Seminar on epidemics, infodemics and mobility

Classic SIR Model

$$\dot{S} = -\beta I S\,,$$
$$\dot{I} = \beta I S - \gamma I\,,$$
$$\dot{R} = \gamma I\,.$$

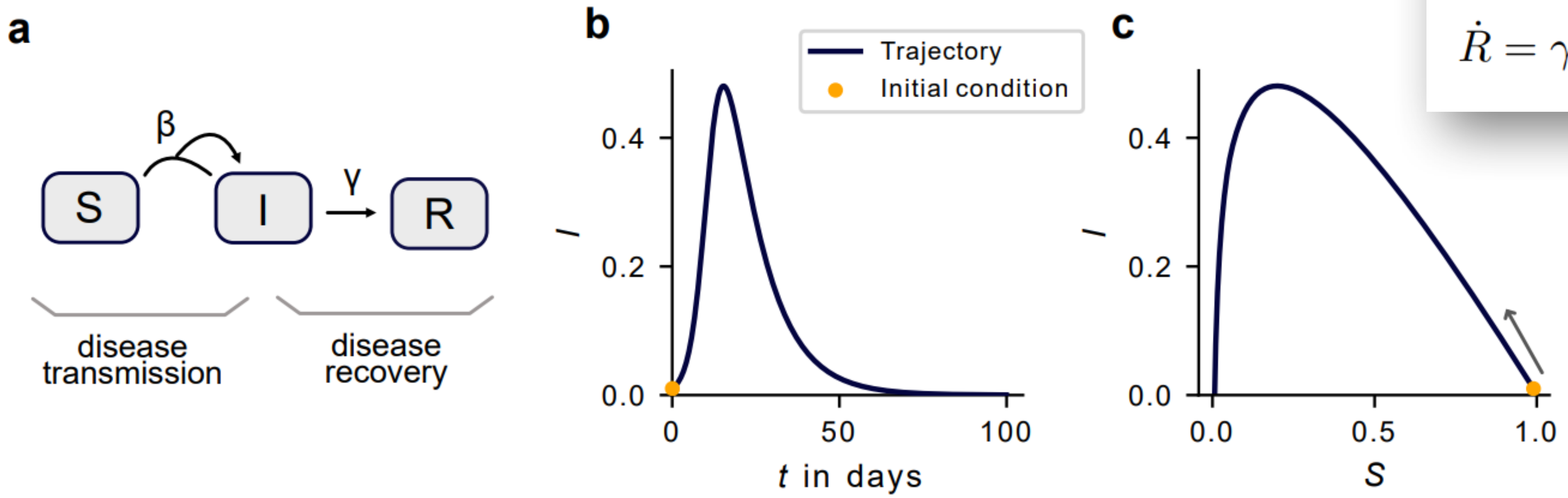


Figure 2: Overview of the SIR model. a) A sketch of the model compartments and transitions in the SIR model. The interaction of susceptible and infected people yields new infected people at rate β who subsequently recover at rate γ. **b)** The infected part of the population I displayed as a timeseries for $R_0 = 5$. **c)** A trajectory of the SIR model in phase space, shown in the S-I plane.

SIRS with vital dynamics

$$\dot{S} = -\beta S I + \mu - \mu S + \nu R$$
$$\dot{I} = \beta S I - \gamma I - \mu I\,,$$
$$\dot{R} = \gamma I - \mu R - \nu R\,.$$

