

Interpretable Neural Networks using EAGGA

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Abstract

- tabular data still difficult for NNs, where it's still outperformed by other ML model classes
 - research suggests NN performance benefits from heavy regularisation - using EAGGA, we regularise an NN and achieve both comparable performance as XGB on EAGGA as well as interpretability - for this, we propose a network architecture specifically suited for the EAGGA algorithm

Keywords: tabular data, multi-objective optimization, interpretability, deep learning

1 Introduction

Tabular Data - most common type of data - still difficult for neural networks - (Borisov et al., 2024, p. 7499) - recent research suggests that strong regularisation is beneficial to NN performance on tab data (Kadra et al., 2021, 8)

- we know regularisation from linear models can come with improvements in interpretability * e.g. LM-LASSO (L1) regularisation as feature selection - reduces # features used in model by setting some coeffs to 0 (Tibshirani, 2018, p. 267) - other forms of regularisation improve performance * e.g. LM-Ridge (L2) on multi-collinear data (Hoerl and Kennard, 1970, p. 55) * e.g. NN dropout, reduces co-adaptation (Hinton et al., 2012, p. 1) * e.g. NN early stopping, reduces overfitting on training data (Finnoff et al., 1993, p. 778f)

- we want to explore if we can use NN regularisation to tackle both interpretability and improved performance (i.e. “comparable” to XGBoost) on tab data - using EAGGA framework, which proved it can improve performance while keeping (already high) performance of XGBoost on tab data - see if interpretability improvements translate to NN + performance can also be on par

2 Background and Related Works

2.1 Interpretability

As there is no clear definition for interpretability, we will consider it as “the ability to provide *explanations* in *understandable terms* to a human”, where *explanations* are logical decision rules and *understandable terms* relate to commonly used terms in the domain of the problem, as suggested by Zhang et al. (2021, chap. 1). Further, we use the term “explainability” in an exchangeable manner with “interpretability”, as is commonly done.

Explainability of a model's reasoning is in many ways desirable. Zach (2019, pp. 3-4) gives a range of examples, amongst which are *gaining trust*, e.g. when doctors rely on medical diagnosis predictions, avoiding *subconscious biases* by making sure loan approvals are non-discriminatory, or *regulatory* reasons, most notably the EU's “Right to Explanation” warranted by the GDPR (Antamis et al., 2024, p. 1) or for approval of drugs discovered

using machine learning models (Zhang et al., 2021, 1B). It can further prove helpful for explaining unexpected drops in model performance, which could arise from optimising a model with respect to loss and then judging its performance on a different metric such as accuracy, a practice known as *model debugging* (Zhang et al., 2021, 1B) or for *scientific understanding* in domains where only models can make sense of increasingly complex data anymore and learnt knowledge encoded in a model needs to be made accessible to humans to be used reliably (Antamis et al., 2024, p. 1).

Commonly, methods for model interpretation are divided into intrinsic methods, where the search space only comprises models with a structure simple enough to be considered “explainable” (such as tree-based or simple linear models), and post-hoc methods, where interpretation methods are applied after model training. Amongst post-hoc techniques, we can further divide the space into model-specific (such as analysing GLM coefficients) and model-agnostic (e.g. partial dependence plots, ALE) methods (Molnar, 2022, chap. 3.2).

Zhang et al. (2021, chap. 2) extend this distinction to a three-dimensional taxonomy, allowing for better categorisation of neural networks (NNs), a model class that in its fully-connected feedforward form is inherently non-interpretable.

Passive vs Active Approaches, where *passive* are all post-hoc methods and *active* methods actively change either the architecture or training process to increase model interpretability.

Type of Explanations, distinguishing between *example* methods, providing concrete examples of what leads to a desired output, *attribution* methods that attribute the effect on the output for a specific feature, *hidden semantic* methods, which explain the types of inputs particular neurons or layers pick up on, and logical *rules*, such as if-then clauses or tree-induced rules.

Local vs Global Interpretability, ranging from *local* methods providing explanations based on individual samples, *semi-local* methods, explaining model behaviour for sets of samples grouped by some criterion, to *global* methods, which explain the network as a whole.

Given its unclear definition, evaluating interpretability can be challenging. Doshi-Velez and Kim (2017, 3) propose a taxonomy to categorise possible evaluation methods based on their rigorosity.

Application-grounded evaluation evaluates interpretations directly with respect to the task, by having human experts evaluate the outcome and is therefore the most expensive and time-consuming of the three approaches.

Human-grounded metrics is similar to application-grounded evaluation in that a human still evaluates the interpretations, but tries to simplify the task so that a layperson can do it. Human-grounded approaches are especially suitable if it’s sufficient to validate the general concepts of a task. A typical evaluation set-up in this category is binary forced choice, where the human evaluator chooses, which of two generated explanations he prefers.

Functionally-grounded evaluation is the least rigorous, but easiest to implement of the three. It assess explanatory quality according to some formally defined proxy for interpretability and is particularly useful in ranking different models if their model-class is already identified (e.g. via human-grounded evaluation) to be interpretable. The main challenge for functionally-grounded evaluation is finding a good proxy.

2.2 Hyperparameter Optimization (HPO)

In contrast to model parameters, which are optimized during training, hyperparameters (HPs) are those describing the machine learning algorithm and are fixed before training. Because they usually have a significant impact on the trained model’s performance, HPs are often optimized, too. Let $\mathcal{D} \subseteq \mathcal{X} \times \mathcal{Y}$ be a dataset consisting of n tuples drawn from the data-generating distribution, i.e. $(\mathbf{x}^{(i)}, y^{(i)}) \stackrel{i.i.d.}{\sim} \mathbb{P}_{\mathbf{x}y}, \forall i = 1, \dots, n$. Furthermore, let $\mathcal{I} : (\mathbb{D} \times \mathbf{\Lambda}) \rightarrow \mathcal{H}, (\mathcal{D}, \boldsymbol{\lambda}) \mapsto \hat{f}$ be an algorithm that maps a given dataset \mathcal{D} and HP configuration $\boldsymbol{\lambda} \in \mathbf{\Lambda}$ to a model \hat{f} . Denote with $\mathcal{I}_{\boldsymbol{\lambda}}$ an algorithm with fixed HP configuration a chosen loss function with L . The goal of model training is, for fixed $\boldsymbol{\lambda}$, to have $\mathcal{I}_{\boldsymbol{\lambda}}$ find model \hat{f} minimising the expected generalisation error $GE(\mathcal{I}_{\boldsymbol{\lambda}}, \mathcal{D}, L) = \mathbb{E}_{(\mathbf{x}, y) \sim \mathbb{P}_{\mathbf{x}y}}[L(y, \mathcal{I}_{\boldsymbol{\lambda}}(\mathcal{D})(\mathbf{x}))]$. The goal of HPO, on the other hand, is to find $\boldsymbol{\lambda}$ minimising the expected generalisation error, i.e. $\arg \min_{\boldsymbol{\lambda} \in \mathbf{\Lambda}} GE(\mathcal{I}_{\boldsymbol{\lambda}}, \mathcal{D}, L)$. For this, there is no analytical expression available, making HPO a blackbox optimization problem. (Karl et al., 2023, pp. 2f)

We will first outline model-free methods to approach blackbox optimization problems, before presenting a model-based variant.

