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Survey

Varieties of agents in agent-based computational economics: A historical and an interdisciplinary perspective

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ABSTRACT

In this paper, we trace four origins of agent-based computational economics (ACE), namely, the markets origin, the cellular-automata origin, the tournaments origin, and the experiments origin. Along with this trace, we examine how these origins have motivated different concepts and designs of agents in ACE, which starts from the early work on simple programmed agents, randomly behaving agents, zero-intelligence agents, human-written programmed agents, autonomous agents, and empirically calibrated agents, and extends to the newly developing cognitive agents, psychological agents, and culturally sensitive agents. The review also shows that the intellectual ideas underlying these varieties of agents cross several disciplines, which may be considered as a part of a general attempt to study humans (and their behavior) with an integrated interdisciplinary foundation.

1. Introduction

Agent-based computational economics (ACE) has experienced steady growth over the past decade. A number of comprehensive edited volumes and journal articles have well articulated its frontiers or its moving directions (Tesfatsion and Judd, 2006). As a part of *computational social science*, a newly integrated and overarching framework of social sciences, ACE does not grow alone, but also evolves with this larger community (Lazer et al., 2009).

The title of this paper suggests two lines of development in this paper. The first line is to place agent-based computational economics (ACE) in a historical context and to trace some of its origins, and the second one is then to focus on the notions of "agents" in each of these origins and to see how these notions have been changing and have been enriched within an interdisciplinary environment. The four origins of ACE considered in this paper are, in chronological order, the *markets origin* (with a long history), the *cellular-automata origin* in the 1970s, the *economic-tournaments origin* (the *game theory origin*) in the 1980s, and the *experimental-economics origin* in the 1990s. These four origins are, of course, not independent of each other. They can be imagined to be four gates to the same castle, with the markets origin being the main gate. While tourists may enter the castle through different gates, their experiences of the castle will be similar if they all explore the castle long enough. The four origins selected above are then very much like the answer to the question, "Where did you start your tour in ACE?" Hence, the answer may be Thomas Schelling's segregation model (Schelling, 1971), Robert Axelrod's simulation of the iterated prisoner's dilemma tournament (Axelrod, 1987), Jasmina Arifovic's simulation of cobweb experiments (Arifovic, 1994), Alan Kirman and Nicolaas Vriend's fish market model (Kirman and Vriend, 2001), or a long list like this. Certainly, these four origins are by no means exhaustive, and different tour guides

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may have different arrangements. However, as long as we have the main gate (the market origin) as the starting point and have the whole castle as the destination, our choices of the other three origins will have little effect.

Leaving the main gate aside, our choice of the three other origins does, however, serve one purpose well, namely, it puts a sharp focus on the *agents* in ACE. Specifically, these three origins of ACE together enable us to better see that agents involved in various ACE models are rather interdisciplinary. They are motivated by different disciplines with different inquiries. This interdisciplinary multi-dimension of agents makes it useful to give a taxonomy of agents or agent paradigms so that all research questions regarding agents in ACE can be dealt with on a general basis. To achieve this purpose, the taxonomy to be given in this paper is not tailored to ACE models with specific applications and techniques. The taxonomy is, hopefully, generic and is only up to the very fundamental level. Hence, only three kinds of thoughts on agents or agent paradigms in ACE are delineated. These three are *simple* (*programmed*) *agents*, *autonomous agents* and *human-like agents*. Each of these three agents fits uniquely into one of the four origins of ACE reviewed in this paper; alternatively speaking, these origins of ACE have uniquely motivated one agent paradigm.

The remainder of the paper is, therefore, organized as follows in order to connect each origin to its associated agent paradigm. We start with the a brief view of the main gate in Section 2. The main gate gives us a broader and holistic picture of agent-based computational economics, specifically, her role in the history and the future of economic analysis. Therefore, we should have a quick look at of it before starting each trial. Then we continue the tour with the cellular automata origin and the simple programmed agents in Section 3, then move to the economic tournament origin and the autonomous agents in Section 4, and end with the economic experiment origin and human-like agents in Section 5. Concluding remarks are then given in Section 6.

2. The markets origin

There are several origins (traditions) of agent-based modeling in the social sciences. For economists, probably the most important one is the historically long pursuit of a real construction (*procurement processes*) and hence a real understanding of *markets*. Since the time that Léon Walras (1834–1910) proposed his competitive general equilibrium model (Walras, 1954) and characterized the economy as a set of a number of equations and unknowns, a seed for the long pursuit has already been planted. Walras himself did not give an actual description of markets. While it was claimed that his t*â*tonnement process was inspired by the Paris stock exchange (Bourse of Paris), the price formation process driven by the Walrasian auctioneer is very different from the process of that institution (Walker, 2001). Hence, the fundamental quest is whether the price discovery process, also known as the t*â*tonnement process, can actually replace the true market process, and the Walrasian auctioneer, or the equivalent advanced supercomputer, can actually do the job of allocating resources as the natural markets do.

Herbert Scarf, one of the founders of the computable general equilibrium model, once stated:

> In my opinion, the major attraction of markets over centralized calculation, for Gorbachev and his economic reformers, is not so much the mathematical difficulty of a single equilibrium calculation; it is rather that these computations must be performed over and over again in real time, in the face of constantly changing economic circumstances. The economy is in continual flux, with new possibilities constantly emerging, and mathematical solutions to the equilibrium equations will at best represent the solutions to yesterday's problem. If we are to be responsive to the novel conditions of daily life–and to engage the energies and skills of millions of self-interested economic actors–it may be necessary to use the market as an algorithm for solving the equilibrium equations rather than solving these equations themselves on the computer. Scarf (1990, p. 379)

In the history of economic analysis, this quest has been pursued by economists in different forms, including the debate on the possibility of a socialist calculation (Boettke, 2000), the silence of "markets" in economic theory (Mirowski, 2007), the aggregation problem over adaptive interacting heterogeneous agents (Kirman, 1992; Stoker, 1993; Blundell and Stoker, 2005; Gallegati and Palestrini, 2006), and the mathematics suitable for social sciences (Velupillai, 2010; Borrill and Tesfatsion, to appear). Regardless of how the issues were presented to economists, they all point to some undesirable consequences by assuming away or oversimplifying the originally decentralized processes in economics. The strong motivation of having decentralized processes as the backbone of economics has then incubated the research of using agent-based models in economics. Examples abound and here are just a few of them: Albin and Foley (1992), Vriend (1995), Kirman and Vriend (2001), McFadzean et al. (2001), Tesfatsion (2002), Riechmann (2002), Gode et al. (2004), Axtell (2005) and Gintis (2006, 2007, 2010). To some extent, they intend to displace the Walrasian auctioneer by a decentralized process or the procurement process (Tesfatsion, 2006):

> To move from Walrasian to agent-based modeling, the Walrasian Auctioneer has to be replaced by agent-driven procurement processes. …this replacement is by no means a small perturbation of the model. (Tesfatsion, 2006, p. 847)

In addition to the oversimplification or the absence of the market mechanism as a decentralized or social (interaction) process, almost the other side of the same coin is the absence of brains, minds, and cultures of agents, or simply, *Homo Sapiens* (Thaler, 2000; Kahneman, 2003; Akerlof and Shiller, 2009). These are the essential ingredients of what makes agents *heterogeneous*. In this regard, the *economics with many and complex heterogeneous agents* may also be attributed to the market origins. Aggregation problems over their interactions can be much harder than those of the representative agents (Kirman, 1992). The rise of ACE can be understood as efforts to bring these two missing dimensions back to economics. Below, we start with the earliest ACE models, namely, cellular automata.

3. The cellular-automata origin

3.1. Cellular automata

One of the pioneering applications of agent-based modeling to social science begins in *city dynamics*. For many, Thomas Schelling's well-known *spatial proximity model* is regarded as the first agent-based computational economic model (Schelling, 1969, 1971). Another similar pioneering application of cellular automata to social sciences is made by Sakoda (1971), who, in effect, already had the checkerboard design in his unpublished dissertation (Sakoda, 1949). Nevertheless, neither Thomas Schelling nor James Sakoda refer to cellular automata; instead, it was then simply known as a *checkerboard model*. The first person who explicitly classifies *checkerboard models* under the cellular automata framework is the economist (Albin, 1975).

The intellectual inquiry behind cellular automata is biological. It is about how cells, as building blocks, can develop into coherent organisms. While a mathematical formalism of the cellular growth processes was already given in Thompson (1917), the speculation that organic development might be susceptible to computation comes much later. Turing (1952) and John von Neumann are the two pioneers, and it is von Neumann who actually demonstrates the first cellular automata, a 29-state self-reproducing cellular automaton, in the 1950s.[1] Von Neumann has drawn some of his inspiration from his colleague in the Manhattan project, Stanislaw Marcin Ulam (1909–1984). At the time, Ulam was studying the growth of crystals. He suggested to von Neumann as early as 1950 that simple cellular automata could be found in sets of local rules that generated mathematical patterns in 2-D and 3-D space where global order could be reproduced from local action.[2]

A cellular automaton is a collection of agents situated on a grid of specified shape, usually a one-dimensional row, or two-dimensional rectangle (checkerboard). Each agent is characterized by a set of *network-based decision rules*, which indicate how an agent's decision or choice is determined by the respective network. While the network can be global and local, agents' behavior, by and large, tends to only be affected by the local network and little by the global network. Specifically, this local network is characterized by a set of *neighbors* whose behavior in the past may affect the agent's behavior in the future.

Cellular automata can arouse widespread interest among both scientists and social scientists because they comprise the earliest popularly used models that demonstrate how complex patterns may emerge from the local interactions of agents who follow very simple rules. One early notable example is given by the famous *Game of Life*, a much simpler architecture (each cell has two states only), which was invented by Cambridge mathematician John Conway in 1970. In the simplest possible way, the Game of Life simulates the essential ingredients of reproduction. Later examples are contributed by Stephen Wolfram, who in 1983 published the first of a series of papers systematically investigating a very basic but essentially unknown class of cellular automata, which he termed *elementary cellular automata* (Wolfram, 1994). Using the concept from dynamical systems, he first classified cellular automata into four classes. The unexpected complexity of the behavior of these simple rules, demonstrated by Class III (chaos) and Class IV (localized structure), led Wolfram to suspect that *complexity in nature may be due to similar simple mechanisms*. This lesson is very influential in the use of simple programmed agents, and even homogeneous ones, in a large class of ACE models.

3.2. Simple programmed agents

Since the focus is never on the individual cell or agent, but the resultant system behavior through interaction, and since highly complex or universal (computation) behavior can be generated from agents following very simple rules,[3] agents in the cellular-automata tradition are naturally simple programmed agents. This becomes the essential idea for a large class of ACE models. To an extreme, one of the most celebrated designs in this vein is that of *zero-intelligence agents*.

3.2.1. Zero-intelligence agents

The zero-intelligence agent was initiated by Gode and Sunder (1993) in a double-auction market environment. Technically, the zero-intelligence agent was characterized as a *randomly behaving agent*. This agent, when making the bid or ask decision, simply randomly picks any price from all those that will not impose a loss on the agent. This constraint is to ensure that *obvious stupidity* can be avoided, but these agents are obviously not able to learn.

[1] Von Neumann started a manuscript entitled "Theory of Automata: Construction, Reproduction and Homogeneity" in 1952. Up until his death in 1957, he was working on the notion that computers through their software could embody a set of rules or instructions that would enable them to reproduce their structure. This work was left on his death in a manuscript form, and was then edited by his student and colleague Arthur Burks (1915–2008) (Von Neumann completed by Burks, 1966).

[2] For a brief review of the history of cellular automata, the interested reader is referred to Wolfram (2002, p. 876).

[3] It has been shown that various versions of cellular automata are capable of universal computation. For example, see Kari (2005).