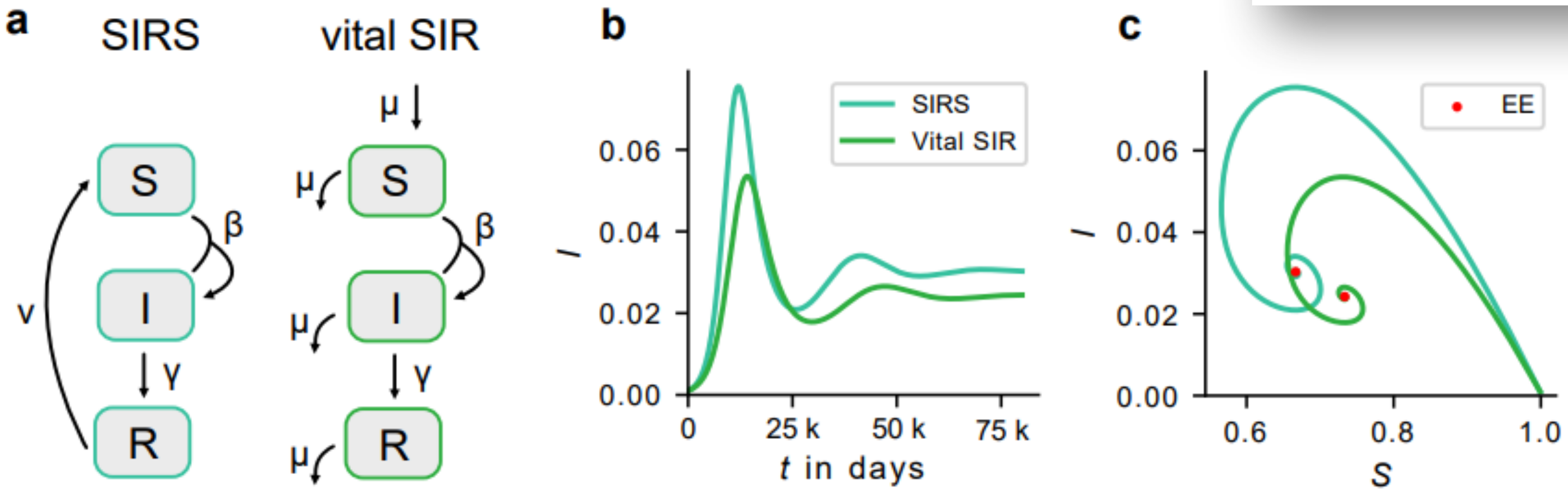
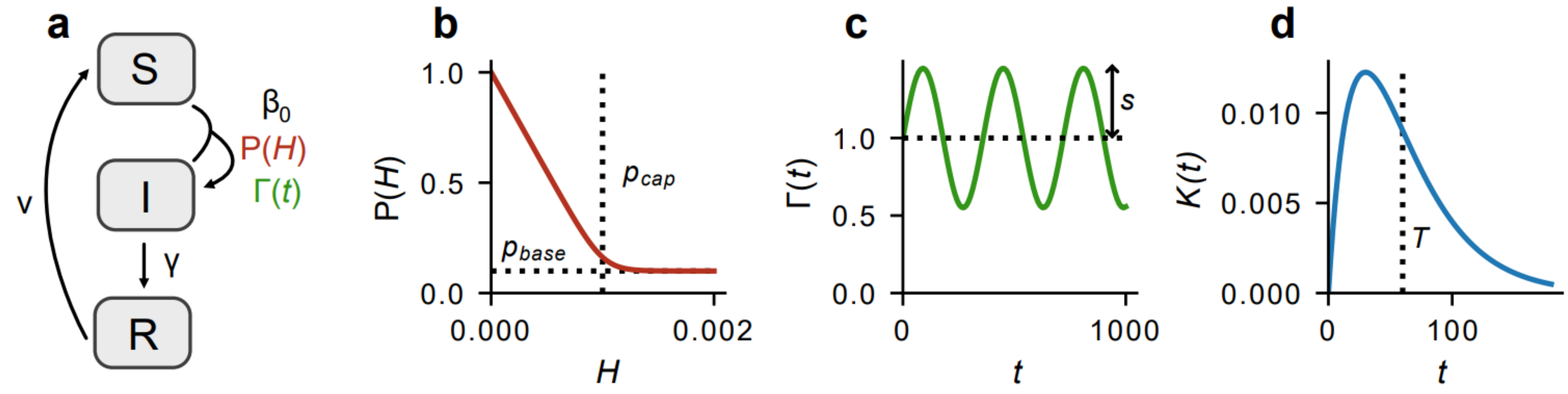


Figure 3: Both the SIRS model and the SIR model with vital dynamics possess to a nontrivial endemic equilibrium. a) A sketch of the model compartments and transitions. In both extensions of the classic SIR model the "pool" of susceptible people is "refilled" which leads to an endemic equilibrium larger than zero as shown in **b)** and **c)**. For this simulation, the rate of waning immunity and the birth-/death rate were set to $\nu = \mu = 1/10\,000\,\mathrm{days}^{-1}$, and the transition rates to $\beta = 1.5/1000\,\mathrm{days}^{-1}$ and $\gamma = 1/1000\,\mathrm{days}^{-1}$, meant to model a disease with slow dynamics where the influence of waning immunity, birth and death cannot be neglected anymore.

