

Msc thesis
Mathematical Modelling and Computation

The dynamics of adaptive neuronal networks: influence of topology on synchronisation

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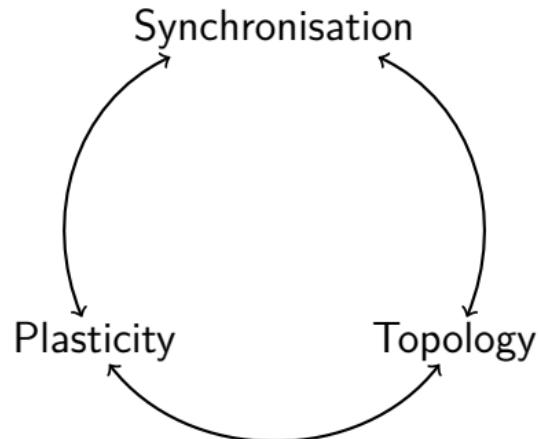
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Department of Applied Mathematics and Computer Science



Contents

Holistic approach



Introduction

Neuron dynamics

How do neurons communicate?

- Neurotransmitters
- Action potential = explosion of electrical activity

How do neurons learn?

- Human brain \sim 100 billion neurons
- *MFR*: average dynamics of the network

How can we capture this behaviour?

- *Fire and wire*: Correlation of neuronal activity

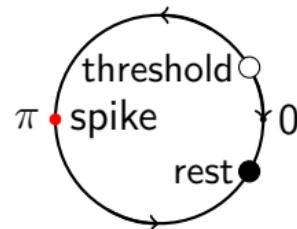
The Theta Neuron Model

Model Description

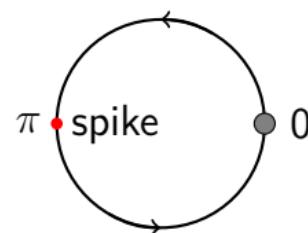
- Formulation

$$\dot{\theta} = (1 - \cos \theta) + (1 + \cos \theta) \cdot I \quad \theta \in \mathbb{T}$$

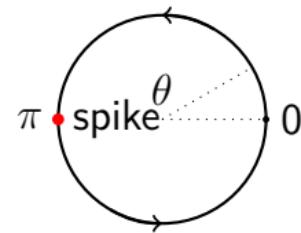
- Normal form of SNIC bifurcation



Excitable regime: $I < 0$



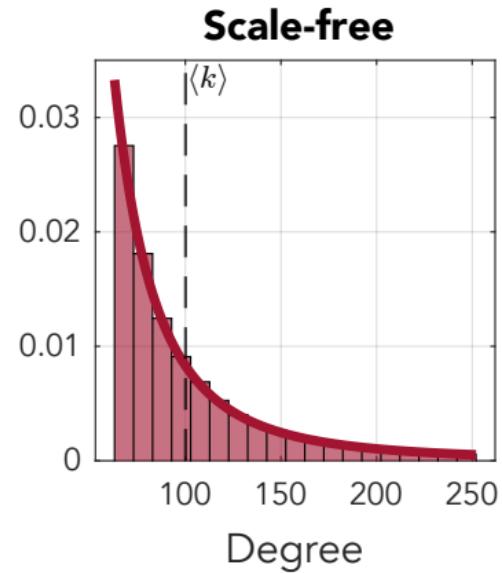
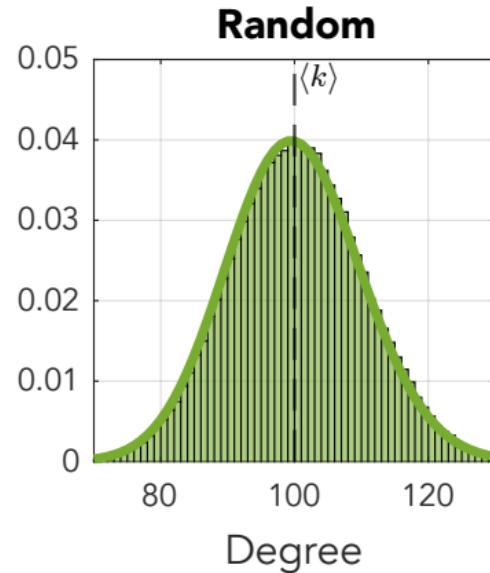
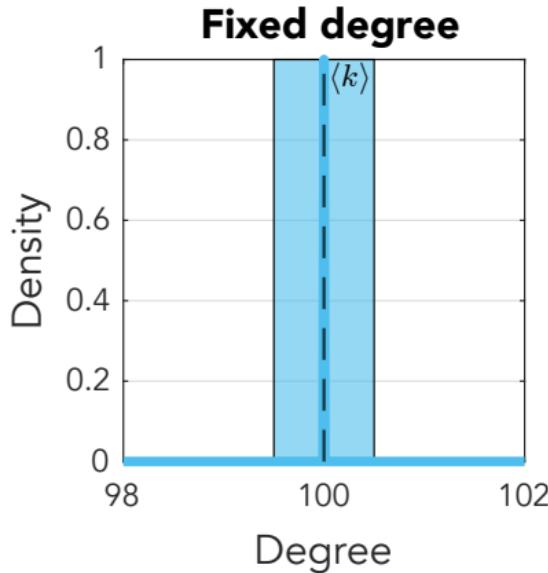
Bifurcation: $I = 0$