Despite this being so, Gode and Sunder have shown that this kind of zero-intelligence agent can perform as well as human agents in the double auction experiments:

> Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality. (Gode and Sunder, 1993, p. 119)

This setting essentially binds agents' behavior only by their *budget constraints*. In other words, given budget constraints and the institutional arrangements, the rationality of the individual may not matter. Hence, zero-intelligence, as pointed out by Gode and Sunder (1993), can be long traced back to Becker (1962), who showed that a budget constraint is sufficient to guarantee the proper slope of the supply and demand curves.

In addition to double auction markets, the ZI agents have also been applied to other ACE models, including those for general equilibrium and financial markets. For the former, the applications are extended from the original partial equilibrium models to general equilibrium models; for the latter the simple continuous double-auction market is extended to order-book-driven markets and other related market mechanisms. While the concrete implementations of the idea of ZI need to be tailored to different applications, the basic idea is almost the same.[4] Recently, it has been further applied to other ACE models, such as prediction markets (see more in Section 3.2.3), macroeconomics (Ussher, 2009a,b), and social networks (Tseng et al., 2008, 2009). Randomly behaving agents are also extensively used in other models where the term zero-intelligence agents may be replaced by others, such as noise traders, irrational traders, etc.

It would be interesting to notice that the design of the zero-intelligence agent is originally *not* motivated by the modeling philosophy of complex science,[5] while it has the intention to show that the self-regulating order can emerge from these simple agents. In this regard, it is very similar to recent studies on *ants* or *termites* in computational entomology, or what is broadly known as *swarm intelligence*.

3.2.2. Swarm intelligence

Starting from the middle of the 1980s, a series of computational studies has been devoted to *mimicking* the global intelligent behavior observed from flocks of birds, school of fish, ant colonies, and bee colonies, etc. The first such kind of study was initiated by Craig Reynolds in his Boids project, which is mainly concerned with the flocking behavior of birds. It has been shown that with each bird homogeneously following a set of simple rules, the birds can together generate an orderly movement pattern (Reynolds, 1987).

The same idea was further extended to the social dynamics of ants. An ant colony was given access to food linked to the nest by two bridges of different lengths. Ants had to choose one way or the other. Experimental observations show that, after a few minutes, most ants used the shortest branch, even though ants are practically blind (Goss et al., 1989). This phenomenon was successfully simulated by Dorigo et al. (1996) using their proposed *ant colony optimization algorithm*. The artificial ant following that algorithm deposits pheromone along the trail while going to the food source and also while returning to the nest. At a branch, they make a probabilistic choice based on the amount of pheromone along each branch. Hence, the behavior or decisions of these ants are also, to some extent, random, which requires no memory or a priori knowledge, and does not involve learning in an explicit way. In this sense, it is similar in vein to the zero-intelligence agent.

To economists, what may be more familiar is the version of the ant algorithm proposed by Kirman (1993). "Ants, faced with two identical food sources, were observed to *concentrate* more on one of these, but after a period they would turn their attention to the other". (Kirman, 1993, p. 137, Italics added). Inspired by observing the behavior of ants, Kirman characterizes the switching potential of each individual by two parameters, namely, *a probability of self-conversion* and *a probability of being converted*. The former gives the probability that an individual will switch to other types of agents (or change direction to the other food source) without external influence, whereas the latter gives the probability that the individual will be *persuaded* by the other individual to whom he/she is randomly matched and then follow him/her. By appropriately setting these two parameters, one can mimic the foraging behavior of ants.

Entomologists have long found that ants or termites can communicate well. The communication is, however, not necessarily direct, but more indirect, partially due to their poor vision. Their reliance on indirect communication has been noticed by the French biologists Pierre-Paul Grasse (1895–1985), and he termed this style of communication or interaction *stigmergy* (Grosan and Abraham, 2006), of which pheromone is a special kind. Stigmergy is a method of communication in which the individuals communicate with each other via modifying their local environment. The price mechanism familiar to economists is also an example of stigmergy. It does not require market participants to have direct interaction, but only indirect interaction via price signals. In this case the environment is characterized as the price, which is constantly changed by market participants and hence constantly invites others to take further actions.

[4] The zero-intelligence agent has been very influential in agent-based computational economics and finance. A general review can be found in Duffy (2006) and Ladley (forthcoming). Also see Barr et al. (2008) for quite an extensive discussion on the use of zero-intelligence agents in the future of ACE.

[5] See Sunder (2004) for how this idea of the zero-intelligence agents was originally motivated and developed by Sunder and Gode in their class assignments. A very interesting process!

3.2.3. Social intelligence

For social scientists, swarm intelligence is known as *social intelligence* or the *wisdom of crowds*, and has become an interdisciplinary research subject that crosses various social and technical disciplines (Bonabeau and Meyer, 2001). Inquiries into the emergent social intelligence are basically concerned with the design of an interaction platform so as to expect and observe the emergent social intelligence. One example is the design of *e-participation* or, more specifically, the *prediction markets* (Wolfers and Zitzewitz, 2004). The social intelligence of prediction markets when applied to predicting election outcomes has already shown the superior performance as compared to the results of polls (Berg et al., 2008).

In talking about social intelligence, one certainly cannot miss mentioning Friedrich von Hayek's influential work (Hayek, 1945). Hayek considered the market and the associated price mechanism to be a way of pooling or aggregating the market participants' limited knowledge of the economy. While the information owned by each market participant is rather limited, the pooling of it can generate efficient prices. The assertion of this article was later on coined as the *Hayek Hypothesis* by Smith (1982) in his double auction market experiments. In fact, it is (Vriend, 2002), who first provided an agent-based model to show the advantages and the limits of information aggregation. From his simulation, what can actually be aggregated, information or ignorance, depends on the statistical decision rules followed by each agent, but that behavior itself co-evolves with the rules followed by other agents. The co-evolution process can sometimes result in inferior outcomes.

The intensive study of the Hayek hypothesis in experimental economics has further motivated or strengthened the idea of prediction markets. Recently, the performance of the prediction markets has been examined using ACE models. Being in the vein of the Hayek hypothesis and probably going even further, most of these ACE models start with the device of zero-intelligence agents. It has been demonstrated that markets populated by these zero-intelligence traders can reach roughly informative price predictions (Othman, 2008; Klingert and Meyer, 2010; Othman and Sandholm, 2010), an outcome analogous to that of Gode and Sunder (1993).

In sum, the device of the zero-intelligence agent gives one of the best illustrations of simple programmed agents. As a first approximation, it replicates both results observed from double auction experimental markets and the real prediction markets, which are the two tests for the Hayek hypothesis. Hence, the powerfulness of simple programmed agents as shown in cellular automata and swarm intelligence is considerably extended to ACE.

3.2.4. Near zero-intelligence agents

Of course, it is not in all cases that zero-intelligence agents are found to work. They can fail to achieve the competitive equilibrium in some extended settings, such as an imbalanced (asymmetric) demand and supply schedule (Cliff and Bruten, 1997), and multiple interlinked markets (Bosch-Domènech and Sunder, 2000), etc. A list of what human subjects can do (in experimental economics) but zero-intelligence agents cannot is given by Duffy (2006). When the ZI agents fail, various enhanced versions to make the ZI agent additionally smart have been proposed. This forces us to answer how much additional intelligence is required to make it work, which leads to various versions of near zero-intelligence agents (Cliff and Bruten, 1997; Duffy and Unver, 2006; Crockett, 2008).

The general idea for making near zero-intelligence agents is to parameterize the zero-intelligence agent. The parameterization depends on the specific application domain. After that, we impose a stronger restriction on the parameter space; in other words, the search space is now restricted or the search is biased. However, the guidance given to this guided search must be minimal as a slight perturbation to the zero-intelligence agents. Nevertheless, it is not entirely clearly on the borderline, and whether the ''intelligence'' of near zero-intelligence agents is really closer to zero may not be that straightforward, as we shall continue discussing in Section 3.3. Despite this being so, the essential message of these efforts remains the same: markets populated by simple agents may be sufficient to perform something remarkable. This feature, therefore, gives quite a strong support for and demonstration of the *KISS* (Keep it simple, stupid) principle, as originally advocated by Axelrod (1997a), and is influential in ACE.[6]

3.2.5. Randomly-behaving (entropy-maximization) agents

The notion of the zero-intelligence agent is not only well received by economists, but also by physicists, specifically, econophysicists (Bouchaud et al., 2002; Farmer et al., 2005), although they are not identically motivated. The reason that physicists favor the device of the zero-intelligence agent is because the strategic behaviors of financial agents are generally poorly known and are difficult to model. Therefore, in the vein of the law of large numbers, they simply assume that these complexifications will cancel each other out so that altogether their aggregate behaviors are observationally equivalent to the randomly-behaving agents (zero-intelligence agents). In a sense, doing this is also generally related to the application of the *maximum entropy principle* to the agent design, i.e., to be ''maximally non-committal with regard to missing information'' (Jaynes, 1957, p. 620).

[6] Having said that, we must also point out the opposition to this principle. The equally well-known alternative is the *KIDS* (Keep it descriptive, stupid) principle, proposed by Edmonds and Moss (2004). It was argued that social simulation models are different from analytical mathematical models, hence the pursuit of simplicity should also change accordingly. The contrast between KISS and KIDS is an on-going research issue in the methodology of social simulation. The interested reader is referred to the special issue on ''The Methodology of Simulation Models'' of the *Journal of Artificial Societies and Social Simulation* (vol. 12, no. 4).

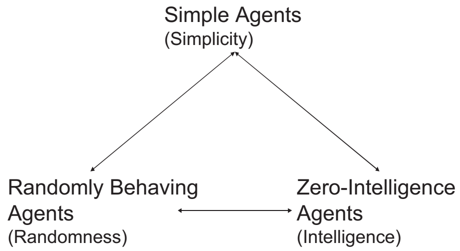

Fig. 1. Relationships among the three notions of agents.

This principle is nicely applicable to Gode and Sunder's zero-intelligence agent. Since normal traders would not propose or accept a deal which would obviously lead to economic loss or not lead to welfare improvement, under no further information on what else they will do, the design of the zero-intelligence agent is *minimally prejudiced* in the sense of Jaynes (1957). Hence, one way to formalize and to generalize the zero-intelligence agent is to explore the applicability of the entropy maximization principle to design the agent. Doing so is also consistent with the KISS principle in spirit since the entropy maximization principle per se is an information-theoretic foundation for model simplicity.

In sum, the idea of the zero-intelligence agent or the randomly behaving agent seems to be a very natural way to start when one has no clue as to how agents actually behave or want to have an easy start without too many complications, or want to have a benchmark as a comparison basis for other more developed models. However, none of the reasons above has anything to do with cognitive salience. Yet the term zero intelligence is a term related to cognitive capability.

3.3. *Simplicity, intelligence, and randomness*

We now attempt to put the several ideas of agents discussed so far together and look more closely at their relationships, which may otherwise be largely taken for granted in many applications. We start with the idea of simple agents or agents following simple rules (automata), then agents with low-cognitive capability (zero-intelligence agents and artificial ants), and finally end up with randomly behaving agents and entropy-maximizing agents. The basic question is: *are they the same*? We have this question simply because in many applications they are assumed to be so. However, here, what we encounter is exactly an example of the agent as an interdisciplinary concept, and the terms, such as ''simple'', ''naive'' and ''random'', cross several disciplines, from computer science, psychology, and neuroscience, to mathematics and physics. Can these different terms or concepts belonging to different disciplines be unified on a higher ground, or are they independent? In the following, we shall reflect upon each of the three bilateral relationships as depicted in Fig. 1.

