Modelling the Co-evolution of Trade and Culture in Past Societies

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ABSTRACT

This paper presents a new framework to study the co-evolution of cultural change and trade. The design aims for a trade-off between the flexibility necessary for the implementation of multiple models and the structure necessary for the comparison between the models implemented. To create this framework we propose an Agent-Based Model relying on agents producing, exchanging and associating values to a list of goods. We first present the key concepts of the framework and two examples of its implementation which allow us to show the flexibility of our framework. We then compare the results obtained by the two models, thus validating the structure of the framework. Finally, we study the price structure obtained to validate one of the implemented model, the trading model.

1 INTRODUCTION

Cultural change comprises the collection of processes that promotes or inhibits the spread of information by social interaction within a population (Boyd and Richerson 2005). An increasing number of social scientists are using an evolutionary framework to model cultural change (Henrich and McElreath 2003). This approach aims at fostering the development of transdisciplinary efforts designed to understand cultural change.

Within the studies done in the evolutionary framework, a cultural phenomenon (such as music) is viewed as a collection of traits (such as musical genres). Multiple biases can explain the fact that certain of the cultural traits are more transmitted than the others. Among the biases studied, some can be explained by the intrinsic properties of a trait (how beneficial it is for the individual using it), while others are explained by the frequency of these traits (how popular a trait is in the culture).

Multiple models can be proposed to represent cultural changes, one of them being the neutral model (Neiman 1995). Within this type of model it is assumed that a trait do not bias the fitness of the individual that acquires it. It therefore means that no bias modifies the rate of transmission of the cultural traits, and that their success will depend only on their frequency in the population. Within analysis of real data, a neutral model produces a distinctive type of frequency distribution of cultural traits termed *power law*.

The *power law* can be replicated within a virtual setup thanks to a simple random copying transmission mechanism (Bentley, Hahn, and Shennan 2004): an individual will copy the traits of a randomly chosen individual with a given probability. This copy can potentially introduce some errors in the acquired trait, which account for innovation processes. The individual will in turn continue to spread these cultural traits which will be further adopted by other individuals. This basic model can be enriched by several

additional processes both in the innovation (Schillinger et al. 2014, Solé et al. 2013, Ziman 2003) and the transmission (Heyes 1994, Henrich and McElreath 2003). Unbiased transmission works as a baseline for identifying frequency-dependent biases: if evidence has higher tendency to copy the most common trait is is known as conformism, while the opposite is defined as anti-conformism.

The archaeological record allow the researchers to identify frequency-dependance biases on cultural transmission over long-term trajectories (Lipo and Madsen 2001, Shennan and Wilkinson 2001, Steele, Glatz, and Kandler 2010). However, the fact that material culture recovered from archaeological contexts is noisy and fragmented presents some challenges on the validity of the method (Kandler and Shennan 2013, Porčić 2014, Crema, Edinborough, Kerig, and Shennan 2014).

This work explores the impact of a crucial element on the transmission of material culture: trade. Networks of good exchanges are being increasingly recognised as key elements that structured ancient societies (Temin 2001, Remesal et al. 2014, Brughmans 2010). The scenarios where this process emerge suggest a complex bias in the selection of cultural traits, which at the same time are also identified as economic goods (Bentley, Lake, and Shennan 2005, Macmillan and Huang 2008). Trasmission is not neutral anymore, as different prices for each good will introduce a dynamic content bias. This affects the frequency of the good within the population, which in turn modifies its price following a co-evolutionary dynamic.

These dynamics are studied using an Agent-Based Model (ABM), a type of simulation particularly useful for studying non-linear dynamics in heterogeneous environments within an evolutionary perspective (Lake 2014). More precisely we propose a framework that can be implemented in multiple ways depending of the model tested. Next section defines the framework, which is based on the basics processes found in evolutionary models of cultural change. Next, we define the implementations used to explore the dynamics of the created framework. Finally, the concluding remarks interpret the results and discuss further possibilities of the presented framework.