Periodic regime: $I > 0$

Network Topologies

Three basic networks



Networks of Theta neurons

- For arbitrary network topology:

$$\dot{\theta}_i = (1 - \cos \theta_i) + (1 + \cos \theta_i) \cdot [\eta_i + I_i(t)] \quad \theta_i \in \mathbb{T}^N$$

$$I_i(t) = \frac{\kappa}{\langle k \rangle} \sum_{j=1}^N A_{ij} \cdot \mathcal{P}_n(\theta_j)$$

- Capture average/mean synchronisation

$$Z(t) = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} \quad Z \in \mathbb{C}_o$$

Predict synchronisation dynamics

The $MFR =$ solution for $Z(t)$

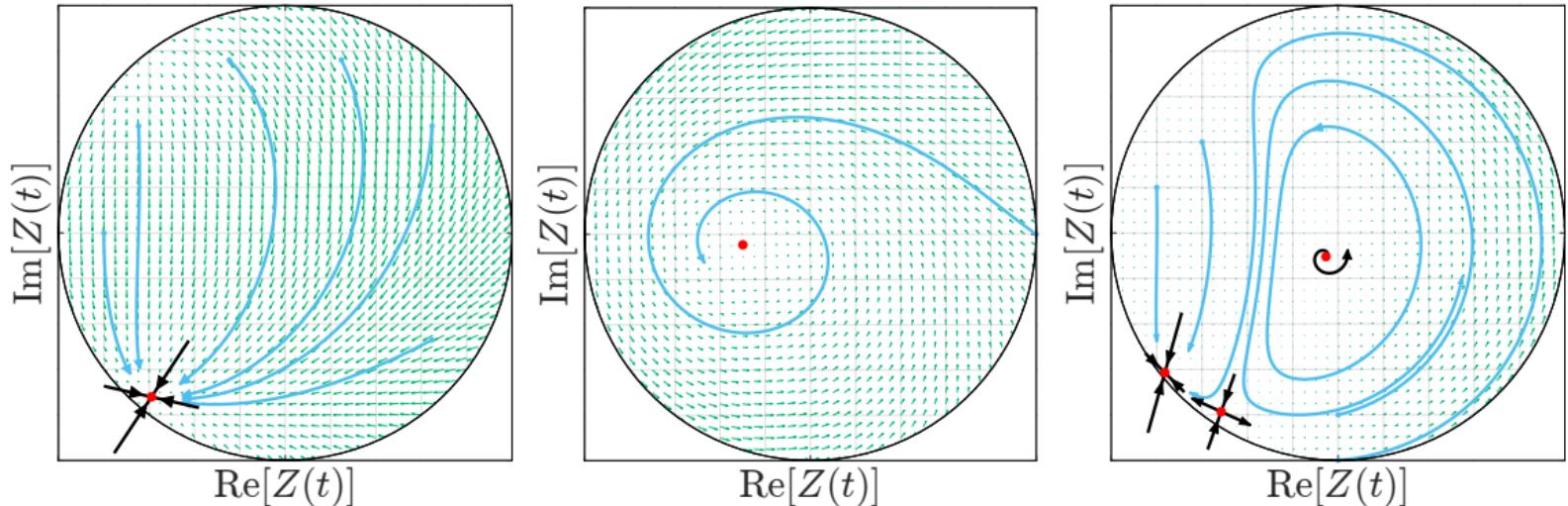
Reduction? Formulate $Z(t)$ per degree $z(\mathbf{k}, t)!$

- $M_k \ll N$ unique node degrees
- Only M_k equations left
- Weighed by $P(\mathbf{k})$

$$\bar{Z}(t) = \frac{1}{N} \sum_{\mathbf{k}} P(\mathbf{k}) z(\mathbf{k}, t) \quad \bar{Z} \in \mathbb{C}_o$$

Problem: M_k still too large when $P(\mathbf{k})$ is bivariate

Mean Field Reduction Fixed-degree networks



Investigation: Mean Field Reductions for undirected graphs

Goals

$Z(t)$ can be measured and predicted: are they the same?

