Chapter Fourteen: Regression Analysis

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In this chapter,we shall look at the following
- The line of best fit
- Regression analysis
- Eye ball method
- Least squares method

In the previous chapter, we looked at how can test the relationships between two variables. We used Pearson and Spearman's correlations for this. We noted the caveats of these methods though and major among them was the fact that they only provide us with the extent of correlation between two interval/ratio variables and nothing else. Well, the world is far more complicated and just knowing that the two continuous variables are positively or negatively correlated is not enough to even begin to solve problems that we face everyday.

Regression analysis is a much more versetile tool than correlation. With regression analysis, we are able to calculate the effects of one variable on another. We can also predict the value of our outcome variable based on the predictor variable and yes we can add many more different types of variables to our analysis. Regression can even get more complicated than that. In fact, most of what is called quantitative social science is regression analysis in different forms and shapes. That's why we spent sometime in this chapter to develop intuitions about regression analysis and then move from bivariate linear regression to multiple regression. It is important that you master the basics of regression analysis because it forms the foundation of most of quantitative analyses in the social sciences.

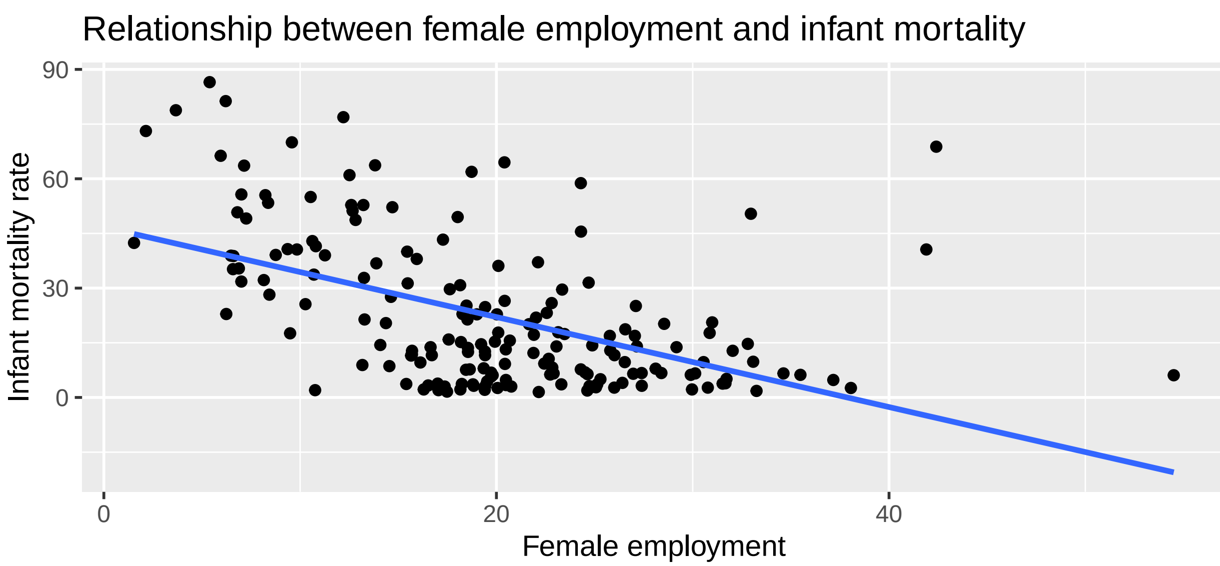
Basics

Regression builds from the correlation analysis that we looked at in the previous chapter but this time around, we will be able to do more with our two continuous variables. In the example of two variables we used in the previous chapter (female secondary enrolment and infant mortality), we were not able to predict the amount of reduction in infant mortality based on the value of female secondary school enrolment. We wouldn't say for example that 10% increase in female secondary school enrolment induces a 15% reduction in infant mortality rate. I am sure you can see how much of a difference being able to make this determination can bring to our understanding of the two variable. Even better, imagine the potential of being able to extend this tool to include different combinations of variables. We could solve a few social, health and policy problems, not so? That is the essence of regression analysis. Being able to predict the value of our outcome variable from one or more predictor variables. But the question is what is the logical foundation of prediction?

It starts from the line that we were able to fit across the scatter plot in the previous chapter. We called it the line of best fit. Well, the other name of the line of best fit is the **regression line**. I didn't want to use that name then to avoid confusions because we had not yet encountered regression analysis. But hence forth, we will be using the regression line. Fitting a line to data to be used to predict values of outcome variables based on the values of predictor variables is also technically called **modelling**. What this entails is that the outcome we want to predict for a particular case or observation can be predicted by whatever model we fit to the data. As you can imagine, there are so many types of models out there that can be used to analyse different types of data but in this chapter we use **linear models**. This is because we are going to analyse variables that have a linear relationship. Remember what linear relationships are: **relationships that can be represented by a straight line on a scatter plot**. This gives a good intuition that you can use in more complex analyses.

