# 1 Introduction

Historical sciences have now a rich trajectory of applying computer simulation to answer research questions. They provide the needed methodological tools to evaluate complex scenarios where non-linear trajectories unfold over long-term time spans and where the contingency of events is central.

These computer models have been often classified in three types based on their goal: a) hypothesis-testing, b) tactical simulations and c) theory-building. Most models follow the first option, which allows the research to evaluate the plausibility of working hypotheses against empirical evidence. The second option is focused on testing methodological challenges of the different discipline.

We focus here in the third type, which explores abstract scenarios with the aim of generating new ideas and knowledge using a stylized model of the studied system. While it is true that any hypothesis needs to be finally tested against evidence, the heuristics utility of these theory-building in other scenarios clearly shows the potential of this approach for any scientific discipline.

This work examines the epistemological and methodological challenges posed by this type of model and why we think they can be fruitfully uses to study History by comparing it with Evolutionary Biology. Its utility is then exemplified by presenting two different Agent-Based Model exploring the co-evolution of trade and culture within the Roman Empire. The first one explore the dynamics of a simple goods exchange economy under different cultural network constraint, the second focus on the impact of the learning abilities on the evolution of a Producer/Consumer system.

# 2 General issues

Incompleteness and uncertainty of the historical and archaeological record affect historical interpretation [1] but *formal modelling* and computer simulation are valuable tools to overcome such limitations.

Evolutionary biologists have successfully embraced these tools long time ago and, as we explain later, the epistemological issues they are confronted to are close to what archaeologists and historians face. We thus suggest that it is a good example to follow.

Indeed, the goal of Evolutionary Biologist is to understand the mechanisms at the origin of the living world as we can observe it. To do so and Assuming the theory of evolution, they characterise the succession of past events that constitute the Story of Life. Starting with Gould [2], several biologists and philosophers have argued that the nature of this research activity *is* historical [3]: the actual biological world does not depend *only* on biological rules, but on the uniqueness of the succession of events. To encompass the issues raised by such historicity, evolutionary biologists use, at least since the Modern Synthesis, formal models to figure out different possible successions of events and the likelihood of such possible historical paths, and they test it against the available data. The more visible example of such use of computer model is exemplified by the major and always growing role played by *Bioinformatics* in a lot of areas in Biology during the last three decades.

This suggest that (i) the problems encountered by evolutionary biologists are close to those archaeologists and historians have to face (ii) the way inferences are made about the history of living beings and the history of human societies fall into a similar epistemological framework and (iii) mathematical and computer models are good candidates to infer, in a statistically plausible and transparent way, missing data and complex hypotheses in both enterprise.

Moreover, this use of computer simulations and modelling is not restricted to biology and evolutionary Biology. It is now widely spread in all branches of Science. People studying computer simulation *per se*, in the field which is commonly called "artificial life" [4], argue that computer simulation are powerful heuristic tools that combine the exploratory power of thought experiments and the logical strength of mathematics [5]. They allow to test quickly a lot of possible "opaque though experiments" that would be impossible to execute mentally. As though experiment, they procure to experimenters a freedom impossible to reach with traditional experiments doing measures directly on the object of study (galaxies in physics, human in psychology,. . . ), while as running on computer, they can handle operation lasting very long timespan and involving range of parameters that one could not handle with his own mind.

Moreover, in complex systems where the interactions of every subcomponents are multiple, such as in human society where the social-cultural environment interacts with biological and economical evolution, the global dynamics are difficult to predict analytically, which make simulation and computer modelling one of the best suitable tool to explore and study those mechanisms.

However, building computational simulations that can provide valuable knowledge about the modelled object still remains a difficult task. Computer scientists have to be aware of every assumptions they could implicitly made and domain experts (as historians or biologists) have to formulate their hypotheses in an epistemological framework yet not clearly specified and far from the one they are used. The communication is thus primordial: knowledge here does not lie in the mathematical models neither in the historical data but emerges from the well articulation of both side [6].

# 3 Evolutionary Model of the co-evolution of a trade and culture

This section presents an exemple of a new framework with wich we want to study the co-evolution of cultural change and trade. This model was designed to propose a trade-off between the flexibility necessary for the implementation of multiple models, that could be based on historical and archaeological assumption, hypothesis and observation; and the structure necessary for the comparison between the models implemented. To create this framework we propose an Agent-Based Model relying on agents producing, exchanging and associating values to a list of goods. We present the key concepts of the framework and two examples of its implementation which allow us to show the flexibility of our framework. Moreover, we compare the results obtained by the two models, thus validating the structure of the framework. Finally, we validate the implementation of a trading model by studying the price structure it produces.

