Msc thesis

Mathematical Modelling and Computation

The dynamics of adaptive neuronal networks: influence of topology on synchronisation

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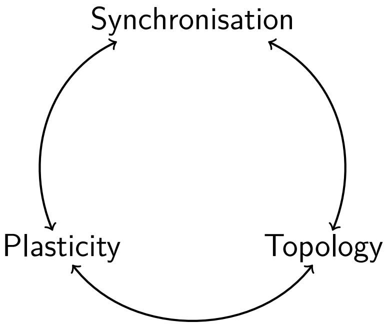
Holistic approach



Neuron dynamics

How do neurons communicate?

- Neurotransmitters
- Action potential $=$ explosion of electrical activity

How do neurons learn?

- Human brain $\sim$ 100 billion neurons
- *MFR*: average dynamics of the network

How can we capture this behaviour?

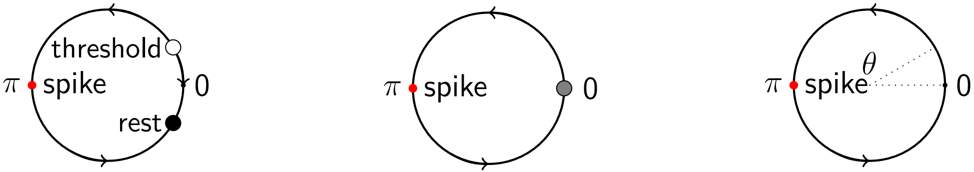
- *Fire and wire*: Correlation of neuronal activity

Model Description

- Formulation

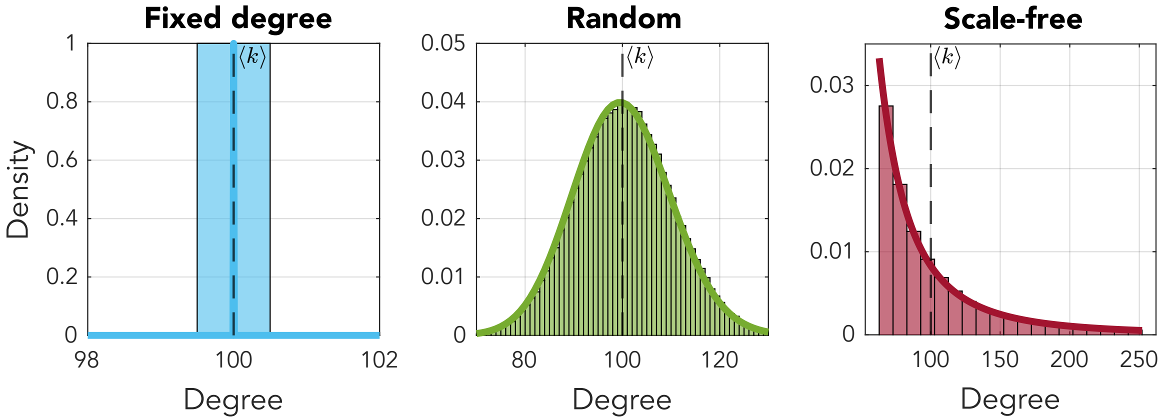
$$\dot{\theta} = (1 - \cos\theta) + (1 + \cos\theta) \cdot I \qquad \theta \in \mathbb{T}$$

- Normal form of SNIC bifurcation



Excitable regime: $I < 0$ Bifurcation: $I = 0$ Periodic regime: $I > 0$

Three basic networks



Networks of Theta neurons

- For arbitrary network topology:

$$\dot{\theta}_i = (1 - \cos\theta_i) + (1 + \cos\theta_i) \cdot [\eta_i + I_i(t)] \qquad \theta_i \in \mathbb{T}^N$$

$$I_i(t) = \frac{\kappa}{\langle k \rangle} \sum_{j=1}^{N} A_{ij} \cdot \mathcal{P}_n(\theta_j)$$

- Capture average/mean synchronisation

$$Z(t) = \frac{1}{N} \sum_{j=1}^{N} e^{\mathrm{i}\theta_j} \qquad Z \in \mathbb{C}_\circ$$