Proposed model

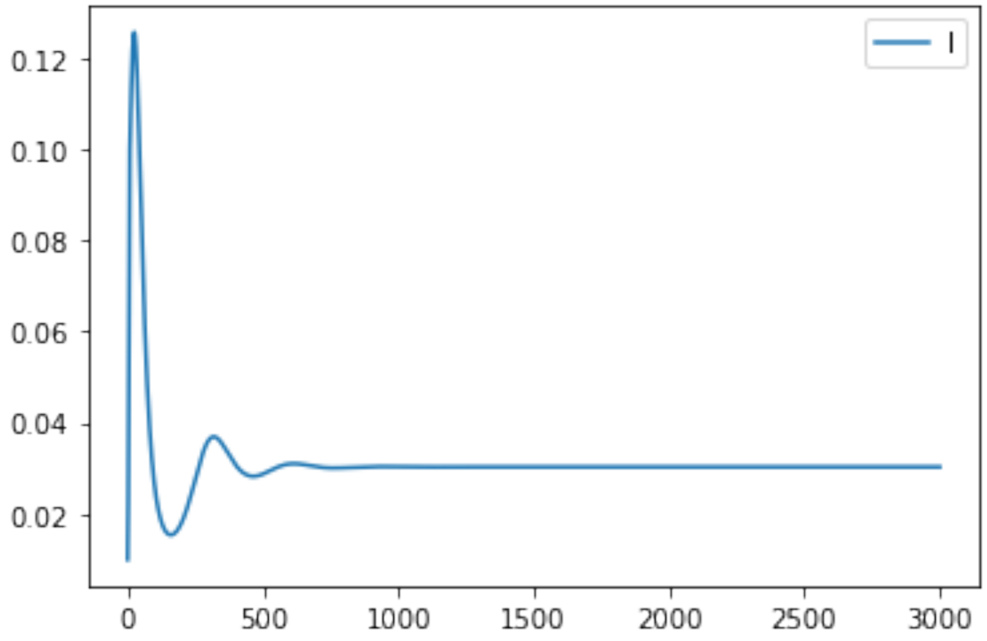


$$\beta(H) = \beta_0 \mathrm{P}(H)$$

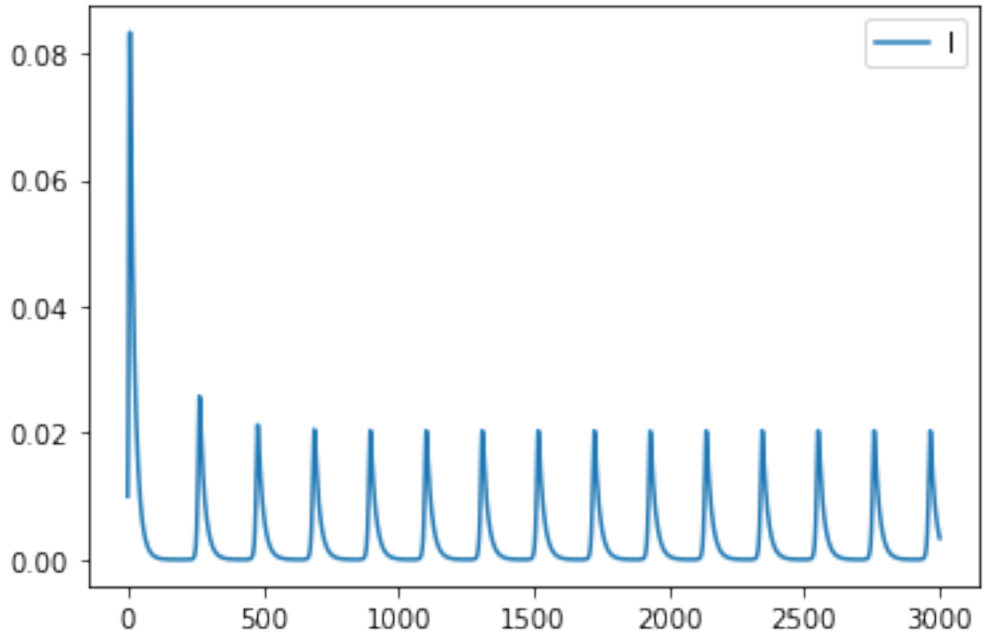
Figure 5: Overview of the proposed model. a) Model compartments and transitions. The transmission rate is modulated by the feedback mechanism and a seasonal forcing shown in **b)** and **c)**, respectively. **d)** Shows the memory kernel given by an Erlang distribution of second order.