3.3.1. *Simplicity and randomness*

For us, a natural technical notion of simplicity required for developing the idea of a simple agent is built upon computer science, i.e., *algorithmic simplicity* or, as more frequently used, *algorithmic complexity*.[7] Put a high concept in a rough but easy-to-understand way: this notion requires us to focus on the program length which we use to describe the agent, also known as the *description length*; the shorter the simpler. Hence, simple agents are agents who are *algorithmically* simple. We can then ask whether the randomly behaving agents are simple in terms of the program length of the underlying pseudo-random number generators. There are several different versions of the pseudo-random number generators, from the initial attempt by Von Neumann (1951) to the breakthrough made by Matsumoto and Nishimura (1998) and to the long-standing challenge of cryptographically secure pseudo-random number generators, each leading to a different quality of the pseudo-random number being generated.[8] The point is that they are not simple like one-line code and tend to get longer the better they are at randomly behaving.

With this background, it is not hard to see that random-behaving agents are not as simple as some might have initially thought. In the context of a double auction, other agents, such as the *truth teller*, simply bid or ask with their reservation price (the redemption value for buyers or the minimum cost for sellers). Given the token value table, one can almost write down such truth-teller programs in one-line code, and it is not hard to see that a market exclusively composed of truth tellers can realize a full market efficiency, i.e., a maximum sum of consumer's surplus and producer's surplus. Hence, is a randomly behaving agent simple? Are there any artificial agents that are simpler or much simpler than randomly behaving agents? How does one make a choice among simple agents? Without answers to these questions, a serious implementation of the KISS principle can become problematic, too.

[7] Other frequently used terms for algorithmic complexity include Kolmogorov complexity, program size complexity, and minimum description length. In computation theory, one way to measure the complexity of an object is based on the minimum program length required for describing or replicating the object. This notion was first introduced independently by Andrey Kolmogorov (1903–1987), Greg Chaitin and Ray Solomonoff in the 1960s, and is now known as *algorithmic information theory*. For an introduction, the interested reader is referred to Li and Vitanyi (2008).

[8] A comprehensive introduction to pseudo random number generators can be found in Gentle (2010).

3.3.2. Simplicity and intelligence

What is the relationship between simplicity (or complexity) and cognitive capability? Are simple agents necessarily related to agents with low cognitive capability (naive agents)? There are several measures of intelligence which have been used in empirical economics and experimental economics (some are to be further reviewed in Section 5.3). These studies provide us with some useful observations to think about the relationship between simple agents and naive agents, in particular, those experiments based on the notion of *iterated dominance*. In the iterated dominance game, the number of steps required to eliminate all dominated strategies is called the *depth of reasoning*, and the interesting feature of these games is that they allow us to develop a *step-by-step* computation in the acquisition of a dominant strategy. How deliberate a strategic behavior is can then be connected to this depth.

Differentiating the complexity of games based on the depth of reasoning or steps of iterated reasoning is not new (Camerer, 2003). Various models of cognitive hierarchy have been proposed to model players' behavior observed in behavioral game experiments (also see Section 5.2.2) that includes the theory of mind (Stahl and Wilson, 1995), Machiavellian intelligence (Ohtsubo and Rapoport, 2006), level-k reasoning (Crawford and Iriberri, 2007), and cognitive hierarchy (Camerer et al., 2004). Within this context, Ohtsubo and Rapoport (2006), in a *beauty contest game* (Nagel, 1998), found a positive relationship between the depth of reasoning and intelligence using a measure known as the Imposing Memory Task. Further research in this direction may shed more light on our maintained hypothesis that naive agents tend to be algorithmically simple and may have difficulties learning or discovering complex rules that can perform a number of iterations or simulate what other agents think (Crawford et al., 2010; Georganas et al., 2010).

The relationship between simple agents and naive agents can become even more complicated when one also incorporates the considerations of learning, adaptation and evolution. In this dynamic setting, agents with high cognitive capability have the potential to be algorithmically complex, but may be satisfied with using simple rules if those are viable strategies.[9] On the other hand, naive agents can learn some complex rules, but learning capability has a tighter limit. To be more precise, let us employ the Chomsky hierarchy from computer science as a metaphor.[10] The Chomsky hierarchy is a hierarchy of formal grammars and the formal languages generated by the grammars. The hierarchy arranges four classes of grammars and the four generated languages in monotonically increasing order, so that any class of grammars (languages) appearing in this order is a *proper subset* of the one coming after it. It also implies a hierarchy of the automata which can recognize the grammars. By the same monotonic order, the automata at a higher level can simulate the automata at a lower level, but not vice versa.

If the cognitive capability of agents (mind and brain) can be arranged in a similar hierarchy as the Chomsky hierarchy of grammar and automata, then agents with higher cognitive capability can simulate what agents with lower cognitive capability can think, but not vice versa. In other words, smarter agents can behave like simple naive agents, even though they are not that simple. In this way, our maintained hypothesis is broken, and simple agents are not necessarily naive.[11]

3.3.3. Intelligence and randomness

Finally, would naive agents be randomly-behaving agents, and random-behaving agents necessarily be naive? By definition, the behavior of the entropy-maximizing (EM) agent should not leave any pattern for observers to detect except those constraints; otherwise destroying these patterns will further contribute to the increase in entropy. From a strategic viewpoint, doing this might not be trivial. Technically speaking, what one needs is a program which would generate any kind of behavior except repeating itself, i.e., any possible pattern will be self-detected and self-annihilated. Based on our discussion earlier about the pseudo-random number generator, it would be surprising to know that the agent who is even unable to memorize and to learn can have all his behavioral patterns *automatically* self-detected and self-annihilated. So far, we are not aware of any experimental studies using human subjects to test whether a great degree of consciousness is required to generate highly patternless behavior. Naive agents may have difficulties making any decision on a sensible basis, but that does not immediately imply that decisions without logic, reasoning, judgments, or fast and frugal heuristics must be patternless. A man who is drunk might walk in a way that appears "random", but in fact he could just follow a sine-like curve. Hence, naive agents and randomly behaving agents are conceptually not tightly coupled.

3.3.4. Three independently motivated agents

Our viewpoint and the discussion about the triangular relationship as demonstrated in Fig. 1 may not be complete. Room for further reflections, of course, exists. Nonetheless, the point is that the equality relationship which is normally assumed in this triangle has not been carefully addressed and soundly founded. Based on the discussion above, the three agents are motivated by different disciplines. The simple agent can be sensibly constructed using computation theory (algorithmic complexity), the randomly behaving agents can be meaningfully generalized using information theory (entropy), and the zero-intelligence agent or agents with low cognitive capability can be much clarified if they are

[9] The *fast and frugal heuristics* as advocated by Gigerenzer (2008) are illustrations of these situations.

[10] An introduction to the Chomsky hierarchy and its relationship to computation theory can be found in Linz (2006). Also see footnote 19.

[11] It seems that we can go further to map the cognitive hierarchy to the Chomsky hierarchy. However, we have reason to hesitate to do so. The cognitive capability as defined and measured by psychologists can become stable with age and hence can be treated as an exogenous variable. However, various models of cognitive hierarchy do not necessarily assume that the cognitive hierarchy is exogenously given; on the contrary, it can be endogenously changed and upgraded when the subjects become more experienced.

grounded in cognitive psychology (intelligence quotient), cognitive neuroscience and computational neuroscience. Then a higher ground to integrate or hybridize the three may exist in an interdisciplinary research incorporating algorithmic information theory and computational neuroscience. However, at this moment, they are very much independent and are three competing options with regard to the benchmarking decision.

3.4. Regime-switching agents

In addition to the zero-intelligence agents, other simple programmed agents are also widely used in constructing ACE models, from financial models, macroeconomic models and large-scale socio-economic models (Epstein and Axtell, 1996). In these models, traders, households, firms, banks, and the government are all programmed as simple agents, who either follow a fixed simple rule or a set of simple rules. The rule may be characterized by few parameters. A typical example is the so-called *H*-type agent-based financial models. In this class of ACE models, agents have a set of *H* behavioral regimes. To decide which regime to activate in each period of time, the agents choose one of these regimes in a stochastic manner which is further connected to agents' learning behavior. Among many possible regime-switching agents, the most standard one is the fundamentalist/chartist model (Kirman, 1991; Brock and Hommes, 1998; Lux, 1998). We will return to the discussion of the regime-switching agents in Section 5.2.3. A survey of regime-switching agents in agent-based financial models can be found in Chen et al. (forthcoming).

4. Economic tournament origin

Simple programmed agents, as the description indicates above, are built upon a set of simple rules. These rules once given are fixed over the whole horizon of simulation. Hence, the emergent complex dynamics are basically generated by a society of *statically* behaved agents. These agents have very limited abilities to learn, to adapt, to discover and, after all, have no incentive to explore the surrounding environment. This notion of agents is very distant from the usual notions of rationality assumed in economics, even the somewhat weaker forms like *ecological rationality* (Gigerenzer and Todd, 1999). One may still argue that these rules are themselves the outcome of a long evolutionary process, but then the missing element is exactly this *process*. In fact, the true virtue of the device of simple programmed agents is to replicate some familiar aggregate dynamics with a minimum cost (complexity) of the model. Obviously, if the concern is the minimum intelligence required for market efficiency and other emergent patterns, then any strategic sophistication should be avoided, at least, at the initial stage. On the other hand, if the inquiry is to know the properties of the most viable strategy in a harsh competitive environment, then agents should be given some degree of freedom to explore or, alternatively, they are *autonomous*.

Autonomous agents are not necessarily complex and they are not necessarily heterogeneous; on the contrary, they can be simple as well as homogeneous. Their essential distinction as opposed to the simple programmed agents is that they are, by and large, *ecologically* constructed, hence the process underlying the rules or the programs is explicitly stated. Tracing the development of autonomous agents in ACE leads us to another pile of literature, which we now consider to be the second origin of ACE. Based on the following citation from Rust et al. (1994), we shall call it the *tournament origin* or, alternatively, the *game theory origin*:

> The use of *computer tournaments* to get insights on *complicated dynamic games* is somewhat unorthodox but not without precedent. Axelrod (1984) sponsored a tournament to study the repeated prisoner's dilemma game. (Axelrod, 1984, p. 63)

The tournament origin started with another kind of agents, known as *human-written programmed agents*, and these then developed into autonomous agents. The human-written programmed agents are best illustrated by two well-known tournaments, namely, the *iterated prisoner's dilemma* (*IPD*) *tournament* and the *double auction* (*DA*) *tournament*. Both tournaments have a fundamental pursuit, i.e., to characterize the form of the effective strategies, as the title of the paper (Rust et al., 1994) indicates. Using tournaments to answer this pursuit is because an analytical closed-form solution is not easily available. That solution may not even exist because the pursuit itself can be open-ended with the presence of co-evolution. Hence, learning from a simulated environment (tournament) composed of professionals, amateurs or software agents becomes a good starting point to tackle the issue.

4.1. Human-written programmed agents

Using the tournament approach to obtain insights on complicated dynamic games was initiated by Robert Axelrod at the University of Michigan (Axelrod, 1984). The tournament approach solicited entries from professionals and amateurs, each trying to develop a strategy for the Prisoner's Dilemma that would do well in the environment provided by all the submissions. Axelrod organized two rounds of the tournament. The first one was organized in 1979, and 14 entries were received with an extra one, a *zero-intelligence player*, being added.[12] The winner was Anatol Rapoport, who submitted the

[12] Axelrod did not use the term zero-intelligence, but the idea is the same. It is a randomly-behaving agent, who defects or cooperates with equal probability.

