MATH1326
Advanced Optimisation with Python

Week 3

- Location Problems
- Integer Programming Formulation
- PuLP Modelling & Solution

Location examples

A location problem typically involves supplies and demands. Some examples:

- Locate warehouses to supply sales centres
- Location of hospitals or schools. In this case the 'demand' are patients or children. The choice of locations must consider the distribution of demand and try to minimise travel distances.
- Location of fire stations. The 'demand' is where fires are most likely to occur and the consequence of those fires.

Classes of Location Models

Set-covering

Find a subset of possible locations to ensure all demand points are covered.

Maximum coverage

Given an upper bound on the number of feasible facilities, maximise the demand covered

Centre models

Like set-covering models but choose locations to minimise the maximum distance of demand points to a facility.

p-median

Unlike the above models each demand centre can be satisfied by a combination of facilities. The objective is to minimise the sum of the distances between demand and facilities, with the constraint that the total demand of the facilities assigned to set of demands does not exceed its given capacity.

Fixed charge facility location models

The cost of installing a facility may depend on the location. In this class of problems the location-dependent fixed charge costs must be minimised while simultaneously meeting the cost of satisfying demand.

Depot Location

A company needs to site up to 12 depots to deliver to 12 service centres

Table 10.3: Delivery costs for satisfying entire demand of customers

	Customer											
Depot	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	100	80	50	50	60	100	120	90	60	70	65	110
2	120	90	60	70	65	110	140	110	80	80	75	130
3	140	110	80	80	75	130	160	125	100	100	80	150
4	160	125	100	100	80	150	190	150	130	∞	∞	∞
5	190	150	130	∞	∞	∞	200	180	150	∞	∞	∞
6	200	180	150	∞	∞	∞	100	80	50	50	60	100
7	100	80	50	50	60	100	120	90	60	70	65	110
8	120	90	60	70	65	110	140	110	80	80	75	130
9	140	110	80	80	75	130	160	125	100	100	80	150
10	160	125	100	100	80	150	190	150	130	∞	∞	∞
11	190	150	130	∞	∞	∞	200	180	150	∞	∞	∞
12	200	180	150	∞	∞	∞	100	80	50	50	60	100

Depot Location

Table 10.4: Fix costs and capacity limits of the depot locations

Depot	1	2	3	4	5	6	7	8	9	10	11	12
Cost (k€)	3500	9000	10000	4000	3000	9000	9000	3000	4000	10000	9000	3500
Capacity (t)	300	250	100	180	275	300	200	220	270	250	230	180

Table 10.5: Demand data

Customer	1	2	3	4	5	6	7	8	9	10	11	12
Demand (t)	120	80	75	100	110	100	90	60	30	150	95	120

Depot Location

- $build_d$ is a binary variable indicating whether a depot is built or not.

- $fflow_{dc}$ is a variable indicating the proportion of the demand of service centre c which is satisfied by depot d.

Depot Location

- Minimise total cost:

$$\sum_{d \in DEPOTS} CFIX_d \times build_d + \sum_{d \in DEPOTS} \sum_{c \in CUST} COST_{dc} \times fflow_{dc}$$

- Fixed costs for each depot to be built
- Costs of supplying each service centre from each depot, multiplied by the fraction actually supplied

Depot Location

- Since it's a ratio:

$$\forall d \in DEPOTS, c \in CUST : \; fflow_{dc} \geq 0$$

$$\forall d \in DEPOTS, c \in CUST : \; fflow_{dc} \leq 1$$

- Also, we need to satisfy demand for all customers:

