

$$\begin{aligned} \textcircled{1} \quad x' &= 6x - 5y \\ y' &= x + 3y \end{aligned}$$

Určete stacionární body a jejich typ

$$6x - 5y = 0$$

$$x + 3y = 0 \Rightarrow x = -3y$$

$$-18y - 5y = 0$$

$$-23y = 0 \Rightarrow y = 0 \Rightarrow x = 0 \Rightarrow [0, 0]$$

$$J = \begin{pmatrix} 6 & -5 \\ 1 & 3 \end{pmatrix} \Rightarrow$$

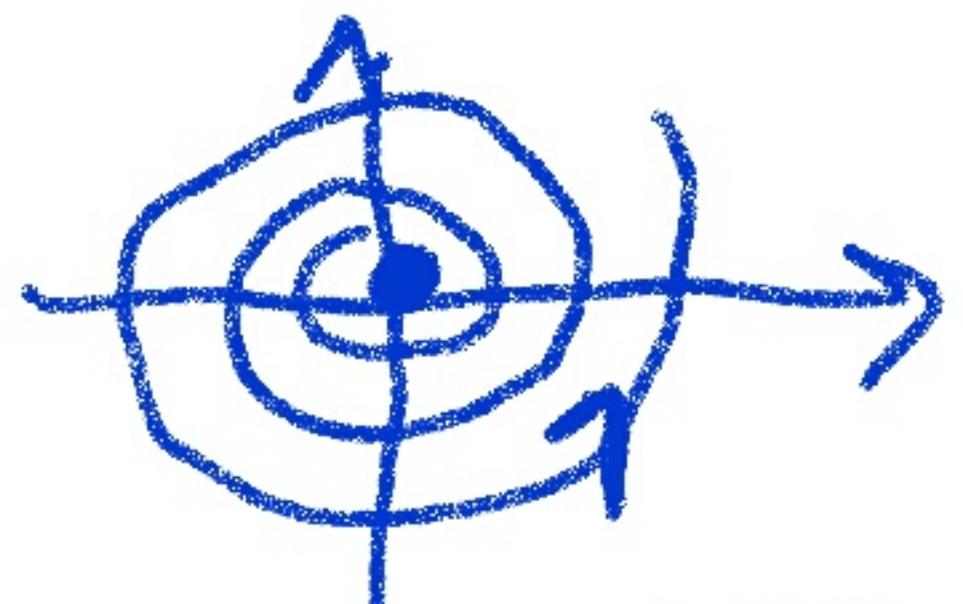
$$\begin{vmatrix} 6-\lambda & -5 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (6-\lambda)(3-\lambda) + 5$$

$$= 18 - 6\lambda - 3\lambda + \lambda^2 + 5$$

$$= \lambda^2 - 9\lambda + 23 = 0$$

$$\lambda_{1,2} = \frac{9 \pm \sqrt{81 - 92}}{2} = \frac{9 \pm i\sqrt{11}}{2}$$



NESTABILNÍ OHNIŠKO

$$\textcircled{2} \quad x' = x^2 - y$$

$$y' = x - y^2 + 2x^4$$

$$J(x,y) = \begin{pmatrix} 2x & -1 \\ 1+8x^3 & -2y \end{pmatrix}$$

$$1) J(0,0) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0 \Rightarrow \lambda^2 = -1 \Rightarrow \lambda = \pm i$$

$$x^2 - y = 0 \Rightarrow y = x^2$$

$$x - y^2 + 2x^4 = 0$$

$$x - x^4 + 2x^4 = 0$$

$$x + x^4 = 0$$

$$x(1+x^3) = 0 \quad \begin{cases} x=0 \Rightarrow y=0 \Rightarrow [0,0] \\ x=-1 \Rightarrow y=1 \Rightarrow [-1,1] \end{cases}$$