The most basic model-free strategies for blackbox optimization are grid and random search. In the latter, the user defines a range of interest for each HP and then evaluating points along a grid within the cartesian product of those. This has the drawback of scaling extremely poorly in both number of HP dimensions and number of query points per range of interest. Random search, on the other hand, randomly samples a value for each HP until it runs out of budget. Its explorative nature, ease of use, and no assumptions about the model makes it a good baseline. It is also superior to grid search in cases where one or more HPs have little impact on model performance, as for a given budget B , each HP will likely be queried with B different values, whereas for grid search each HP will only be queried with $B^{1/N}$ different values for N HPs. (Feurer and Hutter, 2019, chap. 1.3)

Another popular class of model-free optimizers are evolutionary algorithms (EAs), which are conceptually simple and can handle even complex parameter spaces, given appropriate implementation of operators. EAs are iteratively evolving a population of individuals (an individual is simply a hyperparameter configuration) of size μ , where in each iteration (generation), λ^1 offspring are generated via three operators (Goldberg, 1989, pp. 10-14).

Reproduction samples an individual from the population to reproduce with probability proportional to its fitness. For HPO, fitness usually refers to the performance metric the model resulting from the individual’s HP configuration is evaluated on.

Crossover selects two “parents” from the pool of reproducing individuals. An index k is sampled at random and all HP values from index k on are swapped, yielding two new “children” individuals.

Mutation randomly changes single HP values of an individual chosen for reproduction, e.g. by adding Gaussian noise to real values or flipping bits on binary values.

At the end of each generation, the EA keeps the best μ individuals, either only from the offspring (“ (μ, λ) -selection”) or more commonly from population and offspring (“ $(\mu + \lambda)$ -selection”), which is also called an “elitist” strategy, as it’s guaranteed to keep the best individual. (Feurer and Hutter, 2019, chap. 1.3)

1. We distinguish between $\lambda \in \mathbb{N}$ when referring to the offspring size of EAs and $\boldsymbol{\lambda} \in \mathbf{\Lambda}$ when referring to a hyperparameter configuration.

One of the most popular EA implementations is the “Covariance Matrix Adaptation Evolution Strategy” (CMA-ES). It employs a multivariate normal distribution to generate offspring, for which the mean is a weighted average of the previous generation’s individuals and the covariance matrix similarly is the weighted covariance of the previous generation. The weighing scheme for both is done in a way as to reproduce previously successful (i.e. selected) steps (Hansen, 2023, p. 8-11).

Another popular EA approach is Differential Evolution. Its population is randomly initialised so the entire search space is covered. In each generation g , for each $\lambda_{i,g}$ with $i = 1, \dots, n$ it creates “mutant vectors” $\nu_{i,g+1} = \lambda_{r_1,g} + F \cdot (\lambda_{r_2,g} - \lambda_{r_3,g})$, where $r_1, r_2, r_3 \in \{1, \dots, n\}$ are mutually different random indices that also different from i and $F \in [0, 2]$ is constant. For crossover, it then generates a “trial vector” $\mathbf{v}_{i,g+1} = (v_{i,g+1}^{(1)}, v_{i,g+1}^{(2)}, \dots, v_{i,g+1}^{(N)})^T$ with

$$v_{i,g+1}^{(j)} = \begin{cases} \nu_{i,g+1}^{(j)} & \text{if } u \leq \text{CR or } j = R \\ \lambda_{i,g}^{(j)} & \text{else} \end{cases} \quad (1)$$

for a random index $R \in \{1, 2, \dots, D\}$, sampled $u \sim U[0, 1]$, and crossover constant $\text{CR} \in [0, 1]$. Selection is done by picking the better of $\mathbf{v}_{i,g+1}$ and $\lambda_{i,g}$ with respect to fitness. (Storn and Price, 1997, p. 343)

Contrasting to model-free approaches, model-based methods fit a surrogate model on the target function and optimize the surrogate. Currently, one of the most popular model-based approaches is Bayesian Optimization (BO). BO is an iterative algorithm comprising a probabilistic surrogate model \tilde{f} for the blackbox problem f and an acquisition function. It maintains and continually extends a set of points it already evaluated on the target function. In each iteration, the posterior predictive distribution is determined by fitting the surrogate model on this set of points. The acquisition function then retrieves the highest utility point from the surrogate model and evaluates it on the target function, after which it adds the point to the set of queried points (Feurer and Hutter, 2019, chap. 1.3.2) and (Frazier, 2018, pp. 2f). Evidently, neither the target function nor the surrogate model are optimized directly. Rather, the acquisition function trades-off exploration and exploitation of the surrogate and is maximised to yield the next query point most likely to optimize the two objectives as defined by the acquisition function. Common choices for the acquisition function are Expected Improvement (Eq. 2) and Thompson Sampling (Eq. 3).

$$\text{EI}_t(\lambda) := \mathbb{E}_t[(f(\lambda) - f_t^*)^+] \quad (2)$$

Expected Improvement computes the expected minimisation over the incumbent f_t^* for evaluating the target function f at query point λ . \mathbb{E}_t denotes the expectation under the posterior predictive distribution with t query points, similarly, the current incumbent is the minimum value of the target function from previous evaluations $(\lambda_{1:t}, f(\lambda_{1:t}))$. (Frazier, 2018, p. 7)

$$f^{(t)} \sim \tilde{f}_t \quad (3)$$

Thompson Sampling, on the other hand, can conceptually be described as drawing a candidate function $f^{(t)}$ from the posterior predictive distribution and minimising it to obtain the next query point for the target function. (Shahriari et al., 2016, p. 161)

Composing the other part of BO, surrogate models need to be able to model mean and variance of its target function estimate. Commonly chosen models are Gaussian Processes (Eq. 4) and Random Forests.

$$\text{GP}(m(\boldsymbol{\lambda}), k(\boldsymbol{\lambda}, \boldsymbol{\lambda}')) \quad (4)$$

Gaussian Processes (GP) are fully specified by their mean and covariance functions and have closed-form solutions. The mean $m(\boldsymbol{\lambda})$ fits all points already evaluated on the target function, while for the covariance or kernel function $k(\boldsymbol{\lambda}, \boldsymbol{\lambda}')$ it is desirable that points close to each other in HP space $\boldsymbol{\Lambda}$ exhibit stronger correlation than those far apart. The kernel function can be freely chosen, a common choice is the Gaussian (Eq. 5) kernel with hyperparameter α_0 . (Frazier, 2018, p. 5)

$$k(\boldsymbol{\lambda}, \boldsymbol{\lambda}') = \alpha_0 \exp(-\|\boldsymbol{\lambda} - \boldsymbol{\lambda}'\|_2^2) \quad (5)$$