Predict synchronisation dynamics

The *MFR* = solution for $Z(t)$

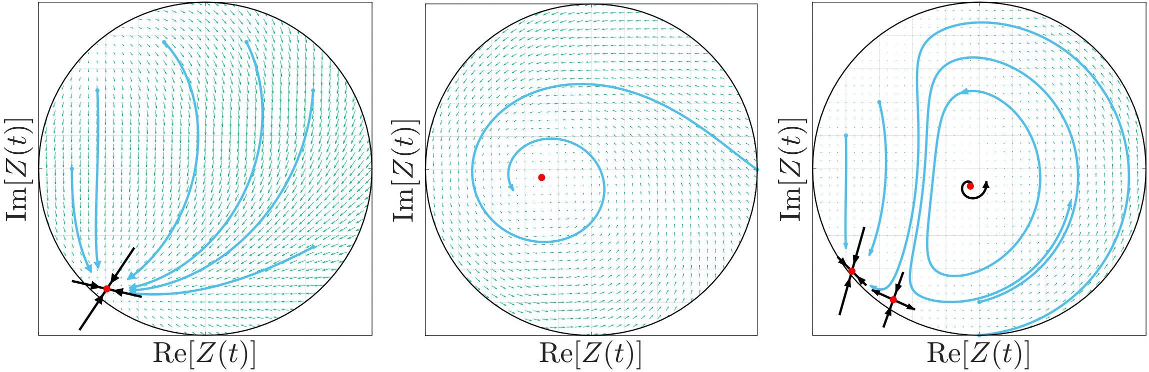
Reduction? Formulate $Z(t)$ per degree $z(\boldsymbol{k}, t)$!

- $M_{\boldsymbol{k}} \ll N$ unique node degrees
- Only $M_{\boldsymbol{k}}$ equations left
- Weighed by $P(\boldsymbol{k})$

$$\bar{Z}(t) = \frac{1}{N} \sum_{\boldsymbol{k}} P(\boldsymbol{k}) z(\boldsymbol{k}, t) \qquad \bar{Z} \in \mathbb{C}_{\circ}$$

Problem: $M_{\boldsymbol{k}}$ still too large when $P(\boldsymbol{k})$ is bivariate

Fixed-degree networks



Goals

$Z(t)$ can be measured and predicted: are they the same?

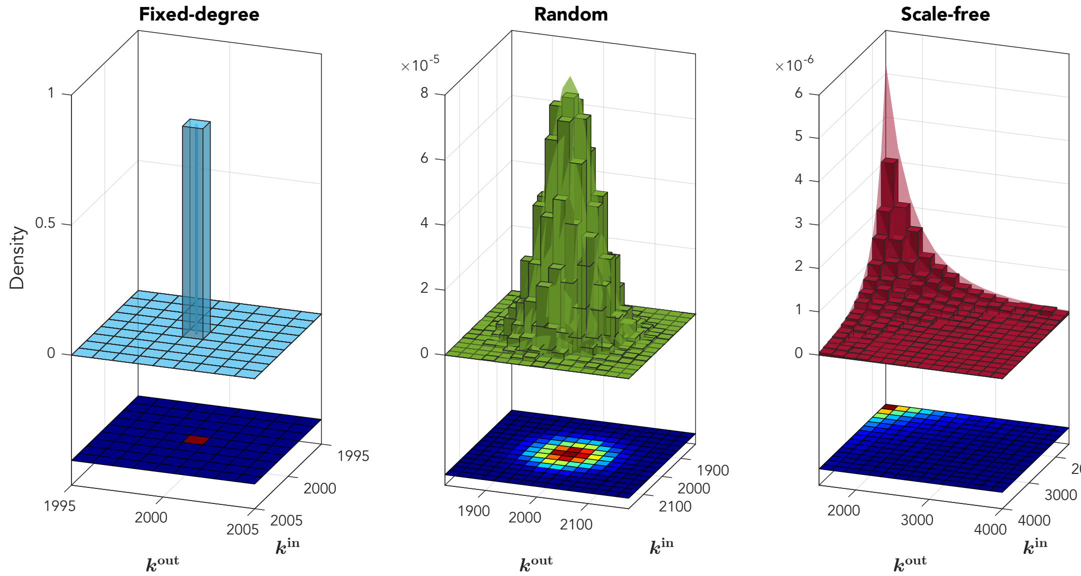
- Formulate directed networks
- Construct adjacency matrix from degree distribution
- Initial and final conditions
- Compare directed and undirected networks

Directed networks

Sample degree vectors from bivariate distribution? Difficult!

