Inhaltsverzeichnis

1 Exercise 1: MA-Processes (Simulation, Estimation, Identification)

1. Simulate realizations of lengths 100 and 200 of the following MA(1)- and MA(5)-Processes

$$\begin{aligned} X_t &= \epsilon_t - 0.7\epsilon_{t-1} \\ X_t &= \epsilon_t + 0.5\epsilon_{t-5} \end{aligned}$$

   where $\sigma^2 = 1$. Hint

   - The command set.seed(n), where $n \geq 0$ is an integer, determines the random-sequence.
   - MA-Processes can be generated directly, by relying on rnorm(length): this command generates a random-sequence ϵ_t of Gaussian noise.
   - Alternatively, MA-processes can be generated by relying on the R-command arima.sim(n = len, list(ma = c(b_1,b_2,...,b_q)), sd = sigma) where $b_1, ..., b_q$ are the MA-parameters.
   - Generate different realizations by varying the random seed.
   - Make a plot of the series. Can you detect differences by eyeballing the series
   - Compute the acf-sequences $\rho(i), i = 0, 1, 2, ...$ of both processes.
   - Does the acf depend on σ^2?
   - Compare the true acf's with the sample acf's generated by R (Hint: use the command acf(x)).
   - Compute the mean μ of the above processes and compare your result with the sample mean mean(x).

2. Parameter estimation and model identification

   - Try to identify MA-models for the realizations in the text-file $ma50$ by relying on the sample acf. Hint: use the command read.table(paste(path," ma50.txt",sep=))
   - Estimate model parameters. Hints:
     (a) (Optional) Use the procedure in section 5.3 of the script (CSS).
     (b) Use the R-function arima(x,order = c(0, 0, q)) and assign the output to a variable x_obj<-arima(x,order = c(0, 0, q)).
     (c) (Optional) Compare estimates.
     (d) (Optional) Verify that differences are unsystematic. Hint: you can access the variance of the estimates by looking at the estimation object x_obj: x_obj$var.coef is the variance-covariance matrix (variances are on the diagonal). If the estimation error is random, then it should lie within a 95% confidence interval centered in zero.

- Check if model assumptions are satisfied: use the R-function tsdiag(x_obj).

3. Same as above but for the series in the data-file $ma300$. The last 50 observations in $ma300$ coincide with $ma50$.

    - Do you identify the same processes (same model-orders)?

2 Exercise 2: MA-Processes (Inversion, Forecasts)

1. **Approximation of a MA(1) by an AR-process**: consider the following MA(1)-process

$$X_t = \epsilon_t + 0.9\epsilon_{t-1} \tag{1}$$

with $\sigma^2 = 4$.

   - Is the process invertible?
   - Is it stationary?
   - Compute the mean and the variance of the process.
   - Compute the acf.
   - (Optional) Try to express the process in terms of ϵ_t, $X_{t-j}, j = 1,2,3,4$ and a rest-term:

$$x_t = a_1x_{t-1} + a_2x_{t-2} + a_3x_{t-3} + a_4x_{t-4} + \epsilon_t + rest - term \tag{2}$$

     Compute the AR-coefficients as well as the rest-tem.
   - (Optional) Compute the variance of the rest-term.
   - (Optional) Compute the percent share of the variance of x_t explained by the rest-term (compute the ratio of both variances).
   - (Optional) Which parameter(s) determine(s) the size of this ratio?
   - (Optional) Can AR(p)-Processes approximate the above MA(1)-process? Which parameter(s) determine(s) the quality of the approximation?

2. Same exercise but for the process

$$x_t = \epsilon_t + \frac{1}{0.9}\epsilon_{t-1} \tag{3}$$

3. Compare the acf's of both processes.
   - What is your conclusion about the identification of the model-equation 1 or 3 based on acf?
   - Which model-equation is suitable for forecasting?

4. Consider the two MA-processes in the previous exercise:

$$\begin{aligned} X_t &= \epsilon_t - 0.7\epsilon_{t-1} \\ X_t &= \epsilon_t + 0.5\epsilon_{t-5} \end{aligned}$$

where $\sigma^2 = 1$.

