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An agent-based simulation model of a primitive agricultural society

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Abstract

The paper describes a model of an agricultural society in which agents live in a single settlement and use the surrounding area to produce essential and non-essential goods. Agents make, and attempt to fulfil, consumption and production plans but markets do not always clear and goods can change hands at different prices between different pairs of agents. The model generates a wide range of agricultural landscapes, including those of a classical von Thünen economy. Demographically, it produces outcomes varying from logistic growth to periodic collapse caused by cyclical famines.

Keywords: Agents; Cells; Simulation; Complex systems; Java; Agriculture; Markets; Land use; von Thünen economy; Demography; Famine

1. Introduction

This paper is concerned with a project in what Epstein and Axtell (1996) call 'generative social science'. It involves (*op. cit.* p.177) the use of

> microspecifications (initial agents, environments, and rules) that are sufficient to generate the macrostructures of interest[1].

The macrostructures in question – the emergent properties of the system – include patterns of agricultural land use around a market town located at the centre of a heterogeneous landscape. The project focuses on interactions amongst agents and between agents and their physical environment.

Epstein and Axtell's (1996) model represents a simple gatherer society, operating in a landscape endowed with two consumable resources. What we have attempted to do is to advance the story to settled agriculture, with the intention of developing it further to explore the evolution of urban form. Our landscape contains a single settlement with constant characteristics (maximum occupancy, etc.) around which the agricultural landscape changes. This is less complex technically than modelling urban evolution because the landscape has no durable features outside the settlement, except for transportation networks. Agents do not construct dwellings or make other semi-permanent marks on the landscape; the slate is wiped clean at the end of each period with respect to the use to which each location can be put. On the other hand, in addition to its demographic features, our model has land ownership, production, transportation, market trading, storage and consumption. Such a foundation is a good basis on which to build a model to simulate changes in urban form.

Some important work has already been done in the urban arena using intelligent agent modelling and allied techniques. A good overview is provided by Benenson and Torrens (2004). Explorations of urban growth and form using cellular automata (CA) have been conducted by Clarke et al. (1996), Batty et al. (1997), White and Engelen (1997, 2000), White et al. (1997), Clarke and Gaydos (1998), O'Sullivan (2001a,b) and others, whilst Takeyama and Couclelis (1997) have extended the basic concepts of CA modelling to address the thorny problem of action at a distance (see also Couclelis (1997). Using intelligent agents, Benenson and Portugali (1997) and Benenson

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(1998) have made notable progress in modelling urban social interaction. Others have experimented with intelligent agents in somewhat different geographical settings (see, for example, Kohler and Gumerman, 2000), whilst Parker et al. (2003) have reviewed a range of models for simulating changes in land use and land cover. What no one has done, so far as we are aware, is to tackle the connected problems of production (and land use) decision-making, consumption decision-making, and market interactions with intelligent agents in a spatial setting. A successful approach to this problem appears to be a prerequisite for an attack on the problem of producing an evolutionary model of an urban economy, which would require these features plus investment decision-making, capital depreciation, a market for land and capital assets and, perhaps, other attributes. Our model does the basics by considering changes in land use around a single market.

This is, of course, the oldest problem in spatial analysis. Our approach to it is new and can be thought of as belonging to the burgeoning field of artificial life (AL). In addition to the intelligent agent and CA approaches to AL mentioned above, the field includes the study of self-reproducing automata, cellular automata, biomorphs and L-systems, autocatalytic networks, bio-inspired multi-agent systems, evolutionary computing, evolving robots, evolvable hardware, and artificial nucleotides. The technical and philosophical issues associated with such systems have been widely discussed (e.g. Langton, 1988; Heudin, 1999; Tesfatsion, 2002; Wooldridge, 2002; Gilbert and Troitzsch, 1999).

2. Growing artificial societies

Epstein and Axtell's Sugarscape has two qualities that make it appealing to geographers. First, the society's agents interact over space; there is a landscape in the form of a lattice across which resources (sugar and spice) are distributed. Second, the relationships between agent and agent and between agent and environment are multidimensional. They are not, for example, purely economic but have biological, demographic, social, economic, political and cultural dimensions. In addition, whilst the agents are relatively simple, agent interactions yield complex patterns and processes with at least a passing resemblance to their real counterparts. Thus, the general modelling strategy appears to be both consistent with basic geographical interests and promising in terms of the macro-behaviour it can generate.

Unlocking that promise is a challenge. There is a substantial gap between the worlds created by Epstein and Axtell and real geographies. For a start, their worlds have no production. There is use of labour in the gathering of resources but no transformation of those resources, except by consumption. As a result, the landscape is not made by agent activity, apart from the rather limited impact of gathering. With no production there is little need for division of labour or for the kind of learning-by-doing that underpins specialisation. The only economic differentiation between agents relates to their physical capacities and to the stocks of goods they accumulate. Trade can occur but only through individual encounters as agents go about the business of gathering. Trade networks (in the sense of networks of contacts) emerge but there are no physical networks. Indeed, there is no production of infrastructure of any kind, just as there is no production of goods for consumption. Consequently, there are no dwellings, no storage facilities, no social facilities and so on. In short, there are locations on the lattice but no places.

Of particular importance is the absence of marketplaces. The reason for this runs deep. Epstein and Axtell's agents operate in the neighbourhood around their current location, where the neighbourhood is limited by the agent's 'vision'. Agents move within their neighbourhood to gather resources. They have no capacity for action at a distance (beyond the neighbourhood boundary). Nor do they have a sense of place other than of their current location, because they have no memory. Furthermore, each cell in the landscape can be occupied by only one agent. This approach to agent behaviour in space owes much to the tradition of cellular automata, from which it inherits some useful features. However, it makes it very difficult to represent marketplaces and places of other kinds.

There are numerous other economic limitations that might be listed and there are comparable shortcomings with respect to the other disciplinary dimensions. This is intended not as a criticism but as a call for action. In this paper, the action focuses on the economic dimension in general and the above limitations in particular. It also gives some consideration to demography because economic and demographic issues are inextricably linked in multi-agent modelling. As Epstein and Axtell put it (ibid. p. 158)

> had agent-based modelling been possible in the days of Thomas Robert Malthus, one wonders whether the fields of economics and demography would have developed so separately.

3. An overview of the model

Our model presumes the existence of a single settlement around which there is a heterogeneous landscape capable of supporting agriculture. The settlement serves as a marketplace and has a fixed number of dwellings, each of which has a storehouse. Agents require shelter (a dwelling), a minimum quantity of an essential good (a food called 'staple'), and some respite from labour (leisure) in order to survive. They can also grow food products other than the essential good and, therefore, have consumption possibilities that include the essential good, these other non-essential goods, and leisure. Preferences over these consumption possibilities vary between agents.

Each agent lives in the settlement and owns one and only one field in which, at any one time, it can produce only one type of good. It chooses which good to produce on the basis of its knowledge of expected yields and market prices. The agent travels from the settlement to its field, produces a harvest, and takes it back to the settlement. It adds the harvest to its store before offering everything it has for sale in the marketplace. Each agent has a memory and, on the basis of its experience, has expectations about the price of each good. Together with its preferences, these expectations govern its behaviour as a buyer and seller in the marketplace.

After the market has closed, each agent returns to its dwelling, consuming part of its holdings and returning the remainder to its store. If the agent is unable to consume at least the survival quantity of staple it dies. On the other hand, if it has more food than it can store, a new agent is born with the excess food as its initial endowment. New agents inherit preferences and acquire knowledge from their parent. To survive, a new agent must find a field to claim as its own. Once claimed, the ownership of a field cannot change unless its owner dies. To find a field, a new agent leaves the settlement and searches, making use of established pathways (and, adding to them as it moves), until it finds a vacant field. If the field is so far away from the settlement that the agent cannot travel there and return with its produce without exceeding its capacity to supply labour, it will die.

As the seasons pass, the agents' society evolves, with land uses changing from season to season, with the agent population changing, and the network of pathways developing. The physical environment in which the drama unfolds has weather that varies from season to season, impacting on crop yields. The productivity of the landscape also varies from place to place, and the accessibility of locations changes with the changing transportation network.

4. Space, time and scale

The assumptions of the model differ from those of Epstein and Axtell in a variety of ways. In particular, there are fundamental differences in the treatment of space, time and scale.

The distribution of agents in space is the feature of Sugarscape on which most attention is focussed. For a geographer, the way these distributions are generated has certain counter-intuitive features. First, there are no boundaries. They are eliminated (so that agents do not have to deal with them) by assuming that the rectangular lattice of the Sugarscape wraps round north on south – to produce a tube – and east end on west end – to produce a torus. This means that all locations have the same spatial attributes (as there are no boundaries there are no central locations). Real landscapes have boundaries with which agents have to deal. To allow for this possibility, it is necessary to modify the landscape specification. In our model:

(1) Locations are represented by a finite, regular, rectangular grid of cells.

