

Msc thesis  
Mathematical Modelling and Computation

# The dynamics of adaptive neuronal networks: influence of topology on synchronisation

Simon Aertssen, s181603

## *Supervisors*

Erik Martens and Poul Hjorth

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**DTU Compute**

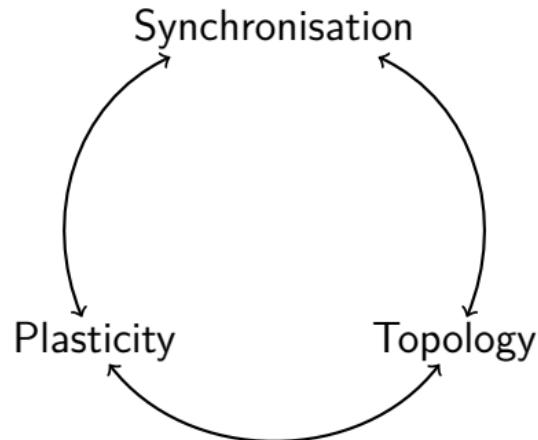
Department of Applied Mathematics and Computer Science

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## Contents

# Holistic approach



# Introduction

## Neuron dynamics

How do neurons communicate?

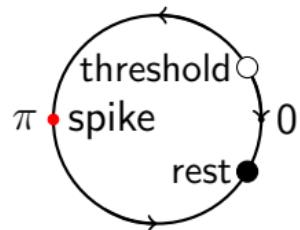
- Neurotransmitters
- Action potential = explosion of electrical activity

How do neurons learn?

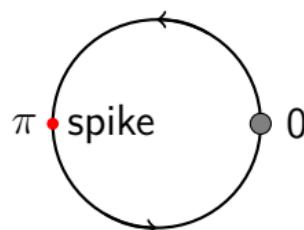
- *Fire and wire*: Correlation of neuronal activity