2 FRAMEWORK DESCRIPTION

To explore the co-evolution between trade and cultural change we have developed a framework where the different agents produce and trade goods to which they assign variable values. The model is composed of a population Pop of m agents, each defined by 2 vectors of size n. The first corresponds to the quantity of each good owned by the agent i:

$$\forall i \in Pop, \quad Q^i = (q_1^i, \cdots, q_n^i)$$

where Q^i is the total list of possessions of agent i, and q^i_j is the number of goods of type j that agent i possesses.

The second vector reflects the estimation of the value of a good made by an agent i:

$$\forall i \in Pop, \quad V^i = (v_1^i, \dots, v_n^i)$$

where V^i is the total list of estimated values of agent i, and v^i_j is the value that agent i associates to the goods of type j.

On top of these elements four processes are used: production, cultural transmission, innovation and trade. The production process describes the creation of goods by the agent. Once a good is produced by an agent i it is added to its quantity vector (Q^i) . The trade process models the exchange of goods between the agents which results in a modification of the quantity vectors (Q^i) . The amount of goods exchanged is computed by the agents concerned by the trade, within the trade process, based on their value vectors (V^i) . Within the cultural transmission process an agent i can copy the value vector (V^j) of an agent j, where $j \neq i$. Finally, the innovation process also modifies the value vector V^i of an agent, but it differs from the cultural transmission process in that the modification is done without reference to the other agents.

The scheduling of the four processes is described in algorithm 1 along with the vectors modified by each of these processes. Between line 2 and 7 all agents of the population are initialised with empty

quantity vectors and random values. Between the lines 11 and 19 is presented the part of the pseudo-code describing the instructions that will be repeated for each iteration of the simulation. One can note that the *trade* process is called at each iteration while the *cultural transmission* and *innovation* processes are executed only every *CulturalStep*. The idea behind this is to perform the *cultural transmission* based on a score that reflects the performance of the agent and not only one lucky or unlucky trading round. The timestep number used in the figures of this article refers to the number of times the *cultural transmission* and *innovation* processes are called.

Algorithm 1 Model

```
1: INITIALIZATION:
 2: for j \in Goods do
 3:
        for i \in \#Pop/\#Goods do
                                                                              Dilitialize the agent with no goods and a random value vector
 4:
            Q^i = (0, \cdots, 0)
 5:
            V^i = (v_0^i, \cdots, v_n^i)
                                                                                                      \triangleright The values of v_i^i are selected randomly
 6:
        end for
 7: end for
 8: SIMULATION:
 9: loop step \in TimeSteps
10:
        for i \in Pop do
11:
            Production(Q^i)
12:
            for j \in Pop do
                TradeProcess(V^i, Q^i, V^j, Q^j)
13:
14:
15:
            if (step \mod CulturalStep) = 0 then
16:
                CulturalTransmission(V)
17:
                Innovation(V^i)
18:
            end if
19:
        end for
20: end loop
```

In order to validate our model we first reproduce common results from the literature on cultural transmission. We then show that it is possible to transform our model to fit processes that are economically sound, i.e. the model should show the convergence optimal values such as shown in (Gintis 2006). To achieve these two goals, we have designed for each one a specific set of implementations of the four core processes (production, trade, cultural transmission and innovation).

2.1 Neutral Configuration

The first scenario is designed to reproduce unbiased transmission, where each good is a cultural trait without intrinsic positive or negative weight (Bentley, Hahn, and Shennan 2004, Bentley, Lake, and Shennan 2005, Mesoudi and Lycett 2009). Under this hypothesis, the processes of *production* and *trade* are not relevant, and as a consequence, they do not modify the content of the quantity vectors of the agents.

Unbiased *cultural transmission* is implemented using "random copy": each agent has a low probability to pick randomly one agent among all and copy its vector of values. The *innovation* process, termed "unbounded", is triggered with a low probability (μ) and draw a new random value to replace an element v_j^i . The probabilities to trigger *cultural transmission* and *innovation* are presented with other parameters in table 1.

