$\varepsilon_{ijk}\varepsilon_{lmn}$

$$= \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

$\varepsilon_{ijk}\varepsilon_{kmn}$

$$= \begin{vmatrix} \delta_{ik} & \delta_{im} & \delta_{in} \\ \delta_{jk} & \delta_{jm} & \delta_{jn} \\ \delta_{kk} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

$\underline{\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}}$

$$\delta_{ik}(\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km})$$
$$-\delta_{jk}(\delta_{im}\delta_{kn} - \delta_{in}\delta_{km})$$
$$= (\cancel{\delta_{jm}\delta_{in}} - \cancel{\delta_{jn}\delta_{im}})$$
$$1 \quad 2$$
$$-(\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm})$$
$$+(\cancel{\delta_{im}\delta_{jn}} - \cancel{\delta_{in}\delta_{jm}})$$
$$2 \quad 1$$

$$= \delta_{in}\delta_{jm} - \delta_{im}\delta_{jn}$$
$$= \varepsilon_{ijk}\varepsilon_{kmn}$$

$$\boxed{\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}}$$

Sign?

$$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{lj}\delta_{mk} - \delta_{lk}\delta_{mj}$$

$$\varepsilon_{jki}\varepsilon_{ilm} = \delta_{mi}\delta_{nj}$$
$$\downarrow\downarrow\downarrow \quad \downarrow\downarrow$$
$$i \; j \; k \quad m \; n \qquad - \delta_{mj}\delta_{ni}$$

$$\varepsilon_{ijk}\,\varepsilon_{lmn}$$

$$= \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

$$\varepsilon_{ijk}\,\varepsilon_{imn}$$

$$= \begin{vmatrix} 1 & \delta_{im} & \delta_{in} \\ \delta_{ji} & \delta_{jm} & \delta_{jn} \\ \delta_{ki} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

$$= \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$$

$$+ \cancel{\delta_{jm}\left(\delta_{kn} - \delta_{in}\delta_{ki}\right)}$$

$$+ \cancel{\delta_{kn}\left(\delta_{jm} - \delta_{im}\delta_{ji}\right)}$$

$$S = \sum_{m=1}^{N} e^{m\left(-\frac{2\pi i x}{L}\right)} = \sum_{m=1}^{N} w^m$$

$$p_m = \frac{2\pi m}{L}$$

$$S - wS = w - w^{N+1}$$

$$S = \frac{w - w^{N+1}}{1 - w}$$

$$= w\left[\frac{1 - w^N}{1 - w}\right]$$

$$\sum_{n=0}^{L-1} e^{-j\frac{2\pi k}{L}n} = \frac{1 - e^{-j2\pi k}}{1 - e^{\frac{-j2\pi k}{L}}}$$

which somehow can also be reduced to

$$L\delta[k]$$

$$S(x) = \sum_{m=1}^{N} e^{m\left(-\frac{2\pi i x}{L}\right)}$$

$$S(k) = \sum_{n} e^{-i\left(\frac{2\pi x}{L}\right)n}$$

$w = 1 \Rightarrow$ Sum is N

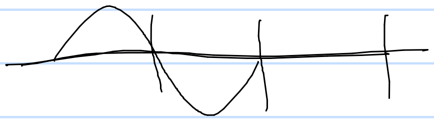
$w \neq 1$

$$1 - e^{-i2\pi\frac{Nx}{L}}$$

$$= 2e^{-i\pi\frac{Nx}{L}} \frac{\left(e^{i\pi\frac{Nx}{L}} - e^{-i\pi\frac{Nx}{L}}\right)}{2}$$

Sin $\frac{\pi N x}{L}$

zero if $\frac{Nx}{L}$ is integer



$$p_m = \frac{2\pi}{L} m$$

$$\psi(p) = \frac{1}{\sqrt{L}} \int dx \; e^{-ipx} \psi(x)$$

$$\psi(x) = \frac{1}{\sqrt{L}} \sum e^{ipx} \psi(p)$$

$$\psi(y) = \int dx \; \underbrace{\frac{1}{L} \sum e^{-ip(x-y)}}_{\delta(x-y)} \; \psi(x)$$

$$[A, BC] = ABC - BCA$$

$$= ABC - BAC$$

$$+ BAC - BCA$$

$$= [A, B]C + B[A, C]$$

$$[A_1, A_2 \dots A_N]$$

$$= A_1 \dots A_N - A_2 A_1 A_3 \dots A_N$$

$+$

$+$

$+$

$$A_2 \dots A_{N-1} A_1 A_N - A_2 \dots A_N A_1$$