ohnísko nebo bod rotace

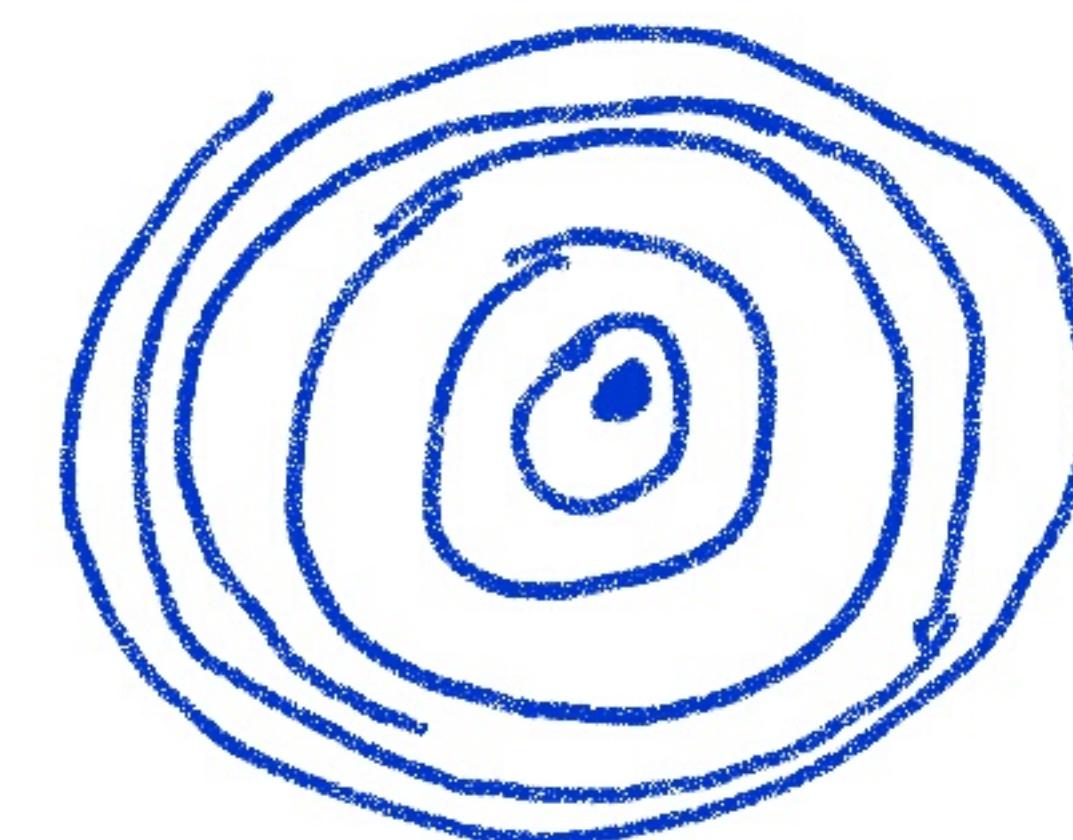
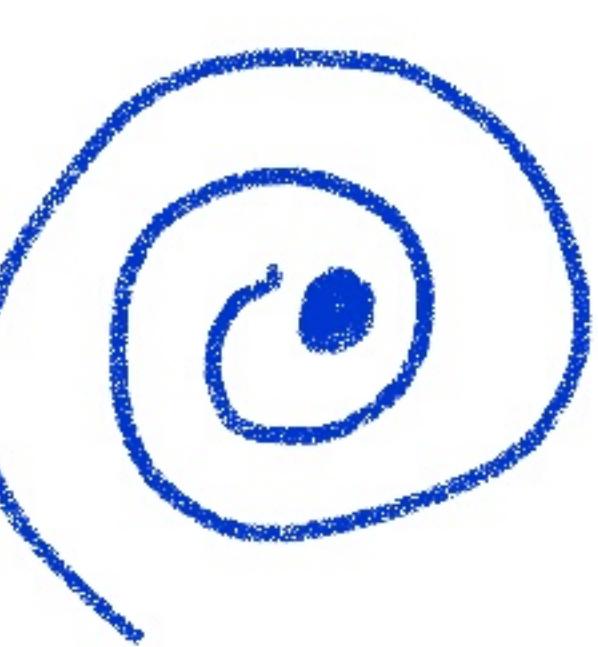
$$2) J(-1,1) = \begin{pmatrix} -2 & -1 \\ -7 & -2 \end{pmatrix} \Rightarrow \begin{vmatrix} -2-\lambda & -1 \\ -7 & -2-\lambda \end{vmatrix} = (-2-\lambda)^2 - 7 = 4 + 4\lambda + \lambda^2 - 7$$

