Msc thesis

Mathematical Modelling and Computation

The dynamics of adaptive neuronal networks: influence of topology on synchronisation

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Supervisors

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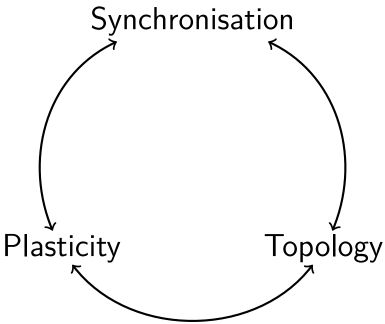
Holistic approach



Neuron dynamics

How do neurons communicate?

- Neurons receive neurotransmitters
- Action potential = explosion of electrical activity
- Synapse releases the neurons' neurotransmitter

How can we capture this behaviour?

- Human brain consists of $\sim$ 100 billion neurons
- The *MFR* yields the average dynamics of the network

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Synaptic plasticity

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- Hebbian learning: *Fire and wire*
- Correlation of neuronal activity
- Change response to incoming and outgoing signals
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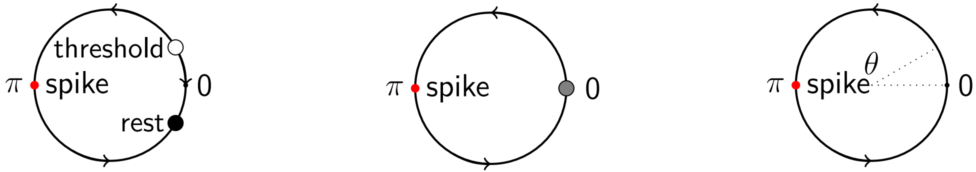
Model Description

- Formulation

$$\dot{\theta} = (1 - \cos\theta) + (1 + \cos\theta) \cdot I \qquad \theta \in \mathbb{T}$$

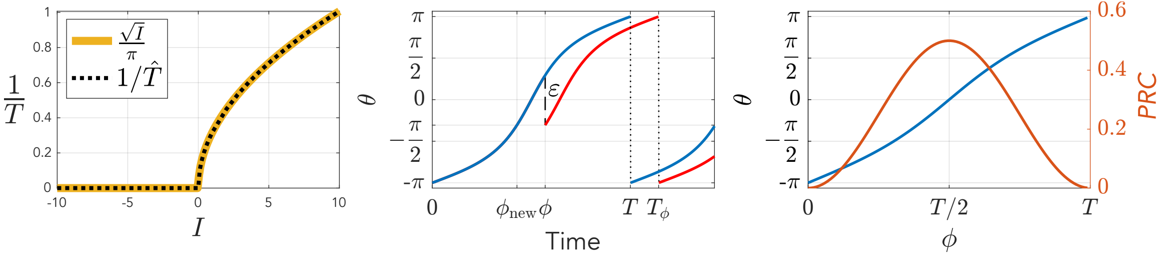
- Normal form of SNIC bifurcation



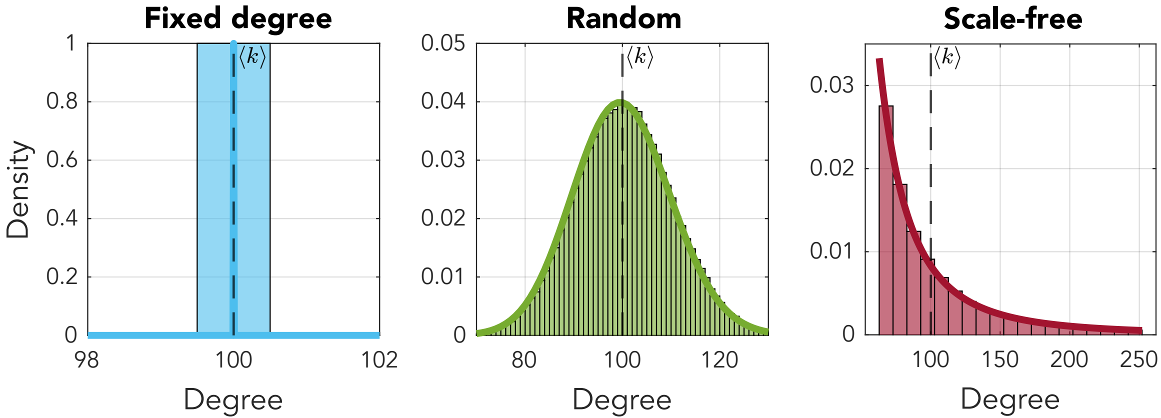
Excitable regime: $I < 0$ | Bifurcation: $I = 0$ | Periodic regime: $I > 0$

