MATH1326
Advanced Optimisation with Python

Week 8

- Routing Problems
- PuLP Modelling & Solution

Paint Production

Table 7.7: Matrix of cleaning times

	1	**2**	**3**	**4**	**5**
1	0	11	7	13	11
2	5	0	13	15	15
3	13	15	0	23	11
4	9	13	5	0	3
5	3	7	7	7	0

Paint Production

Decision variables

$succ_{ij}$: 1 if batch j succeeds batch i

y_i : real variable defined to eliminate subtours

Parameters

DUR_j : Processing time for batch j

$CLEAN_{ij}$: Cleaning time required between consecutive batches i and j

Paint Production

$$\text{minimize} \sum_{i \in JOBS} \sum_{j \in JOBS, j \neq i} (DUR_i + CLEAN_{ij}) \cdot succ_{ij}$$

$$\forall i \in JOBS : \sum_{j \in JOBS, j \neq i} succ_{ij} = 1$$

$$\forall j \in JOBS : \sum_{i \in JOBS, i \neq j} succ_{ij} = 1$$

$$\forall i, j \in JOBS, i \neq j : succ_{ij} \in \{0, 1\}$$

Paint Production

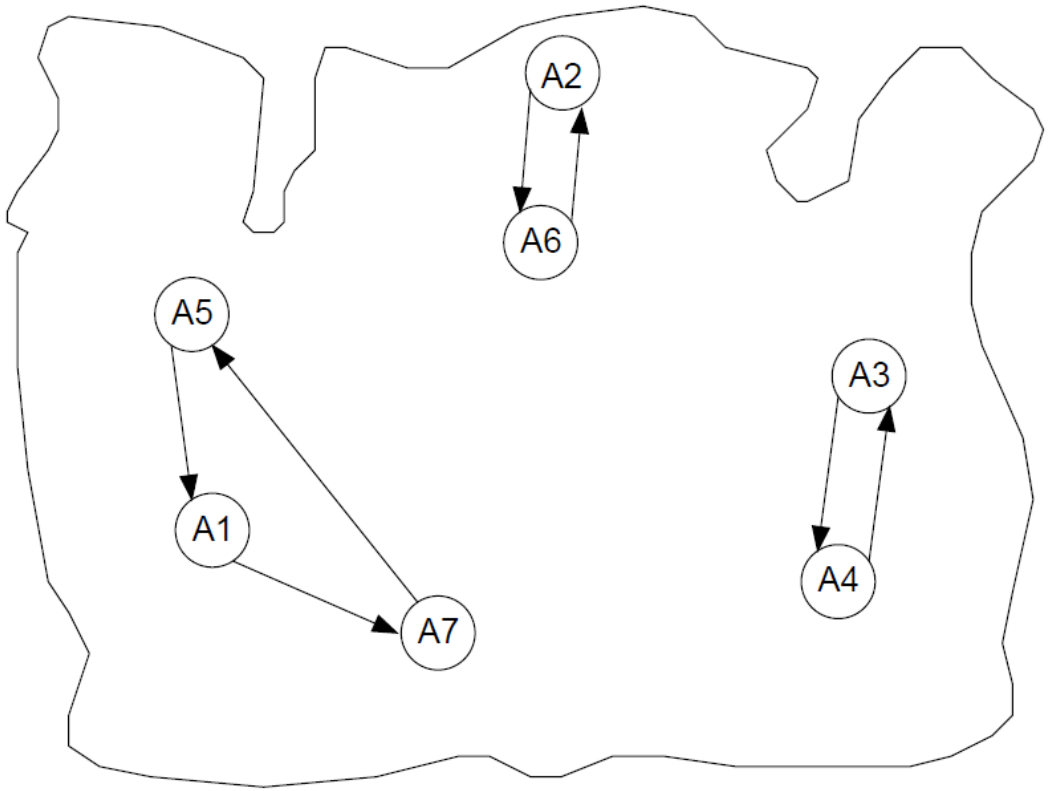

Figure 11.3: Initial solution with three sub-cycles

Paint Production

$$\forall S \subseteq \{2, \ldots, NJ\} : \sum_{(i,j) \in S} succ_{ij} \leq |S| - 1$$

$$\forall j \in JOBS : y_j \geq 0$$

$$\forall i \in JOBS, \forall j = 2, \ldots, NJ, i \neq j : y_j \geq y_i + 1 - NJ \cdot (1 - succ_{ij})$$

Heating oil delivery

Table 10.7: Demands by clients (in liters)

Brain-sur-l'Authion	Craquefou	Guérande	Haie Fouassière	Mésanger	Ponts-de-Cé
14000	3000	6000	16000	15000	5000

Table 10.8: Distance matrix (in km)

	Donges	Brain-sur-l'Authion	Craquefou	Guérande	Haie Fouassière	Mésanger	Ponts-de-Cé
Donges	0	148	55	32	70	140	73
Brain-s.-l'Authion	148	0	93	180	99	12	72
Craquefou	55	93	0	85	20	83	28
Guérande	32	180	85	0	100	174	99
Haie Fouassière	70	99	20	100	0	85	49
Mésanger	140	12	83	174	85	0	73
Ponts-de-Cé	73	72	28	99	49	73	0

Heating oil delivery

Decision variables

$prec_{ij}$: 1 if town *i* immediately precedes town *j* and 0 otherwise

$quant_i$: quantity carried up to client i

Parameters

$DIST_{ij}$: Distance between two towns *i* and *j*

DEM_i : Demand of client i

CAP: Maximum capacity of the tankers

Heating oil delivery

$$\text{minimize} \sum_{i \in SITES} \sum_{j \in SITES, i \neq j} DIST_{ij} \cdot prec_{ij}$$

$$\forall j \in CLIENTS : \sum_{i \in SITES, i \neq j} prec_{ij} = 1$$

$$\forall i \in CLIENTS : \sum_{j \in SITES, j \neq i} prec_{ij} = 1$$

$$\forall i \in CLIENTS : DEM_i \leq quant_i \leq CAP$$

$$\forall i \in CLIENTS : quant_i \leq CAP + (DEM_i - CAP) \cdot prec_{1i}$$

$$\forall i, j \in CLIENTS, i \neq j :$$

$$quant_j \geq quant_i + DEM_j - CAP + CAP \cdot prec_{ij} + (CAP - DEM_j - DEM_i) \cdot prec_{ji}$$

$$\forall i \in CLIENTS : quant_i \geq 0$$

$$\forall i, j \in SITES, i \neq j : prec_{ij} \in \{0, 1\}$$