## Theory: The Theta Neuron Model Model Description

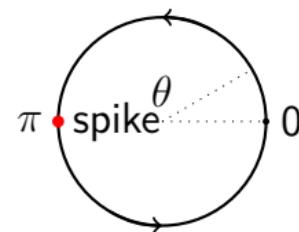
- Normal form of SNIC bifurcation



Excitable regime:  $I < 0$

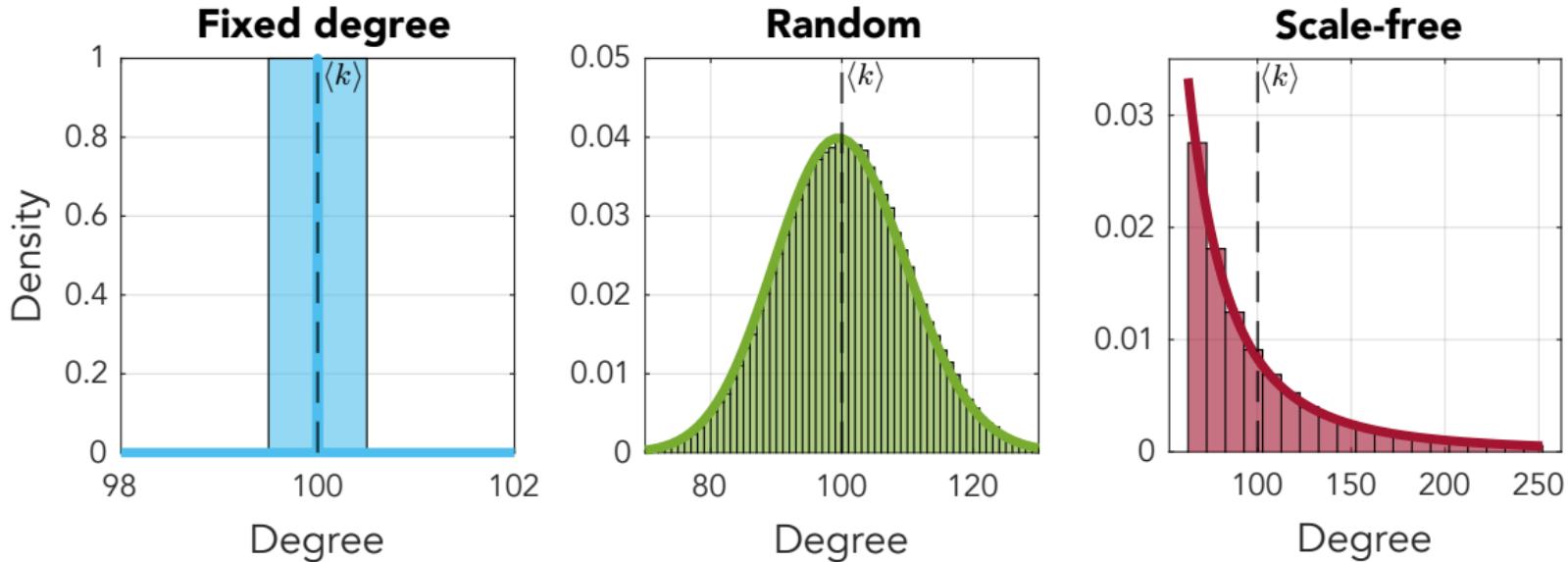


Bifurcation:  $I = 0$



Periodic regime:  $I > 0$

*Theory:* Network Topologies  
**Three basic networks**



## Theory: Network Topologies Networks of Theta neurons

- For arbitrary network topology:

$$\dot{\theta}_i = (1 - \cos \theta_i) + (1 + \cos \theta_i) \cdot [\eta_i + I_i(t)] \quad \theta_i \in \mathbb{T}^N$$

$$I_i(t) = \frac{\kappa}{\langle k \rangle} \sum_{j=1}^N A_{ij} \cdot \mathcal{P}_n(\theta_j)$$

- Capture average/mean synchronisation

$$Z(t) = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} \quad Z \in \mathbb{C}_o$$

*Theory:* Mean Field Reduction

## Predict synchronisation dynamics

The  $MFR =$  solution for  $Z(t)$

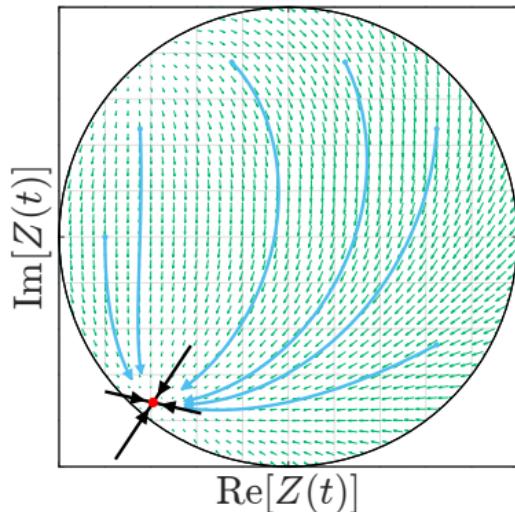
- Simple for fixed-degree networks: one equation

*Solution?* Formulate  $Z(t)$  per degree  $z(\mathbf{k}, t)!$

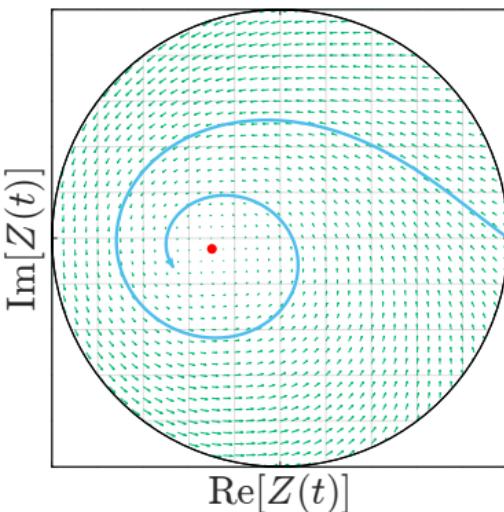
- Less unique degrees than neurons
- = less equations!

$$\bar{Z}(t) = \frac{1}{N} \sum_{\mathbf{k}} P(\mathbf{k}) z(\mathbf{k}, t) \quad \bar{Z} \in \mathbb{C}_o$$