Second, having eliminated the distorting effects of boundaries and, therefore, pre-existing central locations, Epstein and Axtell put the latter back in by the establishment of sugar mountains. The fact that agents appear to cluster or swarm on the Sugarscape hinges on this assumption. Similarly, in the simulated Anasazi settlements in Dean et al. (2000), any clustering is a result of resource distribution. Real people, on the other hand, often cluster for reasons unrelated to the distribution of material resources.

The assumption that each cell can be occupied by only one agent is also problematic. One of the effects of this assumption is to limit the extent to which human-scale clustering – of the kind we find in settlements – can be replicated. In principle, we could address this problem by employing an extremely large lattice and retaining single agent occupancy. Alternatively, we could allow more than one agent in the same cell and devise a scheme to allow co-located agents to relate to one another. This creates a presentational challenge but it does permit the kind of modelling that is required to move from a low-density, mobile, gatherer community to a system with settlements of various densities distributed over an otherwise thinly-populated region.

The treatment of time raises further issues of scale. In Epstein and Axtell, time periods have an interior, which allows negotiations to occur leading to commodity exchanges. More extensive nesting of time periods might be of value in other contexts. In the case of a model of agricultural land use around a settlement, it might be helpful to include, say, daily movements from dwellings to fields, monthly consumption of stored produce, seasonal harvesting and marketing, and so forth. In the longer run, further time scales appropriate to the formation and development of the settlement might be required. As with space, a very large number of time points could be employed, with events of different kinds occurring with different frequencies, but adopting different time scales (with time intervals having interiors) has greater attractions. We assume:

(2) Time is represented by a sequence of equal periods, where each period can be thought of as a year in which there is one growing season.

5. The environment

The focus of attention with the Sugarscape is on the distribution of agents rather than on the changing landscape. The agents have little impact on their environment so there are few changes worthy of note. This cannot be the case in a model of agricultural land use. By definition, such a model has to depict land use changes but it need not show the agents who make those changes. Our grid consists of three types of cell – a single settlement cell, fields and unclaimed land – and the simulator shows the current use of the land in each cell.

(3) There is a single settlement with a fixed number of dwellings, which serves as a place of shelter and a marketplace for agricultural produce.
(4) All other cells are capable of being used as a field (for agricultural production) by a single agent; the fertility of fields can vary over space.
(5) In any time period, only one type of good may be produced in each field.

Assumption (3) allows the settlement cell to contain several agents whereas (4) restricts the use of each field to a single agent. Assumption (4) also allows for the possibility that, at any given time, a cell may be empty (in the sense that it has not been claimed by an agent for agricultural use). It also permits the fertility of each cell to be set to the same value (as in the classical version of the von Thünen model), to have a simple binary form of usability (fertile or unfertile), or to have a more subtle gradation of land quality. It would be easy enough to allow land degradation as well but we have not yet done so.

In addition to having variations in yield over space arising from differences in fertility, variations over time can be introduced through a weather variable. It is not essential to have uncertainty in the weather but its presence enhances the treatment of vulnerability and paves the way for more sophisticated developments, including studies of the effect of climate on settlement patterns and of extreme events such as flooding and storms.

(6) There is a weather variable that influences output.

Finally, there is a network of pathways that connects fields to the settlement. Because this element of the physical environment is agent-made, its evolution will be described in more detail when the agent attributes are considered.

(7) There is a network of pathways. When a new agent claims a previously unused cell as its field, it creates a new pathway connecting the cell to the network. Each new pathway becomes a permanent feature of the landscape. Its capacity to carry traffic extends to cope with every agent that makes use of it, but this implicit 'broadening' of pathways does not affect the yield that can be obtained from the cells through which they pass.

6. Commodities

The nature of the goods produced and the other resources required for their production need to be discussed in relation to the attributes of agents. The essential attributes of the agents that populate the Sugarscape are their metabolism and vision. The metabolic rate of an agent with respect to each type of resource is the amount of that resource which the agent has to consume in each time period to survive. The amount actually consumed is the survival quantity, provided that quantity is available. Any amount that is gathered over and above the survival quantity is stored and carried around by the agent. The vision of an agent is the number of lattice points it can see in the four principal compass directions (its extended von Neumann neighbourhood). This cross-shaped area could, equally well, be regarded as the agent's information field, where vision is the only means of gathering information.

Epstein and Axtell's agents have other attributes that are not immediately relevant to the argument. One thing they do not have is a memory. Thus, if richly endowed locations are re-visited, it is because they are rediscovered not remembered. In contrast, in the hunter-gatherer model of Lake (2000), communication of information on resource (hazel nut) locations is crucial to the development of the model.

Another missing attribute is labour. Agents on the Sugarscape are constrained in their movements by their vision, not by the availability of their own labour. No labour is required to transport stored commodities or to prevent them from deteriorating. No labour is required to obtain information or to participate in exchanges.

From the perspectives of economics and geography, the above features are somewhat unconventional. It has been noted already that the Sugarscape model has no production and the above observations indicate that consumption is rudimentary. There is no choice over consumption quantities; only survival quantities are consumed because the only thing that matters is survival. In the context of settled agriculture and, indeed, in real gatherer systems, other things matter if survival is not immediately threatened. Non-essential commodities are acquired and consumed; leisure (freedom from labour) is valued; saving for the future is important but present consumption is valued as well.

To enable us to capture these possibilities, we make the following assumptions about the commodity space:

(8) Goods are of two types – essentials and non-essentials.
(9) Staple (a basic foodstuff) is an essential good.
(10) Leisure (freedom from labour) is an essential good.

Two other commodities that are added to the mix are shelter and storage. We have adopted a very simple approach to shelter, which would not be adequate if we were seeking to model the evolution of the settlement itself. Assumption (3) establishes that the settlement has a fixed number of dwellings. We make the further assumption that:

(11) Shelter is an essential good, one unit of which is provided in each time period by each dwelling.

Together with assumption (16), this ensures that the maximum number of agents in the system is determined by the fixed number of dwellings in the settlement.

The requirement for storage is related both to long-term survival needs and to the operation of the market. Generally, markets do not clear so surpluses are stored for consumption or sale at a later date. It is assumed that:

(12) Each dwelling has a store, which provides a finite quantity of storage in each time period.
(13) Each storable good has a finite storage requirement per unit per period.

These two assumptions contrast with the Epstein and Axtell approach in which, as noted above, agents can carry an unlimited quantity of goods with them on their travels.

Finally, it is helpful to introduce one more commodity, namely money. It is possible, as on the Sugarscape, to rely on barter but it is both helpful technically and reasonable empirically to assume that:

(14) Money is used as a medium of exchange and as a repository of value.

7. Agent attributes

At this point, it is useful to consider the attributes of agents a little more formally. In contrast to the Sugarscape assumption that agents have a metabolism and a field of vision, we rely on the concepts of labour capacity, survival requirements, information acquisition, memory, and a capacity for computation. We start by specifying that:

(15) Each agent has the capacity to supply a finite quantity of labour in each time period.

The notion of metabolism is wrapped up in the assumption that agents require minimum quantities of the essential goods to survive. In our model, the essentials include shelter and leisure, as well as a foodstuff, so our agent biology is relatively sophisticated; agent survival can be threatened by exposure and overwork as well as by a lack of food. The assumptions already made on essential commodities implicitly capture these attributes but, as a counterpoint to Epstein and Axtell's assumption on agent metabolism, it is helpful to state formally that:

(16) For an agent to survive, it must consume minimum positive quantities of the essentials in each time period (the minimum quantity of shelter being one unit and the minimum quantity of leisure being strictly less than the capacity to supply labour).

Vision is central to agent behaviour on the Sugarscape. Each agent has preferences that relate to its current and possible future holdings of sugar and spice. A welfare function is defined in which the parameters are dependent upon the agent's metabolic rates, such that the choice process reflects the relative threats to survival posed by the exhaustion of its holdings. The agent chooses to move to a site in its field of vision that maximises its welfare function, where the variables are the quantities of sugar and spice that would be available if the quantities at the site were to be added to current holdings. Thus, vision provides information, which informs movement.

In our model, a new agent must leave the settlement and find an unclaimed cell for its field if it is to produce the output that is necessary for survival. One approach to the search process is to assume that agents have vision, which enables them to see vacant cells at a certain distance from their current location, and to specify that they move along existing links in the network of paths until they come to a vacant cell. They can then move into the cell, extending the network to it as they do so (see assumption (7)), and lay claim to it as their field. With respect to the simulator, this movement would not be shown, because the simulator displays the changing pattern of land use from 'year' to 'year', not the movement of individual agents to and from their fields. It would become evident by the appearance of a newly cultivated cell and an extension of the network into that cell (unless the cell had been connected by a previous, now deceased, owner).