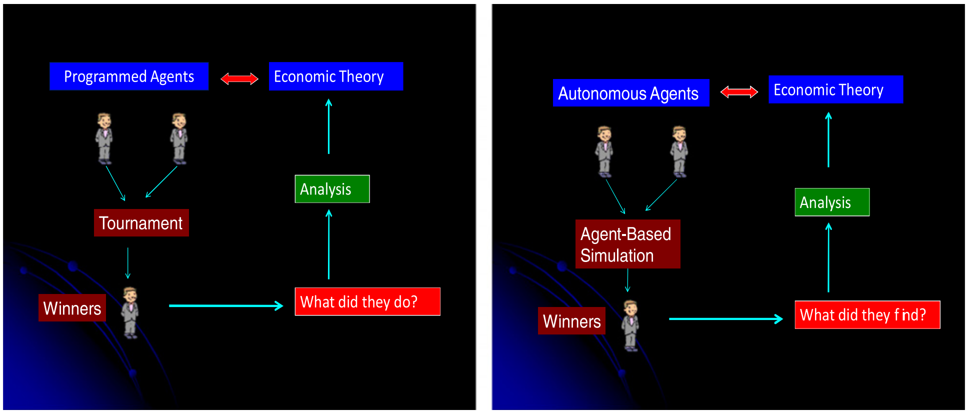

Fig. 2. Economic analysis inspired by programmed agents and autonomous agents.

strategy TIT-FOR-TAT. The strategy is simple: it cooperates on the first iteration, then replicates whatever the opponents did on the previous iteration. In the second tournament, 62 entries were received. The winner was again TIT-FOR-TAT. These two competitions were reported in Axelrod (1984). To celebrate the 20th anniversary, the iterated prisoner's dilemma competition was re-run again in the years 2004 and 2005 separately at the Congress of Evolutionary Computation in 2004 (June 19–23, Portland, Oregon, USA) and Computational Intelligence and Games in 2005 (April 4–6, Essex, UK). There were 223 entries in the 2004 event, and 192 entries in the 2005 event. The results are reported in Kendall et al. (2007).

Almost a decade after Axelrod's iterated prisoner's dilemma tournament, the same idea was applied once again to another familiar economic environment, the double auction (DA) market, in particular after intensive study through experiments with human subjects (Smith, 1991). The first DA tournament was held on the Santa Fe Institute in 1990. The tournament invited participants to submit trading strategies (programs) and tested their performance relative to other submitted programs in the Santa Fe Token Exchange, an artificial market which is operated by the double auction mechanism. A total of 25 different programs based on various design principles had been proposed, and it was shown that the best-performing one was the Kaplan program, submitted by Todd Kaplan, then a student at the University of Minnesota.

The prisoner's dilemma tournament, the double auction tournament, and other similar tournaments are all kinds of agent-based models. They are distinguished by the participation of human subjects, and, in this regard, are related to experimental economics. However, there is a subtle difference. *Usually*, human subjects in a laboratory, say, the double auction experiments, make decisions *on-line*. They are not required to submit their trading strategies beforehand and may not even have any rule, explicitly and implicitly, to follow throughout the experiments.[13] It is quite likely that their decisions at any moment in time can be substantially contingent upon the on-line feedbacks available to them. In addition, the time constraint imposed by each experiment may cause any time-consuming decision process to not be affordable; in many situations, human subjects' decisions tend to be spontaneous. On the other hand, human-written programs are generated *off-line*. Time pressure is not imminent. However, participants receive no immediate feedbacks while writing their programs. Therefore, they have to largely rely on their mind power to develop the program. Sometimes, doing this is essentially to develop models to be able to learn the models of other players (Li, 2007), having the cognitive-hierarchy flavor.

Through the analysis of the winners' programs in these agent-based tournaments, one may gain insights and acquire knowledge of the viable strategies. We call this tournament approach *programmed-agent-inspired economic analysis*, as also demonstrated in Fig. 2 (left panel). The examples of extracted knowledge are *TIT-FOR-TAT* in the prisoner's dilemma tournament and *Wait in the background* in the double auction tournament. Nevertheless, one limitation of these agent-based tournaments is that they are basically *closed* in the sense that reentries and new entries are not permitted. Not only are the human-written programs, once submitted, not able to be revised and resubmitted, but the new submissions are also not acceptable. Under such circumstances, one cannot help but wonder whether we could gain different insights had we been given a different set of submissions. This issue has been both well noticed since the beginning of the tournament approach. For example, Robert Axelrod once wondered "…whether the amount of cooperation I observed was due to the prior expectations of the people who submitted the rules. Axelrod (1997b,p. 6)" Such wondering has been even more systemically raised in the analysis of the double auction tournament (Rust et al., 1993).[14]

[13] Having said that, we are aware that in experimental economics there are various strategy-eliciting methods to elicit subjects' decision rules used in the experiment, such as the strategy method initiated by Brandts and Charness (2011). Depending on how the strategy method is conducted, some, for example, Keser and Gardner (1999) and Leady (2007), are very similar to the tournament approach here.

[14] There, they have stated:

> The open question is whether strategies exist that are capable of dominating Kaplan's "wait in the background" strategy over a nontrivial range of environments. If we were to run another DA tournament, it seems likely that entrants would attempt to beat Kaplan by developing more sophisticated delay and "endgame" strategies rather than reverting to truthtelling mode after a fixed amount of time. (Rust et al., 1993, p. 192).

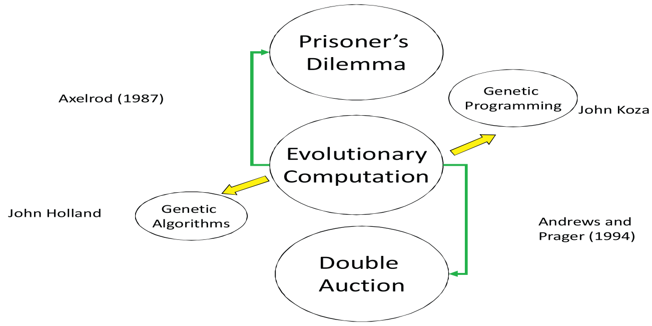

Fig. 3. Automated open tournaments. The diagram shows that genetic algorithms and genetic programming, two branches of evolutionary computation, have been applied respectively by Axelrod (1987) and Andrews and Prager (1994) to automate the iterated prisoner's dilemma tournament and the double auction tournament.

One solution to this conundrum is to institutionalize the tournament so that it can be constantly run and rerun. One example is the *trading agent competition*, held annually since the year 2000 (Wellman et al., 2007). However, this may demand more commitment and devotion. A convenient alternative to introducing the *open* tournament is to *automate* the submissions, as shown in Fig. 3. This attempt was again initiated by Axelrod (1987), which led to the idea of autonomous agents.

4.2. Autonomous agents

Autonomous agents are to be distinguished from the human-written programmed agents. The latter refer to the artificial agents whose behavioral rules or algorithms are written by humans, whereas the former refer to the ones whose behavioral rules are automatically generated by computers. Using computers to generate something which is non-trivial is certainly not a novel experience. For mathematical logicians, the familiar automatic theorem proving, as endorsed by Kurt Godel's well-known completeness theorem, is one example. In this spirit, if a theorem which can be proven by a human can also be proven by a computer, then the idea is to replace humans with computers to generate a set of submissions for the tournaments.[15] By doing so in this way, one can save laborious efforts by humans and hence facilitate automating an open tournament. This is basically what Robert Axelrod did in his pioneering application of autonomous agents in the social sciences.

While automating submissions of strategies may share some fundamental concepts as automated theorem proving, such as recursively enumerable sets, the specific algorithms used by Axelrod, i.e., *genetic algorithms*, were not typical in automated theorem proving in the middle of the 1980s. Put briefly, the genetic algorithm is the application of biological evolution to computer science and artificial intelligence. It was invented by Holland (1975), who with Arthur Burks, Robert Axelrod, and Michael Cohen together are the founders of the well-known BACH group, a group for interdisciplinary research on complex systems, at the University of Michigan.

The applications of genetic algorithms in ACE began with Miller (1988) and Marks (1992). Year after year, genetic algorithms gradually became well accepted by researchers in ACE. Both introductory and survey articles are quite easily accessible (Dawid, 1999; Riechmann, 2001). In one of its most standard forms, each submitted strategy is represented by a binary string (a string of genes on a chromosome) which is initially randomly generated (enumerated) by a computer. To this extent, this is not much different from the randomly behaving agents as discussed in Section 3.2.5.[16] In genetic algorithms, there is a biologically inspired procedure that enables the initially randomly generated strategies to be reviewed and revised *automatically* and that may further lead to some new strategies. This biologically inspired automatic procedure makes the retries or new entries possible, and hence changes the originally closed tournament to an open one (Fig. 3).

4.2.1. Evolutionary and co-evolutionary tournaments

With the participation of autonomous agents, Robert Axelrod considered two conceptually distinct styles of open IPD tournaments, which later on became the stereotypes of automated open tournaments. The first style is based on matching autonomous agents with a fixed set of human-written programmed agents (Fig. 4), whereas the second style is based on leaving autonomous agents to play against themselves. To distinguish the two, we shall call the former the *evolutionary tournament*, but the latter the *co-evolutionary tournament*. In the evolutionary tournament, autonomous agents are adapted to a fixed environment, and the most effective strategy can be considered to be the optimal strategy, *optimal with respect to the fixed set of opponents*. The use of a genetic algorithm in this case is more like the application of a numerical optimization

[15] Of course, this is still a very ideal situation, because by the completeness theorem the resources (time and space) are assumed to be unbounded.

[16] One, however, has to distinguish the random-behaving agents from the random-programmed agents. The former simply randomly behave with no rule as guidance, whereas the latter behave by systematically following a rule which is, nonetheless, randomly generated.

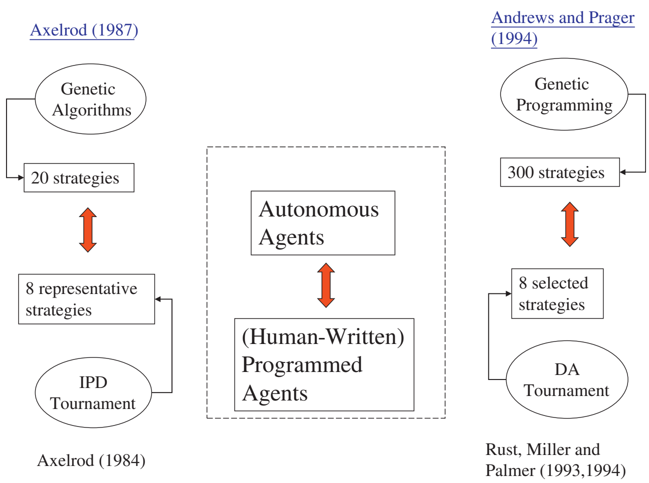

Fig. 4. Automated evolutionary tournament. The evolutionary tournament is one form of open tournament, which seeks to match the autonomous agents with a fixed set of human-written programs, as shown in the middle of the figure. The figure also shows the earliest automated evolutionary IPD tournament and DA tournament, conducted respectively by Axelrod (1987) and Andrews and Prager (1994).

algorithm that searches for the optimal solution. In the co-evolutionary tournament, each agent adapts to the adaptation of other agents. The process may not even converge, and the effective strategy can change over time.

Axelrod presents the results of both styles of tournaments. In his evolutionary tournament, it was found, with a little surprise, that TIT-FOR-TAT was no longer the most effective strategy with respect to a fixed set of eight representative human-written programs which were retrieved out of the 62 entries in his second IPD tournament (Fig. 4).[17] What were found to be even more effective were the autonomous agents who are able to distinguish the exploitable representatives from other less exploitable representatives, and then to exploit the former at a cost of having less cooperation with the latter, but still netting the two with a positive gain. Of course, without a fixed set of opponents and a sufficiently large number of repetitions, this kind of strategy is not readily available. Nevertheless, its existence indicates that the effective strategy derived from the closed tournament may not be complete.