A drawback of Gaussian Processes is its poor scalability in number of data points and number of hyperparameters, although there exist workarounds approximating the full GP with only a subset of the samples. (Feurer and Hutter, 2019, chap. 1.3.2)

Random Forests, unlike GPs, can handle complex hyperparameter spaces even including categorical and hierarchical HPs. Their computational complexity is also far less, making them a popular alternative to Gaussian Processes. (Feurer and Hutter, 2019, chap. 1.3.2)

2.3 Neural Architecture Search

A special case of HPO is Neural Architecture Search (NAS). Because of the flexibility of neural network architectures (number of hidden layers, strength of dropout, each layer can have a different number of neurons, each layer can have a different activation function, etc.), traditional HPO methods cannot efficiently explore the entire space. In our extension of the EAGGA approach we don't employ NAS as research focusses on the NLP and image domains, where features (i.e. tokens or pixels, respectively) exhibit strong correlation amongst themselves, which is usually not the case for tabular data (Borisov et al., 2024, p. 7499). Still, we want to outline two notable approaches for neural architecture search in this paper, one of which are **cell search spaces**. These modularise an NN into a chain-structure of cells, where each cell is a basic building block of the respective architecture. For regular feedforward neural networks, this could be a linear layer of fixed size and with a specific activation function, for CNNs this could be a specific convolutional operation or pooling function, for instance. Instead of tuning every hyperparameter, the sequential placement of the cells is then optimized during NAS, e.g. via random search or Bayesian Optimization. (Elsken et al., 2019, chap. 3.2)

Treating a neural network as a sequence of cells instead of one fixed entity further allows exploiting this structure with special sequential optimization techniques. Zoph and Le (2017, p. 3) and Zoph et al. (2018, pp. 2-4) propose a cell search method using RNNs and reinforcement learning. The cells in their approach are not fixed, but have hyperparameters themselves, such as the filter size and the stride size of a convolutional layer. These hyperparameters are then predicted sequentially, given all the already predicted hyperparameters of previous layers. The RNN is trained using reinforcement learning, where the actions are the different values the network can predict for a given hyperparameter and the reward function is the RNN's performance on held-out validation data.

The second approach to NAS is the **one-shot model**, which trains a “fabric”, which can be understood as a DAG comprising all architectures of a given search space. Each path through the DAG is one specific network architecture, where the nodes represent a layer of neurons and the edges in-between are operations on the layers’ values. The entire DAG has one designated input and one output node. It is then trained just like a regular NN, after which the optimal path, i.e. network architecture, is selected from the fabric. This has the advantage of, albeit being slower than training a single model, being far more efficient and less expensive than training all architectures encoded in the fabric. (Saxena and Verbeek, 2017, pp. 1-2, p.8)

2.4 Multi-Objective Optimization

In most applications, practitioners don’t just want to optimize for performance exclusively but, for instance, also desire an interpretable model, which they may measure via some proxy metric. Let $c_1, c_2, \dots, c_m = \mathbf{c} : \Lambda \rightarrow \mathbb{R}^m$ be the vector of m objectives, the goal of multi-objective optimization (MOO) is to minimise this vector (Karl et al., 2023, p. 11).

2.4.1 PARETO-OPTIMALITY

Optimizing \mathbf{c} usually comes with the challenge of conflicting objectives: improving one objective means deteriorating another. We thus seek to find a set of trade-off solutions, so called *non-dominated* points. A point λ dominates another point λ' ($\lambda \prec \lambda'$) if there is no other point that is strictly better in at least one objective and better or equal in the remaining ones. Formally,

$$\forall i \in \{1, \dots, m\} : c_i(\lambda) \leq c_i(\lambda') \wedge \exists j \in \{1, \dots, m\} : c_j(\lambda) < c_j(\lambda') \quad (6)$$

as defined in (Karl et al., 2023, pp. 7f) and (Goldberg, 1989, pp. 198f).

The set of non-dominated or *Pareto-optimal* points $\mathcal{P} := \{\lambda \in \Lambda \mid \nexists \lambda' \in \Lambda \text{ s.t. } \lambda' \prec \lambda\}$ is commonly referred to as the *Pareto set* and its image as the *Pareto front*. The goal of MOO is to find a set of non-dominated points $\hat{\mathcal{P}}$ approximating the true Pareto set \mathcal{P} well.

There are different techniques to evaluate an estimated Pareto-front. If we have knowledge over the true Pareto-front, we can compute the distance between the approximation and the true front. In most cases, this knowledge cannot be assumed. Then, usually volume-based approaches are used for evaluation, most notably the hypervolume. The hypervolume, or S-metric, of a Pareto-front, computes the volume between the estimated front and some chosen reference point, which is usually the worst point in objective space. The larger the hypervolume, the closer the estimate is to the true Pareto-front. (Karl et al., 2023, pp. 8-10)

2.4.2 A-PRIORI

For optimizing a multi-objective problem, there are largely two general approaches. One of those are *a-priori* methods, of which we will briefly outline two popular techniques.

Scalarization approaches impose an implicit order of priority on the objectives. The simplest one is optimizing a weighted sum of objective functions

$$\arg \min_{\boldsymbol{\lambda} \in \Lambda} \sum_{i=1}^m w_i c_i(\boldsymbol{\lambda}) \text{ s.t. } \sum_{i=1}^m w_i = 1 \wedge w_i > 0, \forall i = 1, \dots, m \quad (7)$$

Weights are chosen a-priori by the user and the solution is very sensitive to them, making this method very reliant on different users’ preferences. (Karl et al., 2023, p. 11) and (Srinivas and Deb, 1994, chap. 3.1)

Another form of scalarization is the ϵ -constraint, which translates all but one objective into constraints and then optimizes the remaining objective subject to the constraints. Without loss of generality, the first constraint can be optimized

$$\arg \min_{\boldsymbol{\lambda} \in \Lambda} c_1(\boldsymbol{\lambda}) \text{ s.t. } c_2(\boldsymbol{\lambda}) \leq \epsilon_2, \dots, c_m(\boldsymbol{\lambda}) \leq \epsilon_m \quad (8)$$

This method is, similarly to the weighted sum, conceptually simple yet also very sensitive to the chosen constraints. (Karl et al., 2023, p. 12)

Alternative to scalarization is the **lexicographic method**. For it, the user assigns a priority to each objective and then greedily optimizes each objective in order of priority, constrained to the solutions of the already optimized, higher-priority objectives. (Riera et al., 2023, p. 13749) Again, solutions are very dependent on the user-defined prioritisation.