- Formulate directed networks
- Construct adjacency matrix from degree distribution
- Initial and final conditions
- Compare directed and undirected networks

Investigation: Mean Field Reductions for undirected graphs

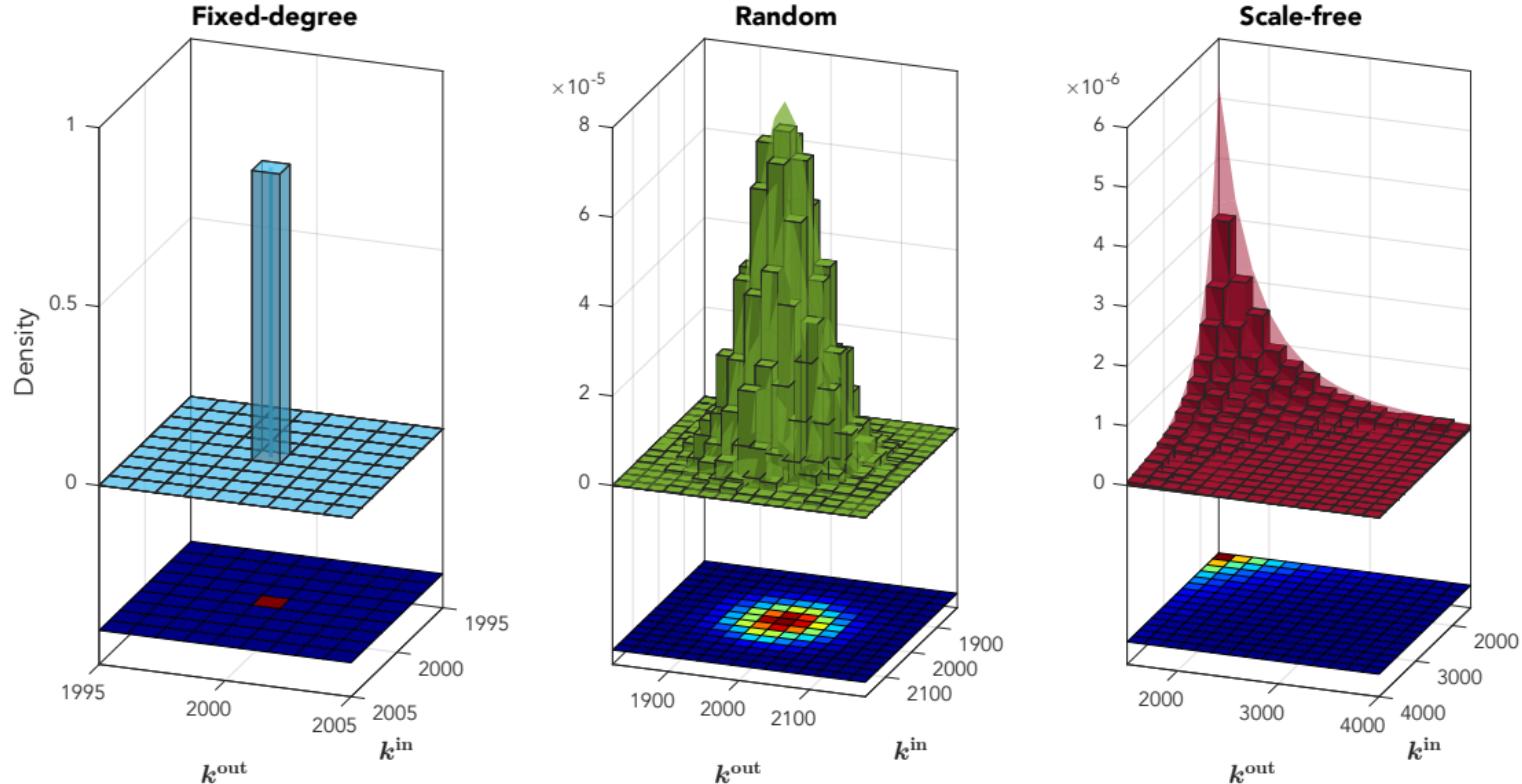
Directed networks

Sample degree vectors from bivariate distribution? Difficult!