From Axelrod's evolutionary tournaments, we can more easily see the meaning and the role of autonomous agents in economics. Autonomous agents are purposive. To achieve their purposes, they can discover and exploit hidden patterns and opportunities to some extent *without the direct external intervention from humans* (*model builders*). In other words, these agents are able to learn and adapt *on their own*. In Axelrod's case above, the autonomous agents were constantly looking for chances, opportunities, and patterns; hence, finding a way to outperform TIT-FOR-TAT or any other fixed strategy is probably just a matter of time. Then, in his simulation of the co-evolutionary IPD tournaments, agents' capability to adapt to the changing environment is further illustrated. In this case, each autonomous agent plays the IPD with each of the 20 agents including its own twin rather than the eight representative human-written programs. Each agent's discovery and exploitation of the environment may bring in a new environment which in turn creates new opportunities for other opponents for further discovery and exploitation. This cycle can be indefinite and echoes well with Alfred Marshall's biological description of the economy as "*constantly changing*" (Marshall, 1924):

> Economics, like biology, deals with a matter, of which the inner nature and constitution, as well as outer form, are constantly changing. (Marshall, 1924, p. 772)

4.2.2. Toward a higher degree of autonomy

At this point, it would be useful to indicate that, although the autonomous agent is quite an important development in ACE, the line between autonomous agents and non-autonomous agents is not clear. In fact, being *autonomous* or not is just a matter of degree. It is very much dependent on the degree of autonomy with which the agents are endowed. Hence, the least-squares learning agents frequently used in macroeconomics can be considered to be autonomous agents because they are able to learn the parameters of the underlying system without further external supervision (Evans and Honkapohja, 2001). However, if the underlying law of motion is not linear but non-linear, these agents may not be able to jump out of the linear trap on their own. With this clarification, it probably would be fair to say that the tools available

[17] At the beginning, Axelrod might have been surprised by this result, as he wrote:
Because TIT FOR TAT had done best in the computer tournament itself, I did not think that it would be possible to do much better with an evolutionary process. But as noted earlier, in about a quarter of the simulation runs with sexual reproduction, the population did indeed evolve substantially better strategies–strategies that were quite different from TIT FOR TAT. (Axelrod, 1997b, p. 26)

for economists to build autonomous agents with a high degree of autonomy were rather limited before the early 1990s. In their pioneering paper introducing autonomous agents to economics (Holland and Miller, 1991), John Holland and John Miller begin with the following:

> Economic analysis has largely avoided questions about the way economic agents make choices when confronted by a perpetually novel and evolving world. …This is so,…, because standard tools and formal models are ill-tuned for answering such questions. However, recent advances…in the subdiscipline of artificial intelligence called machine learning, offer new possibilities. (Holland and Miller, 1991, p. 365)

In either IPD tournaments or DA tournaments, a practical issue of building autonomous agents is the *representation* of strategies. In his application of the GA, Robert Axelrod represented the IPD strategies as history-dependent actions: each agent responds to the two-sided actions of the three previous runs, say, CC|CD|DD ("C" for cooperation and "D" for defection). Since there are a total of $2^6 (=64)$ such histories, a strategy is represented by a 64-bit string, and there are a total of 2^{64} possible strategies. While this number is big, the representation power given by this finite string is still rather limited. One can easily find some human-written programs not belonging to this gigantic set. This shortcoming presents a fundamental challenge to the building of autonomous agents, namely, *how to represent the class of human-written programs as submitted to the tournaments*.

The solution requires expertise beyond statistical learning. In general, it involves areas which are less familiar to economics, such as knowledge representation, formal languages, mathematical logic, etc. One attempt to move forward, from the above fixed-length string, is to consider human-written programs as strings of symbols (alphabets) with variable lengths. This can be quite feasible, for example, if all the programs are written by the program language LISP.[18] If so, they can be placed in the context of *formal languages* in computer science, and can be generated by the appropriate grammars or production rules with a given initial symbol. In each step of the generation process, one of the production rules is applied to a single alphabet of the developing string. This developmental process can be illustrated using the *Backus–Naur Form* (BNF).[19] If, at any stage of this developmental process, we change the production through a different branch of the same non-deterministic production rule, then we may move into a different path and end up with a different string (program).

4.3. Context-free grammars and genetic programming

The idea of using formal languages to represent strategies and hence to build autonomous agents in economic tournaments is first attempted by Andrews and Prager (1994) (Figs. 3 and 4). Andrews and Prager applied the same evolutionary operators used in genetic algorithms to a *context-free language* so that different strings (programs) are connected with each other by evolutionary paths, of which each step is a perturbation to an alphabet of the string at that time. This technique is known as *genetic programming*, invented by Cramer (1985) and further developed by Koza (1992).

A context-free grammar starts with a set of symbols, which can be further separated into a set of terminal symbols and a set of non-terminal symbols. In genetic programming these two sets are also known as the terminal set and function set, respectively. The former normally includes data to be processed, such as variables or constants, whereas the latter have operators to process the data, such as arithmetic and logic operators. As an illustration, Table 1 consists of the kinds of terminal sets and function sets which Andrews and Prager (1994) used to represent and generate bidding strategies in the double auction tournament.

Given the information (Table 1, upper panel) and the way to operate it (Table 1, lower panel), various bidding strategies can be formed. Two examples are the following[20]:

- `(Min PMinBid HT)`
- `(If_Bigger_Then_Else HT CASK CASK+1 Pass)`

In the first example, to decide how much to bid, the buyer simply looks at the minimum bid on the previous day (PMinBid) and his current reservation price (HT), and bids at the minimum of the two. In the second example, the buyer first checks whether his reservation price (HT) is bigger than the lowest ask (CASK) in the previous round. If this condition is met, he will bid by adding one dollar to the current ask; otherwise, he will simply pass. Not all bidding strategies are that

[18] LISP stands for List Processing, which is a high-level computer language invented by John McCarthy in 1958 at MIT. This language is strongly motivated as a practical implementation of the λ calculus or the recursive function theory developed in the 1930s by Alonzo Church (1903–1995) and Alan Turing. See Abelson and Sussman (1996) for details.

[19] The Backus–Naur Form was invented by John Backus (1924–2007) for describing the high-level language ALOGL. Peter Naur was the editor of the report in which it appeared. The class of languages described by the BNF, in terms of the *Chomsky hierarchy* (see also Section 3.3.2), is equivalent to type 2 languages or context-free languages, which are generated by context-free grammars. The class of type 2 languages is a proper subset of type 1 languages (context-sensitive languages), and a proper superset of the type 3 languages (regular languages). For a deeper background, see Linz (2006).

[20] The string representation that the operators in the parentheses are given before the operands is known as the *prefix* representation, invented by the Polish mathematician Jan Lukasiewicz (1878–1956).

Table 1
The terminal and the function sets of the GP traders.

Terminal set	
PMax, PMin, PAvg	The maximum, minimum, and average prices for the previous day
PMaxBid, PMinBid, PAvgBid	The maximum, minimum, and average bids for the previous day
PMaxAsk, PMinAsk, PAvgAsk	The maximum, minimum, and average asks for the previous day
CASK, CBID	The highest bid and the lowest ask in the previous trading step
HT, NT, LT	The first, next, and last reservation prices owned by each trader
TimeLeft, TimeNonTrade	The number of steps left in this trading day, and the number of consecutive no-transaction steps until the current step
Pass, Constant	To give up bidding/asking in this step, or to shout a random number
Function set	
+, −, , /	Basic arithmetic operations to add, subtract, multiply, or divide
Abs, Log, Exp, Sin, Cos, Max, Min	Basic mathematical functions
If-Than-Else, If-Bigger-Than-Else, Bigger	Basic logical operators

simple. A little knowledge of combinatorics or context-free grammar will lead us to see that the formed bargaining algorithm can potentially become complex like the next one and something beyond the scope of this paper:

- `((Min (If_Bigger_Then_Else PMinBid PAvgBid CASK PAvgBid)`
  `(If_Bigger_Then_Else HT PAvgBid PAvgBid CASK))`

Generally speaking, the terminal set and the function set are the set of primitive ingredients to enable genetic programming to get started, and the universe of allowable compositions of these ingredients defines the search space for a run of genetic programming. The degree of autonomy not only depends on the size of the search space, but also on the efforts required to get the primitive ingredients. On the one hand (the output side), we hope that the search space is large enough to accommodate many hidden novelties; but, on the other hand (the input side), we want to achieve this goal with minimal efforts. Intuitively speaking, the supply of primitive ingredients should involve human efforts which are relatively much smaller in comparison to the scale of the resultant automated operation.

Take Andrews and Prager (1994) as an example. Their set of terminals is effortlessly obtained from the information given to the participants of the SFDA tournaments and commonly used in the human-written programs. Yet, in this way, many strategies submitted to the SFDA tournaments are available in the search space "spanned" by the primitive ingredients in Table 1. Hence the built autonomous agents can even have a chance to re-discover some high-performance strategies used in the tournaments, such as Kaplan's "wait in the background" strategy.[21]

Like Axelrod (1987), Andrews and Prager (1994) also automate the SFDA tournament as an evolutionary open tournament (Fig. 3), in which autonomous agents are matched with a fixed set of human-written programs (Figs. 4 and 5, left panel). The human-written programs were also selected from the original SFDA tournament. One advantage of the automated tournaments is that their scale grows with the power of the CPU. Compared to Axelrod (1987), Andrews and Prager (1994) increase the number of entries from 20 to 300, and lengthen the duration of the tournament from 50 times (generations) to 300 times (generations). Since what we will learn from open tournaments is generally open ended, the increasing scale of tournaments is certainly a plus for this experimental way of studying economics.

Despite their larger scale of tournaments, Andrews and Prager (1994) did not analyze the effective strategies discovered by the autonomous agents, in the way that Axelrod (1987) did. While their performance statistics do indicate these discoveries, exactly what they are is not transparent. This "invisibility" is not unique in their work only, but quite generally shared in that of many others using more expressive representation such as genetic programming. This indicates a fundamental difficulty of using autonomous agents. When agents become more autonomous, they can become less tractable while evolving into more complex syntactic forms for which the associated semantics are less straightforward.

To be able to analyze the strategies learned by these agents with a higher degree of autonomy, Chen and Yu (2011) simplify the Andrews–Prager DA market by replacing all human-written programmed agents with truth tellers. In their proposed evolutionary tournaments, the autonomous agents (driven by genetic programming) play against a fix set of truth tellers in three different market topologies (demand-and-supply schedules), each associated with a different kind of market equilibrium (or equilibria), as shown in Fig. 6. A size of 10 or 50 entries (automated programs) is maintained for each tournament, and, like Andrews and Prager (1994), the tournament is run for 300 times (generations). Ninety runs of this evolutionary tournament are carried out. This scale of simulation enables them to examine a set of 18,000–90,000 strategies for each market topology. By crossing comparisons of the results from different market

[21] The chance to discover them, however, is another story. In some applications, it has been shown that genetic programming is able to discover the Black-Scholes option pricing formula (Noe and Wang, 2002), the Bayesian learning rules (Lensberg, 1999), etc. Ample engineering examples are archived in Koza et al. (2005), where it is claimed that genetic programming can now routinely deliver high-return human-competitive machine intelligence.

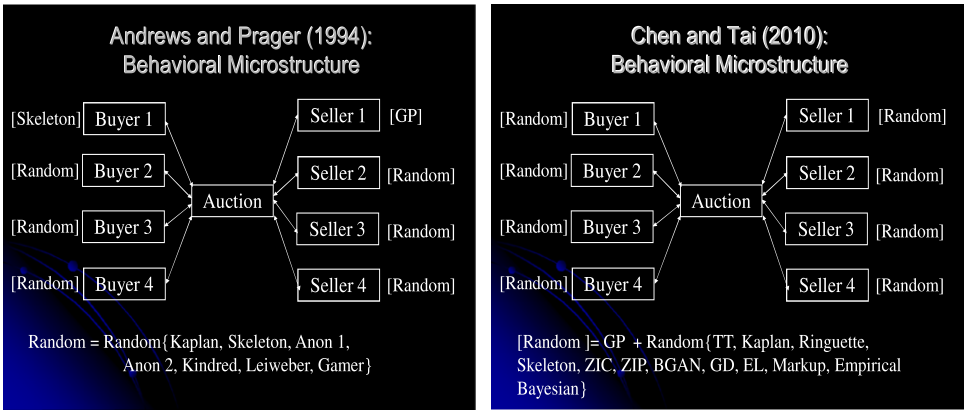

Fig. 5. The behavioral microstructure of two evolutionary tournaments: Andrews and Prager (1994) and Chen and Tai (2010).