2.4.3 A-POSTERIORI

The disadvantage of a-priori methods is the missing knowledge of the interplay between a hyperparameter configuration and the trained model’s performance across objectives: we can either restrict the search space to enforce some will, such as only using a maximum of 50% of features, optimize for performance and use the resulting model, or leave the search space unrestricted, adjust the loss function to incorporate all objectives and take its optimum. In neither case do we know the impact our a-priori trade-off has on the final performance. In practical applications, the impact on performance across all objectives is what matters. This is the main advantage of a-posteriori methods: instead of implicitly defining a trade-off prior to training, we still evaluate multiple HP configurations but keep the set of non-dominated solutions instead of just one that satisfies the trade-off criterion. This is beneficial as it makes the relationship visible, a practitioner can now see, for instance, that a slight decrease in one objective might translate to a significant improvement in another that would have been missed if the problem was optimized with a-priori methods. Aside from the usual baselines grid and random search there are multi-objective Bayesian Optimization adaptations, mainly using one of two approaches:

1. Fitting a single surrogate model on scalarized objectives. Knowles (2006, pp. 54-56) propose *ParEGO*, which employs the augmented Tchebycheff function as scalarization to ensure the Pareto front is explored sufficiently.
2. Fitting one surrogate per objective, then either using one acquisition function per surrogate to return a set of promising candidate query points, or using one overall acquisition function to aggregate the surrogates, such as *EHI* as proposed by Emmerich et al. (2006, pp. 8f), which maximises the expected improvement of hypervolume.

Lastly, there is also a multitude of multi-objective evolutionary algorithms (MOEA) to explore the Pareto front. A key challenge for MOEAs is determining the fitness of an individual, as there are now multiple objectives that need to be incorporated instead of one as in single-objective EA (SOEA). We will outline the popular NSGA-II (Nondominated Sorting Genetic Algorithm II), which improves upon its predecessor NSGA (Srinivas and Deb, 1994) by making it parameterless, ensuring elitism, and reducing the computational complexity of ranking of individuals (Deb et al., 2002, p. 182). NSGA-II uses all the regular operators reproduction, mutation, and crossover that are already known from SOEA. The difference is in the fitness function, which for NSGA-II comprises two parts: non-dominated sorting and crowding distance.

Non-dominated sorting, as described by Goldberg (1989, p. 201) and Deb et al. (2002, pp. 183f) is an iterative procedure to ensure elitist selection through ranking individuals by their fronts. First, it assigns all individuals of the Pareto front a rank of 0. It then temporarily removes these individuals from the population and repeats this procedure with an incremented rank number until no individuals are left.

The second part, *crowding distance* ensures sufficient diversity, i.e. exploration of the Pareto front. The algorithm assigns each individual a score depending on how “crowded” the area in objective space around it is. Crowding distance is computed as the mean side length of the cuboid spanned by its nearest neighbours as vertices in the objective space, where individuals without two neighbours along some dimension are assigned an infinitely high distance value. The less the crowding distance, the more an area is by other individuals. (Deb et al., 2002, p. 185)

NSGA-II then ranks individuals by their non-dominated sorting front ranks in ascending order and by their crowding distance in descending order as tie breaker. This ranking is then used for selecting the μ best individuals to keep for the next generation. Reproduction follows a binary tournament selection, where two individuals are randomly drawn from the population and the better one according to the ranking gets included in the reproduction pool for mutation and crossover.

Conceptually, the reasoning behind using non-dominated sorting to rank individuals is intuitive: the closer an individuals front is to the Pareto front, i.e. the smaller its front rank, the better it approaches the true Pareto front. The usage of crowding distance, on the other hand, is not as straightforward. To understand it, we recall the goal of MOO, which is not merely optimizing hypervolume, but to approximate the true Pareto front well. Considering the scenario of only having individuals of one area in the objective space included in the Pareto front estimate makes it very unstable: removing this area (i.e. the solutions from this area) would collapse the entire front estimate. Considering now the opposite, having multiple areas making up the Pareto front estimate, makes it stable: removing one area would not impact the estimate a lot, as it’s still held up by other individuals. (Goldberg, 1989, p. 185, pp. 189-192)

3 EAGGA

- NSGA-II inspired evolutionary + genetic algorithm - uses AUC-ROC as performance metric and 3 interpretation metrics * NF: rel. # of features used in model * NI: rel. # of pairwise feature interaction effects * NNM: rel. # of non-monotone feature effects -

aims to find high performing models (AUC) with low NF, NI, and NNM - \mathcal{I} ensures better interpretability than simply optimising for performance but creates very large objective space $\tilde{\Lambda}$ to be optimised over with tuples $(\lambda, s, I_s, m_{I_s}) = \tilde{\lambda} \in \tilde{\Lambda}$ with $\lambda \in \Lambda$ “regular” model HP config $s \in \{0, 1\}^p$ vector denoting feature usage $I_s \in \{0, 1\}^{p \times p}$ matrix denoting pairwise interactions $m_{I_s} \in \{-1, 0, 1\}^p$ vector denoting monotonicity constraint of each feature (-1 decreasing, 0 none, 1 increasing) - thus would require to solve $\arg \min_{\tilde{\lambda} \in \tilde{\Lambda}} (GE(\mathcal{I}_{\tilde{\lambda}}, \mathcal{D}), NF(\hat{f}_{\mathcal{D}, \tilde{\lambda}}), NI(\hat{f}_{\mathcal{D}, \tilde{\lambda}}), NNM(\hat{f}_{\mathcal{D}, \tilde{\lambda}}))$ - (Schneider et al., 2023, pp. 540f) - introduces equivalence relation R “allowed to interact” on features that significantly reduces search space for more efficient optimization - thus: augmented search space $\tilde{\Lambda} = \Lambda \times \mathcal{G}$ comprising model HP space and group structure space \mathcal{G} \ast each group structure $G \in \mathcal{G}$ consists of g groups: - G_1 set of excluded features - $\forall k = 2, \dots, g : G_k = (E_k, M_{E_k})$ tuple $\ast E_k$ set of features allowed to interact with each other $\ast M_{E_k}$ monotonicity constraint of entire group k - EA on model hyperparameters Λ - GGA on group structure space \mathcal{G} with adapted mutation and crossover operators - (Schneider et al., 2023, pp. 541f) - further introduces special initialisation of group structures to increase sample-efficiency of EAGGA algorithm compared to random init - (Schneider et al., 2023, pp. 542f)