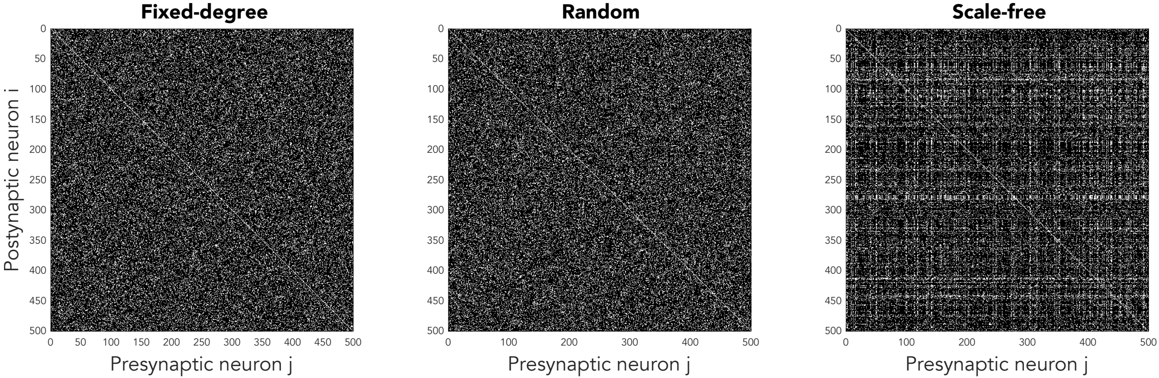
- Use independant univariate distributions as marginal
- In- and out degree vectors are found as a permutation

Directed networks

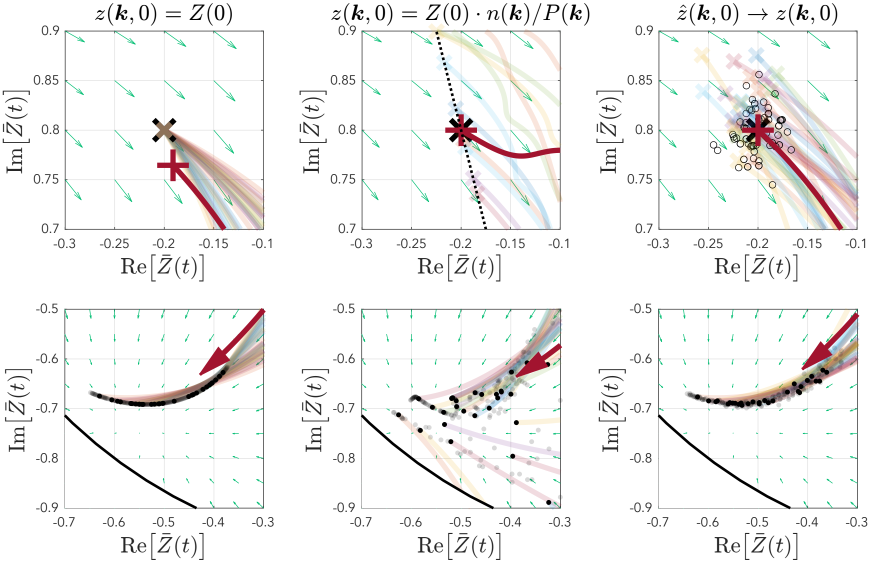


Adjacency matrix

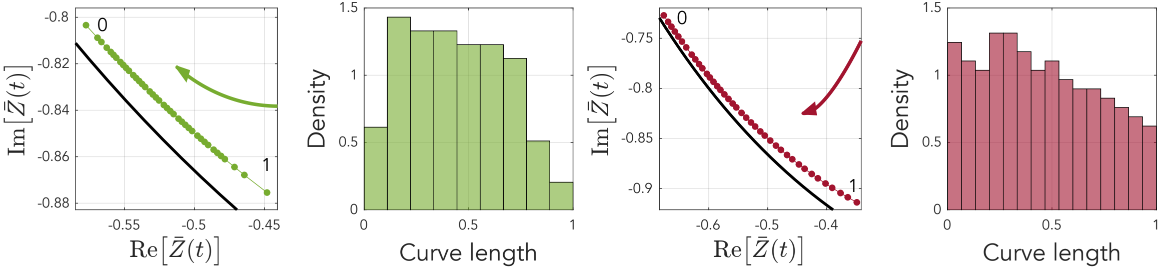
Find a probable solution by sampling from the in- and outdegrees



