

Msc thesis  
Mathematical Modelling and Computation

# The dynamics of adaptive neuronal networks: influence of topology on synchronisation

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## *Supervisors*

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**DTU Compute**

Department of Applied Mathematics and Computer Science

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# Introduction

## Neuron dynamics

How do neurons communicate?

- Neurons receive neurotransmitters
- Action potential = explosion of electrical activity
- Synapse releases the neurons' neurotransmitter

How can we capture this behaviour?

- Human brain consists of  $\sim 100$  billion neurons
- The *MFR* yields the average dynamics of the network

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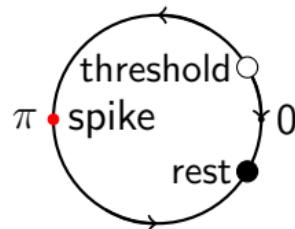
# The Theta Neuron Model

## Model Description

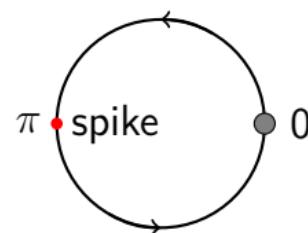
- Formulation

$$\dot{\theta} = (1 - \cos \theta) + (1 + \cos \theta) \cdot I \quad \theta \in \mathbb{T}$$

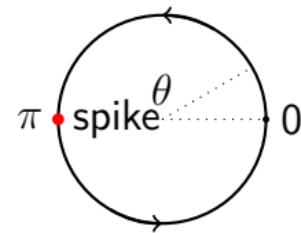
- Normal form of SNIC bifurcation



Excitable regime:  $I < 0$



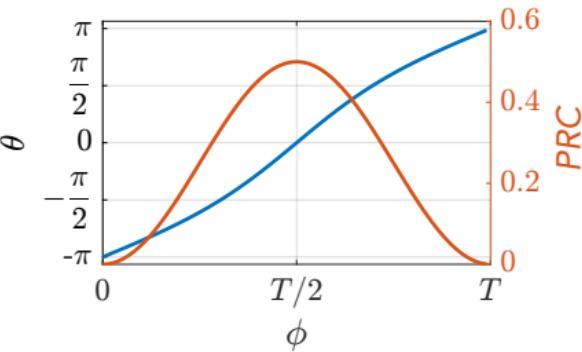
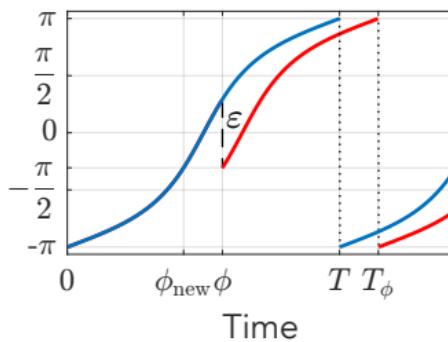
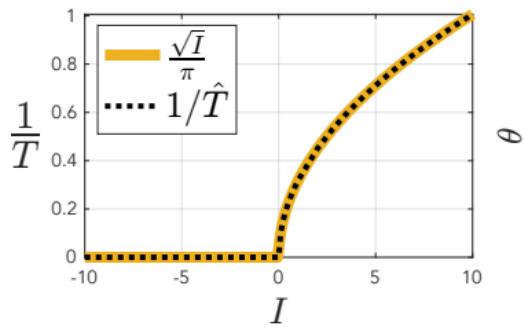
Bifurcation:  $I = 0$