- Use independant univariate distributions as marginal
- In- and out degree vectors are found as a permutation

Investigation: Mean Field Reductions for undirected graphs

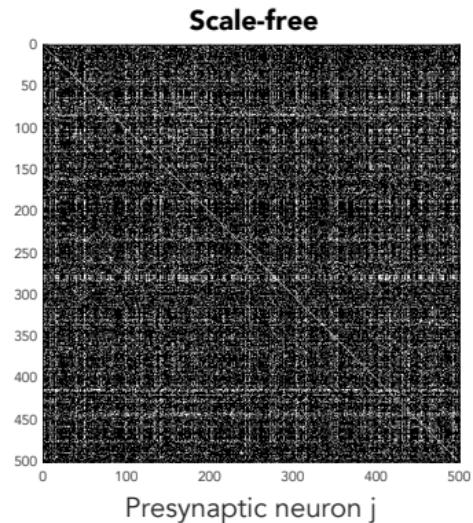
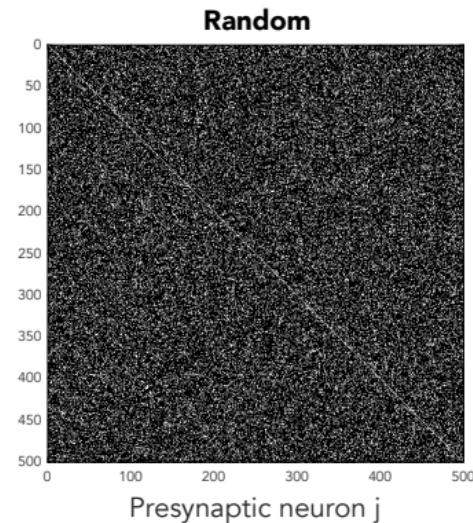
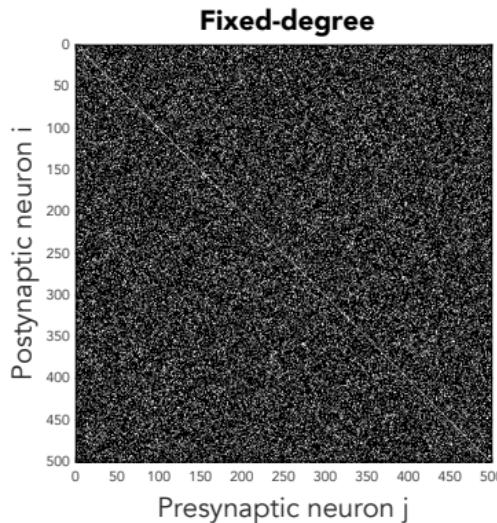
Directed networks