The neutral hypothesis states that the "random copy" transmission and the "unbounded" innovation process used under a fixed population size leads to a distribution of frequency of cultural variants termed *power law*. This distribution is characterized by a small number of very frequent traits and a large number of very rare traits. More formally, this distribution is formalised as:

$$P(v) = C/v^{\alpha}$$

where v is the number of time a variant has been repeated, P(v) the probability to find that variant, C a constant, α a variable describing the slope of the curve obtained. We will therefore attempt to fit as well as

possible the results obtained with this set of implementations to the "power law" distribution by modifying the α parameter.

2.2 Trading Biased Configuration

The second scenario of the four processes of the framework aims at producing economical dynamics. We are interested in the exchange of goods between agents in a barter process where each agent can choose its prices of exchange. We want to implement simple processes leading to the convergence of all prices to values acceptable by all agents, i.e. we would like to observe, at the end of an experiment, all the agents using a set of prices which allow them to trade efficiently.

Production Each agent produces one good. The type of good produced by an agent i is assigned to it at the beginning of the simulation, does not change through the simulation and is referred as $produced^{i}$.

Cultural transmission Social learning is here biased towards the agents which are the best at trading, and is therefore termed "success bias". To achieve this bias, the cultural transmission mechanism used takes into account the value vector of the other agents and relies on two new notions: *need* and *score*.

The *need* is a quantity of good that each agent tries to obtain. This quantity is different for each good but the need for a good is the same for all agents:

$$N=(n_1,\cdots,n_r)$$

The *score* s_j^i of an agent i reflects the ability of this agent to obtain the quantity of good j it needs. It is maximum when the quantity q_j^i that an agent i own of the good j is equal to the need n_j for the good j and lowers proportionally to the distance between the need vector and the quantity vector. It is formally computed as follows for agent i and the good j:

$$s_{j}^{i} = \begin{cases} s_{max} = 1 & \text{if } q_{j}^{i} = n_{j} \\ 1 - \frac{\left| q_{j}^{i} - n_{j} \right|}{\sqrt{\left| (q_{j}^{i})^{2} - (n_{j})^{2} \right|}} & \text{if } q_{j}^{i} \neq n_{j} \end{cases}$$
 (1)

This function ensures that each good has the same weight in the final score (i.e.: managing to get only the right amount of a good with a high "need" value will not give a better score to the agent).

Figure 1 depicts the score of an agent, for different need values of one good, with regards to the quantity of the good possessed by the agent. We can see that even if the need for a good is small (for instance when n(g) = 0.01) an agent can have a high score as soon as it manages to get a quantity close to the need. This insures that an agent needs to have for each of the good a quantity equal to the need and that no agent could reach a better score just by getting the right quantity for a good with a high need.

The complete score of an agent i is termed s^i and corresponds to the sum of the s^i_j . The agents will choose the model from whom the price vector should be copied among the agents that produce the same good and have the highest score. In practice, the worst two percent of the agents (in terms of score) will copy the prices of the best two percent.

Trading During the trading phase the value associated to a good by an agent corresponds to the subjective price of the good for this agent. Briefly summarised, for each good that it does not produce, an agent will trade with the first partner that offers an acceptable trade, i.e. an agent that proposes a satisfiable ratio between the other good and the good produced by the agent.