$$= \lambda^2 + 4\lambda - 3 = 0$$

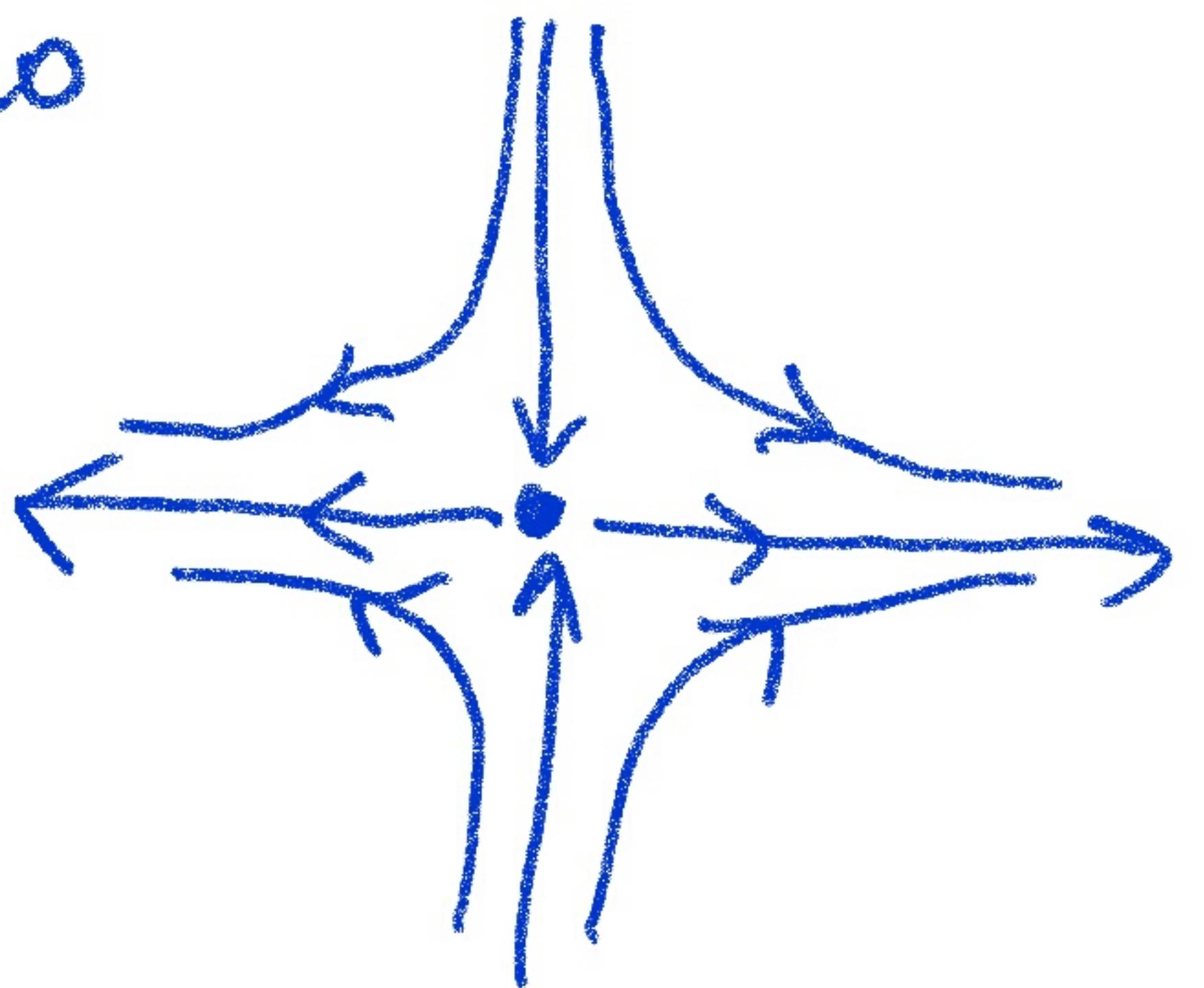
$$\begin{cases} -2 + \sqrt{7} > 0 \\ -2 - \sqrt{7} < 0 \end{cases} \Rightarrow \underline{\text{SEDLO}}$$

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{16+12}}{2} = \frac{-4 \pm \sqrt{28}}{2} = \underline{\underline{2 \pm \sqrt{7}}}$$

