Msc thesis

Mathematical Modelling and Computation

The dynamics of adaptive neuronal networks: influence of topology on synchronisation

Simon Aertssen, s181603

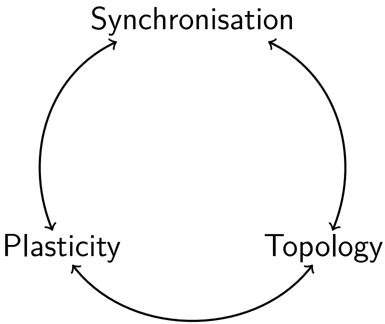
Supervisors

Erik Martens and Poul Hjorth

February 11th 2021

DTU Compute
Department of Applied Mathematics and Computer Science

Holistic approach



Neuron dynamics

How do neurons communicate?

- Neurotransmitters
- Action potential $=$ explosion of electrical activity

How do neurons learn?

- Human brain $\sim$ 100 billion neurons
- *MFR*: average dynamics of the network

How can we capture this behaviour?

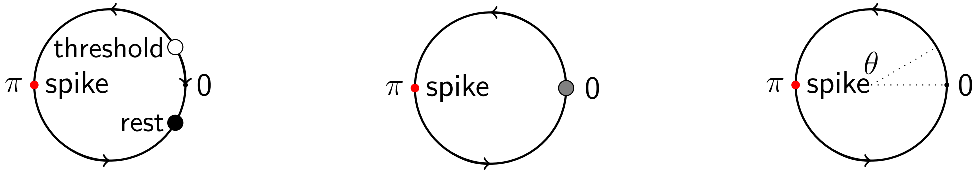
- *Fire and wire*: Correlation of neuronal activity

Model Description

- Formulation

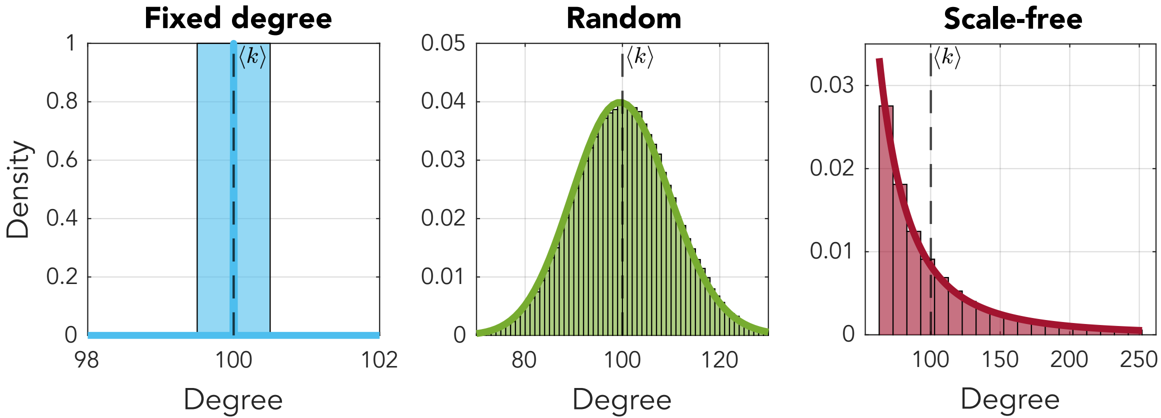
$$\dot{\theta} = (1 - \cos\theta) + (1 + \cos\theta) \cdot I \qquad \theta \in \mathbb{T}$$

- Normal form of SNIC bifurcation



Excitable regime: $I < 0$ | Bifurcation: $I = 0$ | Periodic regime: $I > 0$

Three basic networks



Networks of Theta neurons

- For arbitrary network topology:

$$\dot{\theta}_i = (1 - \cos\theta_i) + (1 + \cos\theta_i) \cdot [\eta_i + I_i(t)] \qquad \theta_i \in \mathbb{T}^N$$

$$I_i(t) = \frac{\kappa}{\langle k \rangle} \sum_{j=1}^{N} A_{ij} \cdot \mathcal{P}_n(\theta_j)$$

- Capture average/mean synchronisation

$$Z(t) = \frac{1}{N} \sum_{j=1}^{N} e^{\mathrm{i}\theta_j} \qquad Z \in \mathbb{C}_\circ$$