4 Extension to neural networks

- original EAGGA applied to XGBoost model proved to vastly outperform union of competitor models w.r.t. dominated hypervolume and comparable or better than ParEGO on extended search space $\tilde{\Lambda}$ (Schneider et al., 2023, pp. 543-545) - why extend it to NNs? \ast NNs notoriously uninterpretable due to complex transformation of the feature space \ast due to (lin alg) non-linear activation functions, interactions can be modelled \ast no monotonicity guarantees - our approach (general algorithm) \ast EAGGA algorithm implemented largely as described in the paper \ast HP init - NN total layers $\in \{3, \dots, 10\}$, i.e. hidden layers $\in \{1, \dots, 8\}$, init from trunc geom with prob=0.5 - NN nodes per hidden layer (for each group) $\in \{3, \dots, 20\}$, init from trunc geom with prob=0.5 - NN dropout % $\in [0, 1]$, init from trunc gamma with shape=2, scale=0.15 \ast group structure init - feature detector \ast as described in paper \ast but instead of fitting 10 trees + taking rel. # of features used as prob for trunc geom, we simply use 0.5 (sklearn dectree examination not straightforward) \ast also in preliminary experiments we found the sampled # of features to be used occassionally to be \mathcal{I} # of non-0 values in normalised information gain filter (e.g. if former is = total # of features and one feature is indep. from target) in these edge cases we simply use all features with non-0 filter values - interaction detector \ast as described in paper \ast same as for feat detect: use prob=0.5 to sample # of interactions used instead of 10 dec trees \ast also for FAST algorithm don’t use RSS (lin reg metric) but mean accuracy, as we fit log reg - monotonicity detector \ast as described in paper \ast use default HPs (max depth 30, minsplitted = 20) of mlr3 classification tree implementation, as this is the library the paper uses - \mathcal{I} both interaction + monotonicity detectors fit their models on 80% of train split from holdout + eval on remaining 20% (implementation details) \ast group structures + datasets - group structures implemented as described in the paper with additional list encoding sign of monotonicity of the individual features as detected by the monotonicity detector - included features are passed to dataset, dataset outputs included features, multiplied by the features’ individual monotonicity (-1 / 1), so that group monotonicity can be encoded with only $\{0, 1\}$, as proposed in paper

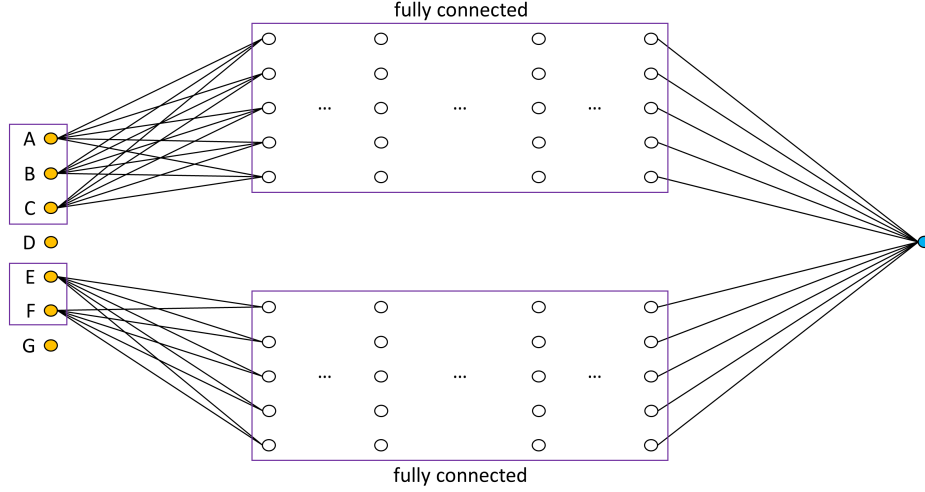


Figure 1: TODO: caption

- entire group structure is passed to NN, where architecture is built accordingly * neural network - "instead of XGB as in the original paper, we apply the method to NNs to examine whether this type of regularisation can achieve interpretability on NNs while outperforming EAGGA on XGB (original paper), ..., this requires special architecture, etc. pp." - NN * comprises of "sub-NNs", one for each equivalence relation - \mathcal{I} basically non-fully connected NN * hidden layers use ReLU activation and dropout afterwards, then a shared output layer with concatenated sub-NNs' outputs as input and sigmoid activation * optimizer is AdamW with default params, loss is binary cross entropy loss (sigmoid thus implicit in the loss function for better numerical stability, NN itself outputs "logits", which refers to pre-activation output in pytorch) - feature sparsity thus achieved by only training on included feature groups - \mathcal{I} goes somewhat against notion of deep learning where model is supposed to decide itself, which feature to "use" / put importance on - \mathcal{I} ELABORATE - feature interaction achieved by grouping, max-operation in ReLU induces interaction effect (different kind of interaction than e.g. in LM with multiplication) - \mathcal{I} equation why the interacting features need to be grouped together when using ReLU, cf. photos * given layer input \mathbf{x} and w.l.o.g. omitting bias * $\text{ReLU}(\mathbf{x}) = \max(0, \sum_{j=1}^p w_j x_j)$ * interaction not of the form $x_j \cdot x_{j'}$ as used to from LMs but of a form where the sum $w_j x_j + w_{j'} x_{j'}$ decides whether $x_j, x_{j'}$ pass through the activation or not * in other words: $\max(0, w_j x_j + w_{j'} x_{j'}) \neq \max(0, w_j x_j) + \max(0, w_{j'} x_{j'})$ and thus there is an interaction - monotonicity constraint achieved by clipping weights to $[0, \infty)$ after each epoch for restricted equivalence relations (monotonic decrease achieved via dataset object multiplying features with their individual signs), bias clipping not necessary (constant additive term) - \mathcal{I} equation how monotonicity is enforced with this * consider vector form computation of hidden neuron $\mathbf{o}_{\text{in}}^{(l)} \in \mathbb{R}^{p^{(l)}}$ of layer l - $\mathbf{o}_{\text{in}}^{(l)} = \mathbf{W} \mathbf{o}_{\text{out}}^{(l-1)} + \mathbf{b}$ -

