5.4. Suppose that a trucking company owns a large fleet of well-maintained trucks and assume that breakdowns appear to occur at random times. The president of the company is interested in learning about the daily rate λ at which breakdowns occur. (Realistically, each truck would have a breakdown rate that depends possibly on its type, age, condition, driver, usage, etc. The breakdown rate for the whole company can be viewed as the sum of the breakdown rates of the individual trucks.) For a given value of the rate parameter λ, it is known that the number of breakdowns y on a particular day has a Poisson distribution with mean λ:

$$p(y|\lambda) = \frac{e^{-\lambda}\,\lambda^y}{y!}, \quad y = 0, 1, 2, \ldots$$

1. Suppose that one observes the number of truck breakdowns for n consecutive days—denote these numbers by $y_1, \ldots, y_n$. If one assumes that these are exchangeable measurements (conditionally independent given λ), find the joint probability distribution of $y_1, \ldots, y_n$.
2. The numbers of breakdowns for 5 days are recorded to be 2, 5, 1, 0, and 3. Find the likelihood function $L(\lambda)$ of the rate parameter λ for these observations. Graph this function. (You may either use R or do it "by hand" by calculating the likelihood for the values $R = 0.1, 0.5, 1, 2, 4, 8$, and 16 and connecting the points with a smooth curve.)
3. Use calculus to find the mle of λ. Then use the `poisson.test` function in R to confirm the mle and to obtain a 95% frequentist confidence interval for λ.