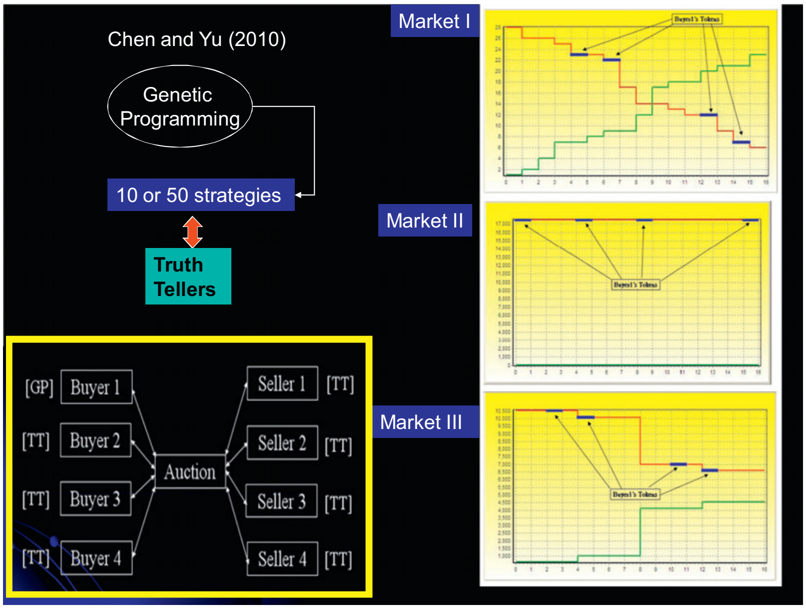

Fig. 6. The evolutionary tournament in Chen and Yu (2011). The left-lower panel gives the microstructure of the DA tournament. In this case, entries of automated programs (Buyer 1) play against a set of truth tellers (TT). The automated submissions are generated by genetic programming, as indicated in the left-upper panel of the figure. The tournaments are held in three different market topologies as displayed in the right panel of the figure. The reservation prices (redemption value) of the four tokens for Buyer 1 are emphasized.

topologies, a general pattern of the autonomous agents in these DA markets is observed, which is termed *optimal procrastination*.

Of course, the optimal procrastination strategies may become complicated or may not even be sustained when different sets of opponents are considered, but the idea of using autonomous agents as a way to inspire economic analysis remains (Fig. 2, right panel). In fact, Chen and Tai (2010) have considered a DA tournament closer in spirit to SFDA and more complex than Andrews and Prager (1994). In their evolutionary tournament, a total of 100 entries play against eleven human-written programs, which were sampled from the literature on double auctions (Fig. 5, right panel).[22] With extensive trials, Chen and Tai (2010) are able to show that genetic programming can build the autonomous agent that outperforms all the 11 human-written programs in a broad variety of environments.

The lesson from Axelrod (1987) to Chen and Tai (2010) is that one can find the effective strategies in any evolutionary tournament by trying the idea of autonomous agents. Nevertheless, if a fixed set of strategies tends to be associated with an Achilles' heel, then there is no reason why opponents should constantly follow a fixed strategy, no matter how

[22] This includes some well-performing strategies in the SFDA tournaments (the Kaplan strategy, the Ringuette strategy, the skeleton strategy), the GD strategy (Gjerstad and Dickhaut, 1998), the BGAN (Bayesian Game Against Nature) strategy (Friedman, 1991), the EL strategy (Easley and Ledyard, 1993), the zero-intelligence-constrained strategy (Gode and Sunder, 1993), and the zero-intelligence-plus strategy (Cliff and Bruten, 1997).

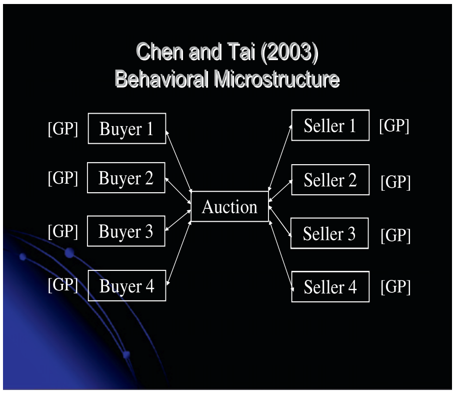

Fig. 7. The behavioral microstructure of two co-evolutionary DA tournaments (Chen and Tai, 2003).

sophisticated it is. Therefore, the essence is that the evolutionary tournament is just the beginning. The autonomous agents used there may help us to have quick grasps of the nature of the tournament, but the co-evolutionary tournament is really what one should expect for the next.

Chen and Tai (2003) then extend the co-evolutionary tournament initiated by Axelrod (1987) to the double auction tournament. In this case, all human-written programmed agents are replaced with autonomous agents driven by genetic programming (see Fig. 7). Their simulation shows that the market composed of these autonomous agents is able to find the equilibrium price. The price converges to a small neighborhood of the equilibrium price and fluctuates around there, a pattern which is similar to those observed in the DA experimental markets with human subjects.[23]

5. Experimental economics origin

5.1. Natural and artificial automata

As discussed above, the idea of autonomous agents is strongly motivated by the observation of human behavior (human-written programs). Broadly speaking, this idea is part of the comparative study of artificial automata (artificial systems) and natural automata (natural systems), which has already drawn considerable attention from those pioneers in machine intelligence. For example, Alan Turing, with his famous *Turing test* (Turing, 1950), considered that the success of machine intelligence lay in its ability to *mimic* human intelligence, and he firmly believed that there was nothing the brain could do that a well-designed computer could not.[24] John von Neumann was more cautionary on this mimicking capability than Turing. Instead, he considered a bi-directional relationship (Von Neumann, 1956):

> Natural organisms are, as a rule, much more complicated and subtle, and therefore much less well understood in detail, than are artificial automata. Nevertheless, some regularities, which we observe in the organization of the former may be quite instructive in our thinking and planning of the latter; and conversely, a good deal of our experiences and difficulties with our artificial automata can be to some extent projected on our interpretations of natural organisms. (Von Neumann, 1956, pp. 2070–2071)

His comparative study (VonNeumann, 1958) of the nervous systems (the brain) and the digital computer serves as an illustration of this bi-directional relationship.

If one treats *human subjects* as natural automata and *software agents* or autonomous agents as the corresponding artificial automata, then a bi-directional study between the two, termed the *experimental-economics origin of ACE*, appeared in the middle of the 1990s. Jasmina Arifovic is one of the pioneers. In her series of studies involving both human subjects and artificial agents, Arifovic used agent-based models to mirror the macroeconomic experiments with human subjects in the cobweb models (Arifovic, 1994) and in the overlapping-generation models (Arifovic, 1995, 1996). The aggregate economic variables, such as commodity prices, inflation rates or exchange rates, are first generated from human-subject experiments, and then the purpose is to see whether these experimental results can be replicated by the designed artificial agents. For example, among many possible equilibria, she examined whether both systems, systems composed of human agents and systems composed of artificial agents, converged to the same equilibrium, and, if so, whether the convergence

[23] In their designed co-evolutionary DA tournament, each autonomous agent is represented by a set of 50 entries. Each co-evolutionary tournament is run 100 times, and 20 co-evolutionary tournaments are run, covering a set of 20 different market topologies.

[24] For a more comprehensive survey of the Turing test, the interested reader is referred to Saygin et al. (2000).

paths were comparable. Since then agent-based economic modeling has often been used to simulate or replicate the behavior observed in the human-subject experiments and provide possible explanations for observed behavior.[25]

5.2. Calibrated artificial agents

A central issue pursued by ACE in light of human-subject experiments concerns the empirical ground of artificial agents. This issue arose partially because of the "wilderness" of the possible characterization of the boundedly rational agents. *Mirroring* is a focus of the issue, as put by Arthur (1993):

> An important question then for economics is how to construct economic models that are based on an actual human rationality that is bounded or limited. As an ideal, we would want to build our economic models around theoretical agents whose rationality is bounded in exactly the same way human rationality is bounded, and whose decision-making behavior *matches* or *replicates* real human decision-making behavior. (Arthur, 1993, p. 2; Italics added)

Arthur (1993) further provides two guidelines for this mirroring function. The first one is a *statistical* one, which is to parameterize the artificial agents and to calibrate them using the empirical data. The second one is an *artificial-intelligence* one, which involves the implementation of a Turing test. Arthur himself gave an example of the first one (but not the second one). Of course, the AI one is much broader than the statistical one. As Arthur himself mentioned:

> The ideal would be algorithmic behavior that could pass the Turing test of being indistinguishable from human behavior with its foibles, departures and errors, to an observer who was not informed whether the behavior was algorithm-generated or human-generated (Turing, 1956). Calibration ought not to be merely a matter of fitting parameters, but also one of building human-like qualitative behavior into the algorithm specification itself. (Ibid, p. 3)

Nonetheless the implementation may be harder; hence, it draws much less attention from ACE economists. Ecemis et al. (2005) and Arifovic et al. (2006) are the only two studies known to us which have this flavor. Despite this being so, later in Section 5.3, we shall see that there is a recent research trend to build human-like qualitative attributes into artificial agents.

The *statistically* calibrated artificial agents were the ones discussed at length in Arthur (1993) and later used in many ACE models. In Arthur (1993) *reinforcement learning* (RL) is chosen as the parametric learning algorithm. Reinforcement learning is a learning behavior originally proposed by psychologists in animal experiments. Its essence is very simple: choices that have led to good outcomes in the past are more likely to be repeated in the future. In the 1950s, it became a formal model of learning in mathematical psychology and has been applied to many areas, including artificial intelligence, neuroscience, and engineering. It was introduced to economics in the 1970s (Cross, 1973) and has received attention only since the 1990s. However, very quickly, it has served as an important class of learning models in ACE. Its applications went beyond individual experiments and had come to cover games (Roth and Erev, 1995) and various markets, such as energy markets (Nicolaisen et al., 2001), fish markets (Kirman and Vriend, 2001), financial markets (Kluger and McBride, 2011), etc.[26]

Arthur (1993) proposed a two-parameter *reinforcement learning* model and applied it to build the artificial agent in an individual choice experiment known as the *two-armed bandit problem*. The two parameters control the conflict between exploration and exploitation. They specify how fast the agent can stop exploring other alternatives and hence the agent is locked into a specific choice. The two parameters are calibrated by using the data from Laval Robillard's experiments with human subjects at Harvard in 1952–1953, which was reported in Bush and Mosteller (1955).

5.2.1. Calibrated heterogeneous agents

The idea of using data for individuals observed in human-subject experiments to calibrate or estimate the parameters of a parametric artificial agent became popular in the middle of the 1990s, particularly in the literature on learning in normal-form games. The parametric artificial agents frequently built by calibration or estimation are agents who follow reinforcement learning, belief learning, and experienced-weighted attraction (EWA) learning. The experienced-weighted attraction learning was proposed by Camerer and Ho (1999) as a hybridization of reinforcement learning and belief learning. Since it generalizes what is reinforced in learning, including expected payoffs derived from players' beliefs regarding various scenarios, but still keeps the fundamental formulation of reinforcement learning unchanged, it can be regarded as a generalized version of reinforcement learning.

While this direction of research started with the homogeneous-agent device by using all of the individuals' data to build a single parametric agent (Arthur, 1993), the later development did consider the empirical significance of *heterogeneous agents* or *individual differences*. Cheung and Friedman (1997) first found considerable heterogeneity among individual behavior in the experiments involving normal-form games.

In Cheung and Friedman (1997), the artificial players are presented with a binary choice in the context of normal-form games. Their binary choice is stochastically determined by the familiar logit model. The main input to the logit model is related to the

[25] For an excellent survey, see Duffy (2006). While Duffy (2006) does suggest a bi-directional relationship between experimental economics and agent-based computational economics, he also points out and explains why there are a lot of ACE studies conducted in light of human-subject experiments, but few in the reverse direction.