An alternative approach is to think of the new agent acquiring information about a vacant cell before it leaves the settlement. It could then follow instructions to reach the cell, making no use of 'vision'. Again, all that would be seen would be a newly cultivated cell and new network link, or the cell of a dead agent being brought back into cultivation. We make the general assumptions that:

(17) A newborn agent has the capacity to search for a cell to use as a field, either by gathering information from fellow agents or by searching independently.
(18) Once an agent has selected a cell as a field, it remembers it and does not change its selection (or add additional cells to its land holding).
(19) The agent is able to determine the shortest network path from its cell to the settlement and always uses that path.

We have tried two different methods to put assumption (17) into effect. The first involves a rather short sighted search in which each agent moves randomly along branches of the network, but always moving away from the settlement, until it encounters an unclaimed cell (it can 'see' it when it gets there but not beforehand). The second employs a DLA (diffusion limited aggregation) model. DLA was introduced by Witten and Sander (1981, 1983) and has been used widely because it has the attractive property of producing dynamic fractal patterns. It provides a model for a variety of phenomena in which diffusion is the dominant growth mechanism (Halsey, 2001). Because the growth of urban areas resembles the output of a DLA model, the technique has gained wide applications in urban growth studies (e.g. Batty, 1991; Longley et al., 1991; Batty and Longley, 1994; Ward et al., 2003). For us, the use of DLA is a computationally convenient way of implementing the idea that new agents gather information about vacant cells from fellow agents. It depicts the outcome of that process but says nothing about the process itself.

8. Production and consumption

The assumptions in the previous section establish that agents have a memory and a capacity for computation. Both attributes are an essential part of the process of production and consumption.

The standard approach of microeconomics is to treat production and consumption decision-making as separate activities; the former is dealt with by the theory of the firm and the latter by the theory of the household. On the production side, firms choose an input–output combination that maximises profits. On the consumption side, labour and any other household resources are sold and the resultant budget is spent on a bundle of consumption goods (including leisure) that maximises household utility. The resource and product markets are left to find prices that make the various production and consumption plans consistent.

In our primitive agricultural society, this approach is problematic. The presumption that markets will clear is, perhaps, the central artifice of economic models built on pre-digital computational foundations. If there is one thing that a multi-agent modelling approach should do it is to dispose of market clearing as a presumption, whilst not ruling it out as a possible outcome. Moreover, the explicit separation of production and consumption decision-making is not necessarily suitable for dealing with agents that resemble peasant households. Peasants, who typically produce for themselves whilst engaging partially in markets, may be drudgery averse, risk averse, profit-maximising or have some combination of these and other attributes (see, for example, Ellis, 1993). They may have minimum requirements for income and leisure, which may depend on the household's demographic structure. In the presence of a labour market they may buy in or hire out labour. A profit-maximising peasant may be treated as a household running a conventional firm but with constraints on its production decisions. A drudgery-averse peasant may be treated in a similar fashion but with a utility function that gives substantial weight to leisure. Risk aversion may be accommodated in a number of different ways including minimising the probability of entering the 'starvation set' (see Sen, 1981; Ravallion, 1987).

To what extent is it desirable and practicable to separate out production and consumption decision making in the context of the assumptions sketched out so far? If there are to be no prices at which markets clear, and on which agents can base their decisions, another mechanism is required. We have considered a number of possibilities, the most promising of which is based on the following supposition:

(20) Each agent has a subjective price for each good and a subjective wage rate.

The subjective prices are the agent's estimates of the prices that it will be able to achieve, and have to pay, in the next time period. The subjective wage rate is an estimate of the value of its labour; it enables the agent to predict its income and to evaluate its consumption of leisure against the consumption of other commodities. With subjective prices and a subjective wage, decisions on production and consumption are immediately separable.

The economic heart of our model, which is presented schematically in Fig. 1 and mathematically in Appendix A, assumes that agents draw up a consumption plan first (step 1) then a production plan (step 2). The connection between these parts of Fig. 1 and the material in Appendix A is presented in Appendix B. Agents attempt to reconcile their consumption and production plans in the light of the actual output produced, their stocks of goods, and the market conditions they encounter (step 3). This leads to actual consumption (step 4) and the revision of assumptions about prices and about the production process (step 5). Step 1 is governed by the following assumptions.

(21) At the start of each time period, each agent draws up a consumption plan based on its preferences and its subjective wage and prices. The plan consists of a decision on the amount of leisure and other commodities to consume. The leisure decision entails a decision on the supply of labour and, given the subjective wage, the agent's expected income. The consumption plan for the other commodities, together with their subjective prices, gives the agent's expected expenditure.

It is assumed that the agent's first objective is to secure its survival. Thus:

(22) If the expected cost of acquiring the survival quantity of staple is less than the agent's holding of cash, or less

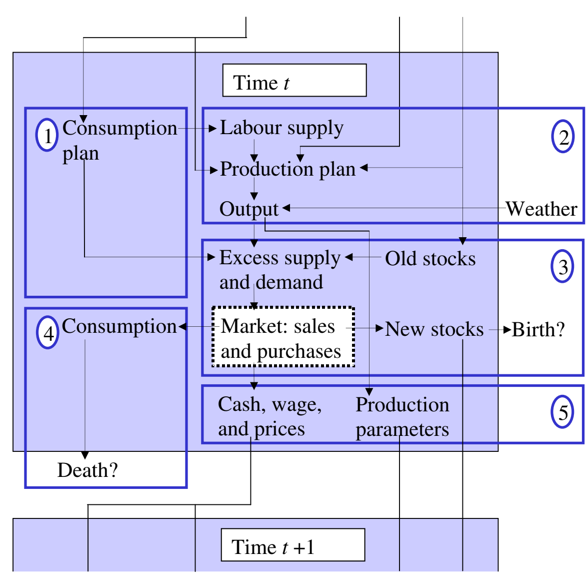


Fig. 1. A schematic representation of the model.

than the agent's maximum expected income (the income obtainable by working up to the survival limit for the supply of labour), then the agent's consumption plan is to have no leisure and consume only staple.

(23) Otherwise, the consumption plan is to maximise utility subject to the constraints that expected expenditure must not exceed expected income, and the consumption of leisure and of staple must be no less than their respective survival quantities.

The production plan (step 2) is derived in a similar fashion, making use of subjective prices, the consumption plan's decision on leisure, and the agent's beliefs about the relationship between labour input and yield (agents do not know precisely what the production function is so they cannot plan with certainty). The production plan is simple in that it specifies only one thing – the crop to be produced. Again, the problem of survival takes precedence because the agent will always grow the essential good if its current holdings are dangerously low.

(24) Having arrived at a consumption plan, each agent makes a production plan, which consists of a decision on which crop to grow in the current period.
(25) If the agent's current holding of staple is below a certain buffer level, it will produce staple.
(26) Otherwise, it will produce the crop that maximises its expected net revenue.
(27) Inputs of labour are required for production, commuting to and from the field, and for the transportation of output, and all of the available labour is utilised except for that set aside for leisure in the consumption plan.
(28) Actual output is determined by the crop chosen, the labour supplied for production, and the environmental conditions (land fertility and weather).

Instead of looking for a (subjective) profit-maximising production decision, one could argue that it would be more consistent with the spirit of multi-agent modelling and more sensitive to the notion of neighbourhoods to have a simple production decision rule based on neighbour behaviour. For example, an agent could decide to grow the crop grown in the previous period by its wealthiest or highest-income neighbour (according to its own subjective prices). It might even forget about subjective prices and grow the crop grown most widely in its neighbourhood in the previous period. The latter possibility is less attractive than the former because it gives no role to market signals and, therefore, disconnects production decision making from the processes of exchange and consumption. Thus far, we have not experimented with these approaches.

9. The market

Once the actual output of an agent is known, it can begin to reconcile its consumption and production plans (step 3), taking account of the stocks of commodities it holds from previous periods. Each agent does this by acting in the market as both a seller and a buyer. The model assumes that:

(29) In each period, each agent adds the output of the current period to its stock then places on the market all of its holdings of each good after subtracting the amounts required by its consumption plan (its excess supplies); the resulting shortfalls from the consumption plan (its excess demands) become the agent's purchasing plan or shopping list.
(30) Agents are price setters for the goods they are selling; the asking price in each case is the seller agent's price forecast for the period.

Note that, as a result of this assumption, different agents will, typically, set different prices for the same commodity. As there are no quality differences in the output of different producers, consumers will discriminate only by price. If all buyers want to pay the lowest price then some device is required to arrange them into an orderly queue.

