LINEÁRNÍ APROXIMACE

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

① $\underline{f(x) = \sin x}$ v okolí $\underline{x_0 = 0}$
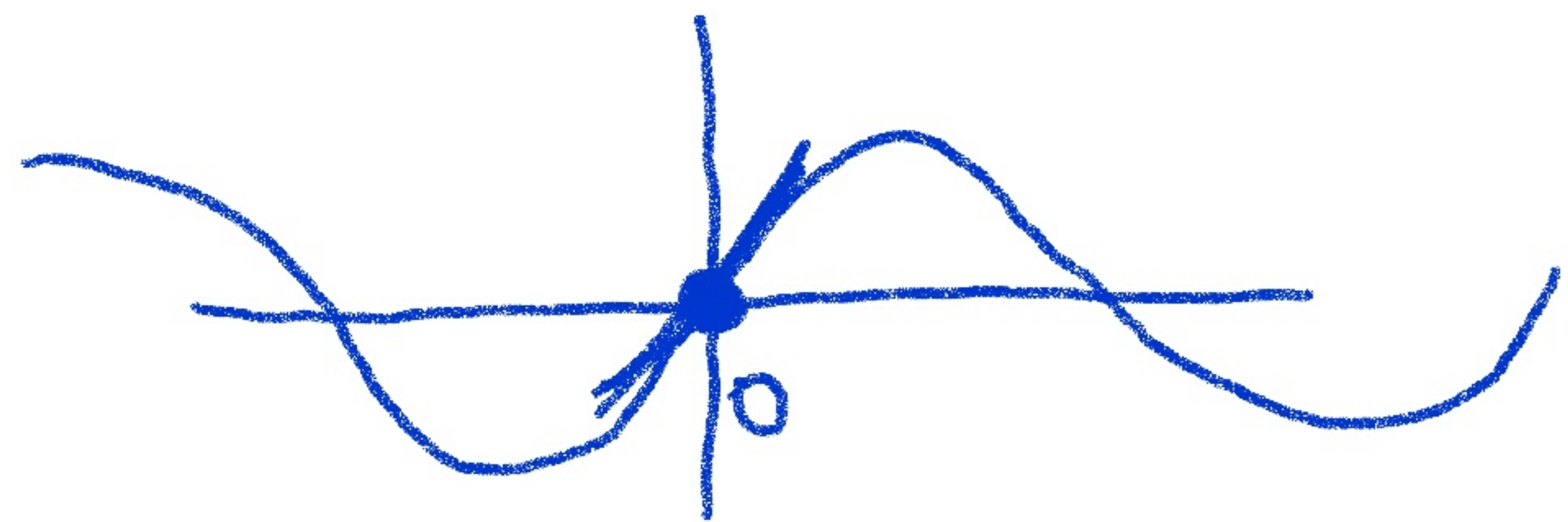
$f'(x) = \cos x$

$f(0) = \sin 0 = 0$

$f'(0) = \cos(0) = 1$

$\Rightarrow \quad \sin x \approx 0 + 1 \cdot (x - 0)$

$$\sin x \approx x$$

② $f(x) = \sqrt{x}$, $x_0 = 1$

$$\boxed{f(x) \approx f(x_0) + f'(x_0)(x - x_0)}$$

$\sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \frac{1}{2\sqrt{x}}$

$f(1) = \sqrt{1} = 1$

$f'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

$\sqrt{x} \approx 1 + \frac{1}{2}(x - 1) = 1 + \frac{1}{2}x - \frac{1}{2} = \frac{1}{2}x + \frac{1}{2}$

$$f(x,y) \approx f(x_0,y_0) + f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0)$$

③ $f(x,y) = x^3y^2 - xy^2 - 3x - y$, $(x_0,y_0) = (1,0)$

$f'_x = 3x^2y^2 - y^2 - 3$ $\Rightarrow f'_x(1,0) = -3$

$f'_y = 2x^3y - 2xy - 1$ $\Rightarrow f'_y(1,0) = -1$

$f(1,0) = -3$

$x^3y^2 - xy^2 - 3x - y \approx -3 - 3(x-1) - 1(y-0) = -3 - 3x + 3 - y$

$= -3x - y$