Periodic regime:  $I > 0$

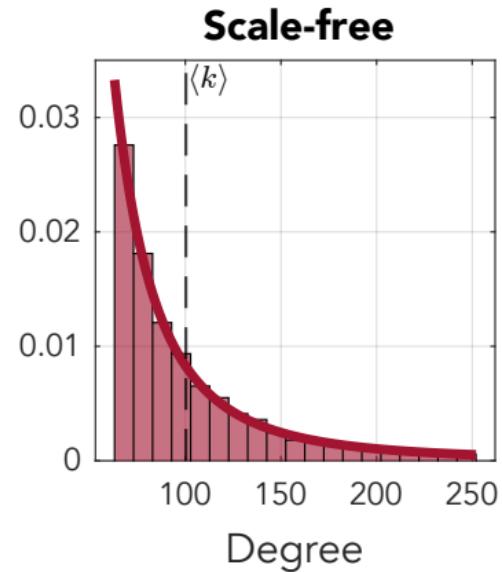
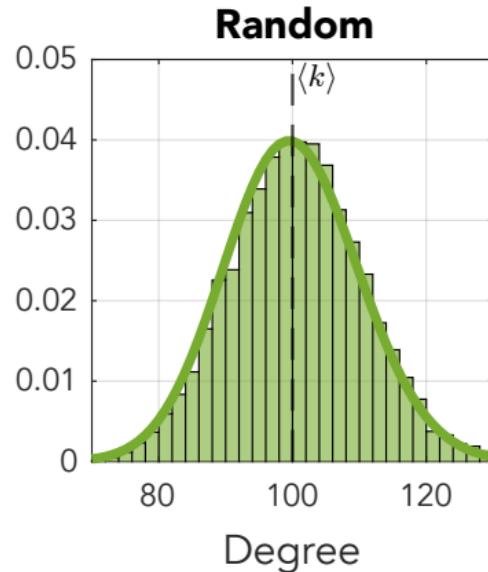
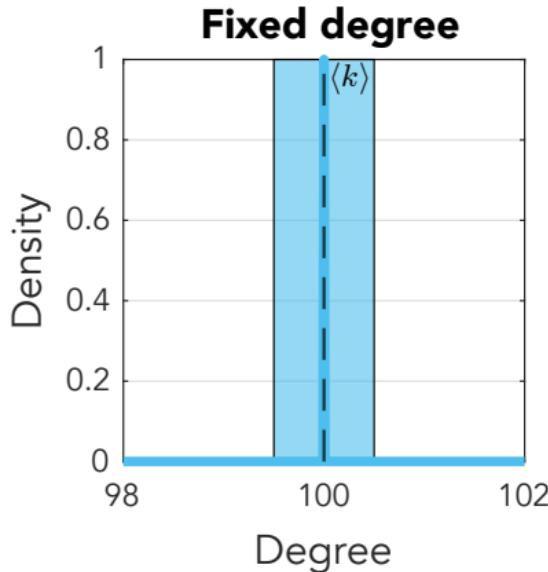
# The Theta Neuron Model Response

- Formulate bifurcations in terms of spiking frequency or phase angle



# Network Topologies

## Three basic networks



## Networks of Theta neurons

- For an arbitrary network topology:

$$\dot{\theta}_i = (1 - \cos \theta_i) + (1 + \cos \theta_i) \cdot [\eta_i + I_i(t)] \quad \theta_i \in \mathbb{T}^N$$

$$I_i(t) = \frac{\kappa}{\langle k \rangle} \sum_{j=1}^N A_{ij} \cdot \mathcal{P}_n(\theta_j)$$

- Capture average/mean synchronisation

$$Z(t) = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} \quad Z \in \mathbb{C}_o$$

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## Mean Field Reductions

# Predict synchronisation dynamics

The *MFR* yields a solution for  $Z(t)$

- $N$  neurons in the network
- $M_k \ll N$  unique node degrees

Then rewrite  $Z(t)$  per degree!

