

For the given matrices

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 4 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -2 & 5 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

evaluate:

1. $AB; BA$ $\left[\begin{pmatrix} 14 & 11 \\ 8 & 4 \\ -7 & -1 \end{pmatrix}; \text{not defined} \right]$
2. $BC; CB$ $\left[\begin{pmatrix} 2 & 1 & 6 \\ 0 & -4 & 10 \\ 8 & 6 & 19 \end{pmatrix}; \begin{pmatrix} 6 & 11 \\ 20 & 11 \end{pmatrix} \right]$
3. $C^2; D^2$ $\left[\text{not defined}; \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} \right]$
4. $(2A - 4I)^T$ $\left[\begin{pmatrix} 0 & 0 & 2 \\ 0 & -6 & 4 \\ 6 & 4 & -8 \end{pmatrix} \right]$
5. $(B^T + C)^T D$ $\left[\begin{pmatrix} 6 & 10 \\ 3 & 6 \\ 29 & 42 \end{pmatrix} \right]$
6. $(D - 3I)^2 + D$ $\left[\begin{pmatrix} 11 & 0 \\ 0 & 11 \end{pmatrix} \right]$
7. $(AB)^T + 2C$ $\left[\begin{pmatrix} 18 & 14 & -5 \\ 11 & 0 & 9 \end{pmatrix} \right]$

Find the rank of the matrix

1. $\begin{pmatrix} 1 & 3 & 4 & -2 \\ 2 & 1 & 3 & 1 \\ 3 & -1 & 2 & 0 \\ 4 & -3 & -1 & 1 \end{pmatrix}$ [4]
2. $\begin{pmatrix} 2 & 1 & 3 & -1 \\ 3 & -1 & 2 & 0 \\ 4 & -3 & 1 & 1 \\ 1 & 3 & 4 & -2 \end{pmatrix}$ [2]
3. $\begin{pmatrix} 2 & 1 & 1 & 2 \\ 3 & 2 & 1 & 1 \\ 1 & 0 & 1 & 3 \\ 4 & 1 & 2 & 5 \end{pmatrix}$ [3]
4. $\begin{pmatrix} 2 & 3 & -1 & 1 \\ 1 & 1 & -2 & 3 \\ 1 & 3 & 4 & -7 \\ 4 & 7 & 1 & -3 \end{pmatrix}$ [2]

Determine whether the given vectors are linearly dependent or independent:

1. $(3, 1, 1, 2), (1, 3, 5, 4), (-1, 5, 9, 6)$ [dependent]
2. $(1, 1, -1, 2), (1, 2, 2, -2), (7, 2, 3, 9)$ [independent]
3. $(2, -1, 3, 4, 1), (3, 1, 3, -1, 2), (5, 0, 6, 3, 3)$ [dependent]
4. $(4, 1, 1, 3), (1, -1, 1, -3), (3, 1, 1, 1)$ [independent]
5. $(1, 3, 1, -1), (2, 2, 1, 1), (3, -1, 2, 9), (1, 1, 1, 2)$ [dependent]