Response

- Formulate bifurcations in terms of spiking frequency or phase angle



Three basic networks



Networks of Theta neurons

- For an arbitrary network topology:

$$\dot{\theta}_i = (1 - \cos\theta_i) + (1 + \cos\theta_i) \cdot [\eta_i + I_i(t)] \qquad \theta_i \in \mathbb{T}^N$$

$$I_i(t) = \frac{\kappa}{\langle k \rangle} \sum_{j=1}^{N} A_{ij} \cdot \mathcal{P}_n(\theta_j)$$

- Capture average/mean synchronisation

$$Z(t) = \frac{1}{N} \sum_{j=1}^{N} e^{\mathrm{i}\theta_j} \qquad Z \in \mathbb{C}_\circ$$

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Predict synchronisation dynamics

The *MFR* yields a solution for $Z(t)$

- N neurons in the network
- $M_{\boldsymbol{k}} \ll N$ unique node degrees

Then rewrite $Z(t)$ per degree $z(\boldsymbol{k}, t)$!

- Only $M_{\boldsymbol{k}}$ equations
- Weighed by $P(\boldsymbol{k})$

$$\bar{Z}(t) = \frac{1}{N} \sum_{\boldsymbol{k}} P(\boldsymbol{k}) z(\boldsymbol{k}, t) \qquad \bar{Z} \in \mathbb{C}_{\circ}$$

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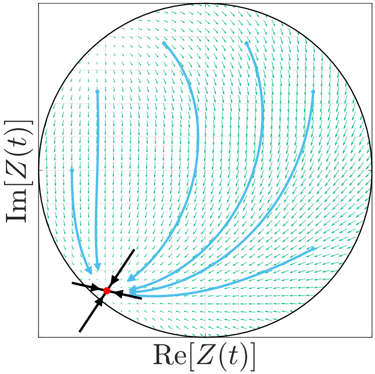
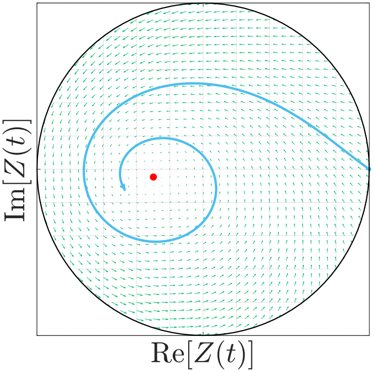
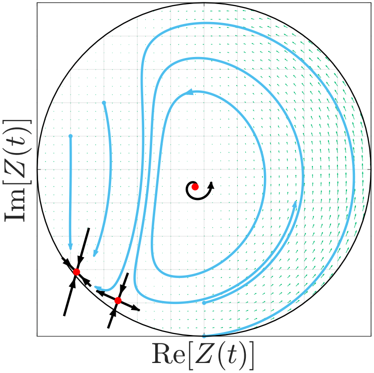
Fixed-degree networks



(a) Partially Synchronous Rest state



(b) Partially Synchronous Spiking state



(c) Collective Periodic wave state

Goals

$Z(t)$ can be measured and predicted: are they the same?

- Formulate directed networks
- Construct adjacency matrix from degree distribution
- Initial conditions
- Results