- Only  $M_k$  equations
- Weighed by  $P(k)$

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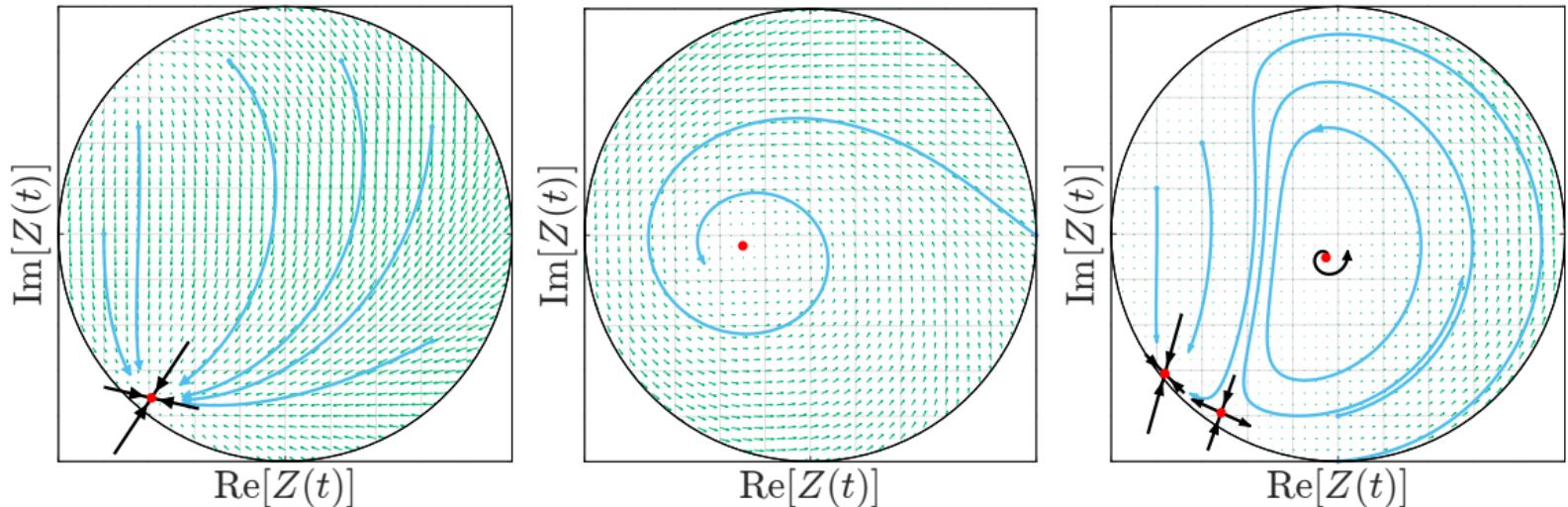
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# Mean Field Reductions

## Fixed-degree networks



# *Investigation:* Mean Field Reductions for undirected graphs

## Goals

$Z(t)$  can be measured and predicted: are they the same?

- Formulate directed networks
- Construct adjacency matrix from degree distribution
- Initial conditions
- Results