   - Generate two realizations of length 100 of the above process (if you want to replicate my results you may set set.seed(10)).

- Define the system-matrix $\mathbf{B}$ used for inversion of the MA-processes into an AR(∞) representation.
- Verify that the two processes are invertible.
- Try to identify the model orders by looking at the sample acf.
- Estimate the parameters by relying on the R-function arima($\cdot$).
- Perform diagnostic checks.
- Plug your estimates into a sample estimate $\hat{\mathbf{B}}$.
- verify that the empirical models (with estimated coefficients) are invertible.
- Compute the weights of the AR-inversion of the empirical models.
- Compute 1-10-steps ahead point forecasts for both series.
- Compute 1-10 steps ahead interval forecasts for both series.
- Compare your results with the R-function predict($\cdot$).

5. Optional: same exercise for the models identified for $ma50$ and $ma300$

3 Exercise 3: AR-Processes (Stationarity, MA-inversion, Moments and Identification)

1. Consider the following AR(3)-process

$$x_t = 6 + 1.6x_{t-1} - 1.4x_{t-2} + 0.5x_{t-3} + \epsilon_t$$

with $\sigma^2 = 1$.

   (a) Define the system-matrix $\mathbf{A}$ of the corresponding multivariate AR(1)-representation.

   (b) Check stationarity of x_t by relying on the characteristic polynomial as well as on the eigenvalues of $\mathbf{A}$.

   (c) Compute the mean $\mu = E[x_t]$.

   (d) Compute and plot the first 100 weights of the MA-inversion. What kind of pattern is visible?

   (e) Can you say something about the quality of the approximation of the above AR(3) by an MA(100)? Hint: the approximation error $\mathbf{A}^{101}\mathbf{x}_{t-101}$ is weighted by $\mathbf{A}^{101}$: how large is $\mathbf{A}^{101}$?

   (f) (Optional) Generate a random sequence ϵ_t of length 100 and generate a corresponding realization of the AR(3) based either on the true AR(3) difference equation as well as on the MA(100)-approximation. Verify that the MA(100) is a good approximation of the AR(3). Hint: see exercises in section 6.1.3. of the script.

We conclude that MA-models of sufficiently large order q can approximate stationary AR(p)-processes arbitrarily well (Wold-Theorem). However, estimating many MA-parameters of a (misspecified) MA(100)-model is problematic for finite samples (recall that10 years of monthly data corresponds to sample-length of 120). Therefore, we generally prefer the simpler AR(3) structure over the more complex MA(100)-model. In exercise 3 below we try to quantify this effect by computing mean-square forecast errors of two model alternatives: an AR(1) and a MA(q) of 'large' model order q.

2. Load the data in the file exercise_3.txt.

   (a) Try to identify the DGP (Data Generating Processes) by relying on acf and pacf (AR and MA are mixed: one of the series is an ARMA).

   (b) Given your hypothetical $\hat{p}, \hat{q}$ based on acf/pacf: estimate model parameters and perform diagnostic checks.

   (c) Revise model orders if *necessary* (model with too few coefficients/misspecification: assumption that $\hat{\epsilon}_t$ is white noise got rejected) or if *possible* (model with too many coefficients: $\hat{\epsilon}_t$ is white noise).

3. Consider

$$x_t = 0.95x_{t-1} + \epsilon_t$$

We model this process with a (true) AR(1)-design as well as with a (misspecified) MA(q)-model, where q is sufficiently large to allow the MA to fit the data. We then perform a simulation study where forecast performances of both models are compared.

(a) Generate a realization of length 100 of the following AR(1)-process and try to identify the true model based on acf/pacf.

(b) Assume you wanted to fit a MA-model to the data (always possible since MA-models of sufficiently large orders are completely general). Which model order would you select?

(c) We now analyze which model performs better in terms of forecasting performances: the AR(1) or the MA(q), with q sufficiently large. For this purpose generate 100 realizations of length 110 of this process (in a for-loop).