## 3.1 Trade & Cultural Evolution

Cultural change comprises the collection of processes that promotes or inhibits the spread of information by social interaction within a population [7]. An increasing number of social scientists are using an evolutionary framework to model cultural change [8]. This approach aims at fostering the development of transdisciplinary efforts designed to understand cultural change.

Within the studies done in the evolutionary framework, a cultural phenomenon (such as music) is viewed as a collection of traits (such as musical genres). Multiple biases (mechanism favouring the use of a cultural trait over an other) can explain the fact that certain of the cultural traits are transmitted more readily than the others. Among the biases studied, some can be explained by the intrinsic properties of a trait (how beneficial it is for the individual using it), while others are explained by the frequency of these traits (how popular a trait is in the culture).

Multiple models can be proposed to represent cultural changes, one of them being the neutral model [9]. Within this type of model it is assumed that a trait does not bias the fitness of the individual that acquires it. It therefore means that no bias modifies the rate of transmission of the cultural traits, and that their success will depend only on their frequency in the population. Within analysis of real data, a neutral model produces a distinctive type of frequency distribution of cultural traits termed *power law*.

The *power law* can be replicated within a virtual setup thanks to a simple random copying transmission mechanism [10]: an individual will copy the traits of a randomly chosen individual with a given probability. This copy can potentially introduce some errors in the acquired trait, which account for innovation processes. The individual will in turn continue to spread these cultural traits which will be further adopted by other individuals. This basic model can be enriched by several additional processes both in the innovation [11–13] and the transmission [8, 14]. Unbiased transmission works as a baseline for identifying frequency-dependent biases: if evidence has higher tendency to copy the most common trait it is known as conformism, while the opposite is defined as anti-conformism.

The archaeological record allows the researchers to identify frequency-dependence biases on cultural transmission over long-term trajectories [15–17]. However, the fact that material culture recovered from archaeological contexts is noisy and fragmented presents some challenges on the validity of the method [18–20]

This work explores the impact of a crucial element on the transmission of material culture: trade. Networks of good exchanges are being increasingly recognised as key elements that structured ancient societies [21–23]. The scenarios where this process emerges suggest a complex bias in the selection of cultural traits, which at the same time are also identified as economic goods [24, 25]. Trasmission is not neutral anymore, as different prices for each good will introduce a dynamic content bias. This affects the frequency of the good within the population, which in turn modifies its price following a co-evolutionary dynamic.

These dynamics are studied using an Agent-Based Model (ABM), a type of simulation particularly useful for studying non-linear dynamics in heterogeneous environments within an evolutionary perspective [26]. More precisely we propose a framework that can be implemented in multiple ways depending on the model tested. Next subsection defines the framework, which is based on the basic processes found in evolutionary models of cultural change. Next, we define the implementations used to explore the dynamics of the created framework. In the following subsection we analyse the results obtained with these two implementations. Finally, the concluding remarks discuss further possibilities of the presented framework.

## 3.2 Framework Description

To explore the co-evolution between trade and cultural change we have developed a framework where the different agents produce and trade goods to which they assign variable values. The model is composed of a population *Pop* of $m$ agents, each defined by 2 vectors of size $n$. The first corresponds to the quantity of each good owned by the agent $i$:

$$\forall i \in Pop, \quad Q^i = (q^i_1, \cdots, q^i_n)$$

where $Q^i$ is the total list of possessions of agent $i$, and $q^i_j$ is the number of goods of type $j$ that agent $i$ possesses.

The second vector reflects the estimation of the value of a good made by an agent $i$:

$$\forall i \in Pop, \quad V^i = (v^i_1, \cdots, v^i_n)$$

where $V^i$ is the total list of estimated values of agent $i$, and $v^i_j$ is the value that agent $i$ associates to one unit of a good of type $j$.

On top of these elements five processes are used: *production*, *consumption*, *cultural transmission*, *innovation* and *trade*. The *production* process describes the creation of goods by the agent. Once a good is produced by an agent $i$ it is added to its quantity vector ($Q^i$). The consumption strategy of these goods is defined in the *consumption* process which decreases the number of goods in the vector ($Q^i$). In this model, all goods are completely consumed at each iteration for all the models tested. The *trade* process models the exchange of goods between the agents which results in a modification of the quantity vectors ($Q^i$). The amount of goods exchanged is computed by the agents involved in the trade, within the *trade* process, based on their value vectors ($V^i$). Within the *cultural transmission* process an agent $i$ can copy the entire value vector ($V^j$) of an agent $j$, where $j \neq i$. Finally, the *innovation* process also modifies the value vector $V^i$ of an agent, but it differs from the *cultural transmission* process in that the modification is done without reference to the other agents.