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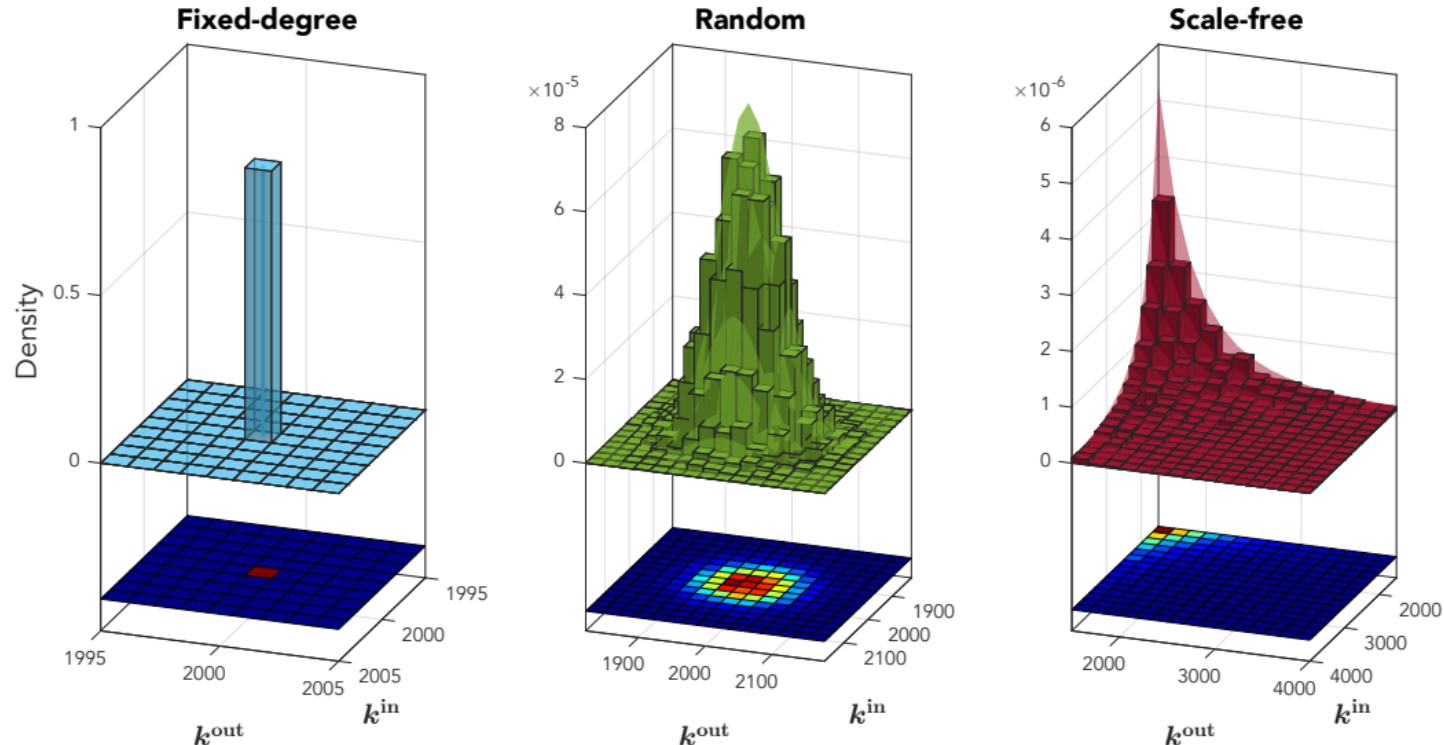
## Directed networks

Use a bivariate degree distribution

- Use identical and independant distributions
- In- and outdegrees are found as a permutation

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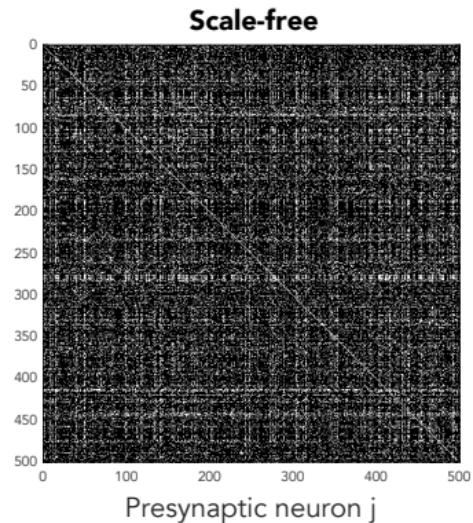
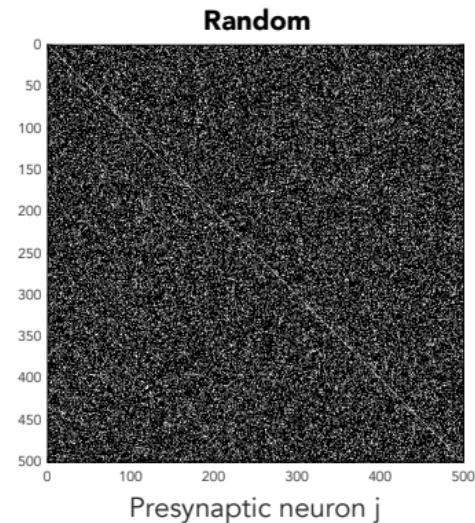
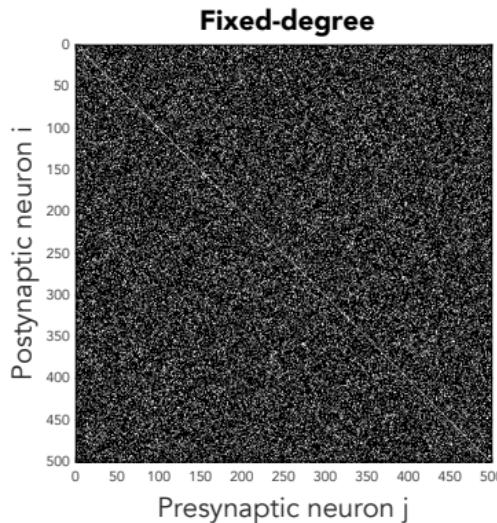
## Directed networks