(31) Agents are selected randomly to enter the market in turn as buyers.

The process of purchasing can be organised in a variety of different ways. First, a buyer might visit only a subset of the sellers' market stalls, as the search for good deals might be thought to involve a cost that increases with the number of stalls visited. For simplicity, we allow buyers to visit every stall and do not assume that they consume leisure time in the process. We also assume that:

(32) Each buyer works down its shopping list, starting with the first item, choosing the seller offering the lowest price, then moving on to the next lowest and so on until it has secured the required quantity. It repeats this procedure for the next item on the list, again starting with the cheapest seller, and so on until it has completed its planned purchases or its cash has been exhausted.

Note that in the latter case, the buyer might fail to purchase quantities of some of the goods towards the bottom of the list. An alternative approach would be to allow the shopping list to consist of planned expenditures on each good rather than quantities. This would enable the purchase of some quantity of every good but the quantities would, typically, be different from those that would have appeared on the shopping list had it been drawn up in terms of volumes rather than expenditures. Another variation would be to require buyers to identify the cheapest seller of the first good on the list, as above, but then require them to purchase all of their other goods from that seller as well, subject to availability and irrespective of price. A further twist would be to make this requirement only if the asking price for a given good was within some percentage of the buyer's expected (subjective) price. It is easy to see that there are a large number of possible variations which might yield rather different outcomes. Variations in the rule for market entry could have particularly significant effects. For example, we would expect less dramatic demographic outcomes if the rule was to allow the poorest agents to shop first and more dramatic outcomes if the rich had priority but we have not tested these variations.

One variation we have tried is to allow buyers and sellers to haggle so that the transaction price is midway between the seller's bid and the buyer's offer. This adds a smoothing effect to the trading process but does not conflict with any of the basic assumptions about the way the market operates.

As well as buyers having shortfalls with respect to their purchasing plans or some surplus cash, sellers will also tend to pack up their market stall with some goods unsold. Thus, it is necessary to add a couple of assumptions:

(33) Any agent with unsold stock returns that stock to its store.
(34) The money received from sales, plus any unspent cash, becomes the cash in hand for the next period.

10. Demography

In addition to the above economic rules, it is necessary to specify the system's demography. The birth of agents could be handled in a number of different ways. It could be assumed that the birth rate is proportional to the size of the agent population. Alternatively, agent birth could be given a behavioural foundation, loosely following the microeconomic theory of fertility (see, for example, Razin, 1995). We have adopted a simpler but theoretically less appealing economic mechanism.

(35) If, at the end of a marketing round, an agent's store is filled to capacity and there is a surplus, a new agent is born; the surplus becomes its initial endowment and it takes half its parent's cash.

This is the final element of step 3. One advantage of our chosen mechanism is that it ensures that each agent has an identifiable parent from whom characteristics are inherited.

(36) The characteristics of each new agent are inherited from its parent with some variation, as are the initial values of its subjective prices, subjective wage rate and output expectations.

The characteristics include such things as the capacity to supply labour and the parameters of the utility function. Thus, for example, drudgery aversion (as captured by a relatively high parameter value attached to the leisure variable) could be passed on. To complete step 3, it is necessary to apply what amounts to an infant survival test.

Survival requires the consumption of a minimum quantity of leisure, shelter and staple. It has been assumed that the capacity to supply labour is greater than the minimum requirement for leisure, and that this requirement is never breached. Thus, each agent will always be able to pass this survival test. As for shelter, there is sufficient for a finite number of agents to survive, each consuming one unit per time period. Provided the number of agents is no greater than this carrying capacity, shelter becomes a survival issue only if an agent's labour capacity, less the survival quantity of leisure, is insufficient to allow a round trip to be made from settlement to field and back again. Thus, it can be assumed that there is a maximum-survivable-distance from home to field and that a choice of field beyond that distance implies immediate death.

(37) A new agent fails to survive its birth period if:
  (a) its addition to the population of the settlement makes the population greater than the number of units of shelter available;
  (b) the distance to the field selected by the agent is greater than the maximum-survivable-distance.

This completes step 3. Step 4 is the consumption process, for which there are only two assumptions, the second of which is demographic:

(38) For each agent, all of the goods bought in the market in a given period, plus those set aside to help meet the consumption plan, are actually consumed.
(39) If an agent is unable to consume a survival quantity of staple in any given period, it dies.

11. Learning

The final step in the model, step 5, is the learning phase. Agents approach the market in step 3 with assumptions about prices. They find that other agents are selling at different prices. In the learning phase, they update their price estimates for the following period.

(40) For each agent, the subjective price of each good is updated using a moving average mechanism in which the new data point is the weighted average of the prices paid in the market; the subjective wage rate is updated in a similar fashion using the value of the agent's latest output at current subjective prices divided by the quantity of labour supplied.

Agents also learn about the production process.

(41) For each agent, the actual output produced is used to update the agent's beliefs about the relationship between labour input and yield for the good in question (the subjective production function), using a moving average mechanism.

12. Simulations

The system has been operationalized using a Java platform, which enables us to visualise model runs directly using an Internet Web Browser. The program is written in the object-orientated Java programming language and the modelling results are presented using Java Applets. Fig. 2 shows the user interface. At the top lies a control panel, which enables the user to select an appropriate simulation speed and the relevant output displays, together with Play, Stop, Reset and Information keys, and a time display frame. The length of runs is limited to 500 periods or years (one tick represents one year in the frame). Below the control panel, the physical environment is shown on the left-hand side in plan form. The environment can be visualised either in a 2-D grid, as shown, or in a 3-D DTM (Digital Terrain Model). The grid has 1140 cells in total (30 rows × 38 columns) and can be used to represent any landscape configuration. The landscape shown has a randomly generated set of agriculturally unproductive hill-cells but it could, equally well, have been sourced from GIS data. On the right-hand side is a panel which displays the values of the system's production, consumption and environmental parameters. In the case of prices, the values shown are initial values for the first agent. The standard system has a single settlement and four crop types, one of which is an essential good (marked (e)). Leisure, which is also an essential good, appears in the consumption part of the control panel. During a simulation run, a variety of other panels can be used to display population size, average prices, the evolving transport network, and other simulation outputs (see Figs. 3–6).

We have run a variety of simulation experiments to test the richness of the emergent economic and demographic properties of the system. In this paper we provide only a brief overview of a set of experiments focussed on the relationship between production and survival. They have generated a rich variety of land use patterns, with considerable dynamic variation, and an equally rich set of demographic histories. The latter may be characterised in terms of the types and effects of famine episodes, and the fate of the poor. In summary, the experiments have shown that:

Production:

- can have a classical von Thünen land use structure (Fig. 3);
- can invert the classical structure, albeit temporarily, giving the essential foodstuff the characteristics of a Giffen good, produced on the margin of cultivation (Fig. 4);
- can exhibit crop rotation, driven by economic rather than environmental considerations;
- can exhibit prolonged instability.

Famines:

- can occur without environmental shocks;
- can be cyclical;
- tend to victimise the poor;
- tend to affect peripheral areas;
- can alter the demographic regime (Fig. 5).

The poor:

- live on the margins and die young (Fig. 6);
- have restricted diets;

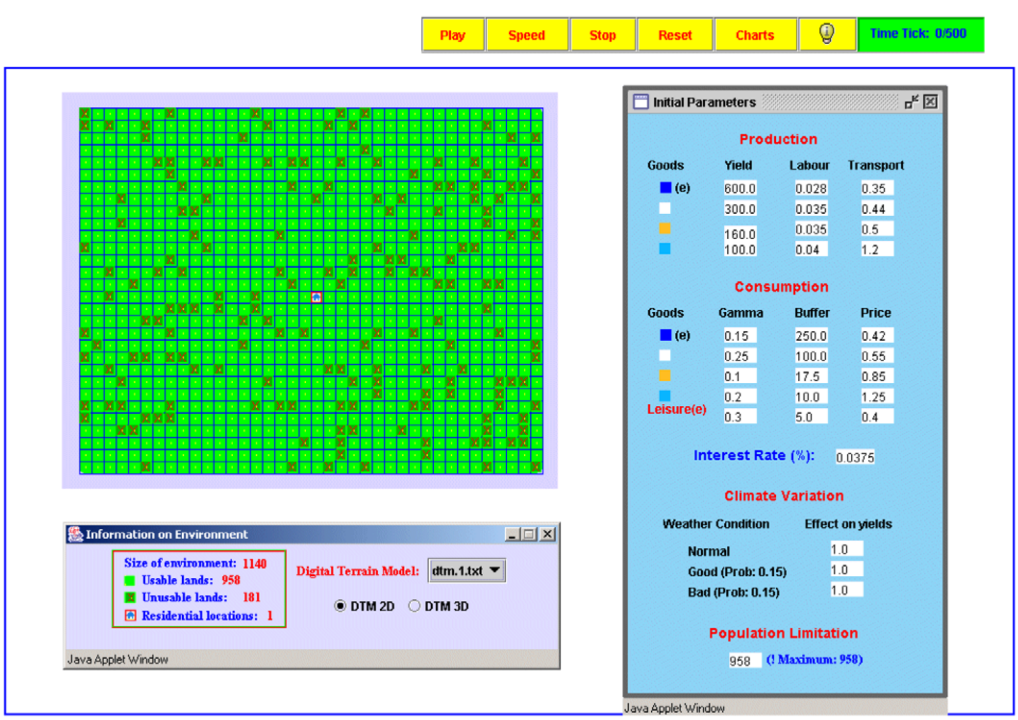


Fig. 2. The current user interface (other figures show simulation runs using an earlier interface).