SIR with Memory

Stable



Unstable



SIRS model with feedback (no seasonality)

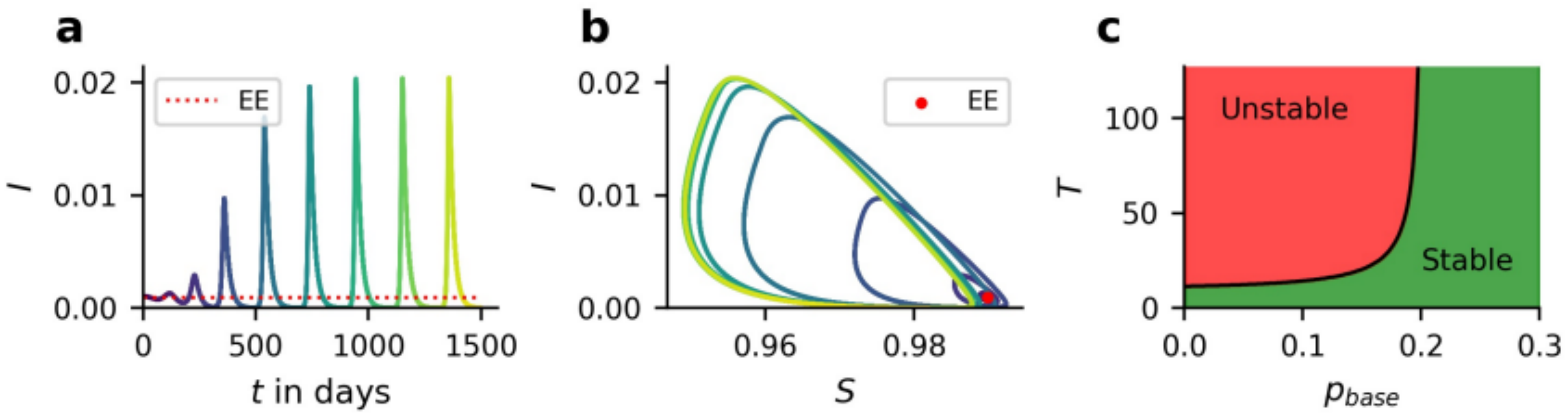


Figure 7: The EE of the SIRS model with a feedback mechanism including memory can turn unstable. a), b) A trajectory initialised close to the endemic equilibrium in the unstable regime of the system spirals outwards onto a stable limit cycle. **c)** The stability diagram using T and p_{base} as bifurcation parameters. In a memory-free system the EE is always stable, whereas in a system with memory the stability can depend on p_{base}. The diagram was computed by calculating the largest eigenvalue of Eq.36.