Predict synchronisation dynamics

The *MFR* = solution for $Z(t)$

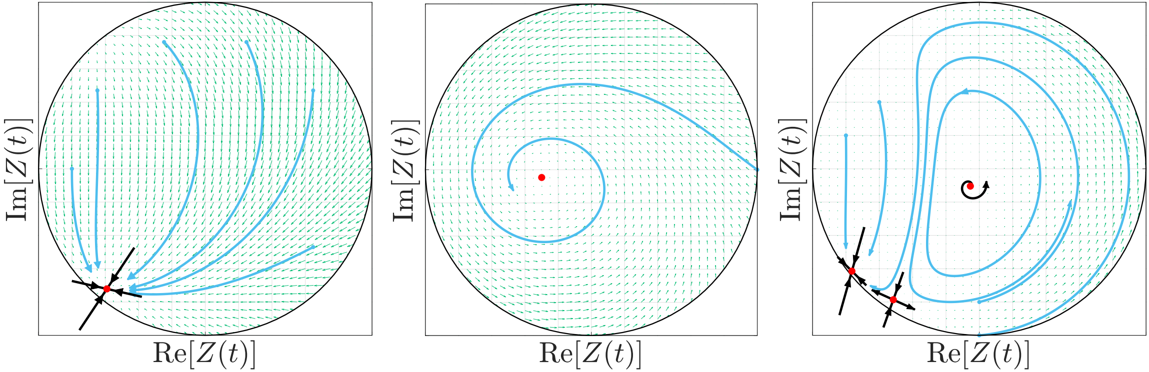
Reduction? Formulate $Z(t)$ per degree $z(\boldsymbol{k}, t)$!

- $M_{\boldsymbol{k}} \ll N$ unique node degrees
- Only $M_{\boldsymbol{k}}$ equations left
- Weighed by $P(\boldsymbol{k})$

$$\bar{Z}(t) = \frac{1}{N} \sum_{\boldsymbol{k}} P(\boldsymbol{k}) z(\boldsymbol{k}, t) \qquad \bar{Z} \in \mathbb{C}_{\circ}$$

Problem: $M_{\boldsymbol{k}}$ still too large when $P(\boldsymbol{k})$ is bivariate

Fixed-degree networks



Goals

$Z(t)$ can be measured and predicted: are they the same?

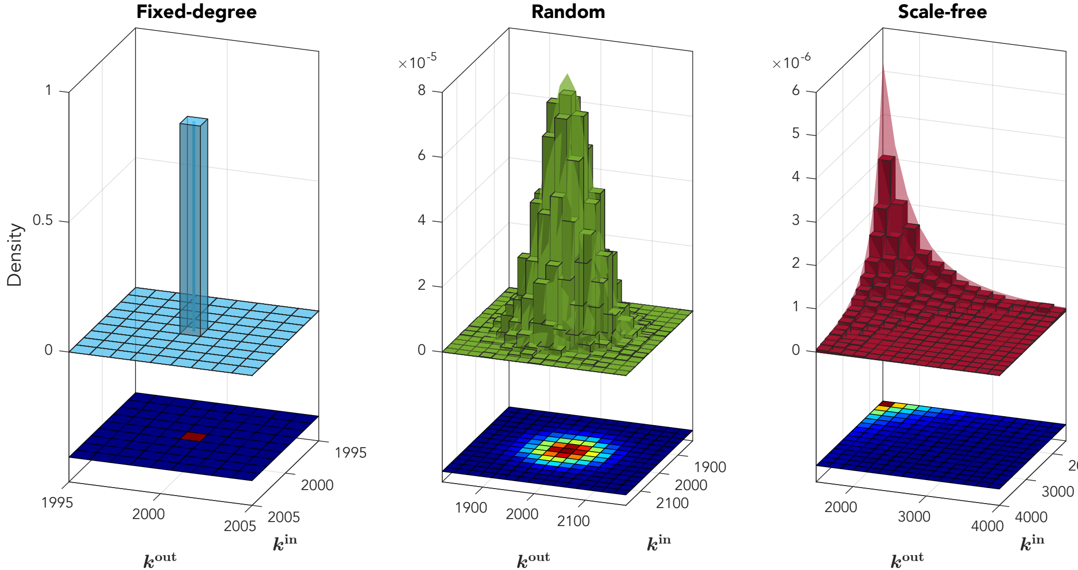
- Formulate directed networks
- Construct adjacency matrix from degree distribution
- Initial and final conditions
- Compare directed and undirected networks

Directed networks

Sample degree vectors from bivariate distribution? Difficult!