In more details, the trading phase starts by the agent looking at a first random agent producing another good. Let o be an agent producing g who proposes a trade and r an agent producing k that receives the proposition. As explained earlier, each has a quantity of good Q^o and Q^r . On the one side, o wants to exchange a quantity w_g^o of the good g for a quantity w_k^o of the good g. On the other side, g wants to exchange a quantity g^o of the good g for a quantity g^o and g and g and g and g are the quantities of goods wanted by agent g and g for one trade proposition and are defined by:

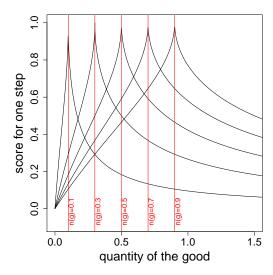


Figure 1: The payoff depending on the quantity owned of a given good and the effective needs n(g) for this good.

$$W^{o} = (w_{g}^{o} = v_{g}^{o}, w_{k}^{o} = \frac{v_{k}^{o}}{v_{g}^{o}}), \quad W^{r} = (w_{k}^{r} = v_{k}^{r}, w_{g}^{r} = \frac{v_{g}^{r}}{v_{k}^{r}})$$
(2)

Where v_j^i are the estimated value of the good j by the agent i as defined earlier. The requested quantity of the non produced good is simply the ratio between the estimated value of the good requested and the estimated value of the produced good.

Once the quantities are defined, the agents declare that the trade is possible if:

$$q_g^o >= w_g^o, \quad q_g^r <= w_g^o, \quad q_k^r >= w_k^o$$
 (3)

$$w_g^o >= (q_g^r + w_g^r), \quad w_k^o <= w_k^r, \quad w_g^o <= w_g^r$$
 (4)

The conditions 3 insure that both agents have enough goods in their inventory to realise the trade while the conditions 4 insure that the quantities of goods fit the will of both agents.

If a trade is possible the two agents will exchange the agreed quantities. If the trade is not possible, the agent will continue to look at random partners for this good until either a partner is found or *TradeThreshold* agents have been tried. At this point the agent will try to trade with agents producing another good. The process goes on until all goods have been tried. This trading process is described in algorithm 2.

Algorithm 2 Trading Process for agent o

```
1: for j \in Goods \& j \neq produced^o do
 2:
        tradeAttempt = 0
 3:
        for r \in Pop \& produced^r = j \& tradeAttempt < TradeThreshold do
 4:
           if acceptableTrade(W_o, W_r) then
 5:
               trade(W_o, W_r)
 6:
 7:
               tradeAttempt = tradeAttempt + 1
 8:
            end if
        end for
10: end for
```

Innovation In a trading environment it seems unlikely that a price will change radically to a very different value. Therefore, a new and more realistic mechanism is proposed. The innovation process,

Parameter	Value
Number of goods	{1,3}
Number of agents	{250,500}
Innovation probability (μ)	{0.004, 0.016, 0.064}
Innovation range (β)	0.005
Cultural transmission probability	.001
CulturalStep	10
TradeThreshold	100
TimeSteps	10000

Table 1: Table of parameters

coined "self referenced", is still triggered with probability μ but modifies the previous price by adding or subtracting a small amount taken randomly from a uniform distribution between $0..\beta$.

Expected outcome

Based on the set of implementations presented and given the equations (1) and (2), it is expected that the prices will converge to value allowing each agent to obtain quantities of resources exactly equivalent to the needs. The best possible price of all good satisfies the equations:

$$\begin{cases} \frac{v_k^o}{v_g^o} = n_k \\ v_g^o = n_g \end{cases} = > v_k^o = n_k \times n_g, \quad \forall k \in Goods, \forall o \in Pop, g = produced^o, k \neq g$$
 (5)

Which means that

$$\forall j \in Goods, \forall i \in Pop \quad \tilde{V}^i = \begin{cases} \tilde{v}^i_j = n_j & \text{if } j = produced^i \\ \tilde{v}^i_j = n_j \times n_{produced^i} & \text{else} \end{cases}$$
 (6)

If such prices are reached, given the exchange rules defined in (2) and the exchange constraints (3) and (4) all exchanges will be optimally achieved, leading to a total score S for each agent of the population :

$$S = \sum_{i=0}^{CulturalStep} s^{i}(\tilde{Q}^{i}) \times ngoods$$

where \tilde{Q}^i is the optimal quantity vector, i.e. the one for which $s^i(\tilde{Q}^i) = s_{max}$. Remember that from equation 1, $s_{max} = 1$.