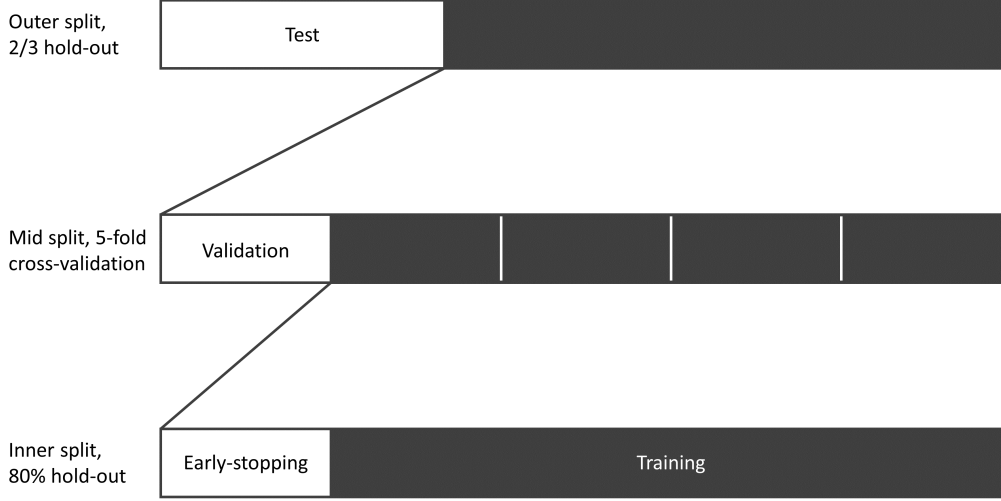


Figure 2: TODO: caption

where $\mathbf{o}_{\text{in}}^{(l)}$ refers to pre-activation values, $\mathbf{o}_{\text{out}}^{(l)}$ to value after applying ReLU - weight matrix $\mathbf{W} \in \mathbb{R}^{p^{(l)} \times p^{(l-1)}}$ - bias $\mathbf{b} \in \mathbb{R}^{p^{(l)}}$ * definition of monotonically increasing function f : for $x_1 \leq x_2$ it must hold that $f(x_1) \leq f(x_2)$ * $\mathbf{W}\mathbf{o}_{\text{out}}^{(l-1)} + \mathbf{b}$ clearly only monotonic if and only if $\mathbf{W} \in \mathbb{R}_{0+}^{p^{(l)} \times p^{(l-1)}}$ - where $\mathbb{R}_{0+} = [0, \infty)$ - hence weight clipping necessary, bias clipping not because only constant additive term * $\mathbf{o}_{\text{out}}^{(l)} = \text{ReLU}(\mathbf{o}_{\text{in}}^{(l)}) = \max(0, \mathbf{o}_{\text{in}}^{(l)})$ monotonic (experimental setup) * evaluation, holdout, cv, early stopping - evaluation via dominated hypervolume along AUC-ROC, NF, NI, NNM as defined in original paper * NF simply rel. # of included features * NI = sum of all possible pairwise interactions in each group over all possible pairwise interactions among all features = $\frac{\sum_g \binom{p_g}{2}}{\binom{p}{2}}$ * NNM = rel. # unconstrained features - outer holdout split: 2/3 train, 1/3 test (as in paper) * run EAGGA on holdout train split, i.e. train + select best $\mu = 100$ individuals based on non-dom-sorting (ascending) + crowding distance (descending) as tie breaker - inner CV split on outer train portion: 5-fold (as in paper) * in each fold fit model on 80% of CV-train portion, use remaining 20% of CV-train for early stopping - early stopping criterion * for each fold, always train for min 200 epochs, keep track of model with lowest loss * after that, use patience of 100: if current model's loss is \downarrow mean of last 100 epochs' losses - \downarrow stop early * if no early stopping, train each fold for max 10 minutes * then stop training, return model with lowest loss - \downarrow refere Figure 2 - then evaluate on last remaining from CV + keep best μ individuals + generate $\lambda = 10$ offspring for new generation + repeat - final evaluation of individuals of pareto set * for each individual, train model on training set of hold-out for max # of epochs taken during CV-training * decided on max # epochs instead of mean after looking at loss graphs in preliminary experiments, also refer

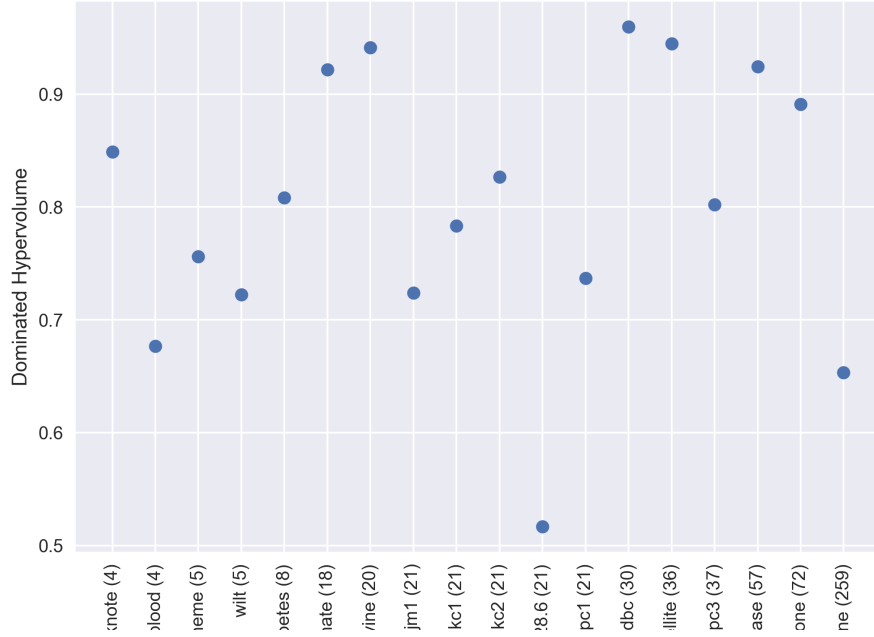


Figure 3: TODO: caption

Figure 4 * those show that there is no uptick in losses on the early stopping portion (dis-junct from training portion) of the set, hence max is reasonable * hardware: training on Sagemaker Notebook instance - initial experiments on ml.g4dn.xlarge instance (2 vCPUs, 16GiB RAM, 1 NVIDIA T4, cf. <https://aws.amazon.com/de/ec2/instance-types/g4/>) using cuda not much faster than on ml.t3.medium (2 Intel Xeon 8000 vCPUs, 4GiB RAM, cf. <https://aws.amazon.com/de/ec2/instance-types/t3/>) - thus decided for more economic + ressourcen-schonend t3.medium * did not train on philippine and gina datasets (308, 970 features, respectively), as they ran out of memory when computing interaction detectors * known bugs - in rare cases (anecdotally once every 5-10 datasets), gga_mutate seems to be generating -1 as monotonicity attribute, despite np.random.randint(low=0, high=2, size=1) * loop crashes at group_structure creation, for these cases subtract previous runtime from 8hrs + load from last generation in output via load_population * so far only happened for madeline in after gen-8.json after 5hrs 45mins (at start of gen-9, which wasn't exported yet) -i loaded this and ran for another 2hrs 15mins * only 18 datasets

5 Experimental Results and Discussion

- preliminary * our implementation of feature + interaction detectors likely tend to include
- more features + interactions for high p datasets - less for low p datasets - remember: original paper samples # of features included from truncated geometric distribution, what is not mentioned is probability of this distribution is determined from fitting 10 trees + looking at relative # of features used (cf. their github repo /R/TunerEAGGA.R function