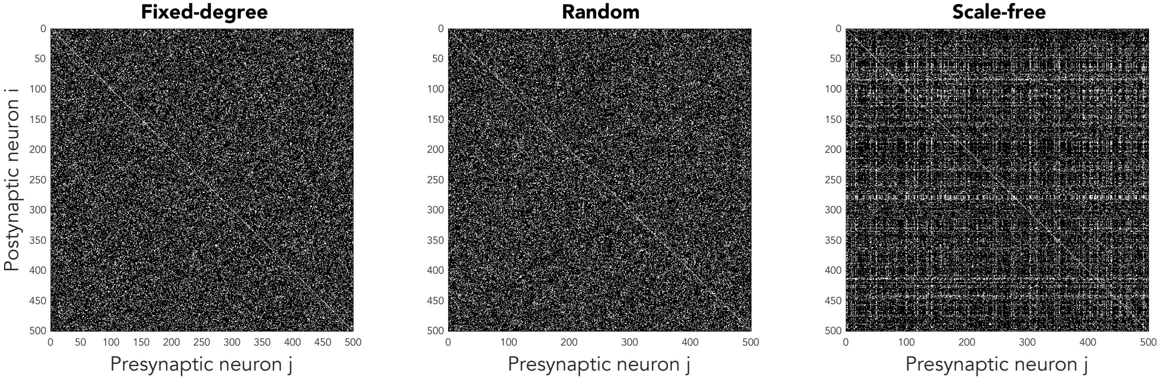
- Use independant univariate distributions as marginal
- In- and out degree vectors are found as a permutation

Directed networks

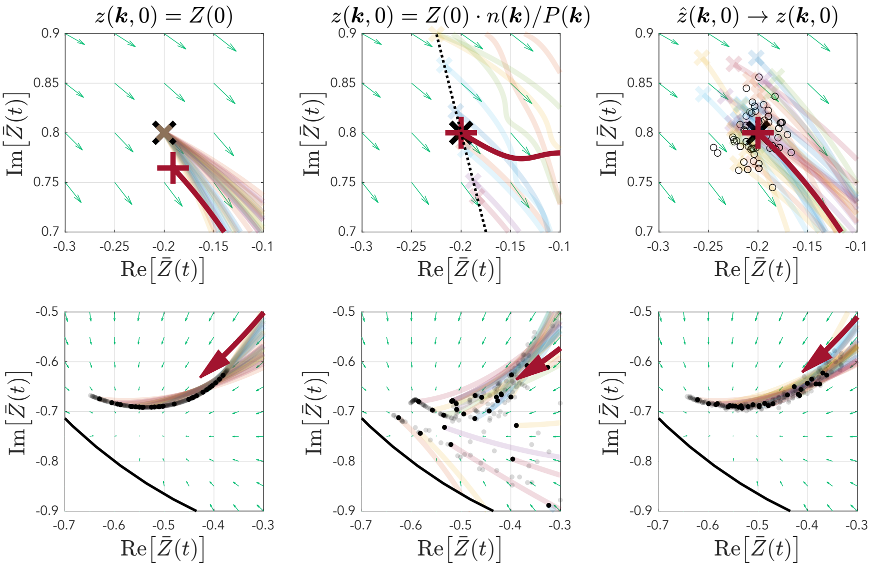


Adjacency matrix

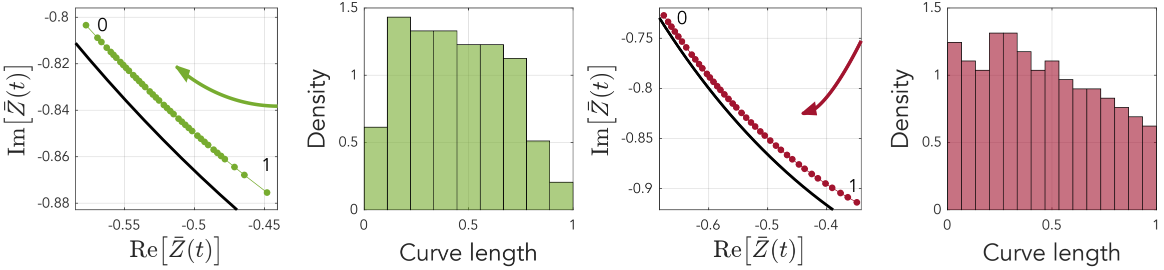
Find a probable solution by sampling from the in- and outdegrees