3 Experimental setup

The neutral model is tested through seven experimental setups. The first six experimental setups are using 1 good, two population sizes (250 and 500 agents) and three values of μ (0.004,0.016 and 0.064). The remaining experimental setup is using 3 goods, 500 agents and μ equal to 0.004. For each setup we have performed 100 runs of 10000 timestep each.

The trading model is tested through four experimental setups using 3 goods, 500 agents and the same four values of μ (0.004,0.016 and 0.064). Again 100 runs of 10000 timesteps are performed.

The complete set of parameters tested on both models is presented in table 1.

4 RESULTS

4.1 Neutral Model

We first analyse the result obtained in the "neutral model". Figure 2 presents the results obtained in the setups using 1 good, two population sizes (250 and 500 agents) and μ varying through three values (0.004,0.016,0.064). The figure represents the average (across all the runs) of the distribution of variants obtained through all experiments. The y-axis shows the frequencies of the variants of the vector values, the x-axis shows how many variant achieves such frequencies. Both axes are in a logarithmic scale. Please note, that here the specific variant achieving high frequency is not studied, only the number of variants achieving such frequencies is presented.

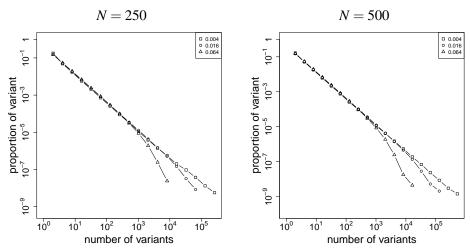


Figure 2: Distribution of frequencies depending on the μ parameter with 250 agents (left) and 500 agents (right). Each plot is the mean obtenaid for 100 runs.

We observe that the lower the mutation rate, the closer from a line the result is. This line corresponds to the "power law" distribution explained in section 2.1 and is typical of the result obtained under the "neutral hypothesis". Nevertheless, under all parameters tested, the value vectors exhibit power law like distribution. The number of agents however has no influence on the distributions obtained.

It is interesting to analyse the slope of these distributions, to compare them to previous experiments testing the "neutral hypothesis". A linear regression is used, for each run, to fit the curve obtained to a function of the form $P(v) = C/v^{\alpha}$. The mean value of all α parameters obtained (across the 100 runs of one setup) and their standard deviation is presented in table 2. We also present side-by-side the results obtained with the same methodology and for the same parameters in (Bentley, Hahn, and Shennan 2004).

Agents Number		Our result		Bentley et. al. 2004	
	μ	α	SD	α	SD
N = 250	0.004	1.53	0.03	1.54	0.02
	0.016	1.57	0.02	1.57	0.01
	0.064	1.66	0.01	1.67	0.01
N= 500	0.004	1.50	0.02	1.53	0.03
	0.016	1.55	0.03	1.61	0.04
	0.064	1.78	0.08	1.81	0.10

Table 2: Mean & SD are calculate on 100 runs for our results, and 5 runs for Bentley et. al. 2004.

We can see in this table that our results are close to those obtained by (Bentley, Hahn, and Shennan 2004). With a similar methodology and parameters (Mesoudi and Lycett 2009) find similar values of α . However, it is difficult to know if the slight differences observed between our work and those previous studies are statistically significant as the two previous studies rely on only five runs for the computation of the mean of α (against 100 in our case).

Additional experiments not presented in this article have shown that the neutral model behaves in the exact same way for two, three and four goods. The value of each good evolves independently and the distribution of values for each good is distributed following the same power law with the same properties than for the experiments performed with one good.

4.2 Distribution of variants

In order to understand the effect of introducing trading mechanisms, we compare first the distribution of values obtained in the "trading model" against the values obtained in the "neutral model". The figure 3 presents the results obtained from 100 runs for each model. All runs rely on the same experimental setup using 3 goods, 500 agents and μ equal to 0.004.