$[0, 0]$  OHVILSEO NEBO BOD ROTACE



$[-1, 1]$  SEDLO



$$\textcircled{3} \quad \begin{array}{l} x' = x - 2y \\ y' = 5x - y \end{array}$$

$$\begin{array}{l} x - 2y = 0 \Rightarrow x = 2y \\ 5x - y = 0 \end{array}$$

$$10y - y = 0$$

$$9y = 0 \Rightarrow y = 0$$

$$J = \begin{pmatrix} 1 & -2 \\ 5 & -1 \end{pmatrix}$$

$$\boxed{\underline{[0|0]}}$$

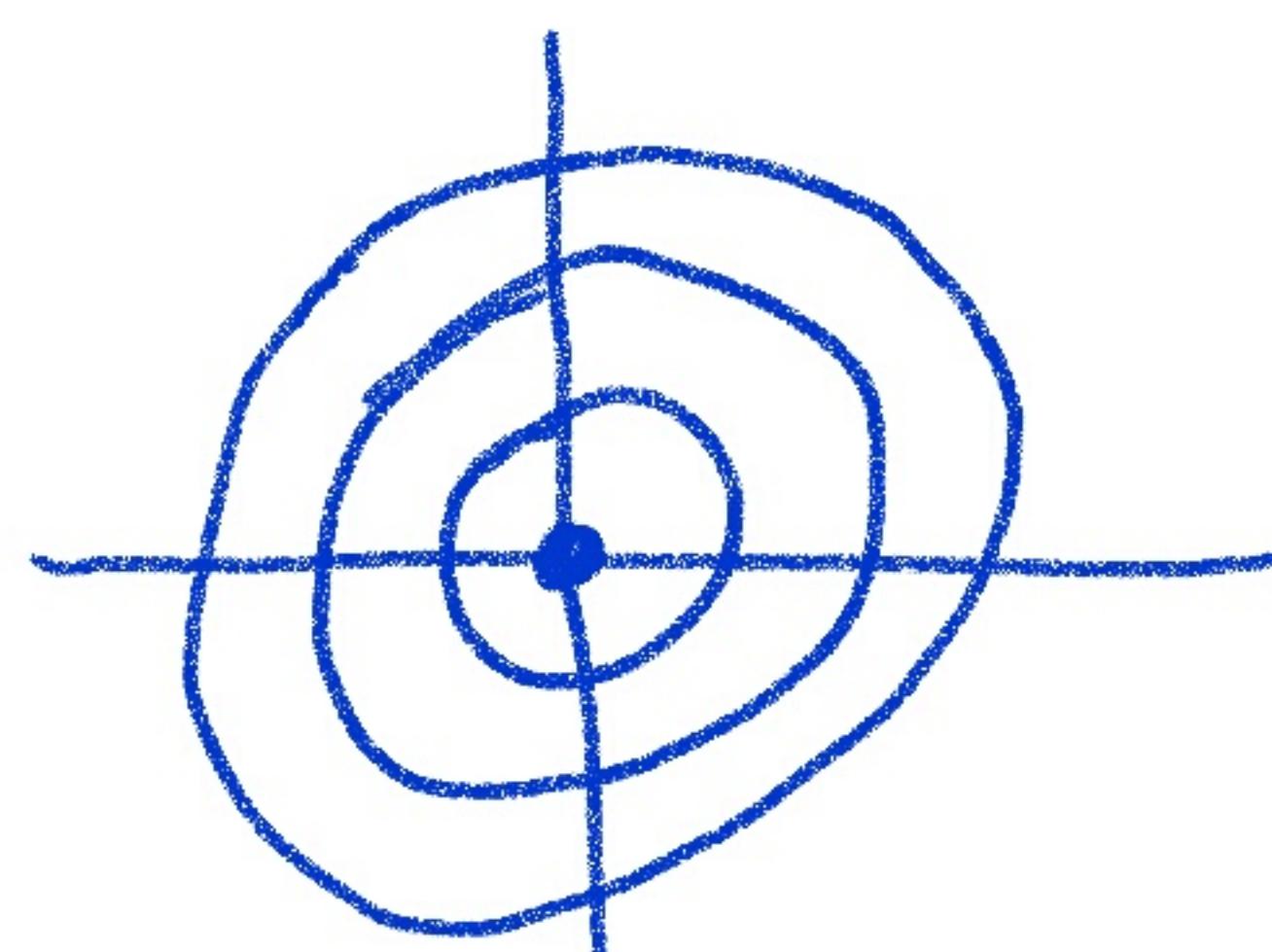
$\Leftarrow$

$$y = 0 \quad \text{and} \quad x = 0$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 5 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) - 5 \cdot (-2) = -1 - \lambda + \lambda + \lambda^2 + 10 \\ = \lambda^2 + 9 = 0 \Rightarrow \lambda^2 = -9$$