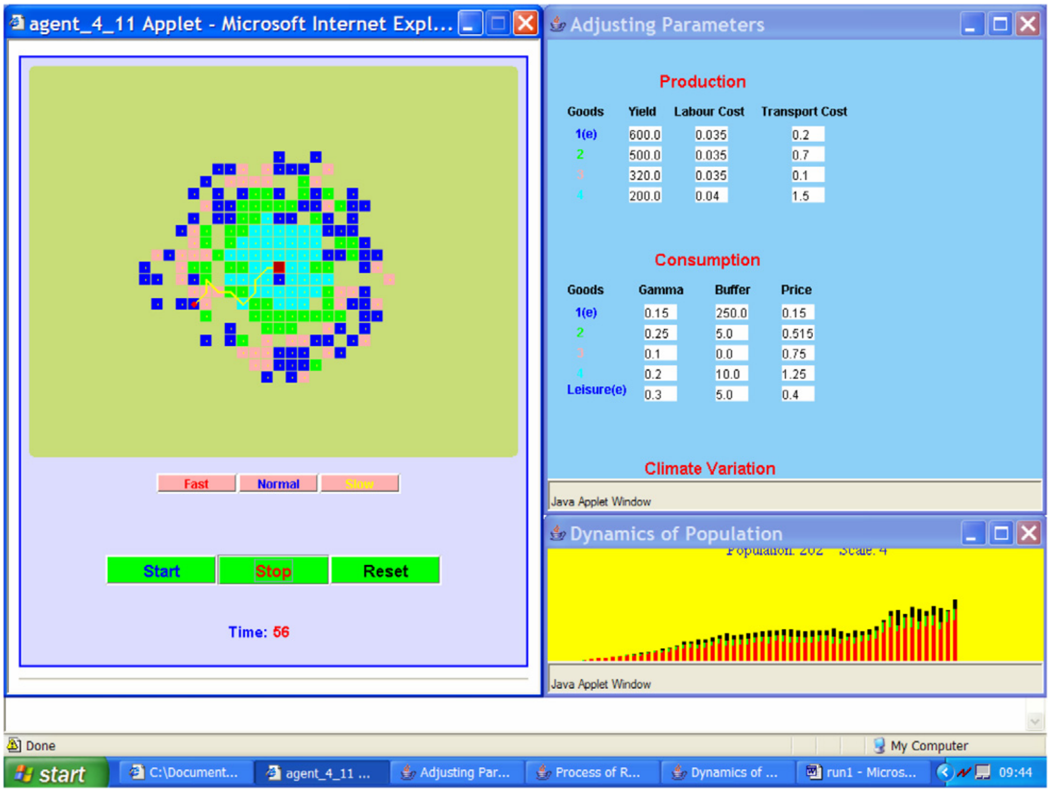


Fig. 3. Classical spatial structure of production: the goods with the highest transport costs are produced closest to the market.

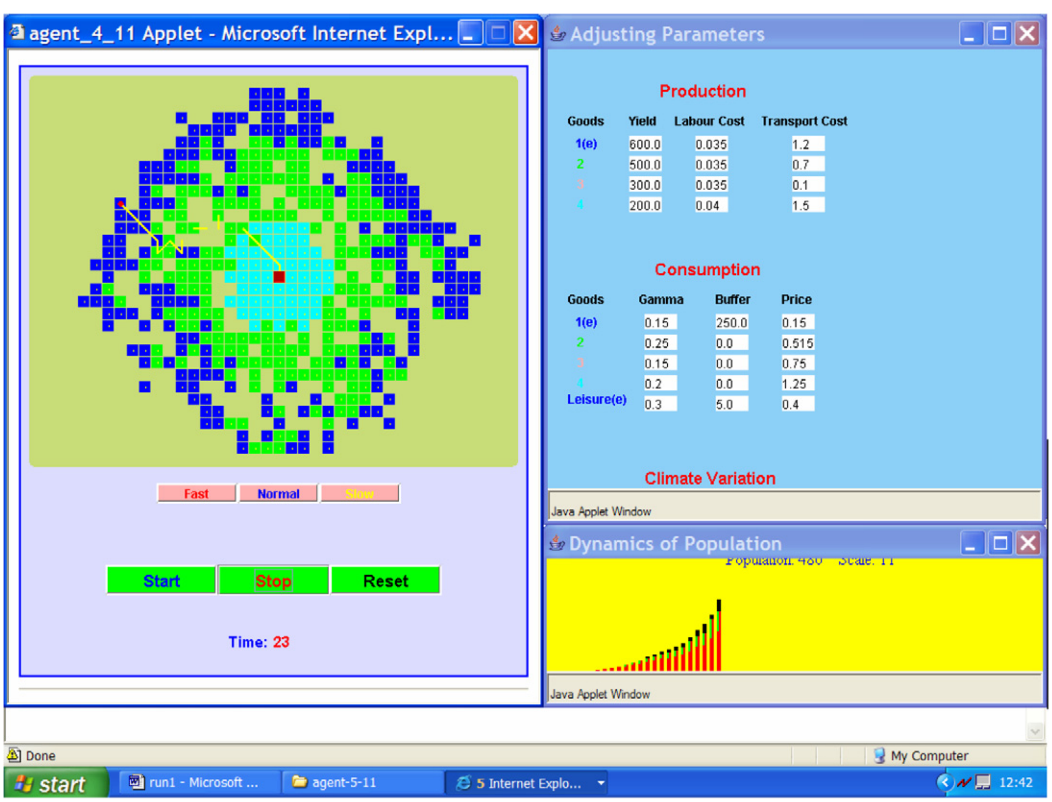


Fig. 4. Non-classical spatial structure of production: the transport cost of the essential good is second highest but it is grown at the furthest distance from the market.

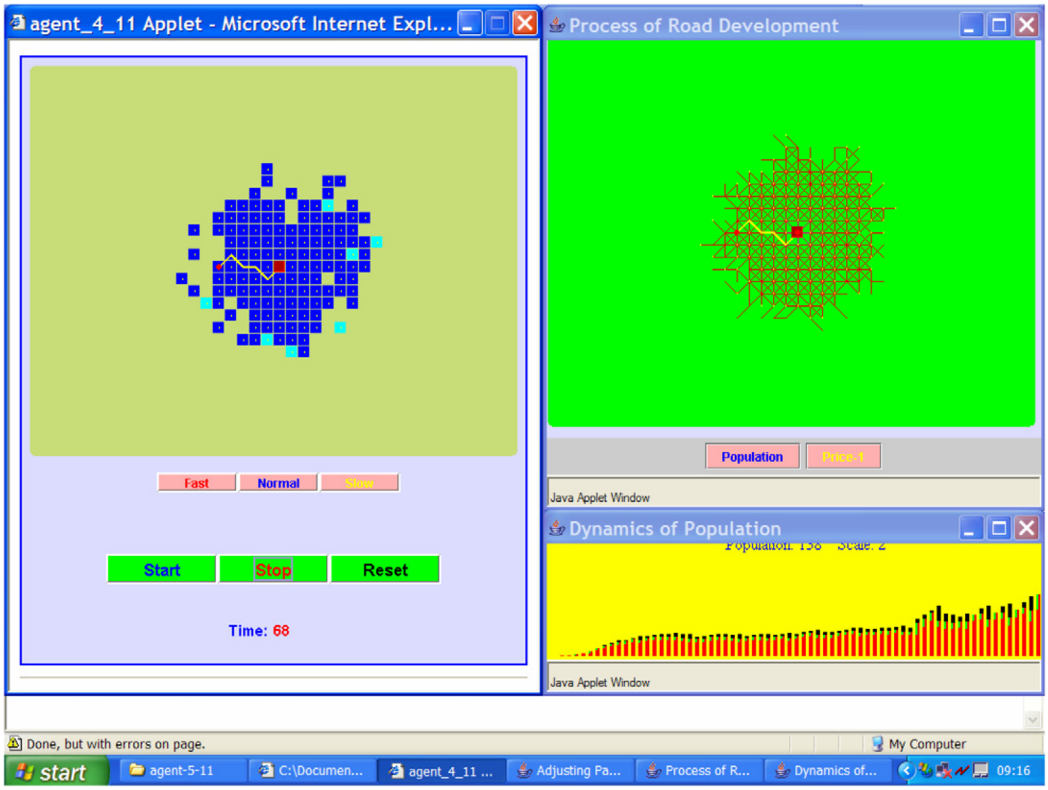


Fig. 5. Change in demographic regime: after a long stable period, the population grows rapidly, then collapses, entering a period of cyclical famines.