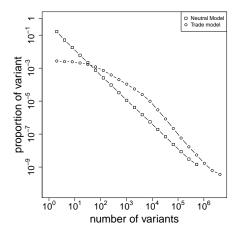
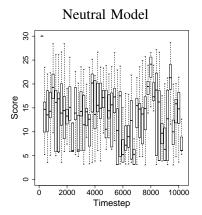


Figure 3: Frequency distribution for the neutral and the trading models. Each points represent the mean for 100 runs.

On this figure it appears that the implementation of trade mechanisms leads to a distribution of prices departing from the neutral hypothesis. In more details, the frequency distribution has a plateau of common prices (a number of prices share similar and high frequency), which shows that, when trade is taken into account, the most common variants are more diverse. Additional investigations (not presented here) have shown that this is due to the *innovation* process used in the trading model which prevents the creation of totally random new price and therefore leads to the creation of few similar prices obtaining high scores.

4.3 Study of scores

We now study the results obtained in more detail by investigating the ability of the population to find the good price to perform exchanges between each other. This is done first by observing the score of all agents in each of the two different models. The figure 4 uses again the results obtained from 100 runs for each model where all runs rely on the same experimental setup using 3 goods, 500 agents and μ equal to 0.004. The figure is showing, as boxplots, the score computed thanks to equation (1) for all agents of all runs. The y-axis shows the score computed and the x-axis shows the timesteps. The left plot shows the results obtained in the "cultural transmission" model and the right plot shows the results obtained in the trading model.



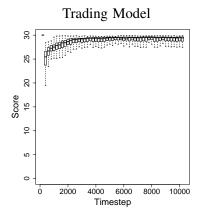


Figure 4: Evolution of the score within the two different models for two typical run with 500 agents and 3 goods evolving during 10000 timestep.

As expected, the scores within the neutral model vary randomly. "Trends" may appear, where a bigger proportion of individuals adopt a better price that allow agents to reach better score (such as around iteration 8000), but such good score fall back as soon as another trend appears. However, with the trading model, the score of all the agents increase. As the selection mechanism allow them to know who has found better vectors of prices, they will progressively adopt prices vector that allow all of them to reach better scores.

The previous figure showed the capacity for the trading model to increase its score but did not analyse the exact prices used. As explained in the section 2.2 we expect that the trade *cultural transmission* and *innovation* processes will produce a convergence toward a set of price for each good that will allow agents to exchange optimally the good they produce with the other goods. To verify this assumption we analyse the prices reached during the simulations. These are presented in figure 5 for the 100 runs relying the experimental setup using 3 goods, 500 agents and μ equal to 0.004. For all runs, all agents and at each iteration we compute the difference between the price used by the agent V_g and the optimal price \tilde{V}_g . Record that the optimal price for a good that an agent does not produce is the product between the need for this good and the need for the produced good, while the optimal price for the produced good is simply the need for it. All the measures performed are presented as boxplots where the results for 100 runs are condensed.

We observe that prices are indeed converging to the optimal prices which means that the agents within the trading model are indeed improving their scores by reaching the optimal prices. Notably, a similar variation of prices was observed in the closely related economical model of (Gintis 2006). This variation of the prices to the optimum offers an additional conformation of the validity of the trading model.

5 CONCLUDING REMARKS

This article has proposed a framework to simultaneously study cultural change and trade dynamics. The development of our framework was first aimed at simplicity which is achieved by the use of two vectors (quantity and value) and four processes (production, trade, cultural transmission and innovation). The second aim of the work conducted was to obtain a flexible framework which is possible since each of the processes can be implemented accordingly to the question studied. We have shown the validity of this approach by reproducing expected results on both the cultural and trade side. On the cultural transmission side we have shown that the implementation of a "neutral model" leads to the expected observations on the variants of the vector value: a power law. When implementing trading mechanisms we observe the convergence of prices to the expected values and the improvement of the scores of the agents.

