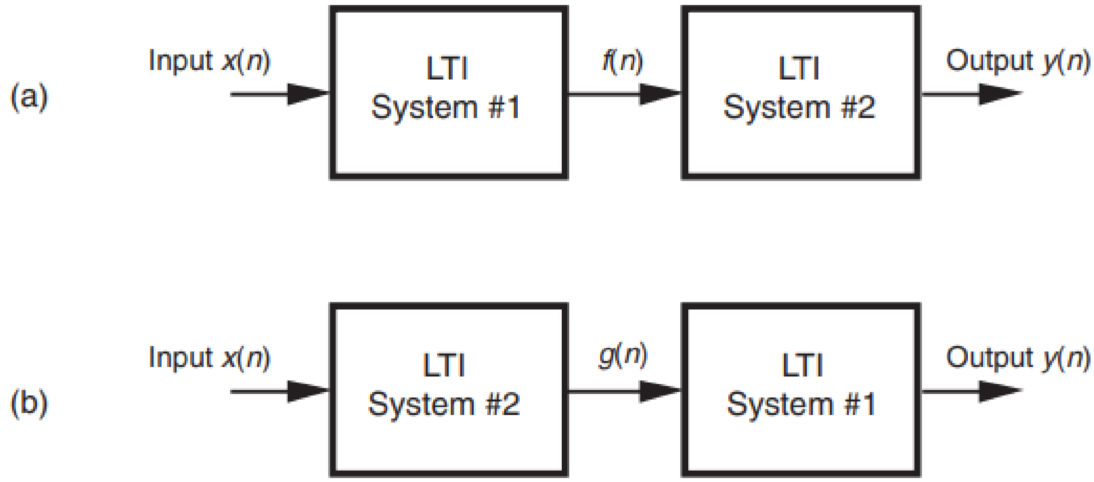
A linear system's output is the sum of the outputs of its parts. Where:
$x(n)_1 + x(n)_2 \rightarrow y(n)_1 + y(n)_2$

# 1.6

A time-invariant system is one where a time delay (or shift) in the input sequence causes an equivalent time delay in the system's output sequence. Keeping in mind that n is just an indexing variable we use to keep track of.

## 1.7

LTI systems have a useful commutative property by which their sequential order can be rearranged with no change in their final output




# Chapitre 2

## 2.1

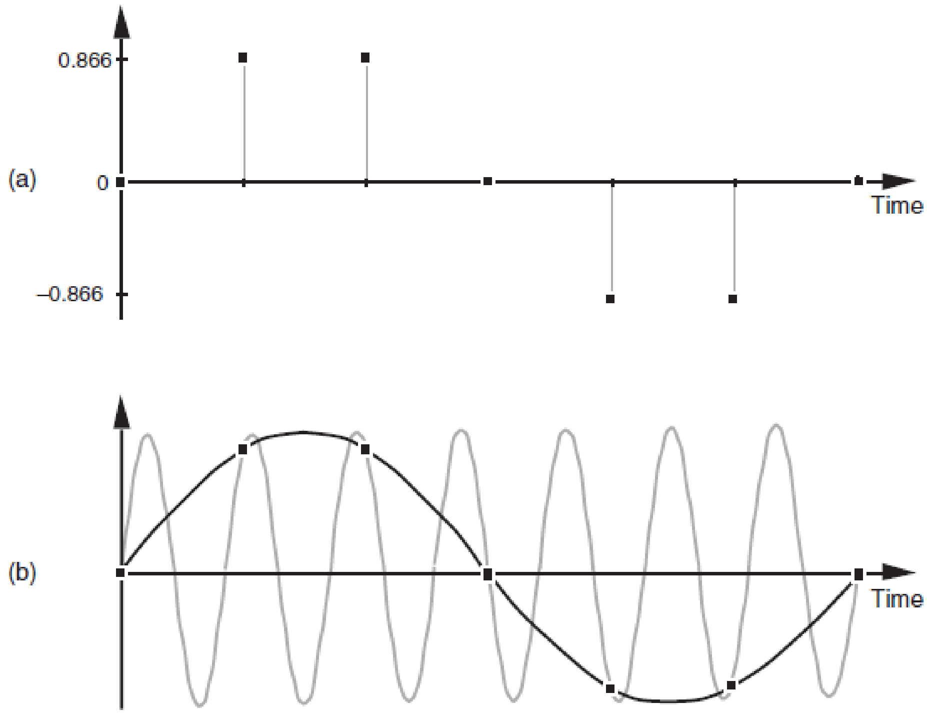



**Figure 2-1** Frequency ambiguity: (a) discrete-time sequence of values; (b) two different sinewaves that pass through the points of the discrete sequence.