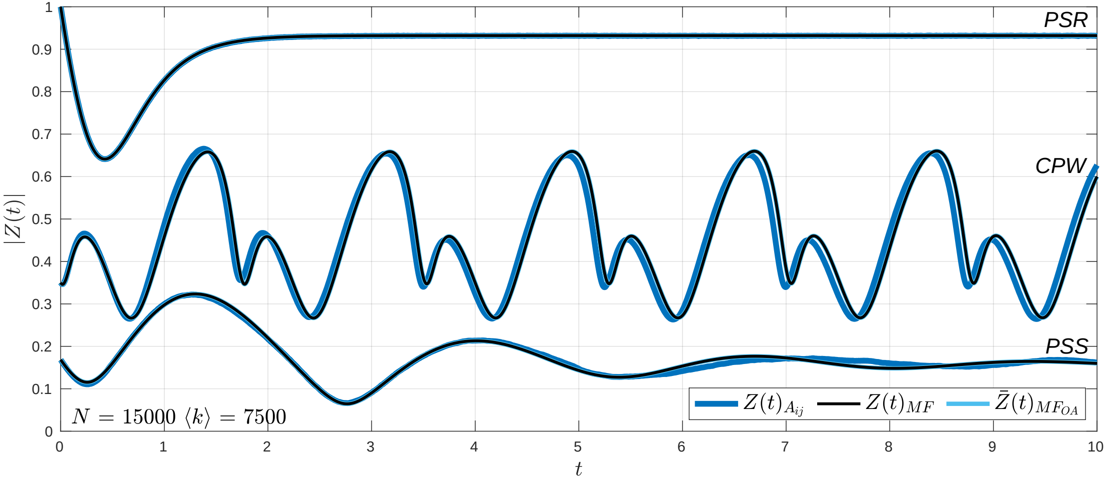
Initial conditions



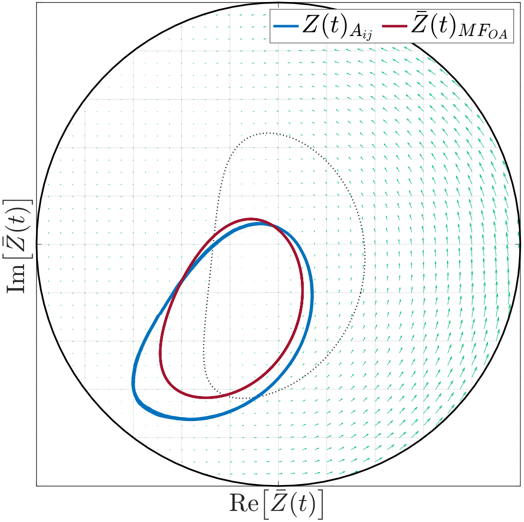
Final conditions



Results


PSR
CPW
PSS
$N = 15000$ $\langle k \rangle = 7500$
$Z(t)_{A_{ij}}$
$Z(t)_{MF}$
$\bar{Z}(t)_{MF_{OA}}$
$|Z(t)|$
t
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10