$$\boxed{\underline{\lambda = \pm 3i}}$$

$\Rightarrow$  **STRED**



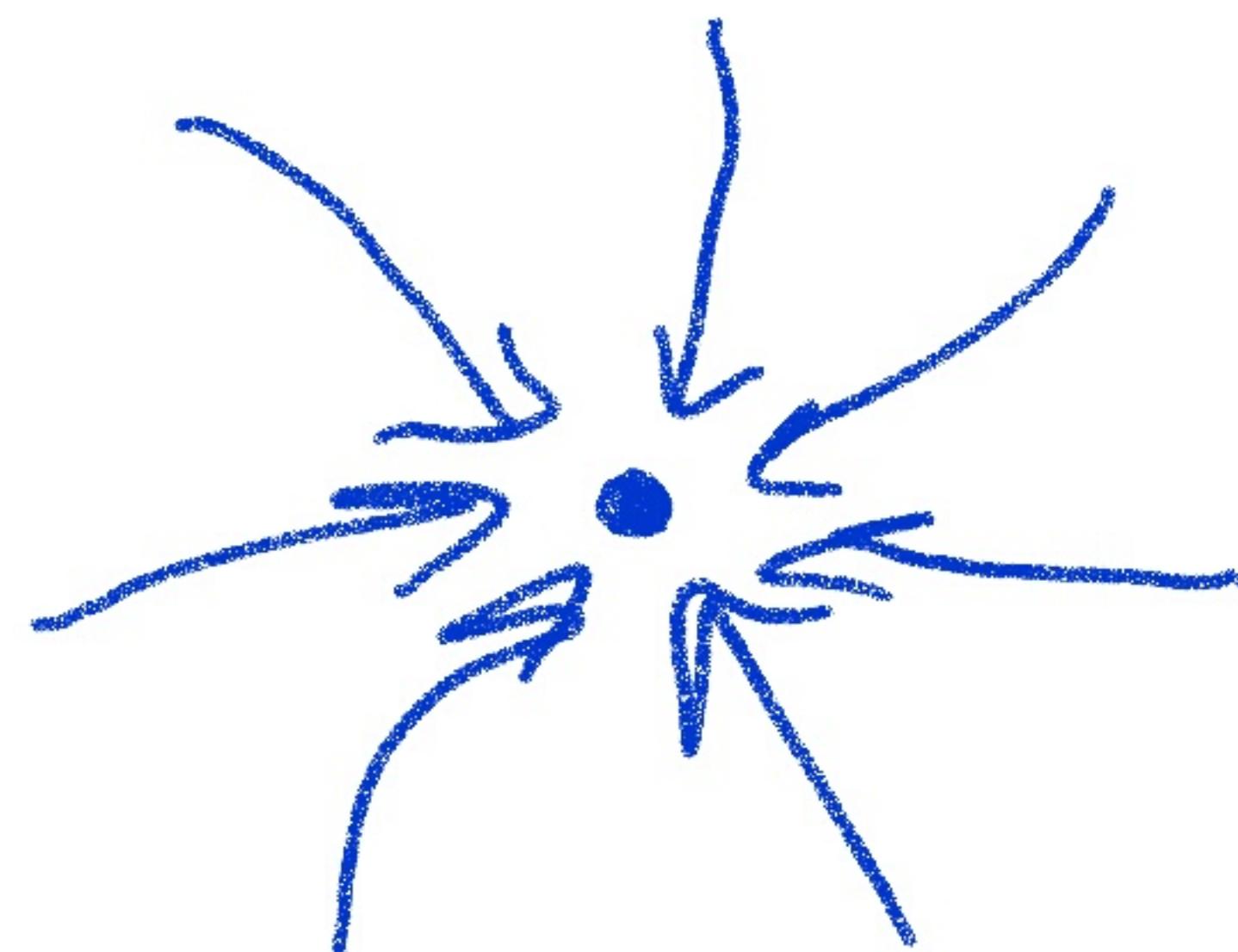
$$\textcircled{4} \quad \begin{array}{l} x' = y^2 - x^2 \\ y' = 1 - y \end{array}$$