The scheduling of the five processes is described in algorithm 1 along with the vectors modified by each of these processes. On lines 3 and 4 all agents of the population are initialised with empty quantity vectors and random values. The code used to update the status of each agent at each iteration is presented between the lines 8 and 22. One can note that each of the five processes is executed synchronously by all agents. Moreover, the *trade* process is called at each iteration while the *cultural transmission* and *innovation* processes are executed only every $CulturalStep$. The idea behind this is to perform the *cultural transmission* based on a score that reflects the performance of the agent and not only one lucky or unlucky trading round. The timestep number used in all the figures presenting the results of this model refers to the number of times the *cultural transmission* and *innovation* processes are called.

In order to validate our model we first reproduce common results from the literature on cultural transmission. We then show that it is possible to transform our model to fit processes that are economically sound, i.e. the model should show the convergence optimal values such as shown in [27]. To achieve these two goals, we have designed for each one a specific set of implementations of the five core processes (production, consumption, trade, cultural transmission and innovation).

### 3.2.1 Neutral Configuration

The first scenario is designed to reproduce unbiased transmission, where each good is a cultural trait without intrinsic positive or negative weight [10, 24, 28]. Under this hypothesis, the processes of *production* and *trade* are

**Algorithm 1** Model

INITIALIZATION:
**for** $i \in \#Pop$ **do** ▷ Initialize the agent with no goods and a random value vector
  $Q^i = (0, \cdots, 0)$
  $V^i = (v^i_0, \cdots, v^i_n)$ ▷ The values of $v^i_j$ are selected randomly
**end for**
SIMULATION:
**loop** $step \in TimeSteps$
  **for** $i \in Pop$ **do**
    $Production(Q^i)$
  **end for**
  **for** $i \in Pop$ **do**
    **for** $j \in Pop$ **do**
      $TradeProcess(V^i, Q^i, V^j, Q^j)$
    **end for**
  **end for**
  **for** $i \in Pop$ **do**
    $ConsumeGoods(Q^i)$ ▷ All goods are consumed
    **if** $(step \mod CulturalStep) = 0$ **then**
      $CulturalTransmission(V)$
      $Innovation(V^i)$
    **end if**
  **end for**
**end loop**

not relevant, and as a consequence, they do not modify the content of the quantity vectors of the agents.

Unbiased *cultural transmission* is implemented using "random copy": each agent has a low probability to pick randomly one agent among all and copy its vector of values. The *innovation* process, termed "unbounded", is triggered with a low probability ($\mu$) and draw a new random value to replace an element $v^i_j$.

The neutral hypothesis states that the "random copy" transmission and the "unbounded" innovation process used under a fixed population size leads to a distribution of frequency of cultural variants termed *power law*. This distribution is characterized by a small number of very frequent traits and a large number of rare traits. The main difference with similar distributions, such as exponential distribution, is that the rare traits are far from being absent of the distribution, i.e. the tail of the distribution is large.

This distribution is formalised as :

$$P(v) = C/v^{\alpha}$$

where $v$ is the number of times a variant has been repeated, $P(v)$ the probability to find that variant, $C$ a constant, $\alpha$ a variable describing the slope of the curve obtained. We will therefore attempt to fit as well as possible the results obtained with this set of implementations to the "power law" distribution by modifying the $\alpha$ parameter.

### 3.2.2 Trading Biased Configuration

In the second scenario, we are interested in the exchange of goods between agents in a barter process where each agent can choose its prices of exchange. We want to implement simple processes leading to the convergence of all prices to values acceptable by all agents, i.e. we would like to observe, at the end of an experiment, all the agents using a set of prices which allow them to trade efficiently.

**Production** Each agent produces one good. The type of good produced by an agent $i$ is assigned to it at the beginning of the simulation, does not change through the simulation, and is referred to as $produced^i$. At each time step, each agent, produces a number of units (of its production good) equal to the number of goods, which ensures that enough is owned to be traded for other goods. Moreover, when an agent consumes its own production good, it does not impact its inventory.

**Cultural transmission** Social learning is here biased towards the agents which are the best at trading, and is therefore termed "success bias". To achieve this bias, the cultural transmission mechanism used takes into account the value vector of the other agents and relies on two new notions: *need* and *score*.