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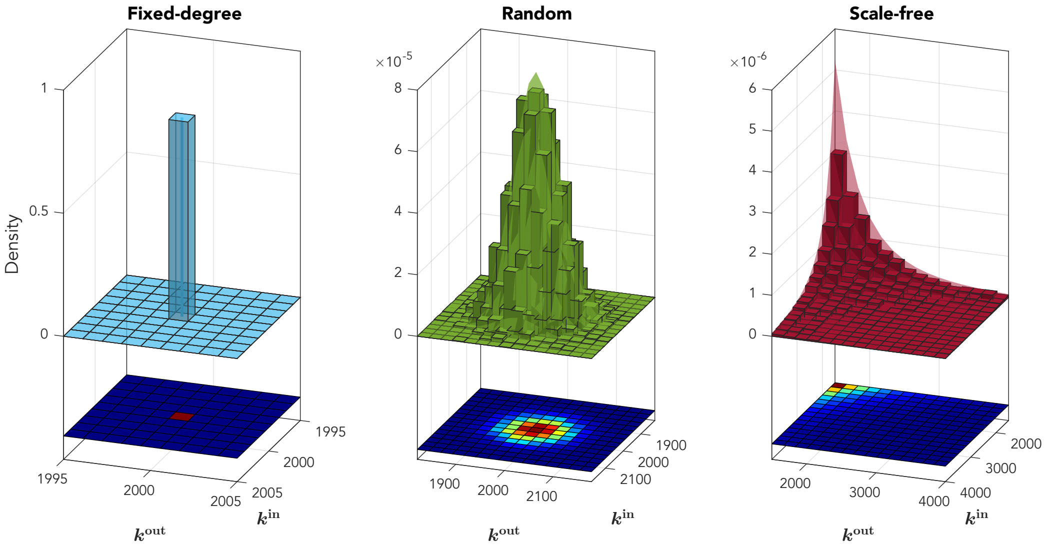
Directed networks

Use a bivariate degree distribution

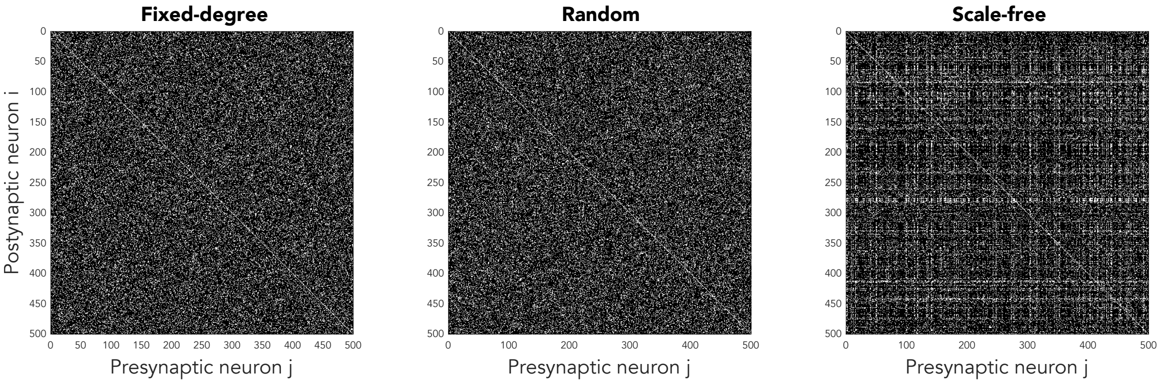
- Use identical and independant distributions
- In- and outdegrees are found as a permutation