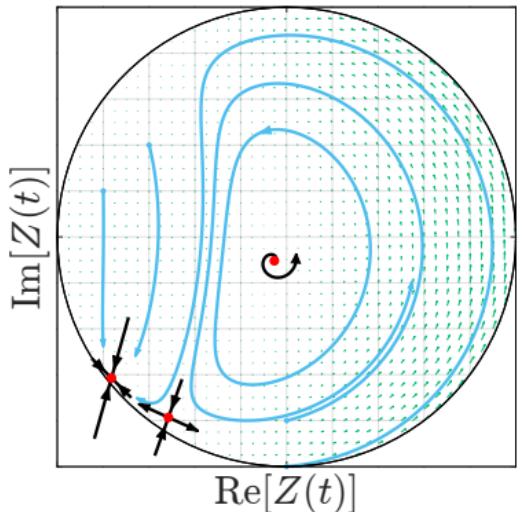
## Theory: Mean Field Reduction Fixed-degree networks



(a) PSR



(b) PSS



(c) CPW

## *Investigation:* Mean Field Reductions for undirected graphs

### Goals

$Z(t)$  can be measured and predicted: are they the same?

- Formulate directed networks
- Construct adjacency matrix from degree distribution
- Initial conditions
- Compare directed and undirected networks

*Investigation:* Mean Field Reductions for undirected graphs

