$$\text{①} \quad 2x_1 + x_2 + 3x_3 = 11$$
$$2x_1 + 3x_2 + x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 5$$

$$\begin{pmatrix} ② & 1 & 3 & | & 11 \\ 2 & 3 & 1 & | & 1 \\ 3 & 2 & 1 & | & 5 \end{pmatrix} \begin{matrix} |\cdot(-1) \ |\cdot(-3) \\ \downarrow \\ |\cdot 2 \end{matrix} \sim \begin{pmatrix} 2 & 1 & 3 & | & 11 \\ 0 & 2 & -2 & | & -10 \\ 0 & 1 & -7 & | & -23 \end{pmatrix} \begin{matrix} \\ |:2 \\ \end{matrix}$$

$$\sim \begin{pmatrix} 2 & 1 & 3 & | & 11 \\ 0 & ① & -1 & | & -5 \\ 0 & 1 & -7 & | & -23 \end{pmatrix} \begin{matrix} |\cdot(-1) \\ \swarrow^{+} \end{matrix} \sim \begin{pmatrix} 2 & 1 & 3 & | & 11 \\ 0 & 1 & -1 & | & -5 \\ 0 & 0 & -6 & | & -18 \end{pmatrix}$$

Rešíme odspodu: $-6x_3 = -18 \Rightarrow \boxed{x_3 = 3}$

$$x_2 - x_3 = -5$$

$$x_2 - 3 = -5 \Rightarrow \boxed{x_2 = -2}$$

$$2x_1 + x_2 + 3x_3 = 11$$
$$2x_1 - 2 + 9 = 11$$
$$2x_1 = 4 \Rightarrow \boxed{x_1 = 2}$$

② $x_1 + x_2 + x_3 - x_4 = 0$

$x_1 + 2x_2 - x_3 + x_4 = 1$

$x_2 - 2x_3 + x_4 = -1$

$2x_1 + 4x_2 - 2x_3 + x_4 = 0$

$$\begin{pmatrix} ① & 1 & 1 & -1 & | & 0 \\ 1 & 2 & -1 & 1 & | & 1 \\ 0 & 1 & -2 & 1 & | & -1 \\ 2 & 4 & -2 & 1 & | & 0 \end{pmatrix} \quad |\cdot(-1), \quad |\cdot(-2)$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & -1 & | & 0 \\ 0 & ① & -2 & 2 & | & 1 \\ 0 & 1 & -2 & 1 & | & -1 \\ 0 & 2 & -4 & 3 & | & 0 \end{pmatrix} \quad |\cdot(-1), \quad |\cdot(-2) \quad \sim \begin{pmatrix} 1 & 1 & 1 & -1 & | & 0 \\ 0 & 1 & -2 & 2 & | & 1 \\ 0 & 0 & 0 & -1 & | & -2 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-1} & | & \cancel{-2} \end{pmatrix}$$

Řešíme odspodu:

$-x_4 = -2$

$\boxed{x_4 = 2}$

$x_2 - 2x_3 + 2x_4 = 1$

$x_2 - 2x_3 + 4 = 1$

$\boxed{x_3 = t, \ t \in \mathbb{R}}$

$\left.\begin{array}{c} \end{array}\right\} \Rightarrow \boxed{x_2 = 2t - 3}$

$x_1 + x_2 + x_3 - x_4 = 0$

$x_1 + 2t - 3 + t - 2 = 0$

$\boxed{x_1 = -3t + 5}$

$$\boxed{3} \quad \begin{aligned} x_1 + x_2 - x_3 + x_4 &= -2 \\ 2x_1 + x_2 - x_3 + 2x_4 &= 2 \\ 3x_1 + 2x_2 - 2x_3 + 3x_4 &= 1 \\ x_2 - 3x_3 + 2x_4 &= -3 \end{aligned}$$

$$\left(\begin{array}{cccc|c} \textcircled{1} & 1 & -1 & 1 & -2 \\ 2 & 1 & -1 & 2 & 2 \\ 3 & 2 & -2 & 3 & 1 \\ 0 & 1 & -3 & 2 & -3 \end{array}\right) \quad |\cdot(-2) \quad |\cdot(-3)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & \textcircled{-1} & 1 & 0 & 6 \\ 0 & -1 & 1 & 0 & 7 \\ 0 & 1 & -3 & 2 & -3 \end{array}\right) \quad |\cdot(-1)$$

↑ LZE VIDĚT UŽ TADY

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & -1 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 2 & 3 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & -1 & 1 & 0 & 6 \\ 0 & 0 & -2 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\underbrace{0 \cdot x_1}_{0} + \underbrace{0 \cdot x_2}_{0} + \underbrace{0 \cdot x_3}_{0} + \underbrace{0 \cdot x_4}_{0} = 1$$

$$0 = 1$$

NEMÁ ŘEŠENÍ

(4) Napište rozšířenou matici soustavy 3 rovnic o 3 neznámých, která

a) má 1 řešení

b) má ∞ řešení

c) nemá řešení

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a) $\begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & 3 & | & 4 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & 3 & | & 4 \\ 0 & 2 & 6 & | & 8 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & 3 & | & 4 \\ 0 & 0 & 0 & | & 3 \end{pmatrix}$

⑤ Soustavu $x_1 \quad + 2x_3 = 1$

$\qquad\qquad 2x_1 + x_2 + 4x_3 = 2$

$\qquad\qquad\qquad\quad x_2 + x_3 = -1$

zapište pomocí maticového násobení

a vyřešte pomocí inverzní matice

k matici soustavy.

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$\qquad\quad A \qquad\qquad \vec{x} \qquad\quad \vec{b}$

$A \cdot \vec{x} = \vec{b}$

$\boxed{\vec{x} = \tilde{A}^1 \cdot \vec{b}}$

$\tilde{A}^1 : \left(\begin{array}{ccc|ccc} ① & 0 & 2 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{c} |\cdot(-2) \\ \nearrow + \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & ① & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{c} |\cdot(-1) \\ + \end{array}$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & ① & 2 & -1 & 1 \end{array}\right) \begin{array}{c} +\!\!\!\!\!\leftarrow \\ \\ |\cdot(-2) \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & -2 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \tilde{A}^1$

$\Rightarrow \vec{x} = \begin{pmatrix} -3 & 2 & -2 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$

$\boxed{\begin{array}{l} x_1 = 3 \\ x_2 = 0 \\ x_3 = -1 \end{array}}$