Simple linear regression

To understand linear relationships (linear models), we have to start simple. What better place to start than **simple linear regression**. It is basically the type of regression analysis that involves two interval/ratio variable, meaning one dependent and one independent variable. In other words, we predict the value of one dependent variable based on the values of one independent variable. How do we do this? Look at the figure below:



We use the regression line to describe the best representation of relationship between the two variables. To make a prediction of the value of the dependent variable based on the independent variable, we observe the value of the independent variable on the x-axis until we hit the regression line and we then follow it until we find the corresponding value on the y-axis. For example, if we want to find the infant mortality rate induced by a 20% female employment levels in the population, we follow the 20% value on the x-axis until we hit the regression line and then find the corresponding value on the y-axis, which in this case is approximately 20. We can then say that countries with a 20% female employment levels in the population are expected to have 20 infant mortality per thousand of the population.

The method we used above to find the corresponding value on the y-axis is called the **eyeball method** and it is just exactly as the name suggests: using your eye balls to draw a line across the data points on a scatter plot and find the corresponding value of the dependent variable given the value of the independent variable. I guess you already have doubts on the accuracy of this method. Yes! your eyes may look pretty good but they probably found themselves on your head for other purposes not to draw regression lines.

However, although the eyeball method is not very effective, we have managed to arrive at an approximate value that we believe represents the corresponding y value from the given x value. This was possible because the graph is two dimensional, owing to the fact that we are working with only two variables. However, things can get much worse when things get more complicated. Can you imagine appproximating the value of your dependent variable when you three or more independent variables. First of all we can not even represent such an analysis graphically. Good luck on drawing a five dimensional graph much less six or more.

Another thing that we seem to have simplified too much is the regression line itself. But is certainly not that simple. Do you know that you can draw many regression lines on the same scatter plot? How do you decide where the line starts and the angle it takes going forward? These are all very legitimate questions. Since this line is so important that if we messy it up it messes up our entire regression analysis, we need some reliable method that we can use to accurately draw it. Enter the **regression equation**

The regression equation

The regression equation offers a much more reliable way of calculating the the dependent of the outcome variable on the independnet variable. So much so that even if we have more relationships to analyse which we can represent on a two dimensional or three dimensional graph, we can just extend it to include them. Also, the regression equation gives accurate starting points in drawing our linear regression line and the angle the line takes is given.

The equation line comes from the equation of a straight line, if you still remember your high school arithmetic. For you to draw a straight line, you requare two pieces of information. Either I give you two points, the first point is where you are supposed to start from and the second is where you should end. Or I give you one point, where the line begins from and the angle or slope it should take. We use the later. Below is the equation of a line

$$Y = mx + c$$

This equation is what has changed by statistician as follows

$$Y = a + bx$$

Where y is the variable on the y-axis and x is the x-axis variable and a is the intercept and m is the slope. The intercept is the point at which the regression line crosses the y-axis. It is usually defined as the value of y when x is zero.

Ordinary least squares

You should have noticed that we did not strike evey data point by our regression line. Meaning that we are very accurate in describing our data using the regression. In fact let me put it this way: it is impossible for us to draw a line that covers all the data points unless we are measuring a variable by itself, also known as perfect correlation. We will always have some gap between the data points and the regression line. That gap is called the **error** or the **error term**. The error is usually inclued in the equation. Thats why you have usually seen the equation line looking like

$$y = a + bx + \varepsilon$$

Simple regression in R

In R, simple regression is calculated by creating using the `lm`() function representing linear model. We start with the dependent variable (mri) followed by a tilde ~ followed our dependent variable (employ) then the dataset same, `data` in our case. The model is stored in the `model` object and we need to run the `summary()` function to see the output.

```
model1 <- lm(mri ~ employ, data = data )
summary(model1)
```

```
##
## Call:
## lm(formula = mri ~ employ, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -31.464 -12.351  -3.038   7.250  74.401
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  46.7513     3.3833  13.818  < 2e-16 ***
## employ       -1.2345     0.1578  -7.823 4.49e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.64 on 177 degrees of freedom
##   (38 observations deleted due to missingness)
## Multiple R-squared:  0.2569, Adjusted R-squared:  0.2527
## F-statistic:  61.2 on 1 and 177 DF,  p-value: 4.493e-13
```

The output shows the call, which is basically the contents in the `lm()` function that we provided. Residuals shows the max value, minimum value and the median. The coefficients section shows the intercept and the independent variable values. For the intercept the value of 46.75 is the intercept and it represents the value of y when x is zero. In countries were there is not employment opportunities for women, infan mortaliy rate is expected to be at 46 per 1000 of the population.

The output also relayes p-values. The immploy