The successful implementation of a trading model within our framework shows that trading models can be viewed as a particular implementation of the cultural evolutionary framework. This aspect which has not been studied before, opens a new line of thinking on the weight of trade mechanisms within cultural

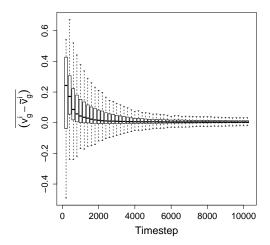


Figure 5: Evolution of the mean of the difference between the estimated value v_g^i and the optimal value \tilde{v}_g^i (as calculated with the equations 6) for a good g and an agent i. As the optimal value \tilde{v}_g^i depends on which good is produced by i, the mean of the difference between the estimated price and the optimal one is computed between all the agents that produce the same good. The figure represents this mean computed at each timestep for each goods, for each groups of agents and for 100 runs in a setup with 500 agents and 3 goods.

change. Notably, we have shown that the implementation of trade mechanisms leads to a distribution of prices departing from the neutral hypothesis, which is the reference in the study of cultural changes. Within the trading model, the frequency distribution has a plateau of common prices (a number of prices share similar and high frequency), which shows that, when trade is taken into account, the most common variants are more diverse. Interestingly, we have not find references pointing at the ability of trade mechanisms to keep a relatively large diversity in the frequency distribution. We suspect that this is mainly because it is not common to compare results of trading models to other cultural evolution models. It would therefore be interesting to compare the frequency distributions obtained by our model and the ones observed in current or past economies. On the side of cultural transmission, the results obtained can also be compared to the ones obtained when prestige biased cultural transmission is used. The idea behind this last comparison, is that trade could be interpreted as a particular prestige biased cultural transmission and therefore be fully integrated within the cultural evolutionary frameworks.

In future works more realistic and complex dynamics can be studied in the same model. First, multiple implementations of the production mechanism can be studied so as to increase the number of goods per agent and include factors such as the meteorology or the various type of goods. Moreover, the introduction of the distinction between "vital" and "common" goods will be useful for the creation of models studying in further details the interaction between trade and cultural transmission. The trading mechanism itself can be naturally implemented in different ways, each reflecting a specific theory, and thus allowing the comparison of the different theories. Moreover, the trade network (which in this work can be interpreted as fully connected) can be modified to study the effect of slow connections or the rupture of certain connections. Finally, the agent themselves could become more complex and be endowed with the ability to learn behaviours which would again produce more realistic simulations.

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The model was created using Pandora (Rubio-Campillo 2014). R was used for figures and statistical analysis (R Development Team 2012). The estimation of the α parameters have been computed with the R package poweRlaw (Gillespie 2015). The simulations have been done in the supercomputer MareNostrum at Barcelona Supercomputing Center - Centro Nacional de Supercomputación (The Spanish National Supercomputing Center). The source code of the model is licensed under a GNU General Public License and will be available for download at publication.