(d) For each realization (each iteration in the for-loop): use the first 100 observations (not the whole sample of length 110!) to estimate the model: estimate AR(1) ar_obj<-arima(x[1:100],order=c(1,0,0)) and MA(q) ma_obj<- arima(x[1:100],order=c(0,0,q)) where we assume $5 \leq q \leq 10$ ($q < 5$ leads to misspecification and $q > 10$ to overfitting).

(e) Compute forecasts from both the estimated AR(1) and the estimated MA(q): forecasts are generated by the function predict(ar_obj,n.ahead=h) and predict(ma_obj,n.ahead=h) where ar_obj and ma_obj are the estimation object and h is the forecast horizon (h=10 here). Hint: the interesting point-forecasts are obtained by specifying the ending $\$pred$ as in predict(ar_obj,n.ahead=h)$pred.

(f) Compute forecast errors by subtracting your 1-10 steps ahead forecasts (obtained for AR(1) and MA(q)) from the retained observations x[101:110] (which were not used for estimation!).

(g) Compute for each forecast horizon $h = 1, ..., 10$ the mean-square forecast error across the 100 realizations of the process and evaluate/compare both models.

This is a so-called out-of-sample forecast exercise: the future observations are unknown when estimating coefficients. This setting replicates typical forecast applications (in contrast, scientific publications are generally 'cheating' since the alleged 'future' data is already known at the time of computing forecasts).

4 Exercise 4: AR-Processes (Moments, Impulse response, dynamics, forecasts)

1. (Optional) Consider the AR(3)-process in the previous series of exercises:

$$x_t = 6 + 1.6x_{t-1} - 1.4x_{t-2} + 0.5x_{t-3} + \epsilon_t$$

   with $\sigma^2 = 1$.

   (a) Compute second-order moments R(k), $k = 0, ..., 100$ based on its system matrix **A** and the Ricatti-equation.

   (b) Generate a series of length 100'000 (one hundred thousand!) and compute the acf-values. Hint: the sample estimates are obtained by specifying acf(x,plot=F,lag.max=100)$acf.

   (c) Compute and plot the *true* acf(k), $k = 0, ..., 100$ as well as the empirical acf (see above) and compare both series. If both series agree (up to sampling error) then the Ricatti-equation does a good job...

   (d) Try to describe the observed pattern of the acf.

2. (Optional) We now analyze the dynamics of the above process

   (a) Compute and plot the impulse response of the AR(3) process defined above.

   (b) How many 'cycles' interfer? Hint: have a look at the eigenvalues of **A**. A cycle of arbitrary frequency $0 < \omega < \pi$ is determined by a *complex conjugate* pair of eigenvalues and 'cycles' of frequency zero or π are determined by positive or negative *real* eigenvalues.

   (c) Determine frequencies, durations (in time units) and persistencies (rate of decay) of the damped cycles.

   (d) Verify these results by visual inspection of the impulse response.

   (e) Plot and compare true acf and impulse response: why do they look similar? Hint: Yule-Walker equations.

3. We now compute forecasts and link our results to previous exercises (see above).

   (a) Generate a realization of length 100 of the above AR(3)-process (my script is based on set.seed(10) but you may try any random seed).

   (b) Identify the process with acf/pacf.

   (c) Estimate model coefficients and perform diagnostic checks.

   (d) verify stationarity of the empirical model.

   (e) Compute a 1-100 steps ahead point forecast.

(f) Compute the weights of the empirical MA-inversion (based on estimated coefficients: not the true ones).

(g) Derive the forecast error variance and compare the estimate obtained for large forecast horizons (say $h = 100$) with the empirical variance $var(x_t)$.

(h) Compare point-forecasts and forecast variances with those obtained by the R-function predict($\cdot$).

(i) Compute and plot a 95% forecast interval for x_t.

(j) (Optional) Can you see similitudes between the impulse function and the point forecast? Try to explain (Hint: it is related to the noise forecast $\hat{\epsilon}_{T+h} = 0$; forecasts are not initialized by an impulse but by the last data points, instead).