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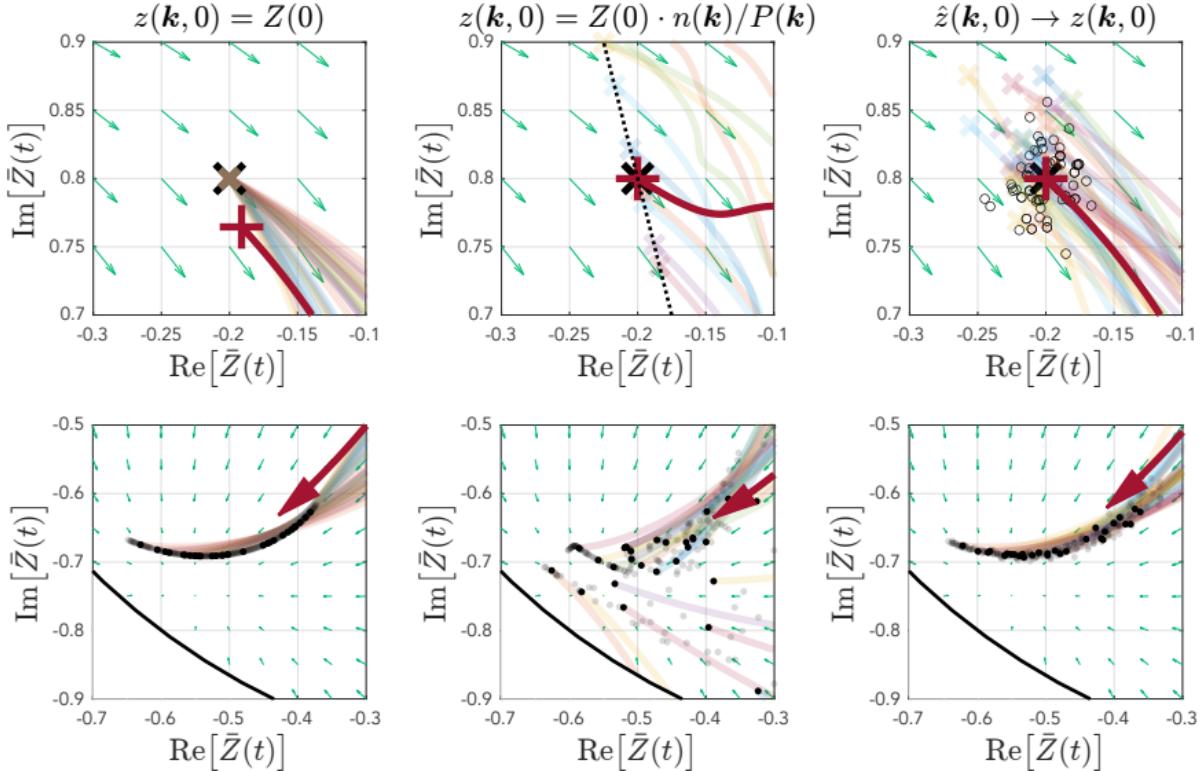
## Adjacency matrix

Find a probable solution by sampling from the in- and outdegrees



# Investigation: Mean Field Reductions for undirected graphs

## Initial conditions



Hebbian Learning and Synaptic Plasticity

# **Hebbian Learning and Synaptic Plasticity**

*Investigation:* Emerging Network Topologies

## **Investigation: Emerging Network Topologies**

## Conclusion and Discussion