`get_n_selected_rpart()` - \downarrow mlr3 default decision tree max depth 30 * i.e. datasets with $p \leq 30$ might use all features, which translates to $\text{trunc geom prob} = 1$ * vice-versa for $p \gg 30$ datasets relative $\#$ features ≤ 0.5 (our trunc geom prob) * similar reasoning for pairwise interactions * individuals not in Pareto sets occasionally exhibit $\text{AUC} \leq 0.5$ - this is not by mistake, usually AUC on binary target can simply be inverted (simply predict the opposite class) - for us this cannot easily be inverted due to monotonicity constraints (weights cannot simply be multiplied by -1 if constraint) - could happen e.g. due to AdamW and weight clipping (employed to enforce monotonicity) * AdamW uses momentum * possible scenario: large momentum would yield negative weights, but after epoch they're clipped to $[0, \infty)$ + training stops due to early stopping * weight clipping very imperfect way of enforcing monotonicity, but currently in pytorch unfortunately only way to implement this - Figure 3 * suggests comparable performance to XGBoost EAGGA * did not compare to unrestricted NN, would not be sensible, as NF, NI, NNM would be 1, hence hypervolume = 0, as values along 3 dimensions would be at reference point * as XGBoost EAGGA, NN EAGGA consistently outperforms union of competitors as evident from (Schneider et al., 2023, Figure 2) * unfortunately due to time constraints no own union comparison with NN instead of XGBoost - overview over pareto sets can be accessed on our github repo at `/code/export/*.csv` * NN HPs - total layers mostly 3-4, most commonly 3, goes as high as 7 (Satellite (36), diabetes (8)) - nodes per hidden layer mostly 3-6, goes as high as 12 (diabetes (8)) - p dropout goes as high as 0.7 (climate (18), spambase (57)), but mostly in the 0.1 to 0.3 range * group structures - great diversity in NF across datasets * phoneme (5) up to 1 * blood (4) up to 1 * banknote (4) up to 0.75 * diabetes (8) up to 0.5 * climate (18) up to 0.67 - \downarrow low p datasets in the benchmark tend to have higher NF - possible consequence of shorter evaluation time - low p datasets have much higher $\#$ generations - group structure space much more likely to be exhausted, i.e. more exploration of NN hps - NI, NNM consequently (bounded by NF, can never be more than max $\#$ for respective $\#$ included features) rather low * dhv contributions - measures contribution of an individual to the hypervolume, i.e. difference between hypervolume of entire pareto front vs hypervolume of pareto front without individual λ : $\text{CON}_{\mathcal{P}}(\lambda) = \text{HYP}(\mathcal{P}) - \text{HYP}(\mathcal{P} \setminus \{\lambda\})$, where $\text{HYP}(\mathcal{P})$ denotes the hypervolume induced by the pareto set \mathcal{P} (Bringmann and Friedrich, 2010, p. 384) - predominately low for fitted models - mostly highest for featureless learner predicting majority class - \downarrow sign of good exploration of pareto front + stable estimate, refer Section 2.4.3 - loss graphs Figure 4 * suggests models could have benefitted from longer training on some datasets, as some loss curves haven't converged when stopping criterion hit * crit was likely triggered by short-term spike, thus could potentially be resolved by comparing average of last k losses against average of $\lfloor \text{patience} \rfloor$ losses prior to that to not be as exposed to short-term spikes in loss * on other datasets, graphs suggest earlier stop would have been totally fine as the networks have long converged, but longer training likely not an issue as loss graphs come from early stopping dataset portion, which is disjunct from training set - dhv over generations Figure 5 * compute on val set, i.e. had 5 folds per individual - NF, NI, NNM always the same for each fold - but 5 different AUCs - \downarrow hypervolume of $(\text{mean}(\text{AUC}_1, \text{AUC}_2, \dots, \text{AUC}_5), \text{NF}, \text{NI}, \text{NNM})$ * artifacts / drops along y likely due to inconsistent computation of Pareto front by third-party library - preliminary experiments on dummy pareto fronts with 10x4 metrics: noticed `nds` function returning different rankings for same front - returned ranking switched back and forth between two only slightly different options

(only 1 or 2 indices were swapped) - not sure what caused this, as made same observation using another library that was planned as alternative - thus unfortunately not fixable for me
 * all but 3 datasets (2 of which only trained for 1 generation, anyway) show improvement of dominated hypervolume over generations * BUT: absolute as well as relative improvement almost negligible -; no large final dhv decrease would we just have evaluated the models gotten from the detectors - this was also the reason we didn't run EAGGA on philippine and gina simply without the detectors (as that's where they crashed) -; evidence suggested that detectors are vital to initial performance - original paper supports this assumption (Schneider et al., 2023, Fig. 4, p. 545)

6 Conclusion and Future Outlook

(Conclusion) - tabular data still difficult discipline for neural networks - research suggests heavy regularisation to improve performance on tabular data - EAGGA proved to be successful in making XGBoost more interpretable while keeping performance on par with unregularised XGBoost - we extend EAGGA to neural networks to see if we can utilise the regularisation induced by it to make NNs both interpretable and performant on tabular data - as consequence propose new architecture allowing to model equivalence relations of EAGGA - ... conforming to Zhang et al. (2021, chap. 2) taxonomy it fits as ..., ..., ... - found overall performance comparable to that of XGBoost fitted using EAGGA, which is a plus, but no outperformance

(Future Outlook) - MO BO on group structure space possible, perhaps via restricted BO?

Appendix A. Software used

for implementation we used openml Vanschoren et al. (2014), Feurer et al. (2021), numpy Harris et al. (2020), pandas pandas development team (2023), Wes McKinney (2010), pytorch Ansel et al. (2024), scikit-learn Pedregosa et al. (2011), scipy Virtanen et al. (2020), pymoo Blank and Deb (2020), and tqdm da Costa-Luis et al. (2024)