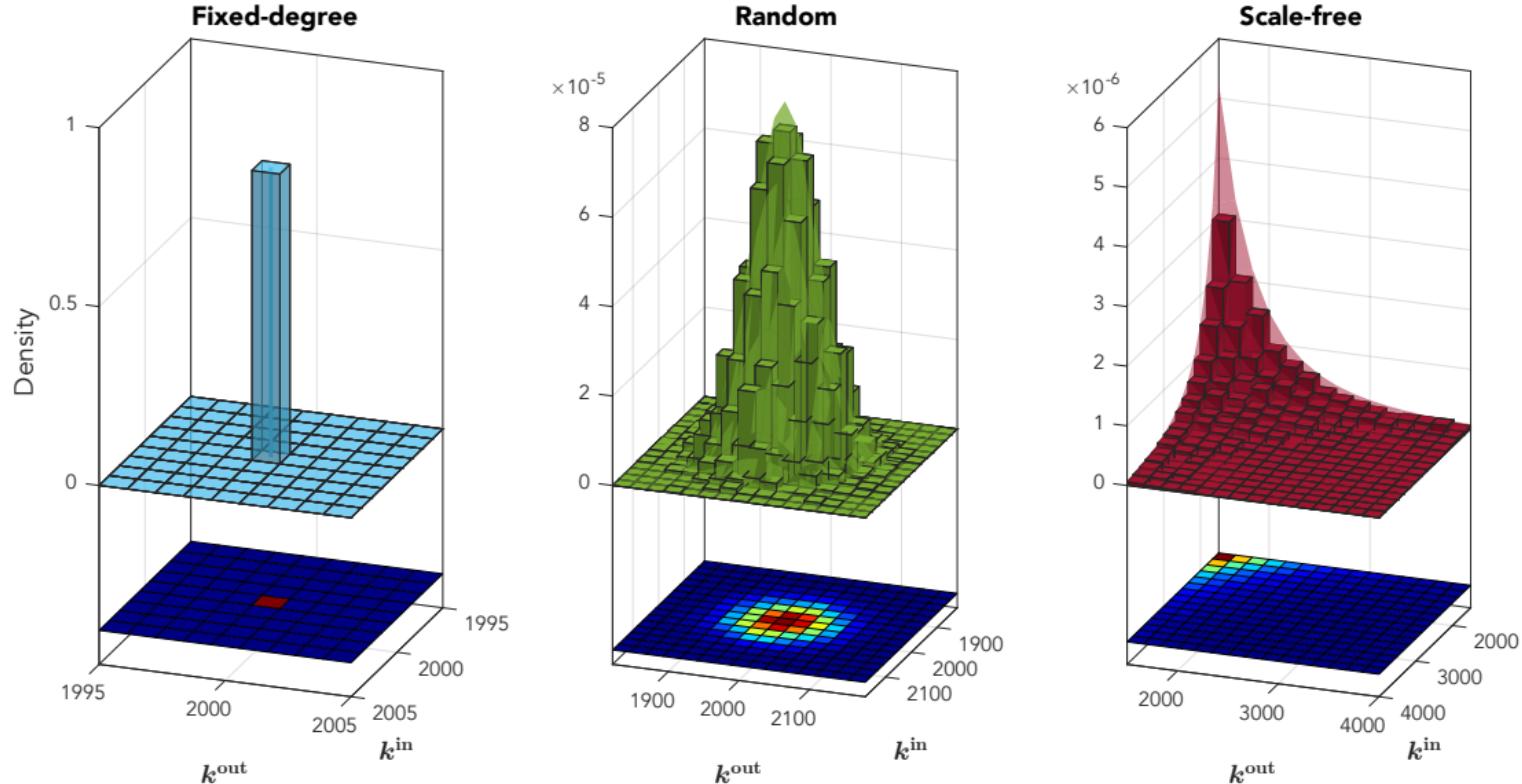
## Directed networks

Sample degree vectors from bivariate distribution? Difficult!

- Use independant univariate distributions as marginal
- In- and out degree vectors are a permutation

# Investigation: Mean Field Reductions for undirected graphs

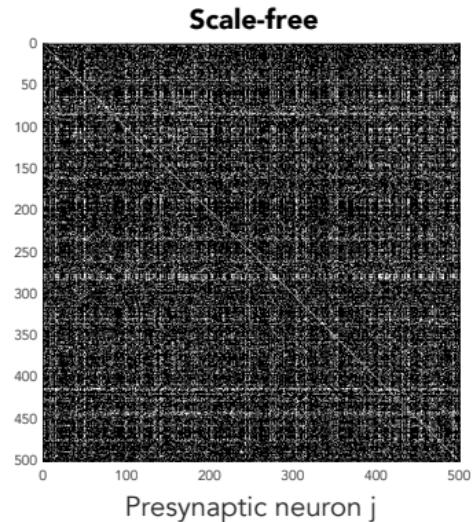
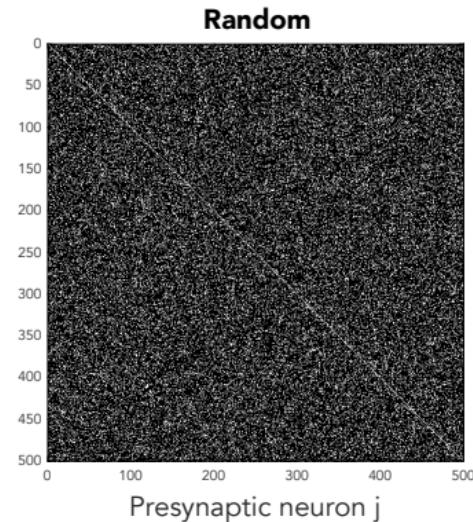
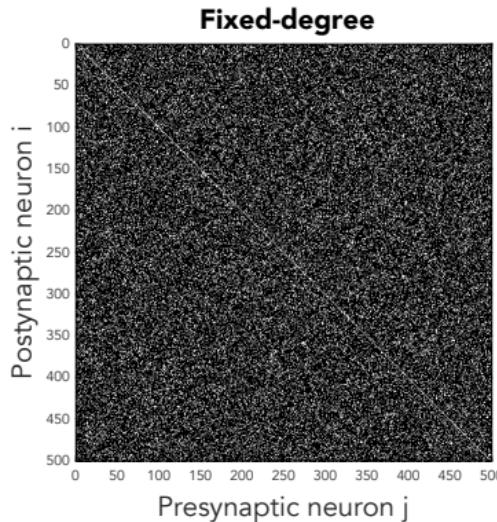
## Directed networks