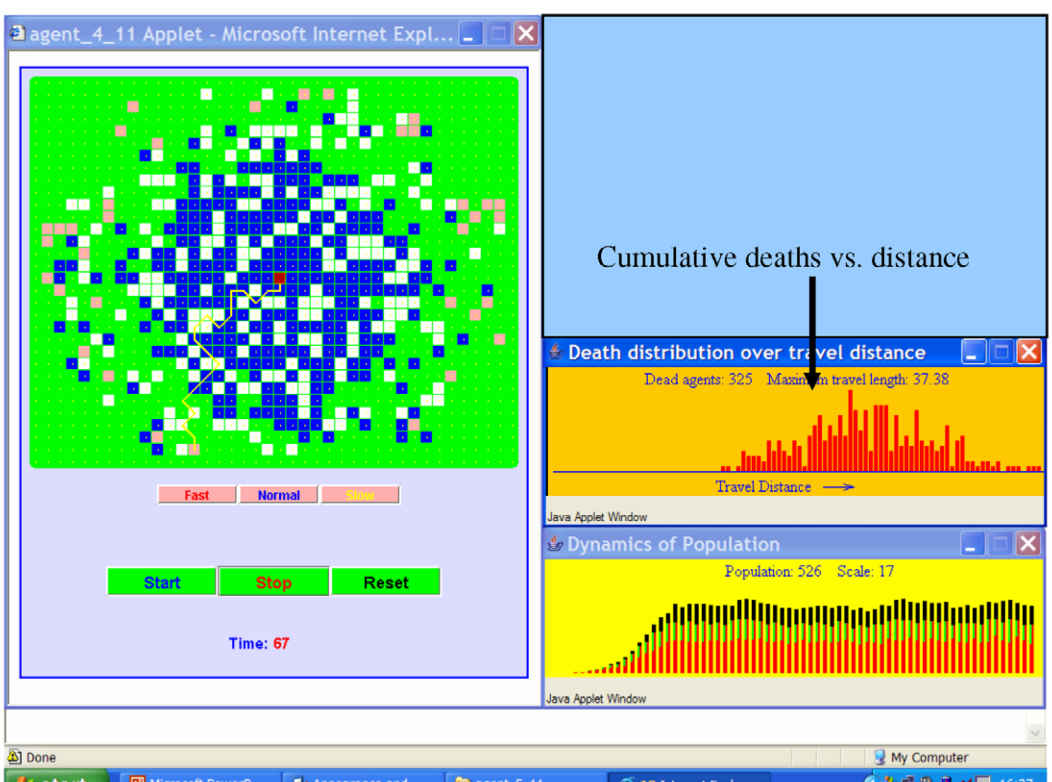


Fig. 6. The poor live on the margins and die young: the transport cost of the essential good is second highest but it is grown at the furthest distance from the market.

- work long hours;
- rely on subsistence production.

Few of these emergent properties would be surprising in real, poor, agricultural societies but it is striking that they are capable of being exhibited by a simulation system with a relatively simple structure and rule-base. This suggests that the system is worth further development.

13. Conclusions: real worlds and familiar models

Our knowledge of 'real geographies' is mediated by generations of geographical models, the structures of which have depended upon the computing techniques available to their authors. The way we think in any quantitative discipline is rooted in the way we compute. For example, the historically important notion that a market price is determined by the intersection of supply and demand schedules is heavily dependent on the ability to formulate and solve a pair of simultaneous equations. Indeed, the idea that there is such a thing as a single market price may well have the same foundation. In rethinking the way we model the world it is prudent not to assume that we can address it afresh, free from computational shackles. We will still be constrained but in different and less familiar ways. A better strategy is to see how the empirical systems we have been trying to model can be remodelled using the new technology. We should ask about the extent to which our current thinking derives from the empirical system itself, the extent to which it is coloured by the particular techniques we have used, and the scope for liberation which the new technique appears to offer.

Agent-based modelling provides a degree of liberation. It offers a new way of conceptualising and simulating interactions amongst agents, and between agents and their physical environments. Like all simulation modelling, it cannot provide analytically neat characterisations of outcomes but it can provide valuable insights into the behaviour of the kind of complex systems that geographers choose to study. The danger is that, to capture the richness of the real world system of interest, it may be necessary to make the simulation model highly complex. It can then be unclear whether the emergent properties are the product of good modelling or mere artefacts of a particular model structure (such as our codification of the workings of the agricultural product market). The only recourse is further extensive testing and refinement.

Appendix A

A.1. Agent behaviour: consumption planning

The consumption planning problem for agent α at time t involves the determination of values $\tilde{x}_\alpha^0(t), \ldots, \tilde{x}_\alpha^N(t)$ for the variables $x_\alpha^0(t), \ldots, x_\alpha^N(t)$, where:

$x_\alpha^0(t)$ is planned demand for leisure;
$x_\alpha^1(t)$ is planned demand for the essential good;
$x_\alpha^2(t), \ldots, x_\alpha^N(t)$ are planned demands for the non-essential goods

by agent α at time t. The decision problem takes the form

$$\max_{x_\alpha^0(t),\ldots,x_\alpha^N(t)} \prod_{n=0}^{1} \left(x_\alpha^n(t) - \delta_{\min}^n\right)^{\gamma_\alpha^n} \prod_{n=2}^{N} \left(x_\alpha^n(t) + \delta^n\right)^{\gamma_\alpha^n}$$

such that

$$\tilde{w}_\alpha(t)x_\alpha^0(t) + \sum_{n=1}^{N} \tilde{p}_\alpha^n(t)x_\alpha^n(t) \leqslant \tilde{w}_\alpha(t)L_\alpha$$

$$x_\alpha^n(t) \geqslant \delta_{\min}^n \quad \text{for } n = 0, 1$$

$$x_\alpha^n(t) \geqslant 0 \quad \text{for } n = 2, \ldots, N$$

where

$$\sum_{n=0}^{N} \gamma_\alpha^n = 1$$

$$\delta_{\min}^0 < L_\alpha$$

The parameters are defined as follows:

$\delta_{\min}^n$ is the survival quantity of essential good n for $n = 0, 1$;
γ_α^n is a utility function parameter for $n = 0, \ldots, N$;
L_α is agent α's capacity to supply labour;
$\tilde{w}_\alpha(t)$ is agent α's subjective wage for time t;
$\tilde{p}_\alpha^n(t)$ is agent α's subjective price for good n for $n = 1 \ldots N$.

The feasible region has an empty interior if

$$\tilde{p}_\alpha^1(t) \geqslant \tilde{w}_\alpha(t)\left(L_\alpha - \delta_{\min}^0\right)/\delta_{\min}^1$$

in which case agent α plans to consume only the survival quantities of the essential commodities:

$$\tilde{x}_\alpha^n(t) = \delta_{\min}^n \quad \text{for } n = 0, 1$$

$$\tilde{x}_\alpha^n(t) = 0 \quad \text{for } n = 2, \ldots, N$$

If the above inequality is not satisfied, then the form of the utility function ensures that planned demand at the optimal solution will exceed the survival quantities for the essential goods and may be zero or positive for each non-essential good. The optimal solution can be shown to be given by the following expressions:

$$\tilde{x}_\alpha^n(t) = \frac{\gamma_\alpha^n}{\tilde{p}_\alpha^n(t)\lambda_\alpha(t)} + \delta_{\min}^n \quad \text{for } n = 0, 1$$

$$\tilde{x}_\alpha^n = \frac{\gamma_\alpha^n}{\tilde{p}_\alpha^n(t)\lambda_\alpha(t)} - \delta^n \quad \text{for } \mathrm{Rank}(n) = R' + 1, \ldots, N - 1$$

$$\tilde{x}_\alpha^n = 0 \quad \text{for } \mathrm{Rank}(n) = 1, \ldots, R'$$

where

$$\lambda_\alpha(t) = \frac{1 - \sum_{\mathrm{Rank}(n)=1}^{R'} \gamma_\alpha^n}{\tilde{w}_\alpha(t)L_\alpha + \sum_{\mathrm{Rank}(n)=R'+1}^{N-1} \tilde{p}_\alpha^n(t)\delta^n - \tilde{w}_\alpha(t)\delta_{\min}^0 - \tilde{p}_\alpha^1(t)\delta_{\min}^1}$$

and the ranking of the non-essential goods is defined as follows:

for all n and m, where $n, m \in \{2, \dots N\}$,
if $\frac{\tilde{p}_\alpha^n(t)\delta^n}{\gamma_\alpha^n} > \frac{\tilde{p}_\alpha^m(t)\delta^m}{\gamma_\alpha^m}$ then $\text{Rank}(n) < \text{Rank}(m)$

and, to break ties,

if $\frac{\tilde{p}_\alpha^n(t)\delta^n}{\gamma_\alpha^n} = \frac{\tilde{p}_\alpha^m(t)\delta^m}{\gamma_\alpha^m}$ when $n \neq m$, then $\text{Rank}(n) < \text{Rank}(m)$ if $n < m$.