The *need* is a quantity of good that each agent tries to obtain. This quantity is different for each good but the need for a good is the same for all agents:

$$N = (n_1, \cdots, n_r)$$

The *score* $s^i_j$ of an agent $i$ reflects the ability of this agent to obtain the quantity of good $j$ it needs. It is maximum when the quantity $q^i_j$ that an agent $i$ owns of the good $j$ is equal to the need $n_j$ for the good $j$ and lowers proportionally to the distance between the need vector and the quantity vector. It is calculated in such a way that each good has the same weight in the final score, i.e. managing to get only the right amount of a good with a high "need" value will not give a better score to the agent and is formal description is given in the Annex

The complete score of an agent $i$ is termed $s^i$ and corresponds to the sum of the $s^i_j$. An agent will choose from whom the price vector should be copied among the agents that produce the same good and have the highest score. In practice, the worst (in terms of score) 20 percent of the agents producing the same good will copy the prices of the best twenty percent producing the same good. The algorithmic description of this selection process is given in the Annex with the algorithm 2.

**Trading** During the trading phase the value associated to a good by an agent corresponds to the subjective price of the good for this agent. Briefly summarised, for each good that it does not produce, an agent will trade with the first partner that offers an acceptable trade, i.e. an agent that proposes a satisfiable ratio between the other good and the good produced by the agent.

If a trade is possible, the two agents will exchange the agreed quantities. If the trade is not possible, the agent will continue to look at random partners for this good until either a partner is found or $TradeThreshold$ agents have been tried. At this point the agent will try to trade with agents producing another good. The process goes on until all goods have been tried. This trading process is described in the algorithm 3 in the Annex session.

**Innovation** In a trading environment it seems unlikely that a price will change radically to a very different value. Therefore, a new and more realistic mechanism is proposed. The innovation process, coined "self referenced", is still triggered with probability $\mu$ but modifies the previous price by adding or subtracting a small amount taken randomly from a uniform distribution between $0..\beta$.

**Expected outcome** Based on the set of implementations presented and given the equations (3) and (4), it is expected that the prices will converge to value allowing each agent to obtain quantities of resources exactly equivalent to the needs. The best possible price of all good satisfies the equations :

$$\begin{cases} \frac{v^o_k}{v^o_g} = n_k \\ v^o_g = n_g \end{cases} \quad => v^o_k = n_k \times n_g, \quad \forall k \in Goods, \forall o \in Pop, g = produced^o, k \neq g \tag{1}$$

Which means that

$$\forall j \in Goods, \forall i \in Pop \quad \tilde{V}^i = \begin{cases} \tilde{v}^i_j = n_j & \text{if } j = produced^i \\ \tilde{v}^i_j = n_j \times n_{produced^i} & \text{else} \end{cases} \tag{2}$$

If such prices are reached, given the exchange rules defined in (4) and the exchange constraints (5) and (6) all exchanges will be optimally achieved, leading to a total score $S$ for each agent of the population :

$$S = \sum_{i=0}^{CulturalStep} s^i(\tilde{Q}^i) \times ngoods$$

where $\tilde{Q}^i$ is the optimal quantity vector, i.e. the one for which $s^i(\tilde{Q}^i) = s_{max}$. Remember that from equation 3, $s_{max} = 1$.

#### 3.2.3 Experimental setups

The neutral model is tested through 15 experimental setups. The first six experimental setups are using 1 good, two population sizes (250 and 500 agents) and three values of $\mu$ (0.004,0.016 and 0.064). The remaining experimental setups are using 500 agents, 3 number of goods (3,6 and 9) and three values of $\mu$ (0.004,0.016 and 0.064). For each setup, we have performed 100 runs of 10000 timestep each. The trading model is tested on an experimental setups using 3 goods, 500 agents and $\mu$ equal to 0.004. Again 100 runs of 10000 timesteps are performed. The experiments, as well as the parameters used to run those experiments, are available online [29] for the Pandora simulator [30].

### 3.3 Results

#### 3.3.1 Neutral Model

We first analyse the result obtained in the "neutral model" with one good. The figure 1 presents the results obtained for two population sizes $N$ (250 and 500 agents) and $\mu$ varying through three values (0.004,0.016,0.064). The figure is a log-log plot of the average (across all the runs) of the distribution of variants obtained through all experiments. The y-axis shows the proportions of the variants of the prices used during the simulation, the x-axis shows how many variant achieves such proportions.