REFERENCES

- Bentley, R. A., M. W. Hahn, and S. J. Shennan. 2004. "Random drift and culture change". *Proceedings of the Royal Society of London. Series B: Biological Sciences* 271 (1547): 1443–1450.
- Bentley, R. A., M. W. Lake, and S. J. Shennan. 2005. "Specialisation and wealth inequality in a model of a clustered economic network". *Journal of Archaeological Science* 32 (9): 1346–1356.
- Boyd, R., and P. J. Richerson. 2005. The origin and evolution of cultures. Oxford University Press.
- Brughmans, T. 2010. "Connecting the dots: towards archaeological network analysis". *Oxford Journal of Archaeology* 29 (3): 277–303.
- Crema, E., K. Edinborough, T. Kerig, and S. Shennan. 2014. "An Approximate Bayesian Computation approach for inferring patterns of cultural evolutionary change". *Journal of Archaeological Science* 50:160–170
- Gillespie, C. S. 2015. "Fitting Heavy Tailed Distributions: The poweRlaw Package". *Journal of Statistical Software* 64 (2): 1–16.
- Gintis, H. 2006. "The emergence of a price system from decentralized bilateral exchange". *Contributions in Theoretical Economics* 6 (1): 1–15.
- Henrich, J., and R. McElreath. 2003. "The evolution of cultural evolution". *Evolutionary Anthropology: Issues, News, and Reviews* 12 (3): 123–135.
- Heyes, C. M. 1994. "Social learning in animals: categories and mechanisms". *Biological Reviews* 69 (2): 207–231.
- Kandler, A., and S. Shennan. 2013. "A non-equilibrium neutral model for analysing cultural change". *Journal of Theoretical Biology* 330:18–25.
- Lake, M. W. 2014, June. "Trends in Archaeological Simulation". *Journal of Archaeological Method and Theory* 21 (2): 258–287.
- Lipo, C. P., and M. Madsen. 2001. "Neutrality, "style", and drift: building methods for studying cultural transmission in the archaeological record". In *Style and Function: Conceptual Issues in Evolutionary Archaeology*, edited by T. D. Hurt and R. Gordon F.M., 91–118. Bergin and Garvey.
- Macmillan, W., and H. Huang. 2008, March. "An agent-based simulation model of a primitive agricultural society". *Geoforum* 39 (2): 643–658.
- Mesoudi, A., and S. J. Lycett. 2009, January. "Random copying, frequency-dependent copying and culture change". *Evolution and Human Behavior* 30 (1): 41–48.
- Neiman, F. D. 1995. "Stylistic Variation in Evolutionary Perspective: Inferences from Decorative Diversity and Interassemblage Distance in Illinois Woodland Ceramic Assemblages". *American Antiquity* 60 (1): pp. 7–36.
- Porčić, M. 2014. "Exploring the Effects of Assemblage Accumulation on Diversity and Innovation Rate Estimates in Neutral, Conformist, and Anti-Conformist Models of Cultural Transmission". *Journal of Archaeological Method and Theory*:1–22.
- R Development Team 2012. "R: A language and environment for statistical computing".

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- Remesal, J., A. Díaz-Guilera, B. Rondelli, X. Rubio-Campillo, A. Aguilera, D. Martín-Arroyo, A. Mosca, and G. Rull. 2014. "The EPNet Project. Production and distribution of food during the Roman Empire: Economics and Political Dynamics". In *Information Technologies for Epigraphy and Cultural Heritage: Proceedings of the First EAGLE International Conference*, 455–464. Paris, France: Sapienza Universitá.
- Rubio-Campillo, X. 2014. "Pandora: A Versatile Agent-Based Modelling Platform for Social Simulation". In *Proceedings of SIMUL 2014, The Sixth International Conference on Advances in System Simulation*, 29–34. IARIA Publishing.
- Schillinger, K., A. Mesoudi, and S. Lycett. 2014. "Copying error and the cultural evolution of "additive" vs. "reductive" material traditions: an experimental assessment". *American Antiquity* 79 (1): 128–143.
- Shennan, S. J., and J. R. Wilkinson. 2001. "Ceramic Style Change and Neutral Evolution: A Case Study from Neolithic Europe". *American Antiquity* 66 (4): 577.
- Solé, R., S. Valverde, M. R. Casals, S. A. Kauffman, D. Farmer, and N. Eldredge. 2013. "The evolutionary ecology of technological innovations". *Complexity* 18 (4): 15–27.
- Steele, J., C. Glatz, and A. Kandler. 2010. "Ceramic diversity, random copying, and tests for selectivity in ceramic production". *Journal of Archaeological Science* 37 (6): 1348 1358.
- Temin, P. 2001. "A market economy in the early Roman Empire". Journal of Roman studies 91:169–181.
- Ziman, J. 2003. Technological innovation as an evolutionary process. Cambridge University Press.