Results

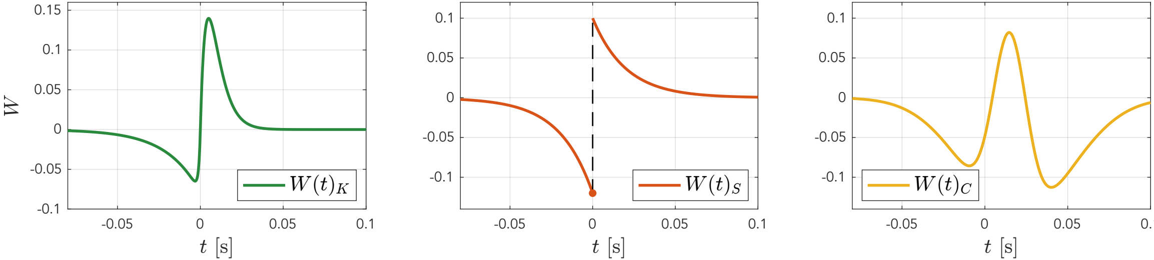


Temporal Interpretation: *STDP*

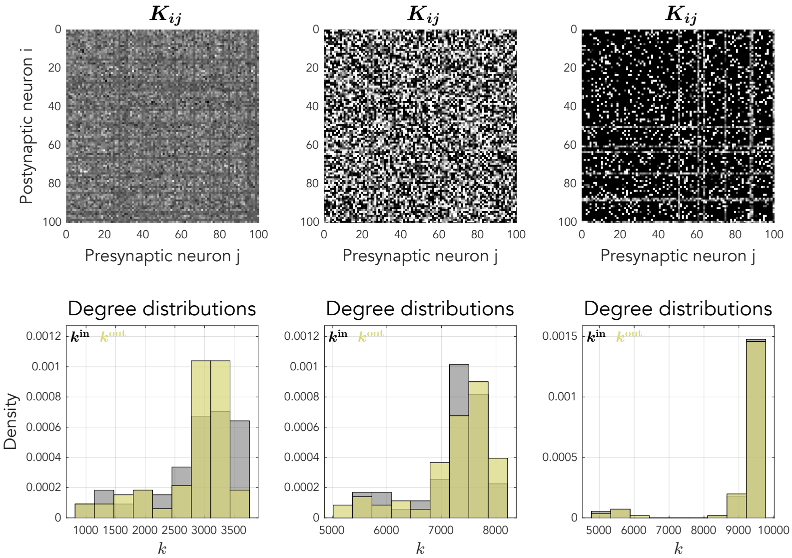
- *Fire and wire*: correlate successive action potentials

$$\Delta\text{Synaptic strength} \sim \sum_{t_j^f, t_i^n \in \mathcal{T}} W\left(t_j^f - t_i^n\right)$$

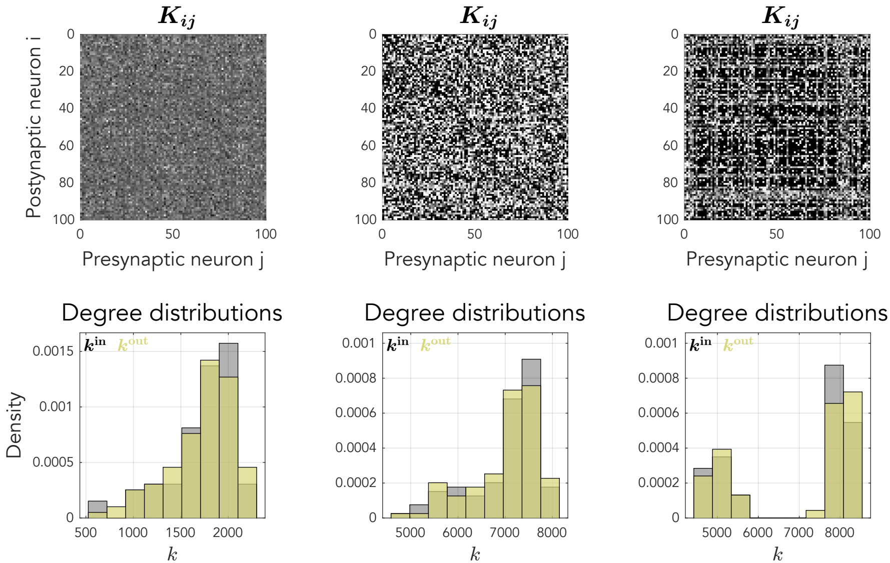
- *IP*: adjust neuron sensitivity to incoming action potentials



STDP


K_{ij}
K_{ij}
K_{ij}
Postsynaptic neuron i
Presynaptic neuron j
Presynaptic neuron j
Presynaptic neuron j
Degree distributions
Degree distributions
Degree distributions
k^{in} k^{out}
Density
k
k
k

STDP + IP


K_{ij}
K_{ij}
K_{ij}
Postynaptic neuron i
0
20
40
60
80
100
0
50
100
Presynaptic neuron j
Presynaptic neuron j
Presynaptic neuron j
Degree distributions
Degree distributions
Degree distributions
k^{in} k^{out}
Density
0.0015
0.001
0.0005
0
500
1000
1500
2000
0.0008
0.0006
0.0004
0.0002
5000
6000
7000
8000
k

Conclusion and Discussion