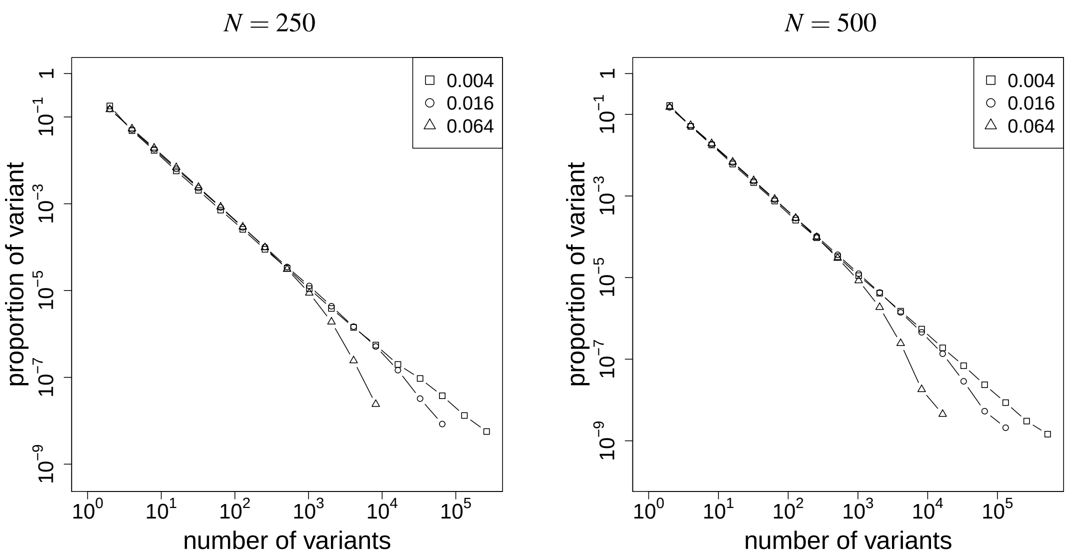


Figure 1: Distribution of proportions depending on the $\mu$ parameter with 250 agents (left) and 500 agents (right). Each plot is the mean obtenaid for 100 runs.

We observe that the lower the mutation rate, the closer to a line the result is. This line corresponds to the "power law" distribution explained in subsection 3.2.1, and is typical of the result obtained under the "neutral hypothesis". In order to verify if the distribution is in fact a power-law, we follow the method proposed by [31] and the R implementation proposed by [32]. Briefly, two values are returned: a) the estimation of the $\alpha$ parameter of the power law equation $P(v) = C/v^{\alpha}$; b) a $p$-value testing the null-hypothesis that our data could have been generated from a power law distribution.

The table 1 summarizes the results obtained on all setups. Each value shown in the table is the mean values of 100 simulations. We see that in almost every case the null hypothesis cannot be rejected, which means that indeed the repartition of the price follows a power law. The only exception is for $\mu$ equal to 0.064, where the $p$-value is less than 0.05. In this last case the null hypothesis is rejected, and we therefore assume that the distribution does not follow a power law.

Table 1: Mean $\alpha$ & *sd* are calculated on 100 runs for our results, and 5 runs for Bentley et. al. 2004. *pr* is the percentage of run for which the *p*-value is less than 0.05, i.e. the percentage of runs for which we rejected the null-hypotheses stating that the distribution follow a power law.

| | | Our result | | Bentley et. al. 2004 |
|---|---|---|---|---|
| N | $\mu$ | $\alpha$ (*sd*) | *p*-value (*sd* - *pr*) | $\alpha$ (*sd*) |
| 250 | 0.004 | 1.53 (0.03) | 0.58 (0.24 - .01) | 1.54 (0.02) |
| | 0.016 | 1.57 (0.02) | 0.35 (0.28 - .05) | 1.57 (0.01) |
| | 0.064 | 1.66 (0.01) | 0.0 (0.00 - 1) | 1.67 (0.01) |
| 500 | 0.004 | 1.50 (0.02) | 0.59 (0.28 - .02 ) | 1.53 (0.03) |
| | 0.016 | 1.55 (0.03) | 0.15 (0.17 - .10 ) | 1.61 (0.04) |
| | 0.064 | 1.78 (0.08) | 0.0 (0.00 - 1) | 1.81 (0.10) |

For comparison purposes the results obtained by [10] (which tested the "neutral hypothesis" with the same methodology) are added in the last column of table 1.

It allows us to statistically test our model and to compare it to other already existing in the literature. We show here that our model is consistent with the existing literature and thus with already observed and known social phenomenon.