Chaotic systems - SIR with Memory and Seasons

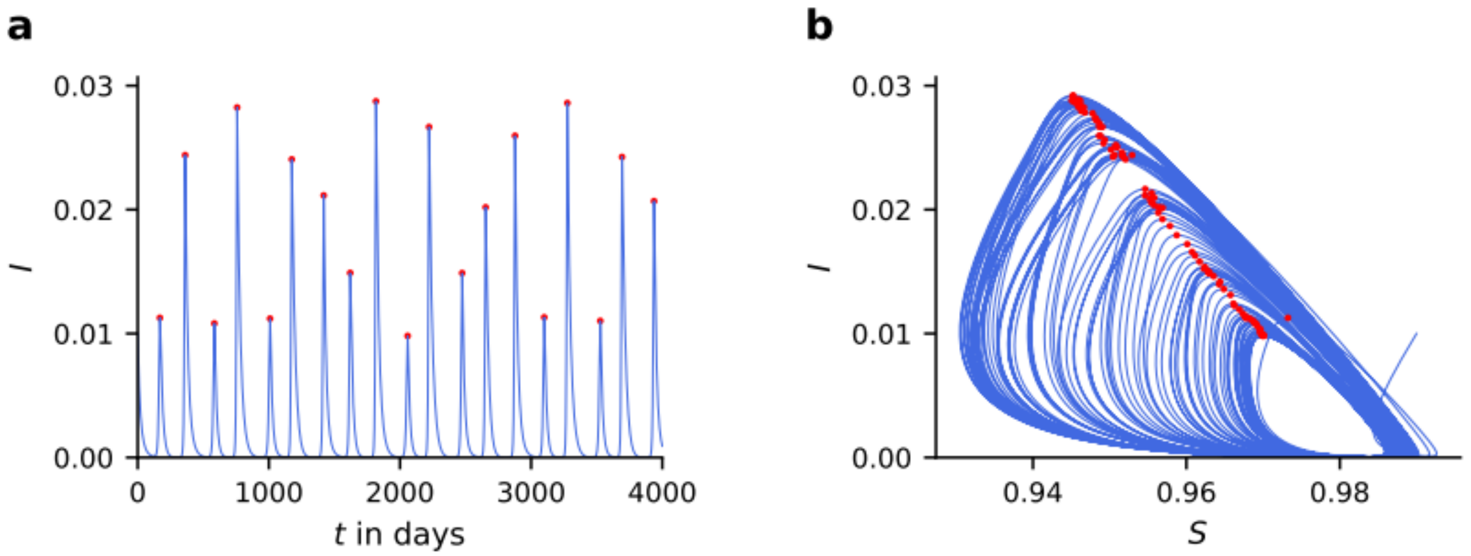


Figure 11: The model displays chaotic-looking dynamics when coupled with a seasonal forcing. a) A timeseries of the infected compartment under the influence of seasonality with $s = 0.3$. **b)** A trajectory in the S-I plane of phase space shows similarities to trajectories of chaotic systems.

Proposed model

Table 1: Parameters and default values of the proposed model.

Parameter	Meaning	Default value	Source
β_0	Transmission rate	$0.5\,\text{days}^{-1}$	[35]
γ	Recovery rate	$0.1\,\text{days}^{-1}$	[36]
ν	Waning rate	$0.01\,\text{days}^{-1}$	[37]
T	Mean memory time of the system	60 days	Chosen
p_{base}	Strongest possible factor that reduces the transmission rate	0.1	Chosen
p_{cap}	Perceived risk beyond which no further reduction of the transmission rate takes place	10^{-3}	Estimated in Fig. 6
ϵ	Curvature parameter	10^{-4}	–
s	Amplitude of seasonal forcing	–	[8]
ω	Frequency of yearly seasonal variation	$\frac{2\pi}{360}\,\text{days}^{-1}$	–

$$\dot{S} = -\beta_0 \mathrm{P}(H)\Gamma(t) I S + \nu R\,,$$

$$\dot{I} = \beta_0 \mathrm{P}(H)\Gamma(t) I S - \gamma I\,,$$

$$\dot{R} = \gamma I - \nu R\,,$$

$$\dot{H}_1 = \frac{2}{T}(I - H_1)\,,$$

$$\dot{H} = \frac{2}{T}(H_1 - H)\,,$$

$$\mathrm{P}(H) = p_{\text{base}} + \frac{(1 - p_{\text{base}})}{p_{\text{cap}}}\epsilon \log\left(1 + \exp\left(\frac{1}{\epsilon}(p_{\text{cap}} - H)\right)\right)$$

$$\Gamma(t) = 1 + s\cos(\omega t)\,.$$

Stability analysis

$$J^* = \begin{pmatrix} -\beta_0 \mathrm{P}(H^*) I^* - \nu & -\beta_0 \mathrm{P}(H^*) S^* - \nu & 0 & -\beta_0 \mathrm{P}'(H^*) S^* I^* \\ \beta_0 \mathrm{P}(H^*) I^* & \beta_0 \mathrm{P}(H^*) S^* - \gamma & 0 & \beta_0 \mathrm{P}'(H^*) S^* I^* \\ 0 & \frac{2}{T} & -\frac{2}{T} & 0 \\ 0 & 0 & \frac{2}{T} & -\frac{2}{T} \end{pmatrix}$$

$$\mathrm{P}(H) = p_{\mathrm{base}} + \frac{(1 - p_{\mathrm{base}})}{p_{\mathrm{cap}}} \epsilon \log \left(1 + \exp \left(\frac{1}{\epsilon} \left(p_{\mathrm{cap}} - H \right) \right) \right)$$

$$\mathrm{P}'(I) = -\frac{\frac{(1 - p_{\mathrm{base}})}{p_{\mathrm{cap}}} \epsilon \exp \left(\frac{1}{\epsilon} \left(p_{\mathrm{cap}} - I \right) \right)}{1 + \exp \left(\frac{1}{\epsilon} \left(p_{\mathrm{cap}} - I \right) \right)} < 0 \quad \forall I$$

$$I^* = \frac{\nu}{\gamma + \nu} \left(1 - \frac{\gamma}{\beta_0 \mathrm{P}(I^*)} \right)$$

$$S^* = \frac{\gamma}{\beta_0 \mathrm{P}(I^*)},$$

$$R^* = \frac{\gamma}{\nu} I^*,$$