$$y^2 - x^2 = 0 \\ 1 - y = 0 \Rightarrow \underline{\underline{y = 1}}$$

$$1 - x^2 = 0 \\ x^2 = 1 \Rightarrow \underline{\underline{x = \pm 1}}$$

$$\Rightarrow \underline{\underline{[1,1]}}, \underline{\underline{[-1,1]}}$$

$$J(x,y) = \begin{pmatrix} -2x & 2y \\ 0 & -1 \end{pmatrix}$$



$$\textcircled{1} \quad J(1,1) = \begin{pmatrix} -2 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\begin{vmatrix} -2-\lambda & 2 \\ 0 & -1-\lambda \end{vmatrix} = (-2-\lambda) \cdot (-1-\lambda) = 0 \\ \underline{\underline{\lambda_1 = -2}}, \underline{\underline{\lambda_2 = -1}}$$

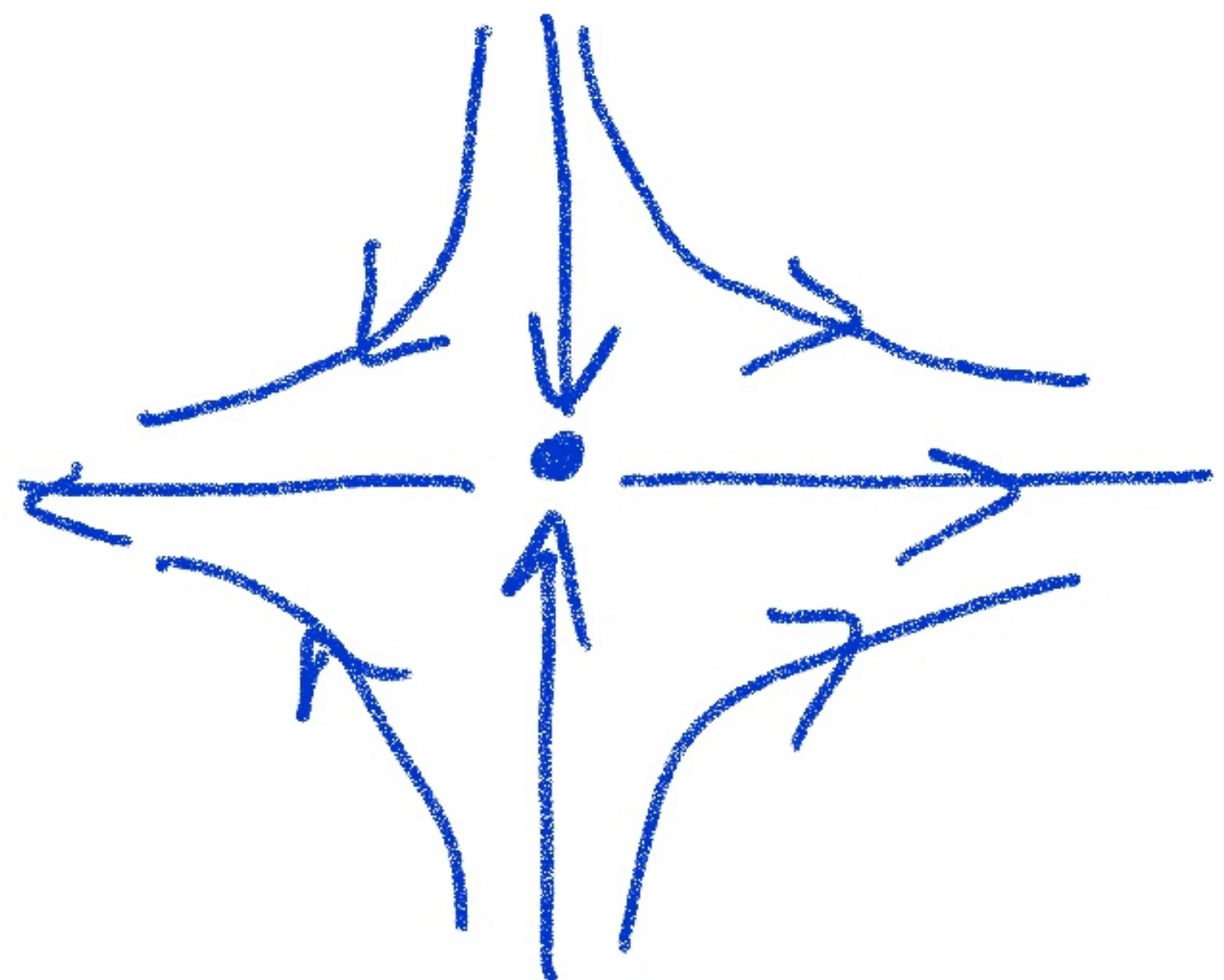
STABILITÄT U2EL

②  $J(-1, \lambda) = \begin{pmatrix} 2 & 2 \\ 0 & -1 \end{pmatrix}$

$$\begin{vmatrix} 2-\lambda & 2 \\ 0 & -1-\lambda \end{vmatrix} = (2-\lambda)(-1-\lambda) = 0$$