(k) (Optional) Compare the width of the intervall forecast with $\sqrt{\hat{R}(0)}$ (obtained by plugging $\hat{\mathbf{A}}$ as well as $\hat{\sigma}^2$ into the Ricatti-equation); compare the long-term forecast (100-steps ahead) with the empirical mean $\overline{x}$ as well as with $\dfrac{\hat{c}}{1-\hat{a}_1-\hat{a}_2-\hat{a}_3}$ (based on the estimated coefficients).

5 Exercise 5: ARMA-Processes (Moments, Parsimony, Identification)

1. Consider the ARMA(2,2)-process:

$$x_t = 6 + 1.6x_{t-1} - 0.9x_{t-2} + \epsilon_t - 0.3\epsilon_{t-1} + 0.4\epsilon_{t-2}$$

with $\sigma^2 = 1$.

   (a) Determine the expectation μ.

   (b) Specify the system-matrices **A** and **B** of the MA- and AR-inversions. Is the process stationary? Is it invertible?

   (c) Compute the first 100 weights of the MA-inversion and derive the second-order moments R(k), $k = 0, ..., 10$ based on these weights.

   (d) Compute the acf-function based on $R(k)$ and compare your results either with the empirical acf based on simulated data or on the output of the R-function ARMAacf(ar = a_vec, ma = b_vec, lag.max = 10, pacf = FALSE), where a_vec and b_vec collect AR- and MA-coefficients.

2. ARMA-models are parsimonious in the sense that for a fixed number of parameters $p+q$ one can fit more general time series dynamics than with pure AR($p+q$)- or pure MA($p+q$)-models.

   (a) Read the data in the file y_data_exercise5.txt and try to identify the DGP by 'pure' AR or 'pure' MA or 'mixed' ARMA-models. Compare the number of parameters necessary for fitting the data for each model-approach.

3. (Optional) Consider the following ARMA-specifications

$$\begin{aligned} x_t &= 2 + 0.5x_{t-1} + \epsilon_t - 0.5\epsilon_{t-1} \\ x_t &= 2 + 0.5x_{t-1} + \epsilon_t + 0.5\epsilon_{t-1} \\ x_t &= 2 + 0.9x_{t-1} - 0.2x_{t-2} + \epsilon_t - 0.5\epsilon_{t-1} \\ x_t &= 2 + x_{t-1} - 0.25x_{t-2} + \epsilon_t - 0.7\epsilon_{t-1} + 0.1\epsilon_{t-2} \end{aligned}$$

   (a) Verify whether the above equations are in normal form and simplify, if possible.

6 Exercise 6: ARMA-Processes (Information Criteria, Forecasting)

Identification: Information Criteria

1. Apply automatic identification techniques based on information criteria.
   (a) Determine model orders of the data in the file y_data_exercise5.txt of the previous series of exercises. Hint: use AIC, BIC as well as the 'R-AIC' computed by arima (do not exceed model orders 5 i.e. try $p, q \leq 5$).
   (b) Check pertinence of the estimated model-orders by applying diagnostic tests to the corresponding models.

2. **Forecasting I**. Consider the ARMA(1,1)-process

$$x_t = 0.9x_{t-1} + \epsilon_t + 0.9\epsilon_{t-1}$$

   (a) Compute and plot the first 100 weights of AR- and MA-inversions. Interpret both curves.
   (b) At which forecast horizon h_0 does the $h-$step ahead forecast variance exceed 90% of the variance of the process?
   (c) Which simple forecast-rule is likely to perform well for forecast horizons exceeding the previous h_0?

3. **Forecasting II**: Consider the series in the file recession_data.txt. The series is a smoothed business-survey indicator based on a broad industrial panel. Currently, the indicator is deeply negative (recessive phase: contraction of the industry). An analyst is interested in forecasting the trough (the minimum or dip of the recession) as well as the time point of positive growth (recovery).
   (a) Select an ARMA(p, q)-model of suitable orders p, q and compute a 1-36 steps ahead forecast (3 years ahead).
   (b) Determine approximate time-points for the dip (minimum) and the recovery (positive growth) based on point forecasts.
   (c) Determine in which time-span the dip will appear with 90% probability.