Investigation: Mean Field Reductions for undirected graphs

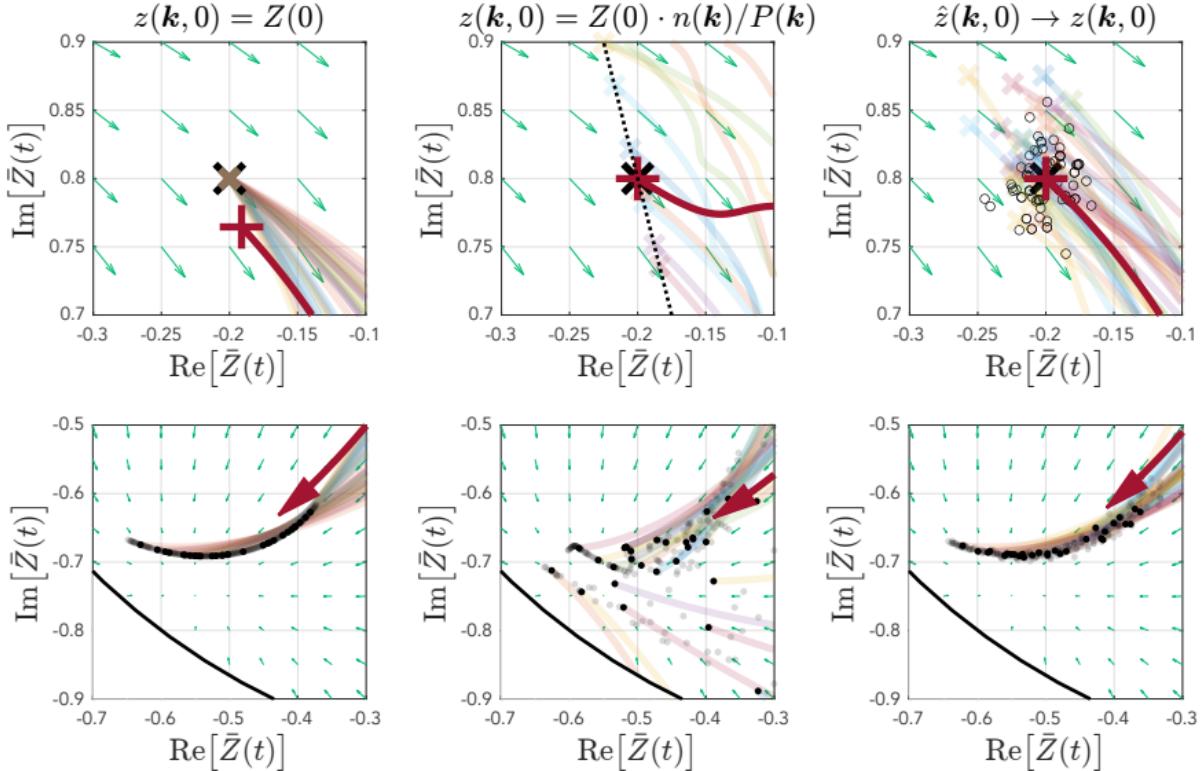
Adjacency matrix

Find a probable solution by sampling from the in- and outdegrees



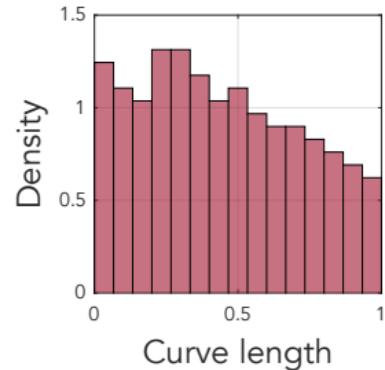
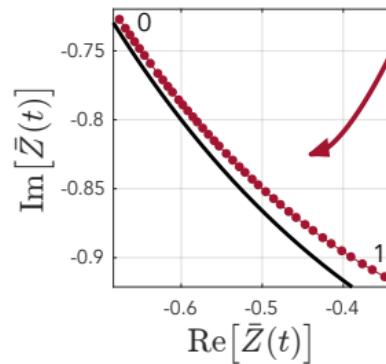
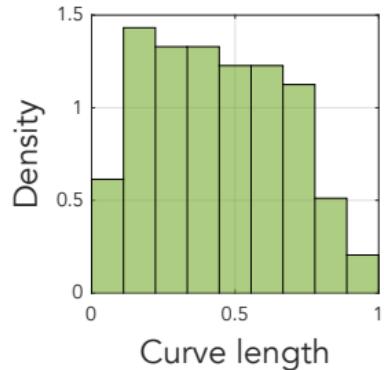
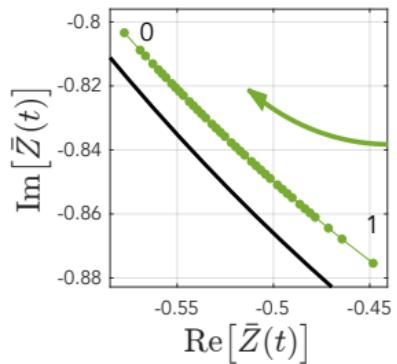
Investigation: Mean Field Reductions for undirected graphs

