

RESEM' Bonus 2

(1)

$$\text{a)} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & 1 & -1 & 3 \\ 1 & 1 & -2 & 3 \end{array} \right) \xrightarrow{\substack{1 \cdot (-2) \\ + \\ +}} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -3 & 3 \\ 0 & 3 & -3 & 3 \end{array} \right) \xrightarrow{1:3} \downarrow$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & -3 & 3 \end{array} \right) \xrightarrow{1 \cdot (-5)} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & -2 \end{array} \right)$$

$$2x_3 = -2$$

$$x_2 - x_3 = 1$$

$$x_1 - 2x_2 + x_3 = 0$$

$$\underline{x_3 = -1}$$

$$x_2 + 1 = 1 \Rightarrow \underline{x_2 = 0}$$

$$x_1 - 1 = 0 \Rightarrow \underline{x_1 = 1}$$

$$\text{b)} \left(\begin{array}{ccc} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{array} \right) \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$$

$$(2) \left| \begin{array}{cc} 2-\lambda & 3 \\ 2 & 1-\lambda \end{array} \right| = (2-\lambda)(1-\lambda) - 6 = \lambda^2 - 3\lambda - 4 = (\lambda+1)(\lambda-4) = 0$$

$$\underline{\lambda_1 = -1, \lambda_2 = 4}$$

VLASTNÍ VEKTOŘE:

$$\lambda_1 = -1: \left(\begin{array}{cc} 3 & 3 \\ 2 & 2 \end{array} \right) \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} u_1 + u_2 = 0 \\ u_2 = -u_1 \end{array} \Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2 = 4: \left(\begin{array}{cc} -2 & 3 \\ 2 & -3 \end{array} \right) \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} -2u_1 + 3u_2 = 0 \\ u_1 = 3, u_2 = 2 \end{array} \Rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$