7 Exercise 7: Non-stationary (ARIMA) Processes: an Application to Economic Data

1. Download or read the data file called *gdp.txt*: this is quarterly US-GDP from 1992 up to current time (the data can be conveniently downloaded from the quandl-website after suitable registration (authorization code)).

   (a) Fit a suitable *stationary* ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).

   - Try to interpret estimated coefficients.
   - Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
   - Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

   (b) Fit a suitable *I(1)*-ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).

   - Try to interpret estimated coefficients.
   - Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
   - Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

   (c) Fit a suitable *I(2)*-ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).

   - Try to interpret estimated coefficients.
   - Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
   - Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

2. Download or read the data file called *indpro.txt*: this is monthly US-Industrial Production from 1992 up to current time (the data can be conveniently downloaded from the quandl-website after suitable registration (authorization code)).

   (a) Fit a suitable *stationary* ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).

   - Try to interpret estimated coefficients.
   - Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
   - Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

(b) Fit a suitable *I(1)*-ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).
- Try to interpret estimated coefficients.
- Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
- Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

(c) Fit a suitable *I(2)*-ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).
- Try to interpret estimated coefficients.
- Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
- Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

3. Download or read the data file called *sp.txt*: this is daily S&P500 from 1992 up to current time (the data can be conveniently downloaded from the quandl-website after suitable registration (authorization code)).

(a) Fit a suitable *stationary* ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).
- Try to interpret estimated coefficients.
- Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
- Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

(b) Fit a suitable *I(1)*-ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).
- Try to interpret estimated coefficients.
- Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
- Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

(c) Fit a suitable *I(2)*-ARMA-model to the data (use the BIC-criterion for identification and check diagnostics).
- Try to interpret estimated coefficients.
- Compute 1-50-steps ahead point forecasts and plot the original series with the appended forecasts: interpret the result.
- Compute Interval forecasts and plot the original series with the appended forecasts: interpret the result.

8 Exercise 8: Multivariate Forecast Models: Principles

1. Consider the bivariate data set *bivar.txt* with series x_t (first column) and y_t (second column).

    - Plot the time series: are they stationary?
    - Determine a suitable univariate (ARIMA) forecast model for x_t, the series in the first column, and compute a one-step ahead point forecast, the corresponding forecast error variance as well as a 95% forecast interval.

2. Fit a simple (benchmark) bivariate forecast model to the data.

    - Regress x_t on x_{t-1} and y_{t-1}:

    $$x_t = c + a_1 x_{t-1} + a_2 y_{t-1} + \epsilon_t \tag{4}$$

      Analyze the regression summary and analyze model-residuals (are they stationary)?
    - Compare forecast error variances of uni- and multivariate models.
    - Compute a one-step ahead 95% interval forecast based on model 4.

3. We now attempt to improve performances of the bivariate model 4 by exploiting residual information

    - An analysis of the model residuals $\hat{\epsilon}_t$ suggests autocorrelation (quite typical in economic applications). How could the above (bivariate) forecast function be improved in this context?
    - Determine a suitable ARMA-model for the (stationary) residuals $\hat{\epsilon}_t$.
    - Compute a new improved one-step ahead forecast for x_t based on model 4 as well as on the ARMA-model for the residual $\hat{\epsilon}_t$.
    - Determine the forecast error variance of the improved forecast function and compute a 95% forecast interval.

4. Instead of fitting an ARMA-model to the residuals $\hat{\epsilon}_t$ of the bivariate model 4 we now augment the lag-structure of the model.

    - Add lagged explanatory variables $x_{t-k}, k > 1$ and $y_{t-j}, j > 1$ to equation 4. Determine a parsimonious model whose residuals are nearly iid.
    - Determine the forecast error variance of the augmented model (comparisons with previous models).
    - Compute a one-step ahead 95% forecast interval.