# *Investigation:* Mean Field Reductions for undirected graphs

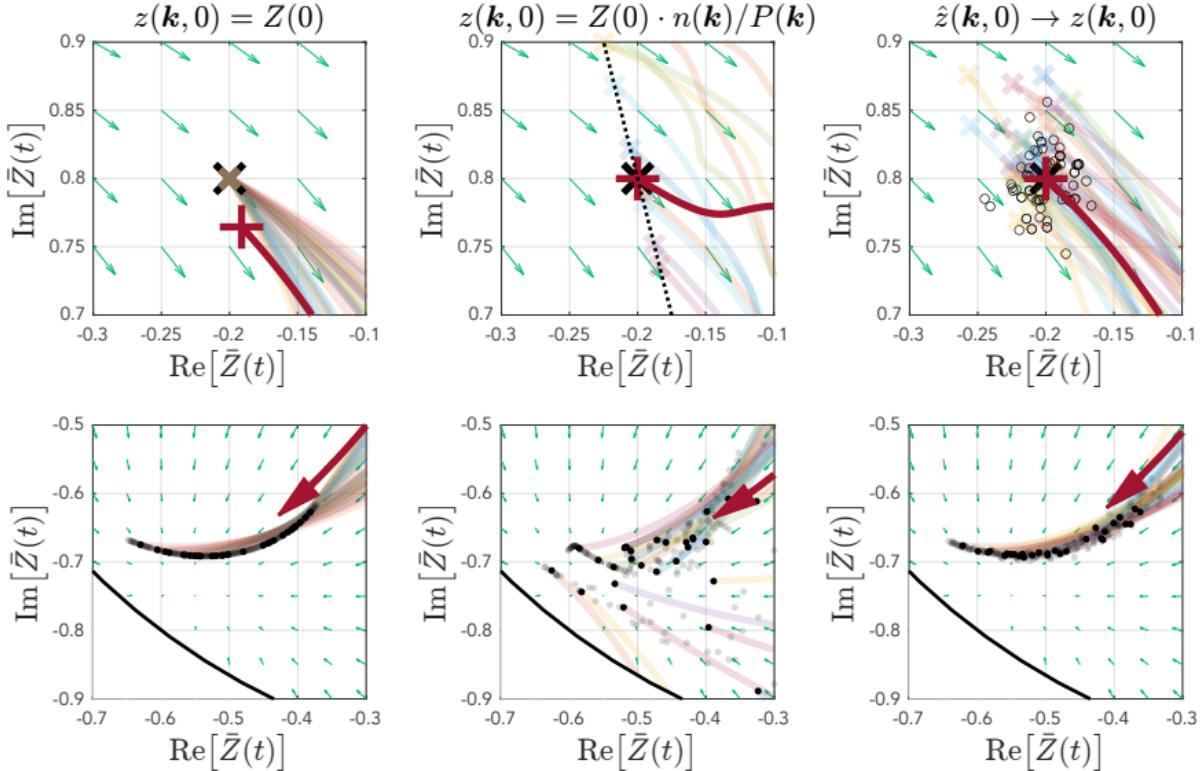
## Adjacency matrix

Find a probable solution by sampling from the in- and outdegrees