Initial conditions



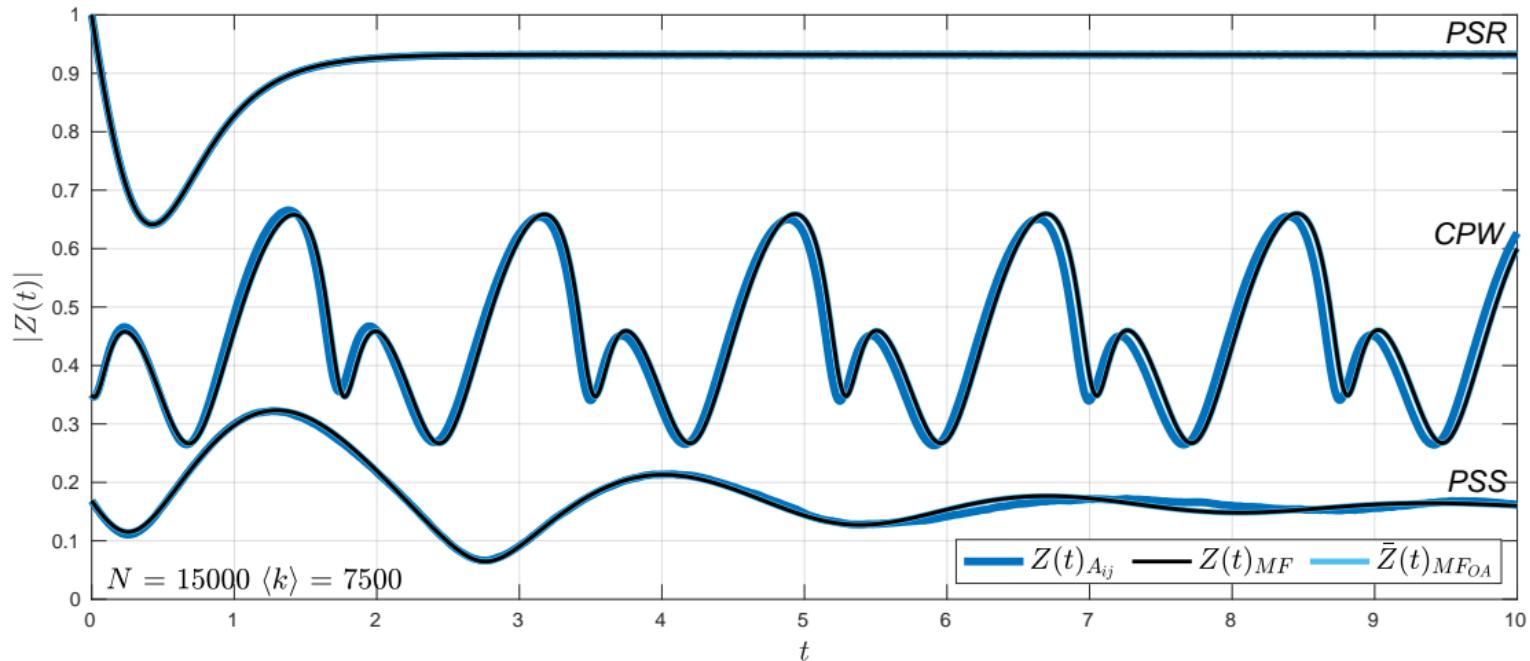
Investigation: Mean Field Reductions for undirected graphs

Final conditions



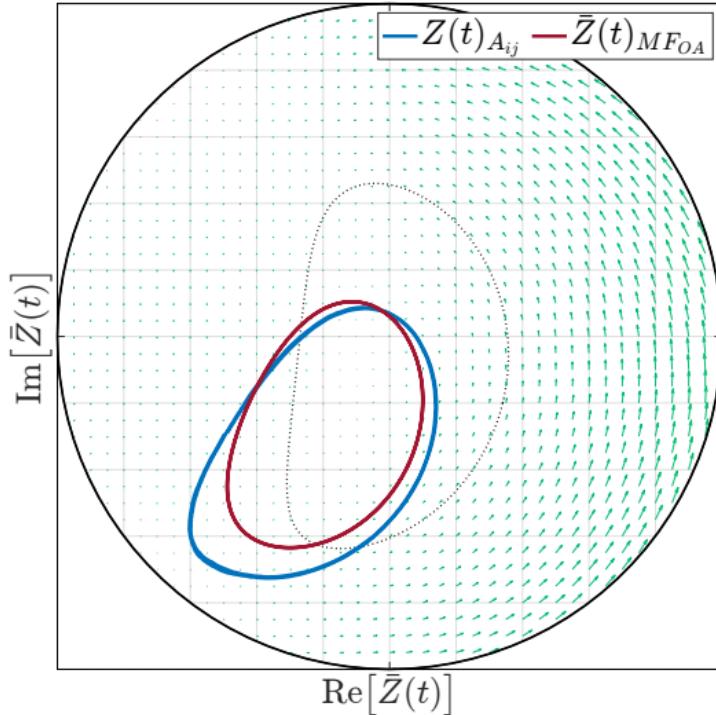
Investigation: Mean Field Reductions for undirected graphs

Results



Investigation: Mean Field Reductions for undirected graphs

Results



Hebbian Learning and Synaptic Plasticity

Hebbian Learning and Synaptic Plasticity

Investigation: Emerging Network Topologies

Investigation: Emerging Network Topologies

Conclusion and Discussion