- An x(n) sequence of digital sample values,
representing a sinewave of fo Hz, also exactly represents sinewaves at other frequencies, namely, fo + kfs.
So you can write:

> $x(n) = \sin(2\pi f_0 n t_0) = \sin(2\pi(f_0 + kF_s)nt_0)$ - Eq. (2-5)

Formally

When sampling at a rate of fs samples/second, if k is any positive or negative integer, we cannot distinguish between the sampled values of a sinewave of fo Hz and a sinewave of (fo+kfs) Hz.

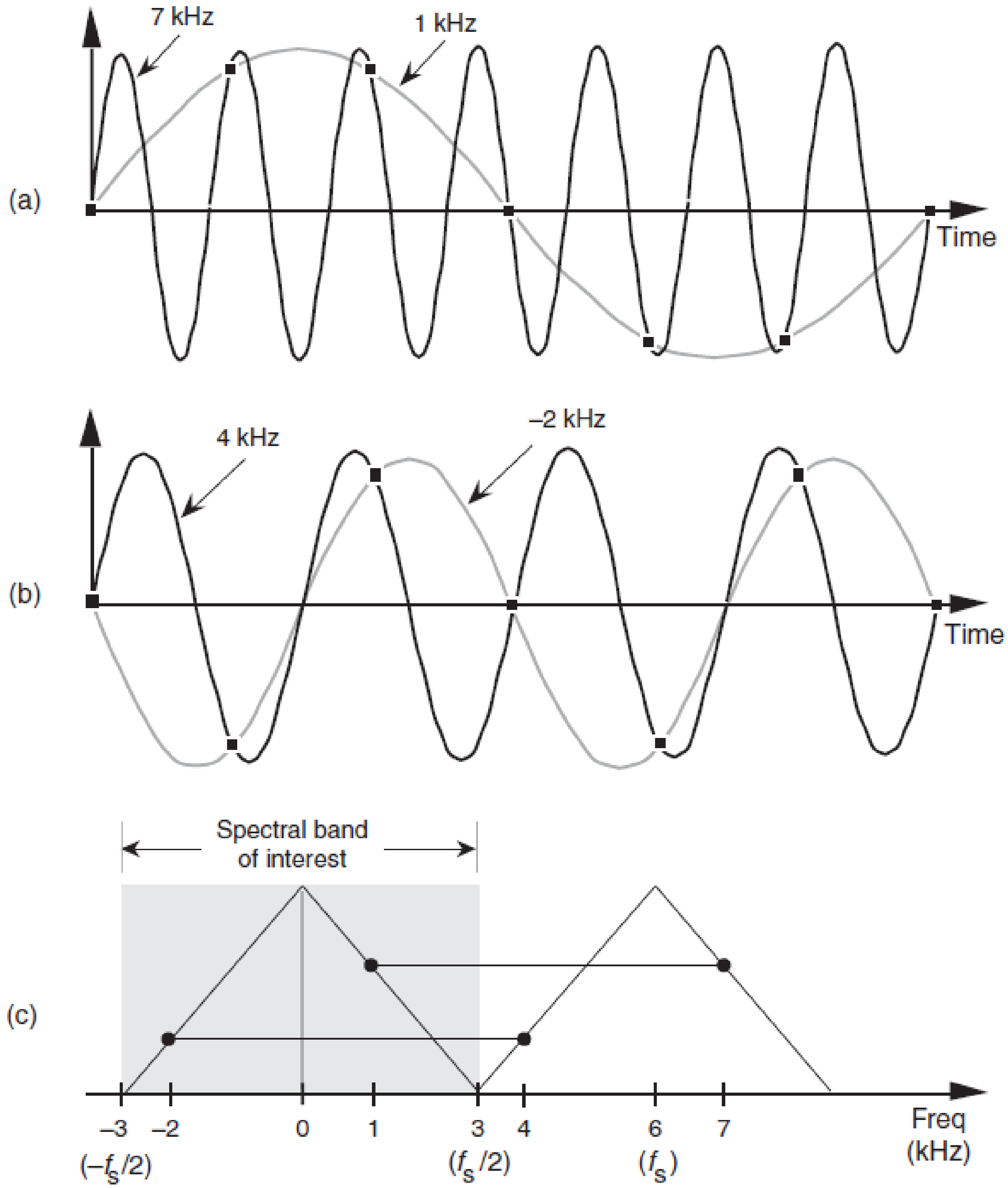



**Figure 2–2** Frequency ambiguity effects of Eq. (2–5): (a) sampling a 7 kHz sinewave at a sample rate of 6 kHz; (b) sampling a 4 kHz sinewave at a sample rate of 6 kHz; (c) spectral relationships showing aliasing of the 7 and 4 kHz sinewaves.

-Figure 2–2(b) shows another example of frequency ambiguity that we'll call aliasing, where a 4 kHz sinewave could be mistaken for a –2 kHz sinewave. In Figure 2–2(b), $f_o = 4$ kHz, $f_s = 6$ kHz, and $k = -1$ in Eq. (2–5), so that fo+kfs = [4+(–1 · 6)] = –2 kHz

$\frac{f_s}{2}$ is an important quantity in sampling theory and is referred to by different names in the literature, such as critical Nyquist, half Nyquist, and folding frequency.