[26] A good introduction to reinforcement learning and a survey of its applications to ACE can be found in Brenner (2006) and Duffy (2006).

difference in the expected payoff of the associated choices, which is derived with respect to the player's belief (subjective probability function). Cheung and Friedman (1997) then parameterized the decision rule (the logit model) and the learning rule (the belief updating scheme) of agents with two and one parameters, respectively. The three-parameter belief learning agent is then estimated based on four kinds of normal-form game experiments, involving a total of 393 human subjects. Cheung and Friedman (1997) then proposed a log-likelihood ratio test to test the null of a homogeneous agent (representative agent) with the alternative of completely heterogeneous agents (all agents are different). The null was rejected, indicating that heterogeneity across subjects in terms of learning and decision parameters is an important feature of their observations. In particular, the observed heterogeneity in the learning parameter allows us to further classify the majority of players, 257 out of 393, into three groups of agents with distinct memory lengths, from the shortest one to the longest one, corresponding to the *Cournot agents* and the *fictitious-playing agents*, respectively, in game theory (Fudenberg and Levin, 1998).

5.2.2. Calibrated agents with incremental cognitive capacity

Evidence on the heterogeneity of human subjects has also been found in other studies (Camerer and Ho, 1998; Stahl, 2000; Ho et al., 2008; Chen and Hsieh, 2011).[27] However, few have ever addressed what may cause the observed heterogeneities of these human agents. In light of the recent trend in cognitive and psychological economic experiments (Section 5.3), for example, one may inquire whether these observed heterogeneities are related to the idiosyncratic differences in subjects' cognitive or psychological attributes. As an empirical foundation of ACE, this query can be quite crucial because the learning models developed in game experiments enable us to think of the heterogeneities of agents from a bounded-rationality perspective, and hence provide a justification for the development of heterogeneous agents in ACE. What is particularly intriguing is that these models may possibly be assembled into a hierarchical framework, from the ones with a lower cognitive capacity to the ones with a higher cognitive capacity. It demonstrates a way to model artificial agents with *incremental* cognitive capacity.

Table 2 proposes a summary on how the cognitive capacity can be incrementally added from the zero-intelligence agent all the way up to the sophisticated experienced-weighted attraction (EWA) agents (Camerer et al., 2002). The key cognitive elements used to distinguish these learning models are *memory*, *consciousness* and *reasoning*. Zero-intelligence agents have none of them, as discussed in Section 3.2. Reinforcement learning agents are endowed with some power of memory, which can help them to recall their past experiences of feeling in each of the same situations. However, they are not given a cognitive capacity that is sufficient for them to be aware of the entire environment with which their decisions are embedded. Essentially, they are not aware of what their players have done and would not try to outguess what they will do. Without enough consciousness and reasoning, decisions made with reinforcement learning can be very *reflexive*, as being automatically driven by the dopamine system.[28]

With the given memory, be it short or long, the belief learning agents are given additional cognitive capacity to be aware of the presence of their opponents. They are conscious of the effect of the opponents' action on their payoffs. Their decisions are, therefore, based on the reasoning by taking these feedbacks into account. In particular, they are able to figure out, in an ex post manner, what would happen had they chosen different actions, and to take these so-called *foregone payoffs* into account. They, however, are not advanced enough to develop a sophisticated model to predict the behavior of their opponents' actions, apart from counting the frequencies of their past actions. The EWA (experience-weighted attraction) agents are a hybridization of the reinforcement learning agents and belief learning agents in the sense that their cognitive capacity to "imagine" all foregone payoffs is only partial. Their consciousness of the environment and the mental capability to engage in counterfactual thinking lies between those of the reinforcement learning agents and belief learning agents.

The sophisticated EWA involves the incorporation of level-k reasoning into EWA learning. There are several different versions of level-k reasoning (Stahl and Wilson, 1995; Camerer et al., 2004; Crawford and Iriberri, 2007), but the basic structure is, roughly, that higher-level agents are able to reason what low-level agents do, and then their response to it. Taking sophisticated EWA learning as example, Camerer et al. (2002) consider two types of agents, level-0 and level-1, with a given distribution of each type. Level-0 agents do not know of the existence of level-1 agents, and behave as the EWA learning model describes. Level-1 agents know of the existence of level-0 agents and their learning behavior, and then form their beliefs by taking into account both level-0 and other level-1 agents' learning, with respect to a belief regarding the distribution of the two types of agents. Therefore, agents with this design are clothed with the additional power to reason what other agents, agents with a lower level of cognitive hierarchy, might think.

5.2.3. Calibrated financial agents

The idea of calibrated artificial agents is also extensively applied in another kind of ACE literature, namely, *agent-based financial markets*.[29] What differs from the calibration in game experiments is that in these applications the data used to calibrate artificial agents are mostly the *aggregate data* from financial markets; only a few exceptions (to be reviewed below) use the *individual data* from laboratory experiments. As for the former applications, the parameters used to

[27] It is not always the case that the null of homogeneous agents can be rejected by the laboratory data. For example, Camerer and Ho (1999) tested the null of homogeneous agents by assuming two types of agents as the alternative, and they found that the null cannot be rejected.

[28] One of the most impressive recent results in neuroscience is the discovery of the relationship between the dopamine neural system and reinforcement learning. See, for example, Montague (2006, Chapter 4).

[29] A number of good surveys on agent-based financial markets can be found in Hommes (2006), LeBaron (2006), and Samanidou et al. (2007).

Table 2
Artificial agents with incremental cognitive capacity.

Models	Memory	Consciousness	Reasoning
Game experiments			
Zero-intelligence	None	None	None
Reinforcement learning	Short to long	None	None
Belief learning	Short to long	Strong	Weak
EWA learning	Short to long	Weak to strong	Weak
Sophisticated EWA	Short to long	Weak to strong	Weak to strong
Agent-based financial markets			
Regime switching	Short to long	Weak	None

characterize financial agents become part of the built agent-based financial models. They are in turn estimated together with other structural or institutional parameters using various econometric techniques. Given the complexity of agent-based economic models, when direct estimation is not available, indirect estimation or simulation-based econometrics is also applied to empirically construct the agent-based economic models.[30] Even though the employed data may not come from a controlled laboratory but from very noisy real markets, the goal to replicate natural automata through artificial automata remains the same.

The artificial agents that are frequently calibrated or estimated are, as mentioned in Section 3.4, the *regime-switching agents*. To decide which regime to activate in each period of time, the agents choose one of these regimes in a stochastic manner which is very similar to the generalized reinforcement learning as discussed in the game experiments. For example, in the agent-based financial models initiated by Brock and Hommes (1998) and Hommes (2002), known as the *adaptive belief systems* (ABS), the logit model is applied for modeling agents' stochastic choice. Another kind, the *herding-based agent-based financial models*, initiated by Kirman (1991) and Lux (1995), employ the idea of the Polya urn process as a basis for the stochastic choice, but this difference may be trivial since financial agents can be reinforced by various kinds of payoffs. In addition to monetary payoffs, they can also be reinforced by peer pressure (herding).[31]

With this similarity, the cognitive capacity of calibrated financial agents can be examined using Table 2. First of all, most calibrated financial agents do take into account the foregone payoffs. For example, in the Brock–Hommes model, the agents who chose to behave like fundamentalists (chartists) would also update the payoffs which they might have received had they behaved like a chartist (fundamentalist). Hence, as with the usual distinction made between reinforcement learning and belief learning, they may be positioned closer to the latter. However, belief learning agents will form beliefs on what other agents may tend to do and react upon these expectations. This degree of consciousness is simply absent in most calibrated financial agents. For example, they will not form expectations of the possible microstructure (percentages of fundamentalists or chartists) in the next period, and make their choice accordingly. This missing awareness is probably because of the overwhelming reliance on the use of simple programmed agents and the pursuit of simplicity in this type of agent-based financial model.[32]

Second, without sufficient consciousness, reasoning capability as characterized by various cognitive hierarchies is also not presented in these calibrated financial agents. While level-k reasoning was partially motivated by Keynes's beauty contest as an influential metaphor of stock markets, its applications to agent-based financial models have not yet been realized.[33] Accordingly, they are not heterogeneous in their cognitive capacity. As a matter of fact, while frequently termed as models of heterogeneous agents, most calibrated agents in financial markets are *homogeneous ex-ante*, i.e., their heterogeneous behavior is just a different realization drawn from the same choice probabilities. The only source of agent heterogeneity is this random component.[34]

Therefore, an inquiry into the relevance of heterogeneity as pursued by game theorists (Cheung and Friedman, 1997; Camerer and Ho, 1999) has rarely been addressed in agent-based financial models. As we have mentioned, since most calibration work in agent-based financial models involves only aggregate data, whether or not heterogeneity is necessary

[30] A lengthy description of this development is beyond the scope of this paper. Nevertheless, a survey on the econometric studies of agent-based economic models is available in Chen et al. (forthcoming).

[31] Having said that, we notice that they can also be reinforced by risk aversion, and forecasting accuracy, etc. The idea of payoffs used in agent-based financial models is, therefore, broader than what has been commonly used in game experiments.

[32] There is another type of agent-based financial models favored by econophysicists, known as *minority games* (Challet et al., 2005). In minority games, financial agents will form expectations on, among the two possible choices, which side tends to be the minority. To form these expectations dynamically, tools such as genetic algorithms are applied; hence, financial agents built in this way are far more complex.

[33] In fact, even the EWA learning also has rather few applications in agent-based financial markets. One exception is Pouget (2007).

[34] Take the fundamentalist-chartist model as an example. The discrete choice model when calibrated gives not just the choice probabilities of individuals, say, a 70% chance of being fundamentalists and a 30% chance of being chartists, but, by the law of large numbers, also gives the exact fraction (distribution) of market participants over these two behavior regimes, i.e., 70% of market participants being fundamentalists and 30% of them being chartists.

to better trace individual traders may require quality individual data that is generally less available than the aggregate data.[35] Controlled experiments, therefore, become indispensable in this situation.

Studies using individual experimental data from laboratory asset markets to calibrated financial agents also exist. Hommes et al. (2005) fit subjects' forecasting behavior using a class of linear models with lags in prices and expected prices, and find that simple forecasting rules, such as naive or adaptive expectations or a simple linear trend-following extrapolation rule, are sufficient for characterizing subjects' behavior. Heemeijer et al. (2009) study subjects' forecasting behavior under two different price feedback mechanisms. The one with positive feedback is typically exemplified by the asset market, whereas the one with negative feedback is a feature of the cobweb model. By fitting subjects' forecasting behavior with a class of "first-order heuristics", they find that adaptive expectations are important in negative feedback markets, while trend-following extrapolation rules are important in positive feedback markets. The regime-switching model or the heuristic switching model was introduced in Anufriev and Hommes (forthcoming). Based on the calibration work in Hommes et al. (2005) and Heemeijer et al. (2009), a set of heuristics was chosen for the artificial adaptive agent. It was shown that their proposed heuristics-switching agents are capable of reproducing various qualitatively different price patterns observed in the experiments, including monotonically converging prices, permanent oscillations, and dampened oscillations. See Hommes (2011) for an overview of calibrating heuristics switching models to experimental data.

Other studies using laboratory data of individuals include Duffy and Unver (2006), who apply the simulated method of moments to calibrate a version of the near-zero intelligence agent,[36] and Chen and Hsieh (2011), who apply the maximum likelihood method to calibrate 3-parameter heterogeneous reinforcement learning agents and find that the performance of agents is not independent of these estimated heterogeneities.