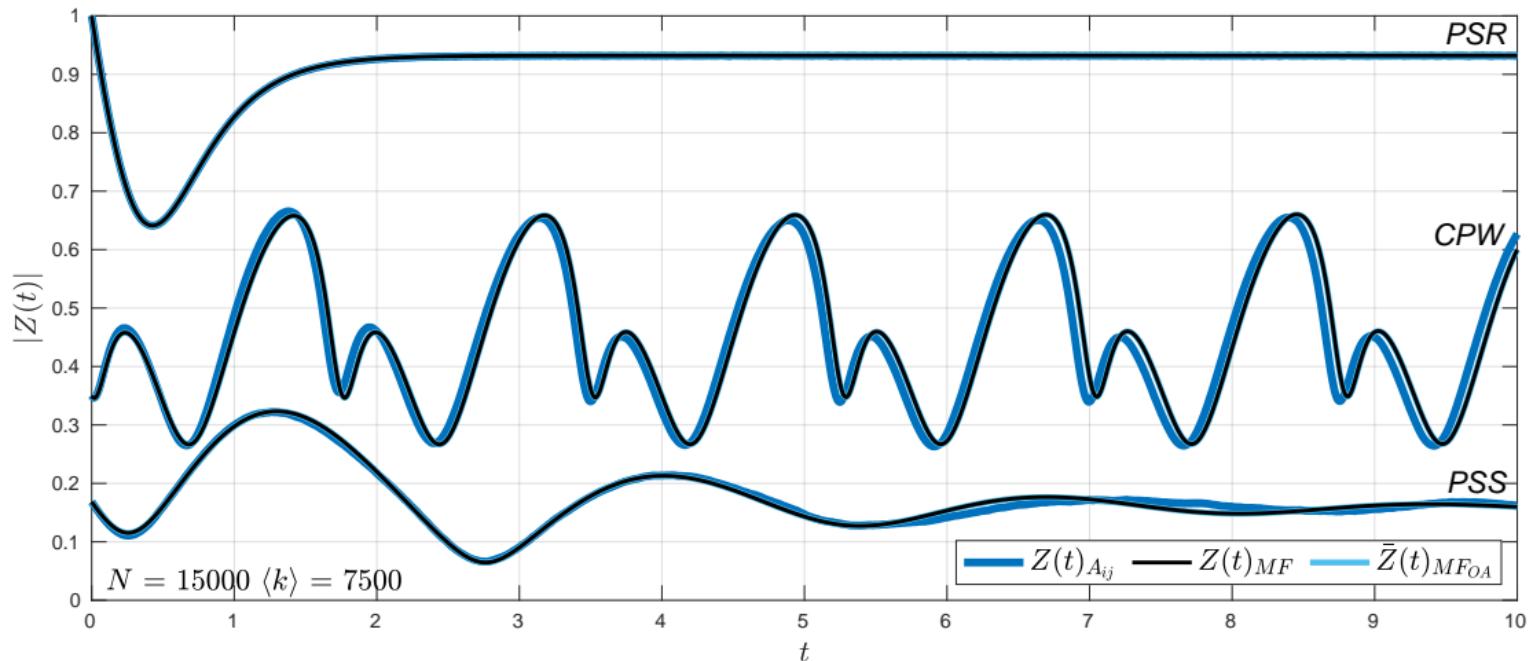
# Investigation: Mean Field Reductions for undirected graphs

## Initial conditions



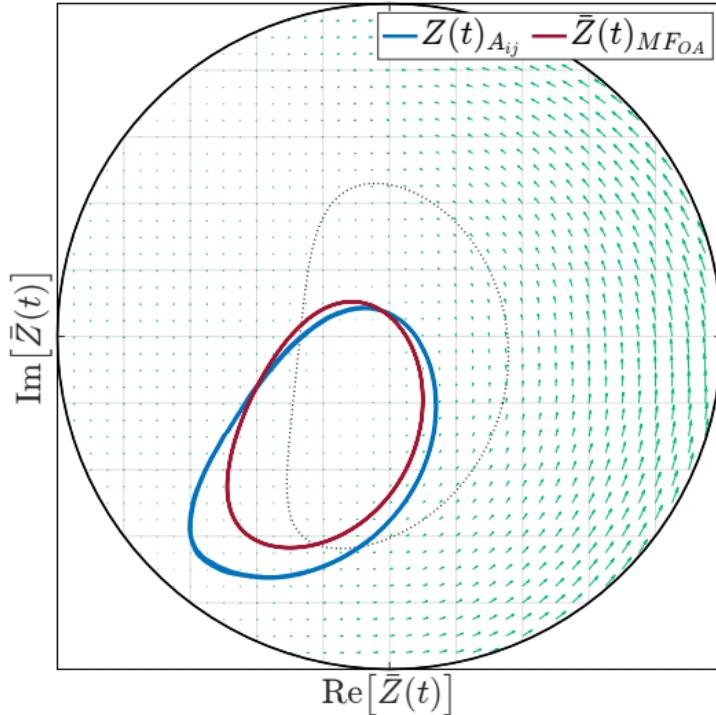
# Investigation: Mean Field Reductions for undirected graphs