# 2.2

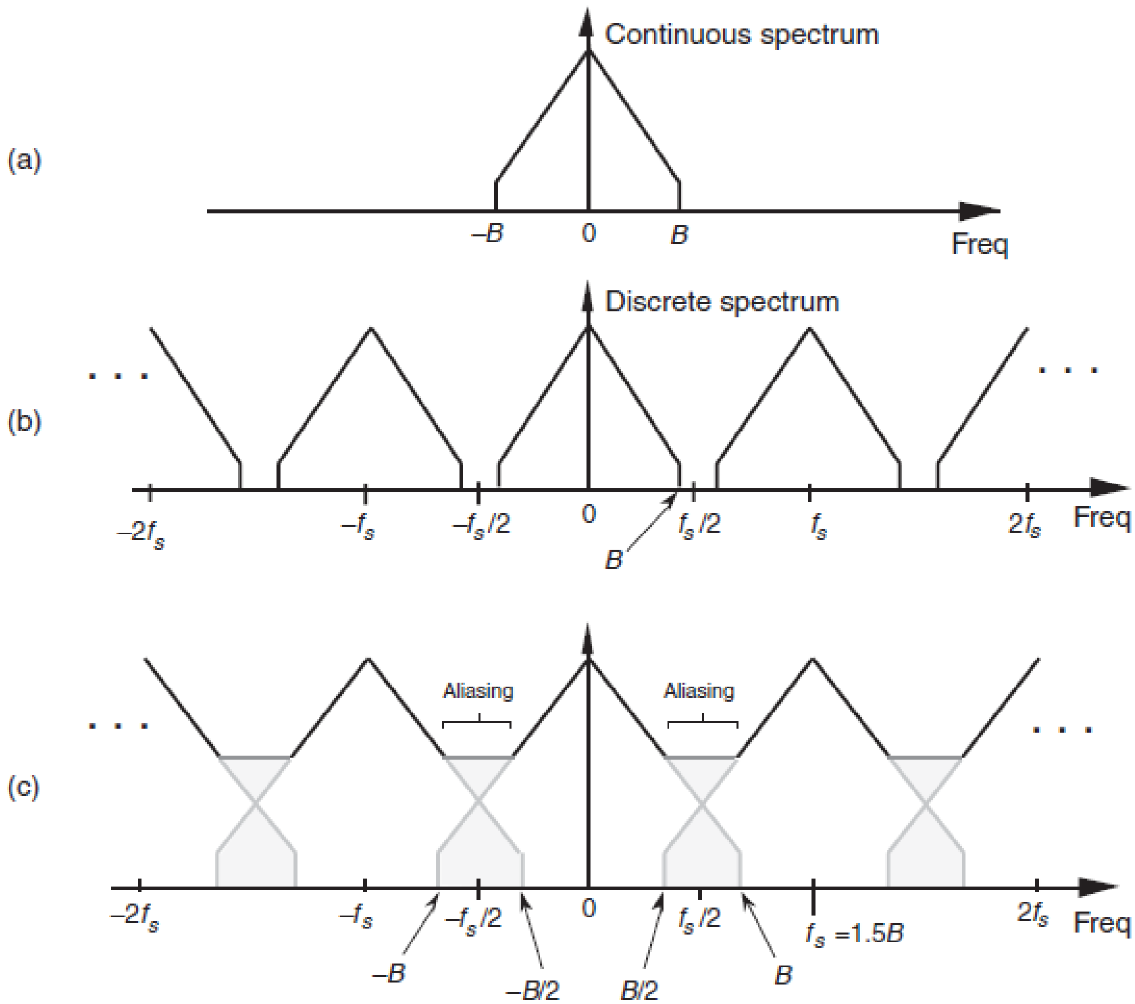



**Figure 2–4** Spectral replications: (a) original continuous lowpass signal spectrum; (b) spectral replications of the sampled lowpass signal when $f_s/2 > B$; (c) frequency overlap and aliasing when the sampling rate is too low because $f_s/2 < B$.

-Given that the continuous x(t) signal, whose spectrum is shown in Figure 2–4(a), is sampled at a rate of fs samples/second, we can see the spectral replication effects of sampling in Figure 2–4(b) showing the original spectrum in addition to an infinite number of replications.To illustrate why the term folding frequency is used, let's lower our sampling frequency to $f_s = 1.5$B Hz. The spectral result of this undersampling is illustrated in Figure 2–4(c)

# Chapitre 3

The DFT is a mathematical procedure used to determine the harmonic,
or frequency, content of a discrete signal sequence.The DFT is useful in analyzing any

discrete sequence regardless of what that sequence actually represents.

-The DFT's origin, of course, is the continuous Fourier transform X( f ) defined as :

> $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j\pi ft}dt$

DFT equation(exponential form): → $X(m) = \sum\limits_{n=0}^{N-1} x(n)e^{-j\pi 2nm/N}$

From Euler's relationship, $e - j\o = cos(\o) - jsin(\o)$, Eq. (3–2) is equivalent to :