To compute $\tilde{x}_\alpha^0(t), \dots, \tilde{x}_\alpha^N(t)$, it is necessary first to determine the rank R' which separates those non-essential goods for which there is positive demand from the rest. To do so, it can be shown that it is sufficient to evaluate

$$\frac{\tilde{p}_\alpha^k(t)\delta^k}{\gamma_\alpha^k}\left(1 - \sum_{\text{Rank}(n)=1}^{k} \gamma_\alpha^n\right) - \sum_{\text{Rank}(n)=k+1}^{N-1} \tilde{p}_\alpha^n(t)\delta^n$$

for $k = 0, \dots, N-1$ and to stop when it is less than

$$\tilde{w}_\alpha(t)\left(L_\alpha - \delta_{\min}^0\right) - \tilde{p}_\alpha^1(t)\delta_{\min}^1$$

at which point $R' = k$. Then $\lambda_\alpha(t)$ may then be calculated from which the optimal solution, $\tilde{x}_\alpha^0(t), \dots, \tilde{x}_\alpha^N(t)$, follows immediately.

A.2. Agent behaviour: production planning

The production decision of agent α in time period t is represented by the variable $n_\alpha(t)$, where $n_\alpha(t) = 1, \dots, N$. The production plan requires the staple good to be grown if its current stock is less than the buffer quantity ($n_\alpha(t) = 1$). Otherwise, it involves choosing the crop that will maximise agent α's expected net revenue in time period t, $\tilde{\pi}_\alpha(t)$, where

$$\tilde{\pi}_\alpha(t) = \sum_{n=0}^{N} \left(\tilde{p}_\alpha^n(t) y_\alpha^n(t) - \tilde{w}_\alpha(t)(y_\alpha^{Ln}(t) + y_\alpha^{LT_n}(t) + y_\alpha^{LT_l}(t))\right)$$

in which, for agent α in time period t,

$y_\alpha^{Ln}(t)$ is labour input to the production of good n;
$y_\alpha^n(t)$ is the expected output of good n;
$y_\alpha^{LT_n}(t)$ is the expected input of labour for the transport of n;
$y_\alpha^{LT_l}(t)$ is the expected input of labour for the journey to work.

The production and transportation functions are as follows:

$$y_\alpha^n(t) = \tilde{a}_\alpha^n(t) y_\alpha^{Ln}(t)$$
$$y_\alpha^{LT_n}(t) = \tilde{a}_\alpha^n(t) y_\alpha^{Ln}(t) a^{T_n} d_{\sigma\alpha}$$
$$y_\alpha^{LT_l}(t) = y_\alpha^{Ln}(t) a^{T_L} d_{\sigma\alpha}$$

where $\tilde{a}_\alpha^n(t), a^{T_n}, a^{T_L}$ are production and transportation parameters; $d_{\sigma\alpha}$ is the distance from the settlement cell to agent α's field cell.

Given the production and transportation functions,

$$\begin{aligned}\tilde{\pi}_\alpha(t) &= \sum_{n=0}^{N} y_\alpha^{Ln}(t)\big(\tilde{p}_\alpha^n(t)\tilde{a}_\alpha^n(t) \\ &\quad - \tilde{w}_\alpha(t)\left(1 + \tilde{a}_\alpha^n(t) a^{T_n} d_{\sigma\alpha} + a^{T_L} d_{\sigma\alpha}\right)\big) \\ &= \sum_{n=0}^{N} y_\alpha^{Ln}(t)\left(\tilde{p}_\alpha^n(t)\tilde{a}_\alpha^n(t) - \tilde{w}_\alpha(t)\tilde{a}_\alpha^{Ln}(t)\right)\end{aligned}$$

where

$$\tilde{a}_\alpha^{Ln}(t) \equiv 1 + \tilde{a}_\alpha^n(t) a^{T_n} d_{\sigma\alpha} + a^{T_L} d_{\sigma\alpha}$$

The above expression may be reduced to

$$\pi_\alpha(t) = \left(L_\alpha - \tilde{x}_\alpha^L(t)\right) \sum_{n=0}^{N} \left(\tilde{p}_\alpha^n(t)\tilde{a}_\alpha^n(t)/\tilde{a}_\alpha^{Ln}(t) - \tilde{w}_\alpha(t)\right)$$

since the total demand for labour (for leisure, production and transportation) is equal to the labour supply, i.e.,

$$\tilde{x}_\alpha^L(t) + y_\alpha^{Ln}(t) + y_\alpha^{LT_n}(t) + y_\alpha^{LT_l}(t) = L_\alpha$$

This implies that

$$y_\alpha^{Ln}(t) = \left(L_\alpha - \tilde{x}_\alpha^L(t)\right)/\tilde{a}_\alpha^{Ln}(t)$$

Thus, defining δ_{buffer}^1 as the minimum storage level required for the essential good, the production plan is given by

$$\begin{cases} n_\alpha(t) = 1 & \text{if} \quad s_\alpha^1(t) < \delta_{\text{buffer}}^1 \\ n_\alpha(t) = n^* & \text{otherwise} \end{cases}$$

where n^* is the value of n which yields $\max_n((L_\alpha - \tilde{x}_\alpha^L(t))(\tilde{p}_\alpha^n(t)\tilde{a}_\alpha^n(t)/\tilde{a}_\alpha^{Ln}(t) - \tilde{w}_\alpha(t)))$, so that

$$\begin{cases} y_\alpha^{Ln^*}(t) = \left(L_\alpha - \tilde{x}_\alpha^L(t)\right)/\tilde{a}_\alpha^{Ln^*}(t) \\ y_\alpha^{Ln}(t) = 0 \quad \text{if} \quad n \neq n^* \end{cases}$$

A.3. Output

Output is determined by crop choice, labour input and a random weather variable:

$$y_\alpha^n(t) = \begin{cases} a^{Mn} y_\alpha^{Ln}(t)/\left(y_\alpha^{Ln}(t) + a^n + \text{randnorm}(0, a^{rn})\right) \\ \text{if} \quad n = n^* \\ 0 \quad \text{otherwise} \end{cases}$$

A.4. Agent behaviour: market supply and demand

The quantity of each crop supplied to or demanded from the market by agent α at time t is determined by adding the output of that crop in t to the stock carried forward from the previous period and subtracting the planned demand. If it is non-negative, it constitutes agent α's supply of the crop at time t. Otherwise, it constitutes its demand. Thus,

$$z_\alpha^{\text{sup}n}(t) = \max\left(s_\alpha^n(t) + y_\alpha^n(t) - \tilde{x}_\alpha^n(t), 0\right) \quad \text{for} \quad n = 1, \dots, N$$
$$z_\alpha^{\text{dem}n}(t) = \max\left(\tilde{x}_\alpha^n(t) - s_\alpha^n(t) - y_\alpha^n(t), 0\right) \quad \text{for} \quad n = 1, \dots, N$$

where for $n = 1, \dots, N$, $z_\alpha^{\text{sup}n}(t)$ is the supply of n by agent α in t; $z_\alpha^{\text{dem}n}(t)$ is the demand for n by agent α in t.

A.5. Market transactions

The market mechanism, described in assumptions (29) to (34), involves agent-to-agent transactions such that

$$\sum_{\beta \in A(t)} z^n_{\beta\alpha}(t) \leqslant z^{\mathrm{dem}n}_\alpha(t)$$
$$\sum_{\beta \in A(t)} z^n_{\alpha\beta}(t) \leqslant z^{\mathrm{sup}n}_\alpha(t)$$

where $z^n_{\beta\alpha}(t)$ is the quantity of n sold by β to α in t; $A(t)$ is the set of agents that exist at t.