#### 3.3.2 Distribution of variants

In order to understand the effect of introducing trading mechanisms, we compare first the distribution of values obtained in the "trading model" against the values obtained in the "neutral model". The figure 2.a) presents the results obtained from 100 runs for each model. All runs rely on the same experimental setup using 3 goods, 500 agents and $\mu$ equal to 0.004. In all following graph a variant is one price of one given good. The distributions are first computed for each good independently and then averaged together.

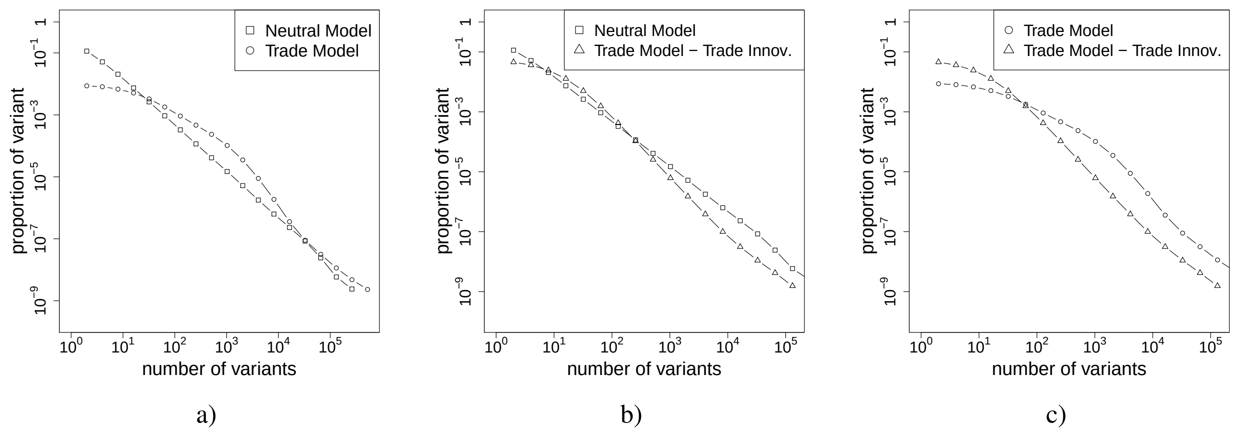


Figure 2: Frequencies distribution, where each points represent the mean for 100 runs, for: a) the neutral and the trading models. b) the neutral model and the trading model without the trading innovation process. c) the trade model and the trade model without the trading innovation process.

On figure 2.a) it appears that the implementation of trade mechanisms leads to a distribution of prices departing from the neutral hypothesis. In more details, the frequencies distribution has a plateau of common prices (a number of prices share similar and high proportions), which shows that, when trade is taken into account, the most common variants are more diverse.

In order to investigate which mechanism influences this departure from the neutral model, additional investigations have been performed. Here is presented the results of the analysis on the effect of the *innovation* process of the trade model. To conduct this analysis the *innovation* process of the trade model has been replaced by the *innovation* process of the neutral model. 100 runs have been performed with this model on the same experimental setup. The results obtained are compared to the neutral model in figure 2.b) and to the trade model in figure 2.c).

On figure 2.b) it appears that the replacement of the innovation process leads to the creation of a distribution close to the one obtained with the neural model. On figure 2.c) we observe a strong reduction in the size of the plateau and an important difference between the two distributions. This analysis highlights the importance of the *innovation* process of the trading model in the distribution of prices. This mechanism, by preventing the creation of totally random new price, promotes the creation of few similar prices.

Moreover, it shows how the implementation of one particular trade mechanism can be tested and compared to an already well know model that is already used to understand available data. This suggests we could easily implement others trade mechanism, inspired or documented by archaeological data or historical sources, and compare their expected impact on the cultural distribution with regard to traditional cultural mechanism or other trade mechanism as those presented here.

### 3.3.3 Study of scores

The results obtained with the trade model are studied in more detail by investigating the ability of the population to find the price most suited for exchanges. This is done by first measuring the score of all agents in each of the two different models. The figure 3 uses again the results obtained from 100 runs for each model where all runs rely on the same experimental setup using 3 goods, 500 agents and $\mu$ equal to 0.004. The figure is showing, as boxplots, the score computed thanks to equation (3) for all agents of all runs. The y-axis shows the score computed and the x-axis shows the timesteps. The left plot shows the results obtained in the "neutral model" and the right plot shows the results obtained in the "trading model".