Appendix B. Plots

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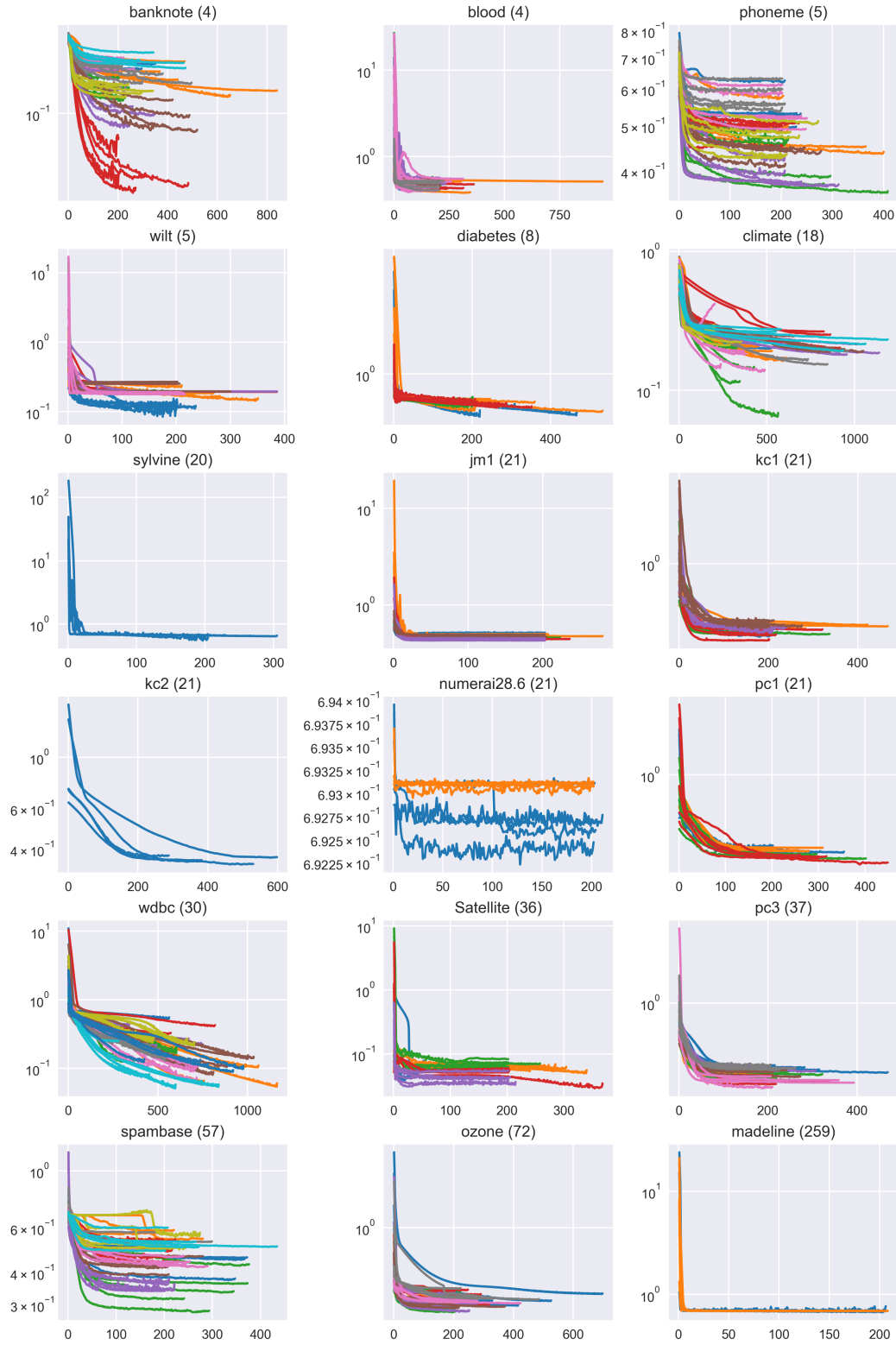


Figure 4: Pareto set loss of all datasets, evaluated on early stopping set. Same colours denote losses coming from folds of the same individual. x-axis portrait epochs, y-axis binary cross entropy loss.

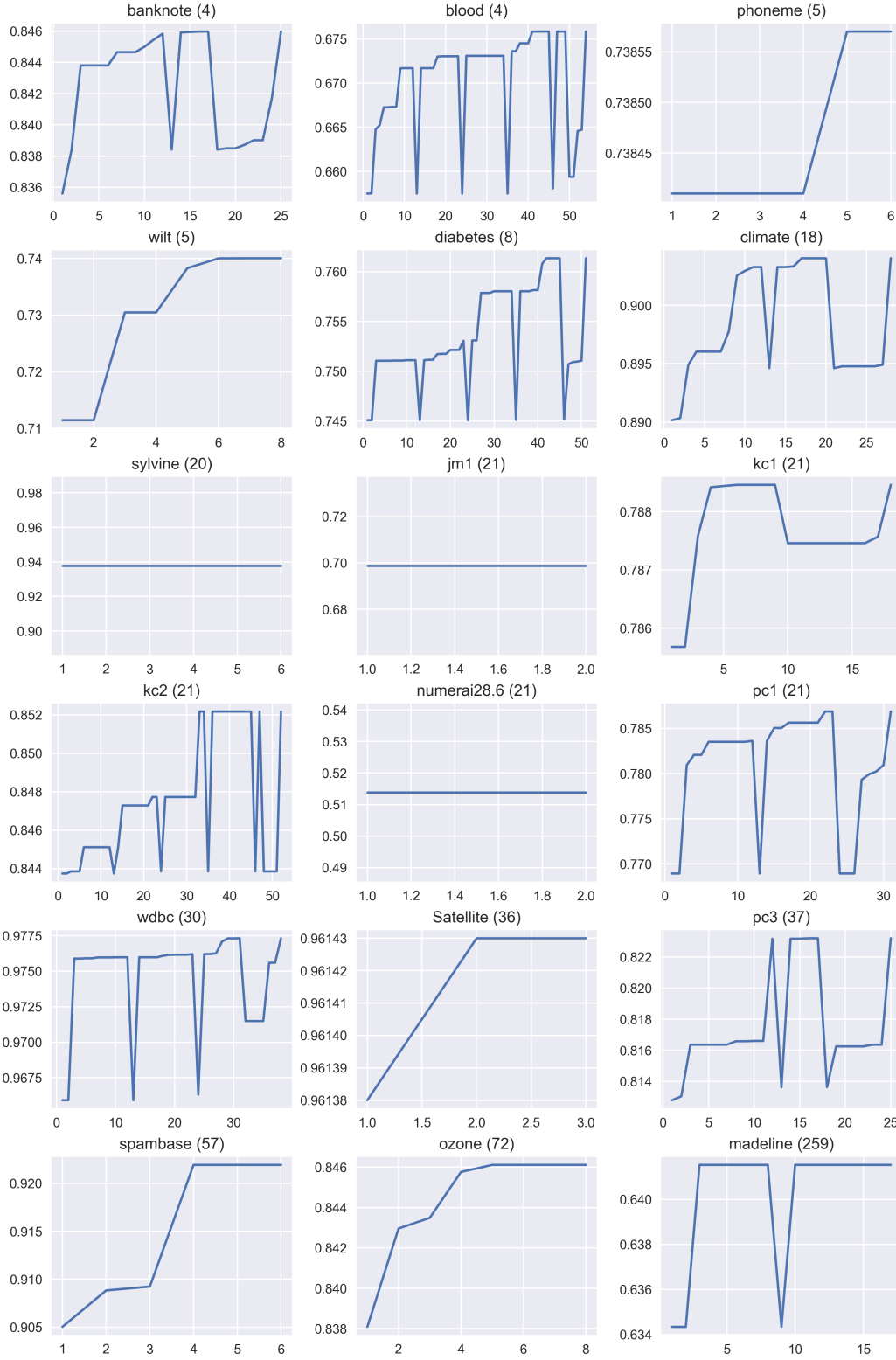


Figure 5: Dominated hypervolume over generations, evaluated on validation set. x-axis portrait generations, y-axis dominated hypervolume using mean AUC over folds.

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