## Results



*Investigation:* Mean Field Reductions for undirected graphs

## Results



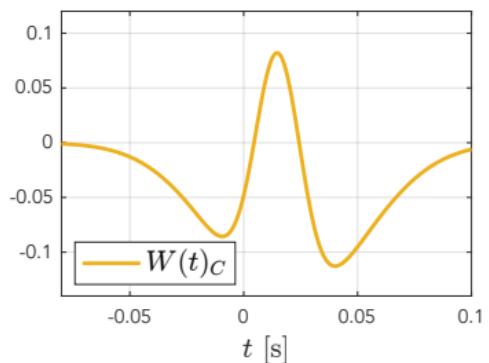
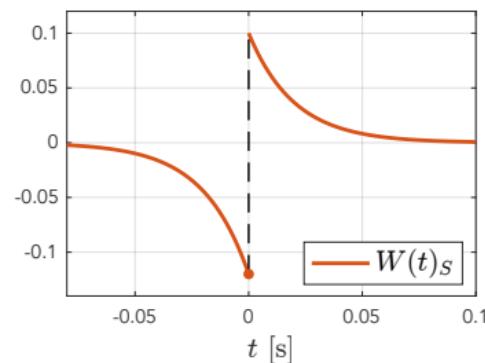
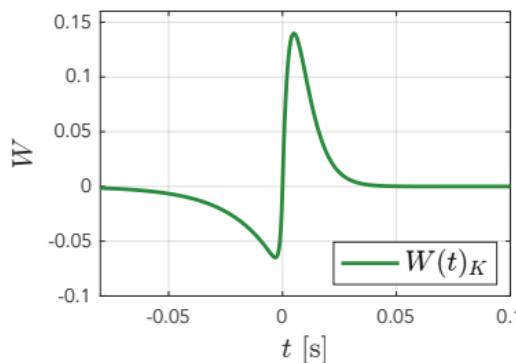
## Theory: Hebbian Learning and Synaptic Plasticity

### Temporal Interpretation: STDP

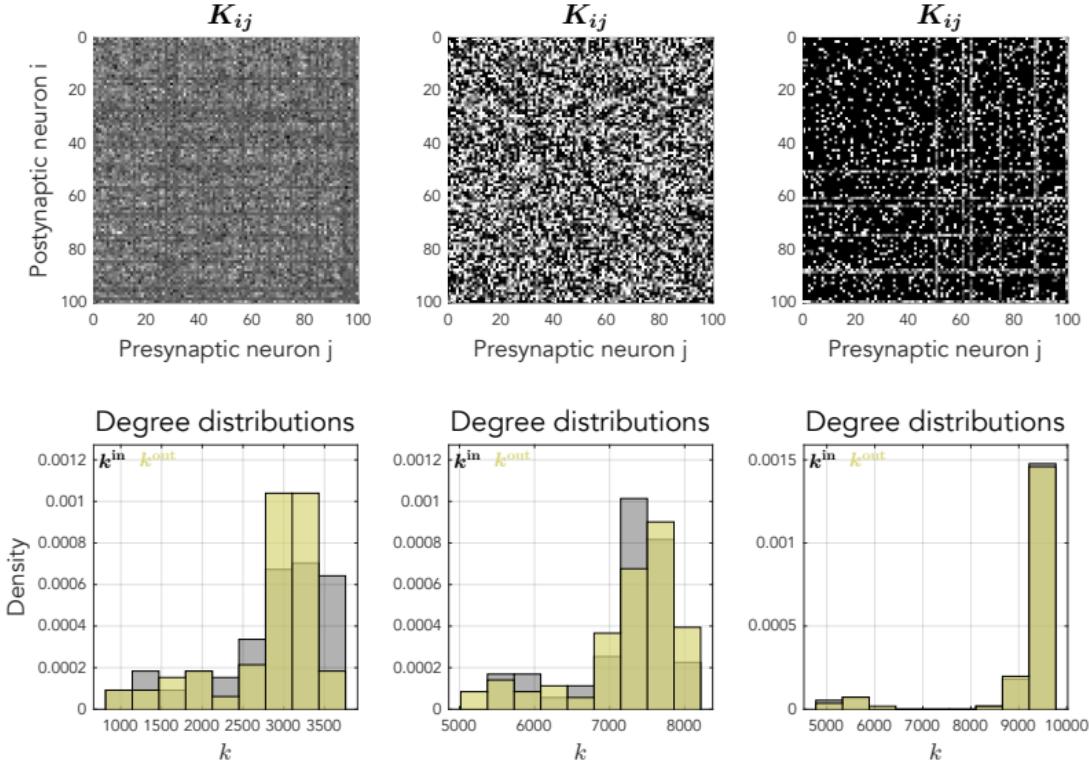
- *Fire and wire*: correlate successive action potentials

