MATH1326
Advanced Optimisation with Python

Week 6

- Partioning Problems
- PuLP Modelling & Solution

Airline Hub Location

Table 11.7: Average quantity of freight transported between every pair of cities

	Atlanta	Boston	Chicago	Marseille	Nice	Paris
Atlanta	0	500	1000	300	400	1500
Boston	1500	0	250	630	360	1140
Chicago	400	510	0	460	320	490
Marseille	300	600	810	0	820	310
Nice	400	100	420	730	0	970
Paris	350	1020	260	580	380	0

Airline Hub Location

Table 11.8: Distances between pairs of cities

	Boston	**Chicago**	**Marseille**	**Nice**	**Paris**
Atlanta	945	605	4667	4749	4394
Boston		866	3726	3806	3448
Chicago			4471	4541	4152
Marseille				109	415
Nice					431

Airline Hub Location

Decision Variables

$flow_{ijkl}$: 1 if freight from city *i* to city *j* flows through hubs *k* and *l,* 0 otherwise

hub_i : 1 if city i serves as a hub, 0 otherwise

Parameters

$cost_{ijkl}$: unit cost for any flow from city *i* to city *j* through hubs *k* and *l*

$quant_{ij}$: Amount of freight that needs to be carried from city *i* to city *j*

Airline Hub Location

$$\text{minimize} \sum_{i \in CITIES} \sum_{j \in CITIES} \sum_{k \in CITIES} \sum_{l \in CITIES} COST_{ijkl} \cdot QUANT_{ij} \cdot flow_{ijkl}$$

$$\sum_{i \in CITIES} hub_i = NHUBS$$

$$\forall i, j \in CITIES : \sum_{k \in CITIES} \sum_{l \in CITIES} flow_{ijkl} = 1$$

$$\forall i, j, k, l \in CITIES : flow_{ijkl} \leq hub_k$$

$$\forall i, j, k, l \in CITIES : flow_{ijkl} \leq hub_l$$

$$\forall i \in CITIES : hub_i \in \{0, 1\}$$

$$\forall i, j, k, l \in CITIES : flow_{ijkl} \in \{0, 1\}$$

Airline Hub Location (revised)

Reduced number of decision variables

Accumulate the values transported between any pair of destinations and only define the decision variables for $i < j$.

Given the locations, there would be one hub in the US, and one in Europe.

Define the decision variable only for those hubs assuming intra-continental flights will only use a single hub.

$$\forall i, j, k \in US, i < j : (i, j, k, k)$$

$$\forall i, j, k \in EU, i < j : (i, j, k, k)$$

$$\forall i, k \in US, j, l \in EU : (i, j, k, l)$$

Airline Hub Location (revised)