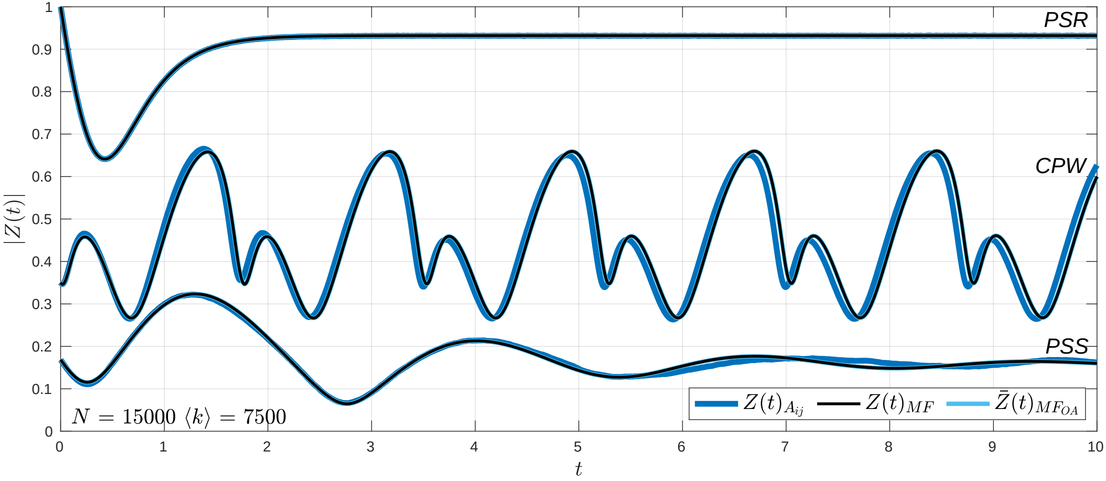
Initial conditions



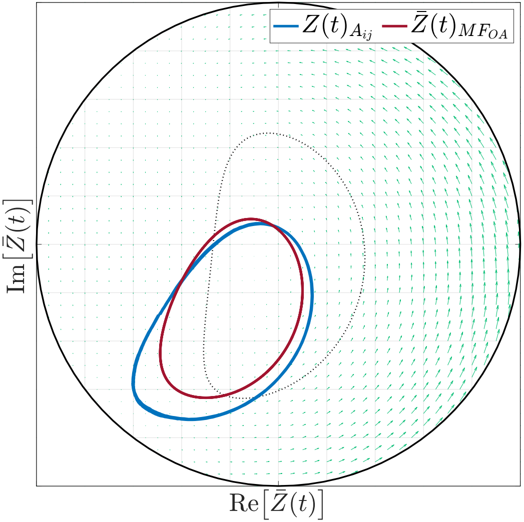
Final conditions



Results


PSR
CPW
PSS
$|Z(t)|$
t
$N = 15000 \ \langle k \rangle = 7500$
$Z(t)_{A_{ij}}$
$Z(t)_{MF}$
$\bar{Z}(t)_{MF_{OA}}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Results

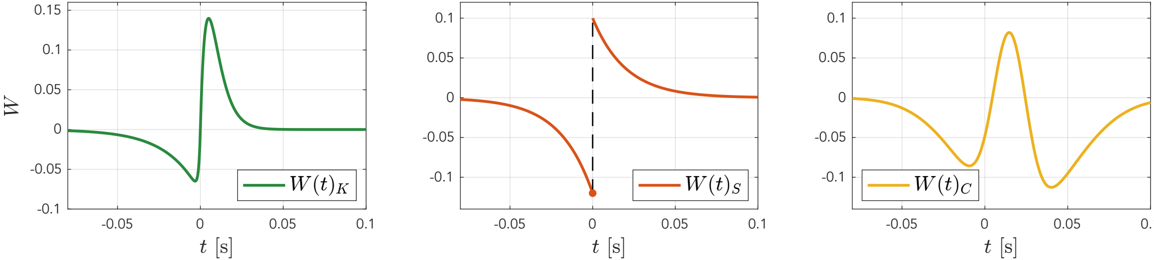

$Z(t)_{A_{ij}}$
$\bar{Z}(t)_{MF_{OA}}$
$\mathrm{Im}\left[\bar{Z}(t)\right]$
$\mathrm{Re}\left[\bar{Z}(t)\right]$