Directed networks

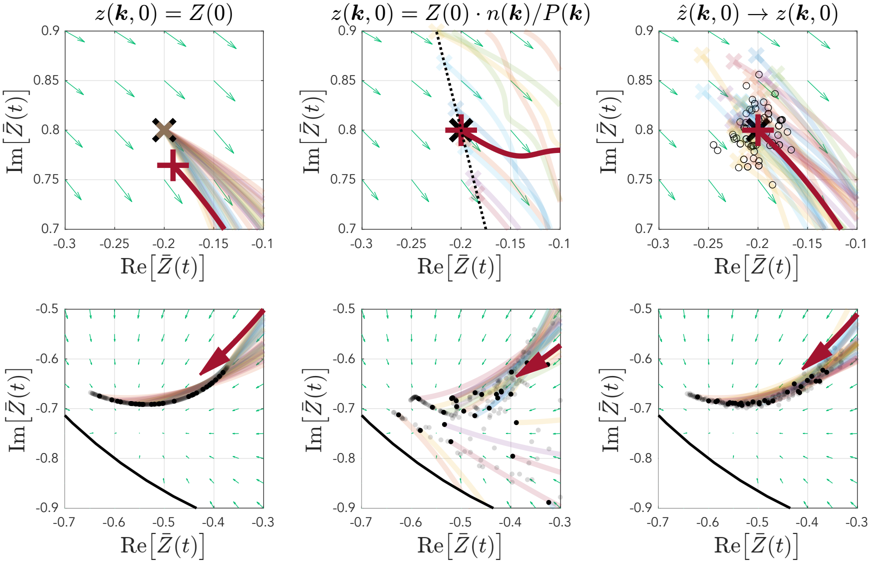


Adjacency matrix

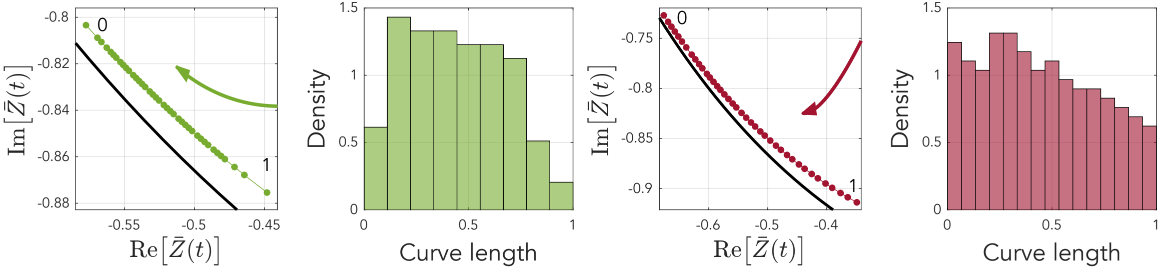
Find a probable solution by sampling from the in- and outdegrees



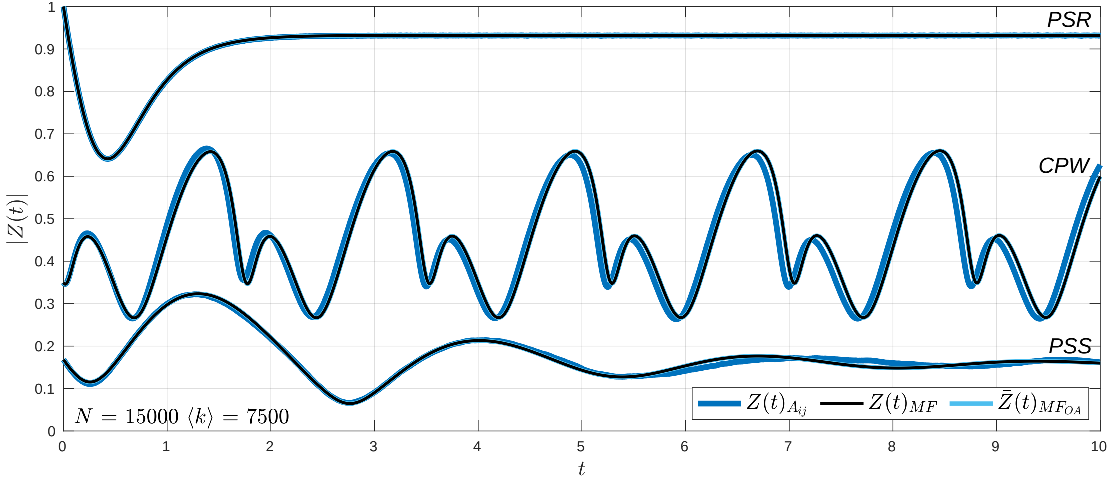
Initial conditions



Final conditions

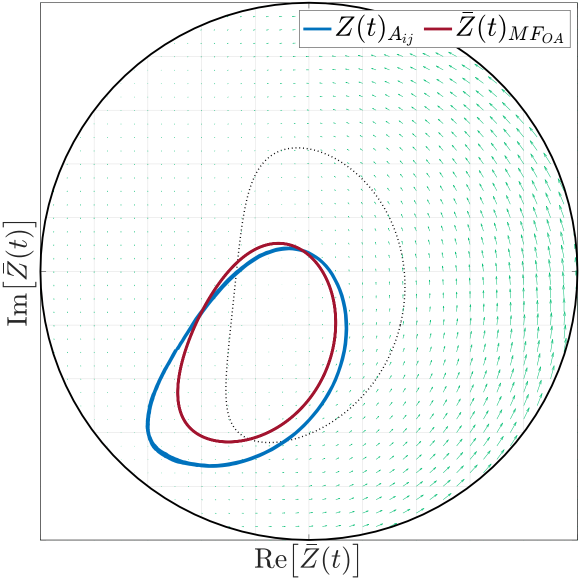


Results



PSR
CPW
PSS
$|Z(t)|$
t
$N = 15000$ $\langle k \rangle = 7500$
$Z(t)_{A_{ij}}$
$Z(t)_{MF}$
$\bar{Z}(t)_{MF_{OA}}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Results



Hebbian Learning and Synaptic Plasticity

Investigation: Emerging Network Topologies

Conclusion and Discussion