$X(m) = \sum\limits_{n=0}^{N-1} x(n)[\cos(2\pi nm/N - j\sin(2\pi nm/N)]$

- X(m) = the mth DFT output component, i.e., X(0), X(1), X(2), X(3), etc.,
- m = the index of the DFT output in the frequency domain,
- m = 0, 1, 2, 3, . . ., N–1,
- x(n) = the sequence of input samples, x(0), x(1), x(2), x(3), etc.,
- n = the time-domain index of the input samples, n = 0, 1, 2, 3, . . ., N–1,
- j= , and
- N = the number of samples of the input sequence

If we plot the X(m) output magnitudes as a function of frequency, we produce the magnitude spectrum of the x(n) input sequence

# 3.2

DFT is called conjugate symmetric. When the input sequence x(n) is real, as it will be for all of our examples, the complex DFT outputs for $m = 1$ to $m = (N/2) - 1$ are redundant with frequency output values for $m > (N/2)$. The mth DFT output will have the same magnitude as the $(N–m)th$ DFT output.

# 3.7

IDTF : $x(n) = \frac{1}{N} \sum\limits_{m=0}^{N-1} X(m)e^{j\pi nm/N}$ and

$x(n) = \sum\limits_{n=0}^{N-1} X(m)[\cos(2\pi nm/N + j\sin(2\pi nm/N)]$

## 3.8

A characteristic known as leakage causes our DFT results to be only an approximation of the true spectra of the original input signals prior to digital sampling

# Chapitre

## 4.3

FFT

$$x(n) = \sum_{n=0}^{(N/2)-1} x(2n)e^{-j2\pi(2n)m/N} + e^{-j2\pi m/N} \sum_{n=0}^{(N/2)-1} x(2n+1)e^{-j2\pi(2n)m/N}$$