Temporal Interpretation: *STDP*

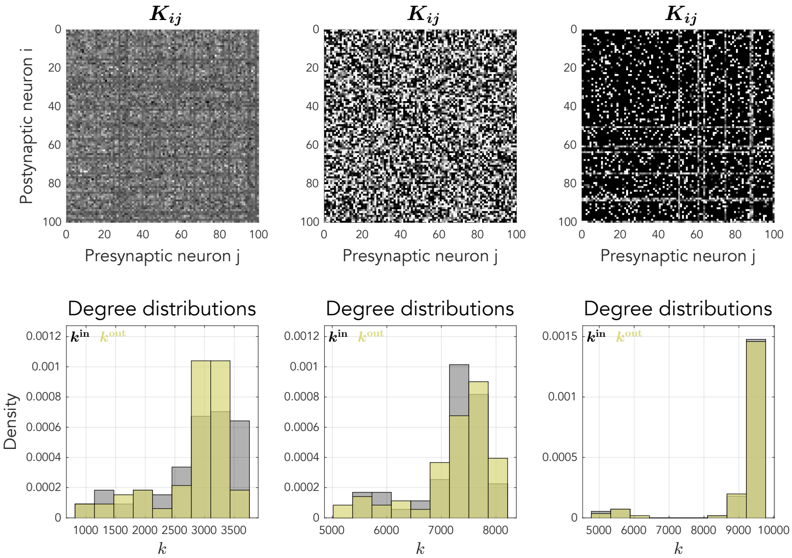
- *Fire and wire*: correlate successive action potentials

$$\Delta\text{Synaptic strength} \sim \sum_{t_j^f, t_i^n \in \mathcal{T}} W\left(t_j^f - t_i^n\right)$$

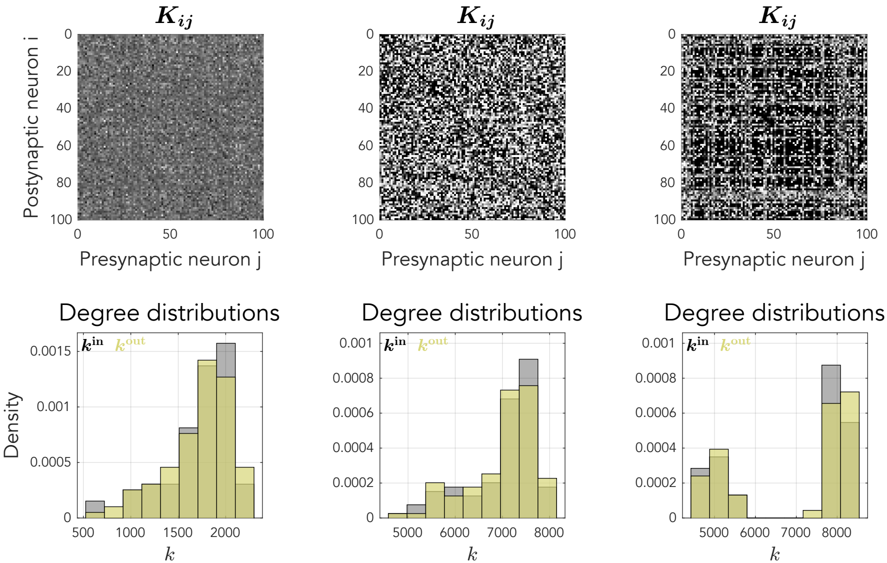
- *IP*: adjust neuron sensitivity to incoming action potentials



Investigation: Emerging Network Topologies
STDP



STDP + IP



K_{ij}
Postynaptic neuron i
Presynaptic neuron j
0
20
40
60
80
100
0
50
100
Degree distributions
Density
k^{in}
k^{out}
0.0015
0.001
0.0005
0
500
1000
1500
2000
0.0008
0.0006
0.0004
0.0002
5000
6000
7000
8000
k

Accomplishments

- Built and simulated directed networks
- Compared simulation and prediction at every timepoint
- Network structure emerges from learning strategy
- Unification of dynamics *on* and *of* networks

Future Work

- Further investigation of initial and final conditions
- Find learning strategy with desirable properties
- Understand symmetry of the learned degree distributions
- Synchronisation or spiking rate?
- Computational challenges