$$\forall c \in CUST : \; \sum_{d \in DEPOTS} fflow_{dc} = 1$$

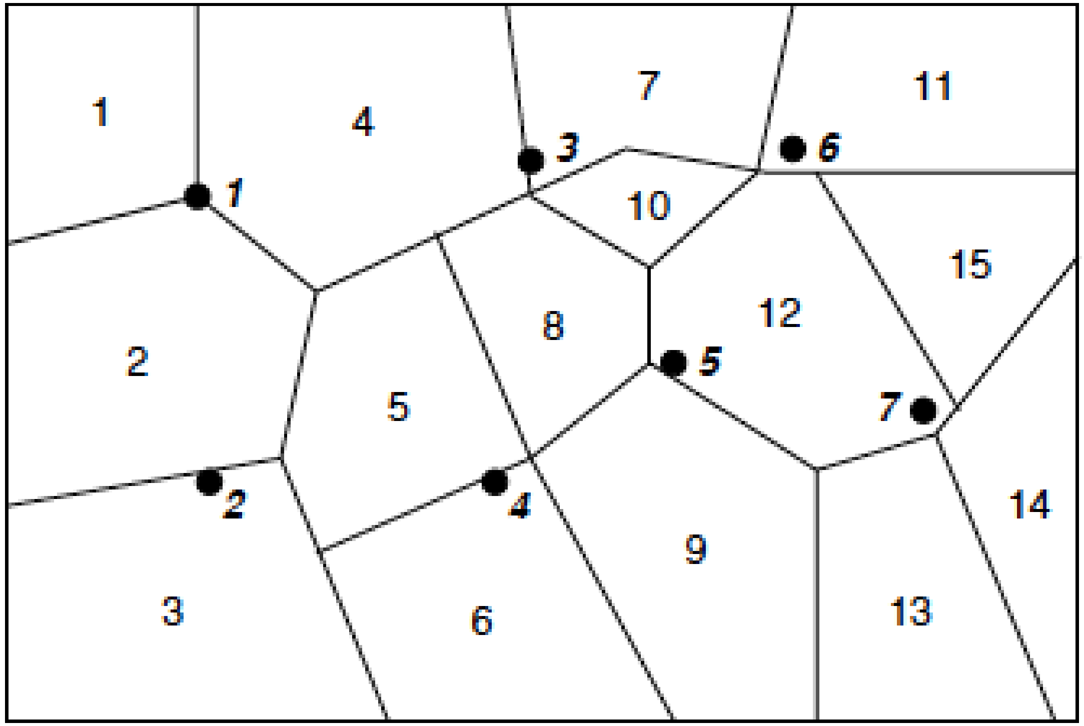
Depot Location

- For each depot, the demand it supplies is less than its capacity.
- If it isn't built, its capacity becomes zero.
- Each customers demand is multiplied by the fraction satisfied by depot d, and this is summed across all customers.

$$\forall d \in DEPOTS: \sum_{c \in CUST} DEM_c \times fflow_{dc} \leq CAP_d \times build_d$$

Location of GSM transmitters

Choose from seven potential transmitter sites, indicated by black dots, to cover 15 communities.



Location of GSM transmitters

Table 12.11: Cost and communities covered for every site

Site	1	2	3	4	5	6	7
Cost (in million €)	1.8	1.3	4.0	3.5	3.8	2.6	2.1
Communities covered	1,2,4	2,3,5	4,7,8,10	5,6,8,9	8,9,12	7,10,11,12,15	12,13,14,15

Table 12.12: Inhabitants of the communities

Community	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Population (in 1000)	2	4	13	6	9	4	8	12	10	11	6	14	9	3	6

Location of GSM transmitters

Decision variables

covered$_c$ 1 if community *c* is covered, 0 otherwise

build$_p$ 1 if a transmitters is build in site *p*, 0 otherwise

Location of GSM transmitters

$$\text{maximize} \sum_{c \in COMMS} POP_c \cdot covered_c$$

$$\forall c \in COMMS : \sum_{p \in PLACES} COVER_{pc} \cdot build_p \geq covered_c$$

$$\sum_{p \in PLACES} COST_p \cdot build_p \leq BUDGET$$

$$\forall c \in COMMS : covered_c \in \{0, 1\}$$

$$\forall p \in PLACES : build_p \in \{0, 1\}$$