5. Compute two-steps ahead Interval forecasts for the above models.

9 Exercise 9: Multivariate Now- and Forecast for the Industrial Production Series

Read the data file *all_data.txt*: this file collects monthly observations for the industrial production index Ipi_t, the unemployment $Unemp_t$ and non-farm payrolls $Payroll_t$ (USA), from Jan 1999 to April 2016. All series are important indicators for assessing the state of the (US-) economy.

1. Univariate ARIMA-model
   - Identify a univariate ARIMA-model for Ipi_t.
   - Compute a one-step ahead (95%) interval forecast for the missing observation at the sample end.
2. Compute a nowcast of Ipi_T in $t = T$ based on data $Payroll_T, Unemp_T$ (naive model without lags)
   - Regress Ipi_t on $Payroll_t$ and $Unemp_t$

     $$Ipi_t = b_0 + b_1 Unemp_t + b_2 Payroll_t + \epsilon_t$$

   - Compute a nowcast of the missing Ipi_t at the sample end $t = T$. Compare that number with the most recent observations (what problem is to be observed?).
   - Plot and analyze the model residuals. Are they stationary?
   - Try to explain the poor performance of the previous nowcast by looking at the model residuals towards $t = T$.
3. Compute an interval-nowcast for Ipi_T based on the above model augmented by the lagged Ipi_{t-1} as an additional explanatory variable.
   - Regress Ipi_t on $Payroll_t, Unemp_t$ and Ipi_{t-1}.

     $$Ipi_t = b_0 + b_1 Unemp_t + b_2 Payroll_t + b_3 Ipi_{t-1} + \epsilon_t$$

   - Compute an interval-nowcast of the missing Ipi_t at the sample end $t = T$. Compare that number with the most recent observations.
   - Plot and analyze the model residuals. Are they stationary? Are they uncorrelated?
4. Compute an interval-nowcast for Ipi_T based on the previous model augmenetd by additional lagged explanatory variables (to resorb the autocorrelation of the residuals).

- Regress Ipi_t on $Payroll_t, Payroll_{t-1}, Unemp_t, Unemp_{t-1}, Ipi_{t-1}$ and Ipi_{t-2}.

$$\begin{aligned} Ipi_t &= b_0 + b_1 Unemp_t + b_2 Payroll_t + b_3 Ipi_{t-1} + b_4 Unemp_{t-1} \\ &\quad + b_5 Payroll_{t-1} + b_6 Ipi_{t-2} + \epsilon_t \end{aligned}$$

- Compute an interval-nowcast of the missing Ipi_t at the sample end $t = T$. Compare that number with the most recent observations.
- Plot and analyze the model residuals. Are they stationary? Are they uncorrelated?

5. Compute an interval-nowcast based on the *log-transformed data*, including additional lagged explanatory variables (to resorb the autocorrelation of the residuals).

- Regress $\log(Ipi_t)$ on $\log(Payroll_t), \log(Payroll_{t-1}), \log(Unemp_t), \log(Unemp_{t-1}), \log(Ipi_{t-}$ and $\log(Ipi_{t-2})$.

$$\begin{aligned} \log(Ipi_t) &= b_0 + b_1 \log(Unemp_t) + b_2 \log(Payroll_t) + b_3 \log(Ipi_{t-1}) \\ &\quad + b_4 \log(Unemp_{t-1}) + b_5 \log(Payroll_{t-1}) + b_6 \log(Ipi_{t-2}) \\ &\quad + \epsilon_t \end{aligned}$$

- Compute an interval-nowcast of the missing Ipi_t at the sample end $t = T$. Compare that number with the most recent observations.
- Plot and analyze the model residuals. Are they stationary? Are they uncorrelated?

6. Compute a one-step ahead forecast-interval based on the *log-transformed data* and the previous model (augmented) model.

- Shift the explanatory variable by one month into the future and apply the regression model in the previous exercise (with the newly shifted dependent variable).
- Compute an interval-forecast of Ipi_{T+1}. Plot the last few observations together with the nowcast interval as well as the forecast interval at the end of the time series.