Additional constraint

Intracontinental flights will use only a single hub

$$\forall i, j \in US, i < j : \sum_{k \in US} flow_{ijkk} = 1$$

$$\forall i, j \in EU, i < j : \sum_{k \in EU} flow_{ijkk} = 1$$

Rigging Elections

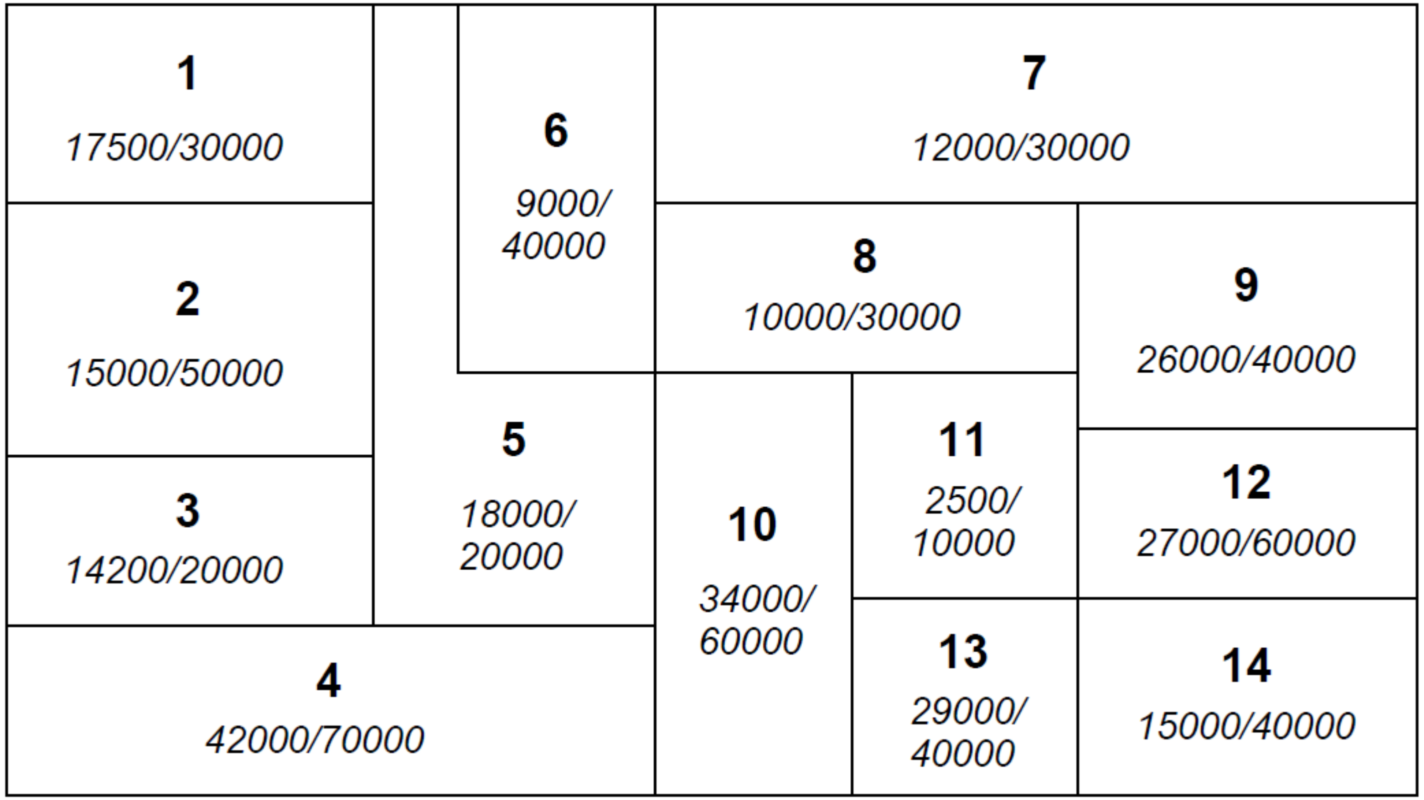

Figure 15.6: Map of the capital and its quarters. Legend: **quarter number**, *supporters/electorate*

Rigging Elections

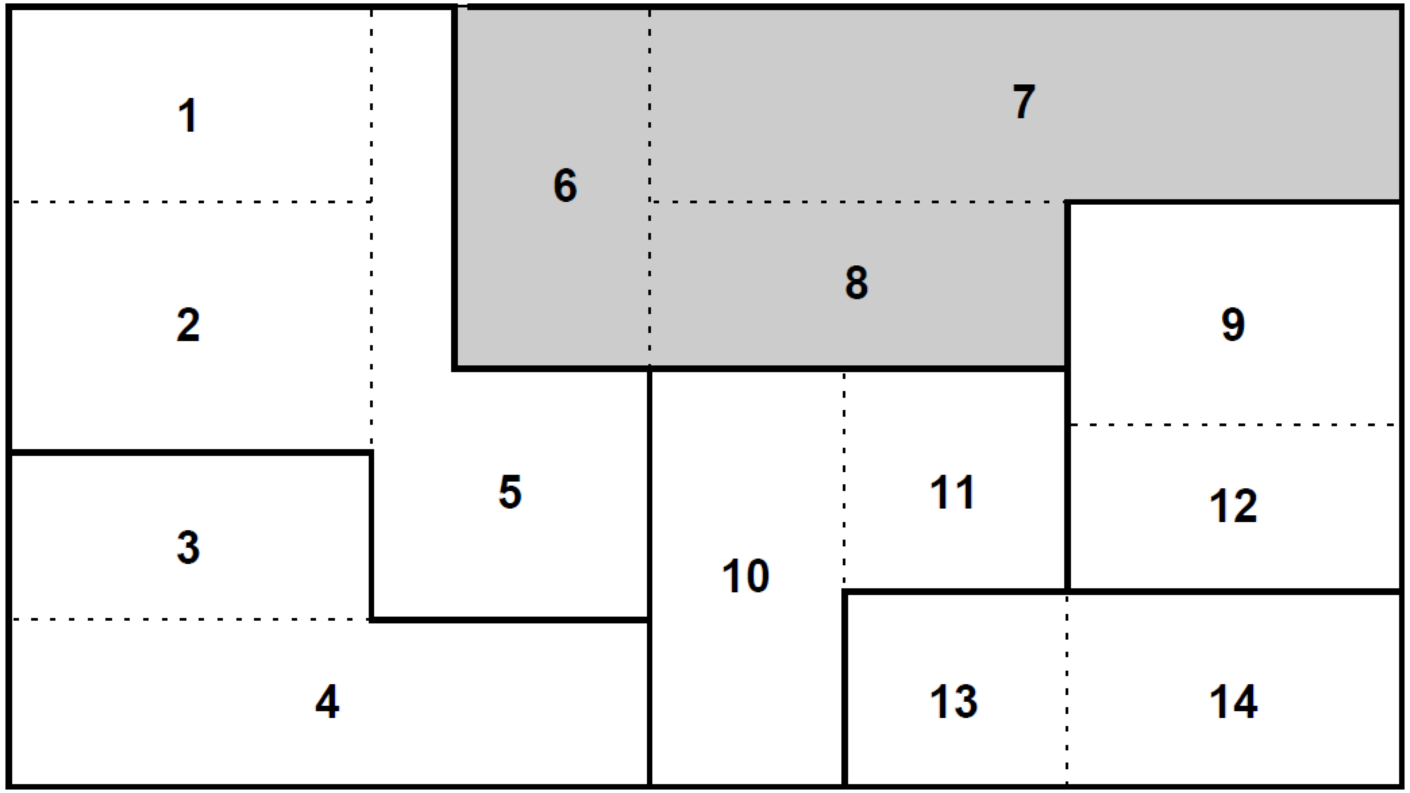

Figure 15.7: Electoral districts for *REQD* = 6

Rigging Elections

Decision variables

$Choose_d$ *:* 1 if district *d* is selected, 0 otherwise

Parameters

Maj_d : 1 if there is majority support in district d, 0 otherwise

$Distr_{dq}$: 1 district *d* includes quarter *q,* 0 otherwise

Rigging Elections

$$\text{maximize} \sum_{d \in RDIST} MAJ_d \cdot choose_d$$

$$\sum_{d \in RDIST} choose_d = REQD$$

$$\forall q \in QUARTERS : \sum_{d \in RDIST} DISTR_{dq} \cdot choose_d = 1$$

$$\forall d \in RDIST : choose_d \in \{0, 1\}$$