The total value of purchases by agent α in time t is also constrained by the cash available:

$$\sum_{n=1}^{N} \sum_{\beta \in A(t)} \tilde{p}^n_\beta(t) z^n_{\beta\alpha}(t) \leqslant C_\alpha(t)$$

where $C_\alpha(t)$ is the cash available to α in t.

A.6. Cash, wages and prices

The cash carried forward by α to $t+1$ is given by the sum of its earnings and any unspent cash:

$$C_\alpha(t+1) = C_\alpha(t) - \sum_{n=1}^{N} \sum_{\beta \in A(t)} \tilde{p}^n_\beta(t) z^n_{\beta\alpha}(t) + \sum_{n=1}^{N} \sum_{\beta \in A(t)} \tilde{p}^n_\alpha(t) z^n_{\alpha\beta}(t)$$

Agent α's subjective prices are updated on the basis of market evidence. The weighted average price paid for n by α in t is

$$\left(\sum_{\beta \in A(t)} \tilde{p}^n_\beta(t) z^n_{\beta\alpha}(t)\right) \Big/ \left(\sum_{\beta \in A(t)} z^n_{\beta\alpha}(t)\right)$$

and the subjective price in $t+1$ is taken to be a weighted average of this figure and the previous price:

$$\tilde{p}^n_\alpha(t+1) = \theta_{\alpha 0}\tilde{p}^n_\alpha(t) + \theta_{\alpha 1}\left(\sum_{\beta \in A(t)} \tilde{p}^n_\beta(t) z^n_{\beta\alpha}(t)\right) \Big/ \left(\sum_{\beta \in A(t)} z^n_{\beta\alpha}(t)\right)$$

for $n = 1, \ldots, N$

where

$\theta_{\alpha 0} + \theta_{\alpha 1} = 1$

Similarly, the subjective wage is updated using the current subjective value of labour:

$$\tilde{w}_\alpha(t+1) = \psi_{\alpha 0}\tilde{w}_\alpha(t) + \psi_{\alpha 1}\left(\tilde{p}^n_\alpha(t) y^n_\alpha(t)\right) / \left(L_\alpha - \tilde{x}^0_\alpha(t)\right)$$

where

$\psi_{\alpha 0} + \psi_{\alpha 1} = 1$

A.7. Production parameters

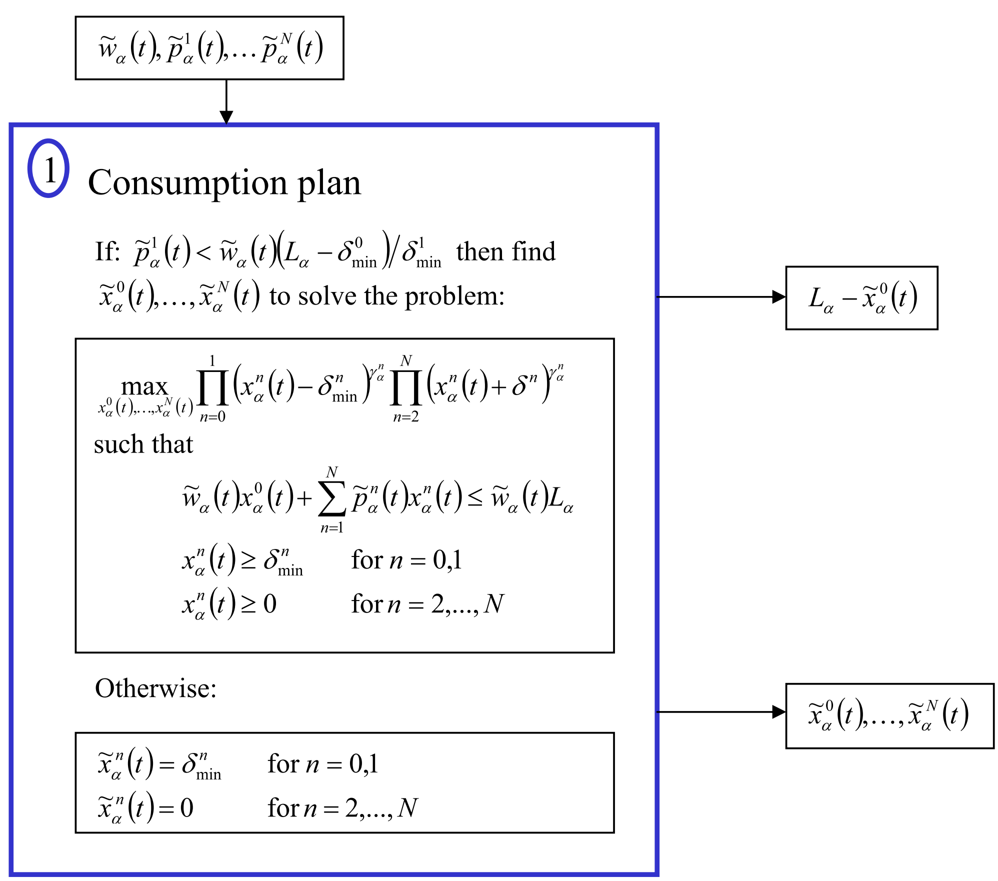
Finally, agent α's estimate of the production parameters is updated as follows for all α, n and t:

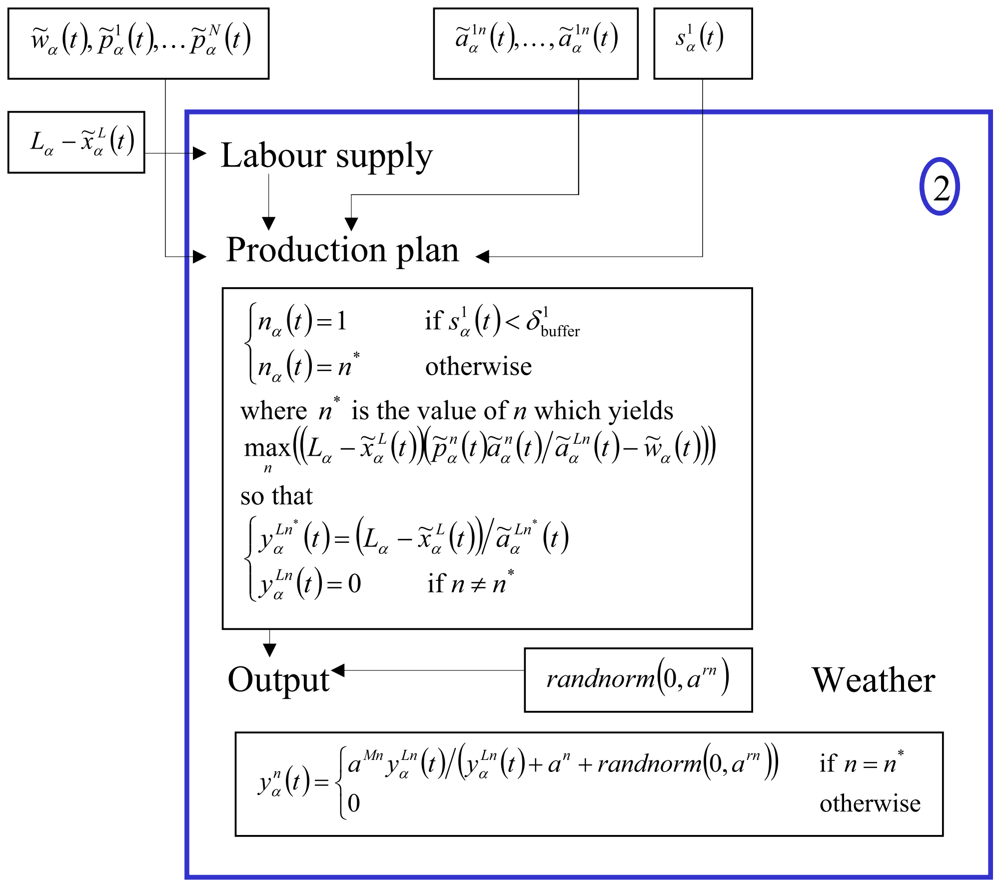
$$\left.\begin{array}{ll} \tilde{a}^n_\alpha(t) = \omega_{\alpha 0}\tilde{a}^n_\alpha(t-1) + \omega_{\alpha 1} y^n_\alpha(t)/y^{Ln}_\alpha(t) & \text{if } n_\alpha(t-1) = n \\ \tilde{a}^n_\alpha(t) = \tilde{a}^n_\alpha(t-1) & \text{otherwise} \end{array}\right\}$$

where

$\omega_{\alpha 0} + \omega_{\alpha 1} = 1$

Appendix B





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