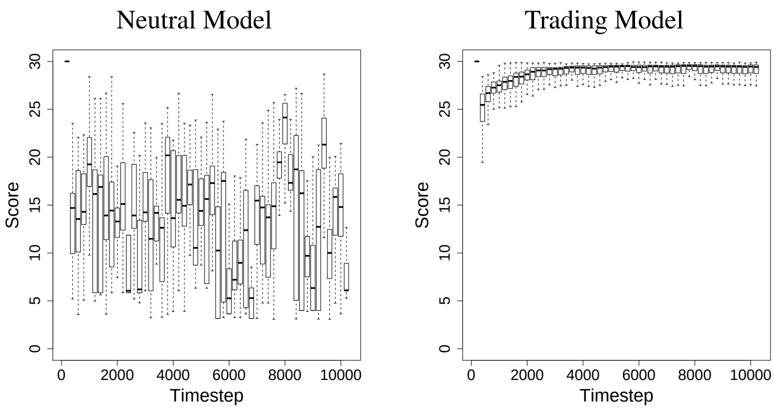


Figure 3: Evolution of the score within the two different models for two typical run with 500 agents and 3 goods evolving during 10000 timestep.

As expected, the scores within the neutral model vary randomly. "Trends" may appear, where a bigger proportion of individuals adopt a better price that allow agents to reach better score (such as around iteration 8000), but such good score fall back as soon as another trend appears. However, with the trading model, the score of all the agents increase. As the selection mechanism allow them to know who has found better vectors of prices, they will progressively adopt prices vector that allow all of them to reach better scores.

The previous figure showed the capacity for the trading model to increase its score but did not analyse the exact prices used. As explained in the subsection 3.2.2 we expect that the trade *cultural transmission* and *innovation* processes will produce a convergence toward a set of price for each good that will allow agents to exchange optimally the good they produce with the other goods. To verify this assumption we analyse the prices reached during the simulations. These are presented in figure 4 for the 100 runs relying the experimental setup using 3 goods, 500 agents and $\mu$ equal to 0.004. For all runs, all agents and at each iteration we compute the difference between the price used by the agent $V_g$ and the optimal price $\tilde{V}_g$ (given by equation 2). The measures performed are presented as boxplots condensing the results for 100 runs.

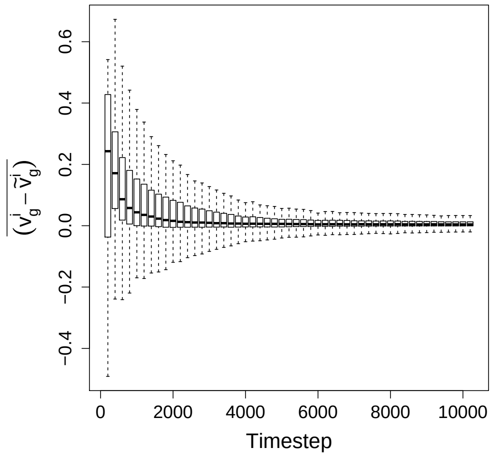


Figure 4: Evolution of the mean of the difference between the estimated value $v_g^i$ and the optimal value $\tilde{v}_g^i$ (calculated with equation 2) for a good $g$ and an agent $i$. As the optimal value $\tilde{v}_g^i$ depends on which good is produced by $i$, the mean of the difference between the estimated price and the optimal one is computed between all the agents that produce the same good. The figure represents this mean computed at each timestep for each goods, for each groups of agents and for 100 runs in a setup with 500 agents and 3 goods.

We observe that prices are indeed converging to the optimal prices which means that the agents within the trading model are indeed improving their scores by reaching the optimal prices. Notably, a similar variation of prices was observed in the closely related economical model of [27]. This variation of the prices to the optimum offers an additional conformation of the validity of the trading model.

### 3.4 Summary

With this model we propose a framework to simultaneously study cultural change and trade dynamics. The development of this framework was first aimed at simplicity which is achieved by the use of two vectors (quantity and value) and five processes (production, consumption, trade, cultural transmission and innovation). The second aim of the work conducted was to obtain a flexible framework which is possible since each of the processes can be implemented accordingly to the question studied. Both objectives were selected to provide a way to implement assumptions, hypotheses and hints made on the cultural and trading environment in the Roman Empire, where the culture dimension was tightly linked to the economics activity. We shown here how simple assumptions made at different level (see our implementation of the production, consumption, trade, cultural transmission and innovation mechanisms described in the Section 3.2.1 and 3.2.2) can be tested and compared. Those simple assumption could then easily be change by other made in the literature [33].