$$\Delta \text{Synaptic strength} \sim \sum_{t_j^f, t_i^n \in \mathcal{T}} W(t_j^f - t_i^n)$$

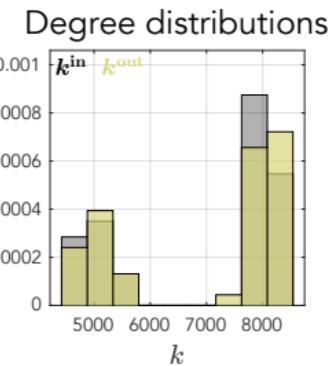
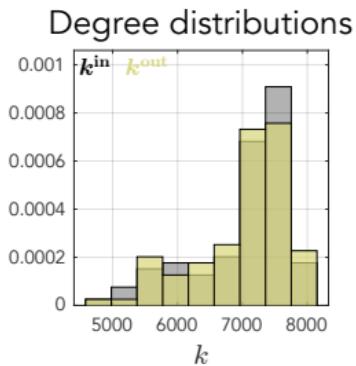
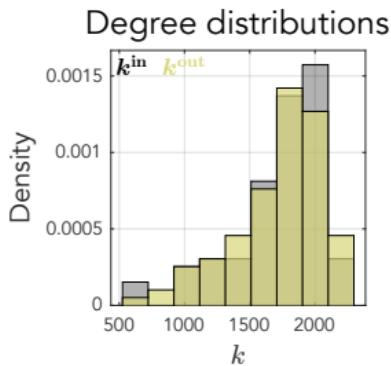
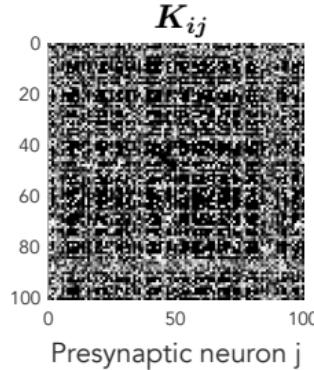
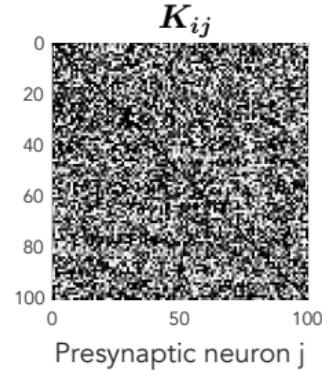
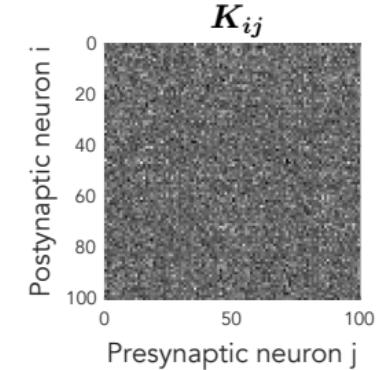
- *IP*: adjust neuron sensitivity to incoming action potentials



# Investigation: Emerging Network Topologies *STDP*



# Investigation: Emerging Network Topologies ***STDP + IP***



## Accomplishments

- Built and simulated directed networks
- Compared simulation and prediction at every timepoint
- Network structure emerges from learning strategy
- Unification of dynamics *on* and *of* networks

## Future Work

- Further investigation of initial and final conditions
- Find learning strategy with desirable properties
- Synchronisation or spiking rate?
- Computational challenges

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