The homogeneous Spatial Possion Process (SPP) is adopted, it is a counting process to model the probability distribution of events count that occur within the a certain region given the density of events λ and the area of the region $A_{Sensors}$ [14]. Here the events are the existence of sinks within in the transmission range of sensors.

The probability that a nanosensor could be covered by a sink via a single hop is the probability that there is at least one nano sink located within the transmission range of the nanosensor. Here the following events are defined:

- S: A nanosensor is connected to at least one sink via a single hop.
- N: No sink exists in the transmission range of the nanosensor.

Therefore, according to SPP, the probability of single hop is obtained by

$$\Pr(Single\ hop) = \Pr(S) = 1 - \Pr(N) = 1 - \lambda_{Sinks} A_{Sensors} e^{-\lambda_{Sinks} A_{Sensors}} \tag{3.8}$$

$$\lambda_{Sinks} = \frac{N_{Sinks}}{A_{Total}} = \frac{N_{Sinks}}{\pi R^2} \tag{3.9}$$

$$A_{Sensors} = \pi r^2 \tag{3.10}$$

where λ_{Sinks} is the density of sinks, $A_{Sensors}$ is the effective area of nanosensors, N_{Sink} is the number of total number of nano sinks.

In this analysis, the border effect is eliminated from the perspective of analysis rather than simulation. The border effect is caused by the assumption of SPP that the events are distributed in an infinite area rather than a bounded area, therefore, the effective area for a nanosensor close to the border is smaller than the effective area for a nanosensor far away from the border. In our scenario, nanosensors close to the border have lower probability to be covered by sinks via one hop due to the smaller effective area. Therefore, the effective area of nanosensors considering the border effect is:

$$A_{Sensors} = \begin{cases} \min(\pi r^2, \pi R^2), \text{ for } x \le |r-R| \\ r^2 \arccos(\frac{x^2+r^2-R^2}{2xr}) + R^2 \arccos(\frac{x^2+R^2-r^2}{2xR}) \\ -\frac{1}{2}\sqrt{[(r+R)^2 - x^2][x^2-(r-R)^2]}, \text{ for } |r-R| < x < r+R \\ 0, \text{ otherwise} \end{cases} \tag{3.11}$$

where x is the relative distance between the center of the sensing area and the nanosensor.

The expectation of the effective area of nanosensors is given by

$$\mathbf{E}[A_{Sensors}] = \int_0^R A_{Sensors}(x) f_{A_{Sensor}}(x)dx = \int_0^R A_{Sensors} \frac{2\pi x}{\pi R^2} dx = \int_0^R A_{Sensors}(x) \frac{2x}{R^2} dx \tag{3.12}$$

where $f_{A_{Sensor}}(x)$ denotes the probability density function of the event that the nanosensor is x meters away from the center of the sensing area.

Combining Eqn. (3.11) and Eqn. (3.12), the expectation of the effective area is given by

$$\begin{aligned} \mathbf{E}[A_{Sensors}] &= \int_0^{R-r} A_{Sensors}(x) \tfrac{2x}{R^2} dx + \int_{R-r}^{R} A_{Sensors}(x) \tfrac{2x}{R^2} dx \\ &= \int_0^{R-r} \tfrac{2\pi r^2 x}{R^2} dx \\ &+ \\ & \int_{R-r}^{R} \tfrac{2x}{R^2} (r^2 \arccos(\tfrac{x^2+r^2-R^2}{2xr}) + R^2 \arccos(\tfrac{x^2+R^2-r^2}{2xR}) \\ & -\tfrac{1}{2}\sqrt{[(r+R)^2-x^2][x^2-(r-R)^2]})dx \end{aligned} \tag{3.13}$$

Consequently, substituting $A_{Sensors}$ in Eqn. (3.8) with $\mathbf{E}[A_{Sensors}]$, the single-hop probability of nanosensors is given by

$$\Pr(Single\ hop) = \Pr(S) = 1 - \Pr(N) = 1 - \lambda_{Sinks}\mathbf{E}[A_{Sensors}]e^{-\lambda_{Sinks}\mathbf{E}[A_{Sensors}]} \tag{3.14}$$

To validate the analysis, a simulation containing 100 nanosensors is adopted. In the simulation, R is set to 0.03 m, N_{Sinks} is varied from 5 to 100, r is varied from 0.0005 m to 0.01 m. For each pair of N_{Sinks} and r, the simulation is repeated 3000 times for different topologies. The results are shown in Fig. 3.11. The black mesh denotes the simulation results. The blue mesh denotes the analytical results using Eqn. (3.8) with the border effect. The red mesh denotes the analytical results using Eqn. (3.14) without the border effect. Table 3.1 shows the differences between the analysis and the simulation. From Fig. 3.11 and Table 3.1, in this analysis, the method adopted eliminating the border effect shows effectiveness.

Given the total sensing area, the single-hop probability can be expressed as a function of the sensor transmission range and the sink density. Therefore, controlling these two variables could influence the data delivery in multi-sink networks.

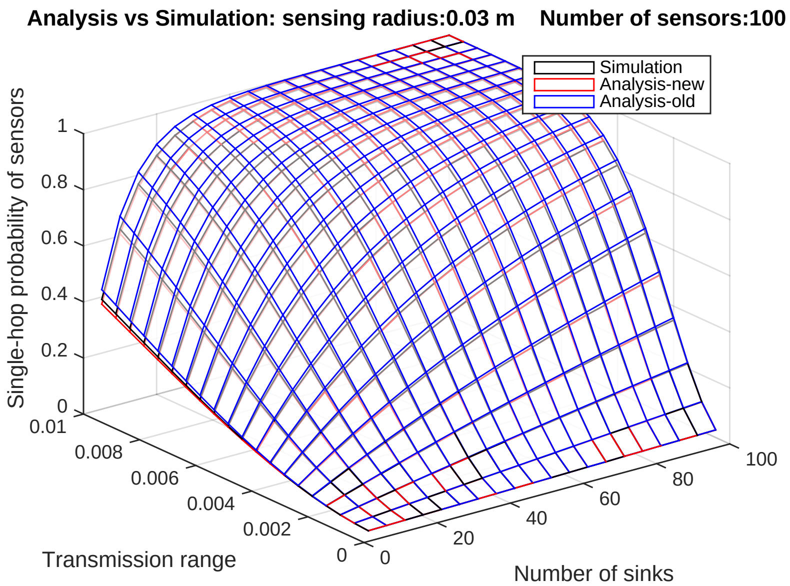

Figure 3.11: Single-hop probability of nanosensors

Table 3.1: Averaged differences between the analysis and the simulation

Analysis	Average $\Pr(Single\ hop)$	Differences between the analysis and the simulation
Analysis without border effect	58.37%	-0.07%
Analysis with border effect	59.92%	1.48%
Simulation	58.44%	/

3.2.2 Future Work

To further explore the benefits of multi-sink networks for EM-WNSNs, the following work will be done in the future:

- Analyzing how the sensor transmission range and the sink density influence the average channel capacity.