MATH1326
Advanced Optimisation with Python

Week 4

- Set Covering Problems

- Integer Programming Formulation

- PuLP Modelling & Solution

Nursing Roster – Question 1

Determine the minimum number of nurses required to cover all the requirements, knowing that a nurse works eight hours per day and that she is entitled to a break of two hours after she has worked for four hours.

Table 14.3: Personnel requirement per time period

Number	Time interval	Minimum number of nurses
0	00am – 02am	15
1	02am – 04am	15
2	04am – 06am	15
3	06am – 08am	35
4	08am – 10am	40
5	10am – 12pm	40
6	12pm – 02pm	40
7	02pm – 04pm	30
8	04pm – 06pm	31
9	06pm – 08pm	35
10	08pm – 10pm	30
11	10pm – 12am	20

Nursing Roster – Question 1

- There are 12 shifts each 24-hour day

- Each nurse works two consecutive shifts then has a break for one shift followed by another two shifts. For example, a nurse starting with shift 1 would work shifts 1, 2, break for 3 and then work for shifts 4 and 5.

- Following on the above, at any shift we will only have the nurses who started their shifts in that shift, the previous shift, and 3 and 4 shifts earlier, hence we can define a set WORK as {0, -1, -3, -4}.

- Every 24 hours we start again with shift 1. So shift 1 follows shift 12. For example, a nurse starting with shift 10 will work 10, 11, break for 12, work 1, and 2. We will need to use mod (%) function to find the right shifts to consider.

Nursing Roster – Question 1

$start_t$: Number of nurses starting work in time period t

$$\text{minimize} \sum_{t \in TIME} start_t$$

$$\forall t \in TIME : \sum_{l \in WORK} start_{(t+l+NT) \bmod NT} \geq REQ_t$$

$$\forall t \in TIME : start_t \in \mathbb{N}$$

Nursing Roster – Question 2

The service only has 80 nurses, which is not sufficient with the given requirements.

Nurses can work two additional hours per day as overtime. These two additional hours

follow immediately after the last four hours, without any break. Determine the

schedule of the nurses in this service that minimizes the number of nurses working

overtime.

Nursing Roster – Question 2

$overt_t$: Number of nurses starting work in time period t and working overtime

$$\text{minimize} \sum_{t \in TIME} overt_t$$

$$\sum_{t \in TIME} start_t \leq NUM$$

$$\forall t \in TIME : overt_t \leq start_t$$

$$\forall t \in TIME : overt_{(t-5+NT) \bmod NT} + \sum_{i \in WORK} start_{(t+i+NT) \bmod NT} \geq REQ_t$$

$$\forall t \in TIME : start_t \in \mathbb{N}$$

$$\forall t \in TIME : overt_t \in \mathbb{N}$$

CCTV surveillance

What is the minimum number of cameras that have to be installed to survey all the streets of this zone and where should they be placed?
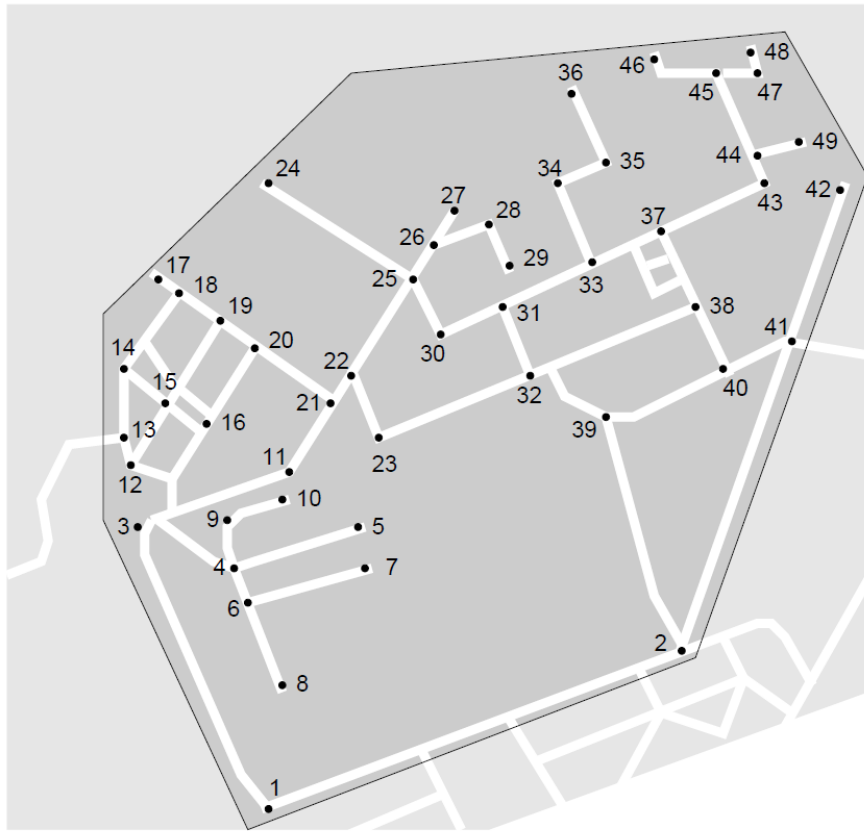

Figure 15.4: Industrial zone in Billston

CCTV surveillance

$place_n$: 1 if a camera is located at location n

minimize $\sum_{n \in NODES} place_n$

$\forall (n, m) \in STREETS : place_n + place_m \geq 1$

$\forall n \in NODES : place_n \in \{0, 1\}$

streets = {
 (1,2), (1,3),
 (2,39), (2,41),
 (3,4), (3,11), (3,12), (3,16),
 (4,5), (4,6), (4,9), (6,7),
 (6,8),
 (9,10),
 (11,21),
 (12,13), (12,15),
 (13,14),
 (14,15), (14,18),
 (15,16), (15,19),
 …
 }