MATH1326
Advanced Optimisation with Python

Week 7

- Scheduling and Timetabling Problems
- PuLP Modelling & Solution

Scheduling Flight Landings

Table 11.4: Characteristics of flight time windows

Plane	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Earliest arrival	129	195	89	96	110	120	124	126	135	160
Target time	155	258	98	106	123	135	138	140	150	180
Latest Arrival	559	744	510	521	555	576	577	573	591	657
Earliness penalty	10	10	30	30	30	30	30	30	30	30
Lateness penalty	10	10	30	30	30	30	30	30	30	30

Scheduling Flight Landings

Table 11.5: Matrix of minimum intervals separating landings

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	–	3	15	15	15	15	15	15	15	15
2	3	–	15	15	15	15	15	15	15	15
3	15	15	–	8	8	8	8	8	8	8
4	15	15	8	–	8	8	8	8	8	8
5	15	15	8	8	–	8	8	8	8	8
6	15	15	8	8	8	–	8	8	8	8
7	15	15	8	8	8	8	–	8	8	8
8	15	15	8	8	8	8	8	–	8	8
9	15	15	8	8	8	8	8	8	–	8
10	15	15	8	8	8	8	8	8	8	–

Scheduling Flight Landings

Decision variables

$land_p$: landing time of plane p

$early_p$: earliness of plane p measured in minutes

$late_p$: tardiness of plane p measured in minutes

$prec_{pq}$: 1 if landing of plane p precedes the landing of plane q, 0 otherwise

Parameters

$START_p$: Earliest arrival time for plane p

$TARGET_p$: Target arrival time for plane p

$STOP_p$: Latest arrival time for plane p

$CEARLY_p$: minute penalty for early landing of plane p

$CLATE_p$: minute penalty for late landing of plane p

$DIST_{pq}$: minimum interval separating the landings of planes p and q

Scheduling Flight Landings

$$\text{minimize} \sum_{p \in PLANES} (CEARLY_p \cdot early_p + CLATE_p \cdot late_p)$$

$$\forall p, q \in PLANES, q < p : land_p + DIST_{pq} \leq land_q + M \cdot prec_{qp}$$

$$\forall p, q \in PLANES, p < q : land_p + DIST_{pq} \leq land_q + M \cdot (1 - prec_{pq})$$

$$\forall p \in PLANES : land_p = TARGET_p - early_p + late_p$$

$$\forall p \in PLANES : START_p \leq land_p \leq STOP_p$$

$$\forall p \in PLANES : 0 \leq early_p \leq TARGET_p - START_p$$

$$\forall p \in PLANES : 0 \leq late_p \leq STOP_p - TARGET_p$$

$$\forall p, q \in PLANES, p < q : prec_{pq} \in \{0, 1\}$$

Sequencing jobs on a bottleneck machine

Table 7.6: Task time windows and durations

Job	1	2	3	4	5	6	7
Release date	2	5	4	0	0	8	9
Duration	5	6	8	4	2	4	2
Due date	10	21	15	10	5	15	22

Sequencing jobs on a bottleneck machine

Decision variables

$rank_{jk}$: 1 if job j has the position (rank) k and 0 otherwise

$start_k$: start time of the job in position k

$comp_k$: completion time of the job in position k

$late_k$: tardiness of the job in position k

Parameters

REL_j : Release date of job j

DUR_j : Duration of job j

DUE_j : Due date of job j

Sequencing jobs on a bottleneck machine

$$\text{minimize } start_{NJ} + \sum_{j \in JOBS} DUR_j \cdot rank_{j,NJ}$$

Makespan

$$\text{minimize } \sum_{k \in JOBS} comp_k$$

Total Completion Time

$$\text{minimize } \sum_{k \in JOBS} late_k$$

Total Tardiness

Sequencing jobs on a bottleneck machine

$$\text{minimize } start_{NJ} + \sum_{j \in JOBS} DUR_j \cdot rank_{j,NJ}$$

$$\forall k \in JOBS : \sum_{j \in JOBS} rank_{jk} = 1$$

$$\forall j \in JOBS : \sum_{k \in JOBS} rank_{jk} = 1$$

$$\forall k \in JOBS : start_k \geq \sum_{j \in JOBS} REL_j \cdot rank_{jk}$$

$$\forall k \in \{1, \ldots, NJ - 1\} : start_{k+1} \geq start_k + \sum_{j \in JOBS} DUR_j \cdot rank_{jk}$$

$$\forall k \in JOBS : start_k \geq 0$$

$$\forall j, k \in JOBS : rank_{jk} \in \{0, 1\}$$

Sequencing jobs on a bottleneck machine

$$\text{minimize} \sum_{k \in JOBS} comp_k$$

$$\forall k \in JOBS : comp_k = start_k + \sum_{j \in JOBS} DUR_j \cdot rank_{jk}$$

$$\forall k \in JOBS : comp_k \geq 0$$

$$\text{minimize} \sum_{k \in JOBS} late_k$$

$$\forall k \in JOBS : late_k = \max(0, comp_k - \sum_{j \in JOBS} DUE_j \cdot rank_{jk})$$

$$\forall k \in JOBS : late_k \geq 0$$