$$\underline{\lambda_1 = 2}, \underline{\lambda_2 = -1}$$

SED LO



$$\textcircled{5} \quad \left. \begin{array}{l} x^1 = x + y - 2 \\ y^1 = x + y^2 - 2 \end{array} \right\}$$

$$J(x,y) = \begin{pmatrix} 1 & 1 \\ 1 & 2y \end{pmatrix}$$

$$\begin{aligned} x+y-2 &= 0 & \Rightarrow x = 2-y \\ x+y^2-2 &= 0 & \\ 2-y+y^2-2 &= 0 & \Rightarrow y^2-y=0 \end{aligned}$$

$$y(y-1)=0$$

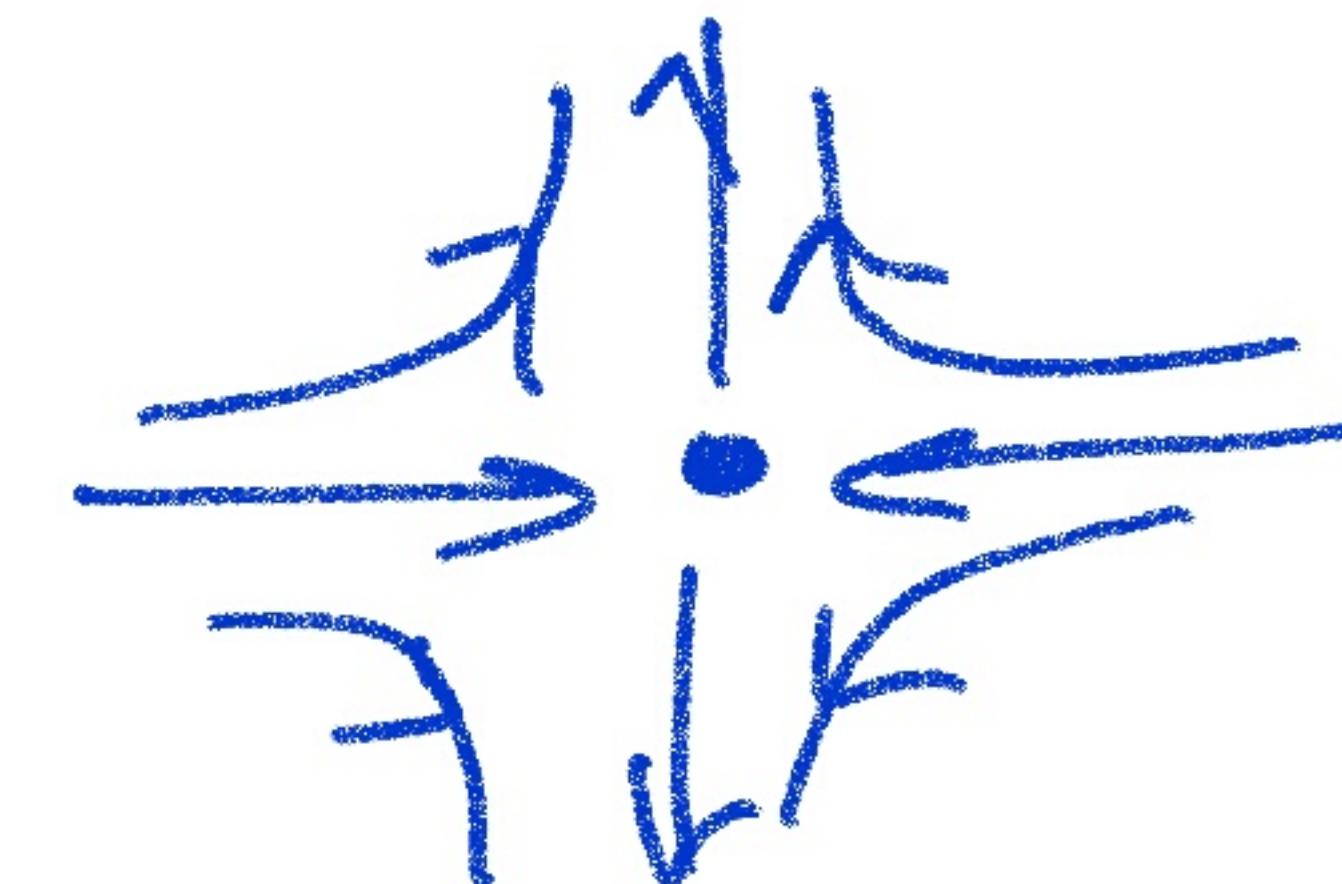
$$\begin{array}{l} y=0 \\ x=2 \end{array}$$