Moreover, we have shown the theoretical validity of this approach by reproducing expected results on both the cultural and trade side. On the cultural transmission side we have shown that the implementation of a “neutral model” leads to the expected observations on the variants of the vector value: a power law. When implementing trading mechanisms we observe the convergence of prices to the expected values and the improvement of the scores of the agents.

# 4 Conclusion

In the this chapter we argued that computer model and simulations are a good candidate to help historians and archaeologists in their quest for understanding the past. We have explain such point by taking the example of Biology and by showing that history is a perfect sample of what kind of problem simulations can help us to solve and how. We have then illustrated that by a model designed to help archaeologist and historians in their task.

# 5 Annex

## 5.1 Score & Selection

Formal computation of the score for agent $i$ and the good $j$:

$$s_j^i = \begin{cases} s_{max} = 1 & \text{if } q_j^i = n_j \\ 1 - \dfrac{\left|q_j^i - n_j\right|}{\sqrt{\left|(q_j^i)^2 - (n_j)^2\right|}} & \text{if } q_j^i \neq n_j \end{cases} \tag{3}$$

**Algorithm 2** Selection Process

$ToGet = 0.2 \times \frac{\#Pop}{\#Good}$<br>
**for** $g \in Good$ **do**<br>
    $ToReplace = \{\}$<br>
    **while** $\#ToReplace < ToGet$ **do**<br>
        $j = SelectRand(Pop, g)$ ▷ Select randomly an agent $j$ among the agents producing $g$<br>
        $X \sim U([0,1])$ ▷ Draw a random number from the uniform distribution between 0 and 1<br>
        **if** $X > ComputeScore(j)$ **then** ▷ Select preferably the agents with the lowest scores<br>
            $ToReplace = \{ToReplace, j\}$<br>
        **end if**<br>
    **end while**<br>
    **while** $\#ToReplace > 0$ **do**<br>
        $j = SelectRand(ToReplace)$<br>
        $i = SelectRand(Pop, g)$ ▷ Select randomly an agent $i$ among the agents producing $g$<br>
        $X \sim U([0,1])$<br>
        **if** $(X < ComputeScore(i))$ **then** ▷ Select preferably an agent $i$ with a high score<br>
            **if** $(ComputeScore(i) > ComputeScore(j))$ **then** ▷ Verify that agent $i$ has a higher score than agent $j$<br>
                $CopyPrice(i, j)$<br>
                $ToReplace = ToReplace - i$<br>
            **end if**<br>
        **end if**<br>
    **end while**<br>
**end for**

## 5.2 Trade Mechanism

The trading phase starts by the agent looking at a first random agent producing another good. Let $o$ be an agent producing $g$ who proposes a trade and $r$ an agent producing $k$ that receives the proposition. As explained earlier, each has a quantity of good $Q^o$ and $Q^r$. On the one side, $o$ wants to exchange a quantity $w_g^o$ of the good $g$ for a quantity $w_k^o$ of the good $k$. On the other side, $r$ wants to exchange a quantity $w_g^r$ of the good $g$ for a quantity $w_k^r$.

The tuples $W^o$ and $W^r$ describe the quantities of goods wanted by agent $o$ and $r$ for one trade proposition and are defined by:

$$W^o = (w^o_g = v^o_g, w^o_k = \frac{v^o_k}{v^o_g}), \quad W^r = (w^r_k = v^r_k, w^r_g = \frac{v^r_g}{v^r_k}) \tag{4}$$

Where $v^i_j$ are the estimated value of the good $j$ by the agent $i$ as defined earlier. The requested quantity of the non produced good is simply the ratio between the estimated value of the good requested and the estimated value of the produced good.

Once the quantities are defined, the agents declare that the trade is possible if :

$$q^o_g >= w^o_g, \quad q^r_g <= w^o_g, \quad q^r_k >= w^o_k \tag{5}$$

$$w^o_g >= (q^r_g + w^r_g), \quad w^o_k <= w^r_k, \quad w^o_g <= w^r_g \tag{6}$$

The conditions 5 ensure that both agents have enough goods in their inventory to realise the trade while the conditions 6 ensure that the quantities of goods fit the will of both agents.

**Algorithm 3** Trading Process for agent $o$

**for** $j \in Goods\ \&\ j \neq produced^o$ **do**
  $tradeAttempt = 0$
  **for** $r \in Pop\ \&\ produced^r = j\ \&\ tradeAttempt < TradeThreshold$ **do**
    **if** $acceptableTrade(W_o, W_r)$ **then**
      $trade(W_o, W_r)$
    **else**
      $tradeAttempt = tradeAttempt + 1$
    **end if**
  **end for**
**end for**

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