In sum, the main difference between the calibrated agents developed in game experiments and those developed in agent-based financial markets lies in the *cognitive heterogeneity* and *cognitive hierarchy*. The former have both of them, while the latter have some consideration of cognitive heterogeneity, but not much of cognitive hierarchy. One of the reasons for the general lack of these two features in the H-type agent-based financial models is partially because of the influence of the traditional cellular automata (Section 3), namely, the use of simple agents to generate or evolve complex phenomena. Hence, one hesitates to introduce agent heterogeneity into the model unless it is favored by empirical evidence, but the kind of econometric tests for heterogeneity, observed in game experiments, has not yet become available in the literature on agent-based financial markets. Nevertheless, as we shall review in the next section, one recent trend in the behavioral economic experiments has been to address the significance of the cognitive heterogeneity of human agents. Moreover, attempts have been made to begin to explore other sources of heterogeneity, such as personality and cultural sensitivity.

5.3. Artificial agents with personal traits

Bounded rationality is about how people actually make decisions without actually optimizing. In their elaboration of Herbert Simon's original concept of bounded rationality, in addition to cognition, Gerd Gigerenzer and Reinhard Selten also consider the role of *emotions* and *culture* in bounded rationality (Gigerenzer and Selten, 2001). Over the last decade, we have seen the rapid development of *behavioral experiments* which treat cognitive capacity, intelligence, personality, emotion, risk attitude, and culture as independent variables. Given the already established connection between economic experiments and agent-based modeling and the relationship between human agents and software agents (Sections 5.1 and 5.2), it is expected that one of the next attempts in ACE is to design economic software agents explicitly with these personal attributes in light of these behavioral experiments. Some initial work on the artificial agents in light of these experiments has been developed. Generally speaking, these agents can be categorized into three types: cognitive agents (agents with working memory), psychological agents (agents with personality traits), and culturally sensitive agents. In this section, we will first review the general idea of designing this kind of human-like artificial agents followed by a brief review on the agents with working memory.

5.3.1. General idea

Work to integrate personal traits into artificial agents and social simulation is developing slowly in ACE. How does one give artificial agents personal traits? Before economists get interested in this issue, the research community of artificial agents has already tackled this issue for some time. Generally speaking, it involves three steps. First, choose one or a few major measure(s) (variables) of the personal traits, which are well studied in their respective fields (cross-cultural studies, psychology, etc.). Second, use these chosen variables to define the internal states of the artificial agents. Third, construct a mapping between these states and agents' behavior through a set of rules.

The first step is relatively easy. Basically, one can follow the literature on behavioral economic experiments, and consider those measures which are employed in human-subject experiments. For example, working memory capacity, Big

[35] The recent availability of more proprietary data has, however, enhanced the transparency of the trading behavior of financial agents, which include both individual and institutional investors. See, for example, Nolte and Nolet (forthcoming). However, this kind of empirical microstructure has not been integrated into the current agent-based financial markets.

[36] The simulated method of moments is one kind of indirect estimation method (simulation-based method). It has also been applied to estimate other agent-based financial models, such as Gilli and Winker (2003). See Chen et al. (forthcoming) for a comprehensive survey.

Five,[37] and the five dimensions of culture[38] are just examples of what artificial agents with personal traits may start with. The second and the third steps may be more intriguing because they involve expertise beyond the conventional domain of economics, and may require interdisciplinary collaboration. In the following, for illustrative purposes, the studies using artificial agents with working memory are briefly reviewed.

5.3.2. Artificial agents with working memory

The game experiments mentioned in Sections 5.2.1 and 5.2.2 did not treat cognitive capacity as an independent variable. Hence, even though it was found in some cases that the human subjects are heterogeneous in the calibrated artificial agents and these artificial agents with generalized reinforcement learning agents may be given a cognitive interpretation, it is not clear whether the estimated (observed) heterogeneity of human subjects is caused by their heterogeneity in cognitive capacity or by other factors. The cognitive game experiments starting in the early 2000s are different: they directly treat cognitive capacity as an independent variable, and formally examine the relevance of cognitive capacity to the required capability of information processing and strategic reasoning in the context of game experiments (Segal and Hershberger, 1999; Casari et al., 2007; Jones, 2008). These studies draw our attention not only to the heterogeneity of human in terms of working memory capacity, or some other related measures, but also to its consequences in relation to economic behavior.

The human's working memory capacity is frequently tested based on the number of the cognitive tasks which humans can simultaneously process (Cappelletti et al., 2008). Dual tasks have been used in hundreds of psychological experiments to measure the attentional demands of different mental activities (Pashler, 1998). This inspires us to model working memory capacity directly through population-based algorithms, which can implement parallel processing, and genetic algorithms (GA) and genetic programming (GP) are such kinds of algorithms. The population size of GA or GP will directly determine the capability of parallel processing. Hence, the population size seems to be an appropriate choice with regard to mimicking the working memory capacity of human agents. A smaller population size, therefore, corresponds to a smaller working memory capacity, whereas a larger population size corresponds to a larger working memory capacity. In this way, an agent-based model composed of agents with different working memory capacities can be developed.

The idea of using *population size* as a proxy variable for working memory is first proposed by Casari (2004). In an agent-based model of the common property resource experiments conducted by Casari and Plott (2003), Casari (2004) represented each artificial agent with a genetic algorithm, which is known as the *multi-population GA* or (*individual learning*), to be distinguished from the *single-population GA* (*social learning*).[39] Casari (2004) literally treated the population size used in the genetic algorithm as being equivalent to the number of chunks that a human can process at a time. According to the famous "7 ± 2" rule proposed by Miller (1956), the capacity lies between five and nine. Casari (2004) then set the population size of genetic algorithms to 6, "which implies that decision-makers have a hardwired limitation in processing information at six strategies at a time." (Casari, 2004, p. 261). It is demonstrated that the agent-based simulation with these artificial agents with the given *artificial* working memory capacity can simulate many patterns observed in common property resource experiments. This is probably the earliest article which connects the *population size* used in genetic algorithms to *working memory capacity* in cognitive psychology. Casari (2004), however, did not show the effect of his chosen working memory capacity (population size). Throughout his simulations, only the results with a population size of six are presented. It is, therefore, not clear whether human subject results are sensitive to working memory capacity (population size).

Later on, Casari's idea was extended by Chen et al. (2008) to genetic programming. In the context of the agent-based double auction market, they used genetic programming to model agents' adaptive behavior. This way of modeling agents is not new; however, they no longer assumed that agents are equally smart; instead, following the series of experiments which provide evidence of heterogeneity in subjects' working memory capacity, they used the population size of GP as the parameter to differentiate the agents' working memory capacity. They then simulated this agent-based double auction market, and examined the emergent properties at both the macro level (the market's performance) and the micro level (the individual's performance). It is found that working memory capacity has a positive and significant impact on the market efficiency, which is measured by the percentage of the realized consumer's and producer's surpluses. A more interesting part of their work is to examine the co-evolutionary dynamics when competing agents become equally smarter. Chen et al. (2008) show that, even in a competing situation like the double auction game, pairs of smarter agents can figure out a way to cooperate (collude) so as to create a win–win situation, whereas this collaboration is not shared by pairs of less smart agents.

[37] The five-factor model of personality was developed by Gordon Allport (1897–1967). The five factors are *openness*, *conscientiousness*, *extraversion*, *agreeableness*, and *neuroticism*. A convenient acronym is "OCEAN". Big Five has also been applied to economic experiments (Ben-Ner et al., 2004a, 2004b; Ben-Ner and Halldorsson, 2007; Hirsh and Peterson, 2009).

[38] Geert Hofstede, based on his extensive survey of IBM international employees, has proposed the well-known *five dimension theory of culture* (Hofstede, 2001). The five dimensions are *individualism* (vs. collectivism), *power distance*, *uncertainty avoidance*, *long-term orientation*, and *masculinity* (vs. femininity). This work is considered very influential in cross-cultural studies and has been cited in behavioral and experimental economics (Hsee and Weber, 1999; Carpenter et al., 2004; Tanner et al., 2005; Statman, 2008).

[39] This distinction was first made by Holland and Miller (1991). The effects of these two different designs were further studied by Arifovic (1994) and Vriend (2000).

In addition to the aggregate outcome, Chen and Yu (2011) also compared the strategies learned from agents with different working memory capacity. They analyzed the relationship between the complexity of profitable strategies learned by the agents and their associated working memory capacity. They found that some strategies, which are more complex but also more profitable, had never been found by agents with a smaller capacity, but could quite frequently be found by agents with a larger capacity. Further analysis of these strategies shows that additional capacity facilitates the combinatoric operation which agents need to cook up with more complex and profitable strategies.

6. Concluding remarks

In this paper, by developing different ideas with the designing of artificial agents as the key thread, we try to present one style in the review of ACE. We provide a short historical review of agent-based computational economics by tracing its four origins, from markets, to cellular automata (the 1970s), to automated tournaments (the 1980s) and further to economic experiments (the 1990s). The origins reviewed in this paper have led to the corresponding development of three agent paradigms, in short, simple-programmed agents, autonomous agents, and human-like agents. The essence is that the agents or agent paradigms currently employed in ACE models are increasingly interdisciplinary, partially because agent-based modeling has the potential to become a universal platform for social sciences and sciences or, as Paul Borrill and Leigh Tesfatsion have suggested, agent-based modeling is the *right mathematics* for the social sciences (Borrill and Tesfatsion, to appear). Therefore, the scope of agent-based modeling has also become broader in accommodating different needs through these varieties of agents.

As the title of the paper suggests, we by no means attempt to provide a general review of ACE, but only one which can demonstrate different agent paradigms. In addition to having agents as the key thread, we can also have the institutions, the embedding structure, or the networks which facilitate interactions as an alternative thread to review the ACE literature. While the interacting agent is one of the distinguishing features of ACE, the structure or the network accommodating the interactions are quite often implicit. It is only very recently that ACE and social networks have been studied as a coherent body (Chen, 2006; Vriend, 2006; Wilhite, 2006). Apart from that, statistical physics provides an alternative model of social interactions, although without explicitly referring to network topologies. In this regard, ACE is also closely connected to the literature on econophysics in terms of interaction platforms.[40]

These two threads, agent paradigms and interaction paradigms, are fundamental to the ACE models since the behavior of the ACE model may be sensitive to the choice of them. This sensitivity issue becomes particularly important when ACE modeling is applied to policy designs or market designs.[41] When general information on what would be the most convincing choice of the agent paradigms is not available, it would be imperative to know the possible range of the result as a gauge of the policy uncertainty or the policy risk. This exploratory modeling was promoted by Leamer (1985) in econometrics and was also further pursued in the agent-based community (Lempert et al., 2003). The varieties of agents can provide a thorough study of design issues.[42]

We would like to close this survey with some speculations regarding the future. The idea of human-like agents has been recently pursued quite intensively by the agent community, which is mainly computer science-oriented. Various models of agents with personality, emotion and cultural backgrounds have been attempted (Gmytrasiewicz and Lisstti, 2002; Schmidt, 2002; Alavizadeh et al., 2008; Hofstede et al., 2008). This research trend has been further connected to computational models of the brain, neuroscience and neuroeconomics (Vromen, 2010). To what extent ACE can benefit from this enlarging interdisciplinary integration of agent research, an open mind is required.

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[40] For example, Vinkovic and Kirman (2006) use a model involving liquid as an analogue of Thomas Schelling's segregation model.

[41] The ACE applications to market designs or policy designs is another pile of literature that our survey does not include. The interested reader is, however, referred to the special issue on *Agent-Based Models for Economic Policy Designs* of the Journal of Economic Behavior and Organization (Dawid and Fagiolo, 2008).

[42] For example, one can first start with a benchmark populated with simple agents to give the role of the institution a basic grasp, then take into account the co-evolving strategic behavior of agents as a robustness check, and lastly further consider various human constraints in order to evaluate whether the policy is portable for stakeholders with different cognitive capacities, personality traits and cultural backgrounds. This process enables us to see when structural dominance gives way to behavioral complexity, and can be particularly useful for those who conduct comparative studies.

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