$$\begin{array}{l} y=1 \\ x=1 \end{array}$$

⇓

$$\underline{\underline{[2,0]}}, \underline{\underline{[1,1]}}$$

$$2) J(2,0) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

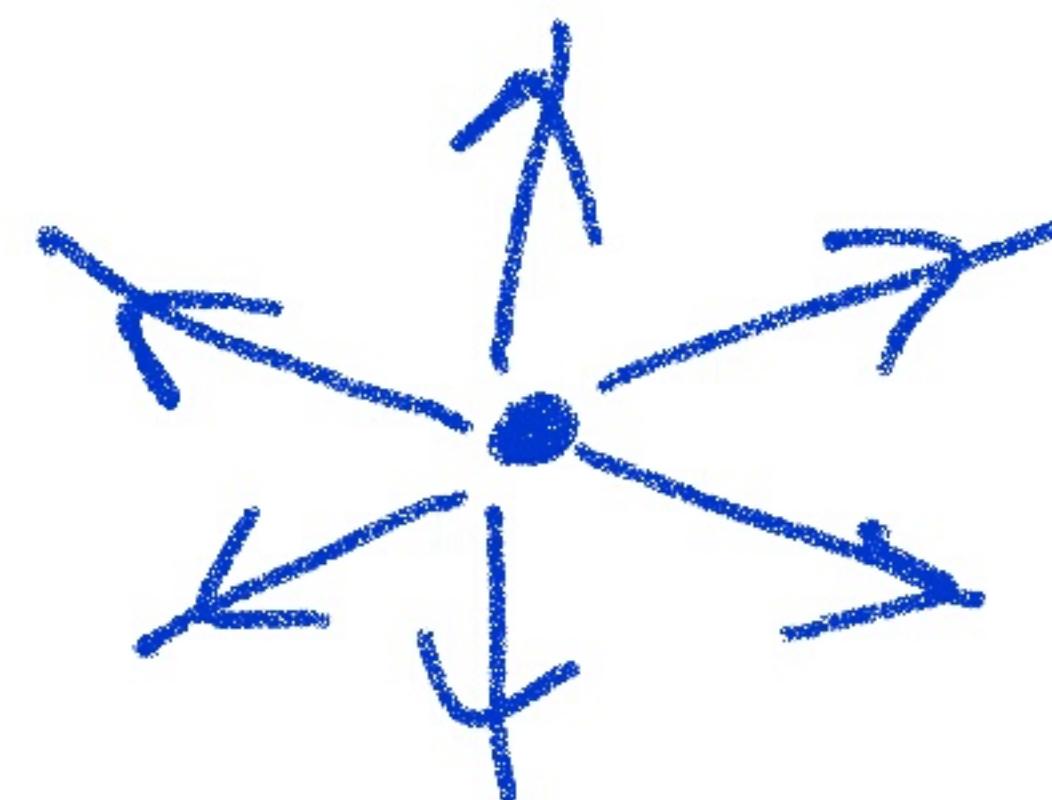


$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = (1-\lambda)(-\lambda) - 1 = -\lambda + \lambda^2 - 1 \Rightarrow \underline{\lambda^2 - \lambda - 1 = 0}$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}, \quad \frac{1+\sqrt{5}}{2} > 0$$

SEDLÖ

$$3) J(1,1) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$



$$\frac{1-\sqrt{5}}{2} < 0 \quad \underline{\underline{}}$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 1 = 2 - \lambda - 2\lambda + \lambda^2 - 1 = \lambda^2 - 3\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2} > 0$$

NESTABILNÍ UZEL