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INNOVATION, DIVERSITY AND DIFFUSION: A SELF-ORGANISATION MODEL*

Gerald Silverberg, Giovanni Dosi and Luigi Orsenigo

The diffusion of new products and new processes of production within and between business enterprises is clearly one of the fundamental aspects of the process of growth and transformation of contemporary economies.

It is well known that the diffusion of new products and processes takes varying lengths of time: some economic agents adopt very early after the development of an innovation while others sometimes do it only after decades. Moreover, during the diffusion process the competitive positions of the various agents (adopters and non-adopters) change. So do the economic incentives to adopt and the capabilities of the agents to make efficient use of the innovation. Finally, the innovation being adopted also changes over time, due to more or less incremental improvements in its performance characteristics which result in part from its more widespread use.

Contemporary analysis of diffusion has been essentially concerned with the following questions: (a) why is a new technology not instantaneously adopted by all potential users? (i.e. what are the 'retardation factors' preventing instantaneous diffusion?), (b) how can the dynamic paths of diffusion be represented?, and (c) what are the relevant variables driving the process?

However, innovation diffusion has rarely been formally treated as part of a more general theory of economic dynamics in which diversity of technological capabilities, business strategies, and expectations contribute to shape the evolutionary patterns of industries and countries (a remarkable exception is the evolutionary approach developed in particular by Nelson and Winter (1982) who, however, are more concerned with the general features of industrial dynamics than with the specific characteristics and implications of the diffusion process).

In this work, we shall analyse the nature of diffusion processes in evolutionary environments characterised by technological and behavioural diversity amongst the economic agents, basic uncertainty about the future, learning and disequilibrium dynamics.

First, we shall identify some fundamental characteristics of technology, innovation and diffusion which, we suggest, must be accounted for in theoretical models. Second, against this background, we shall briefly review

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what we consider the major achievements and shortcomings of the current models of innovation diffusion. Third, we shall present what we call a 'self-organisation' model of innovation diffusion, that is, a model whereby relatively ordered paths of change emerge as the (partly) unintentional outcome of the dynamic interactions between individual agents and the changing characteristics of the technology. Fourth, the main properties and simulation results of the model will be discussed.

I. CHARACTERISTICS OF TECHNOLOGY AND DYNAMIC INDUSTRIAL ENVIRONMENTS

A renewed interest in the economics of innovation over the last two decades has brought considerable progress in the empirical description and theoretical conceptualisation of the sources, characteristics, directions and effects of technical change. We review these topics in Dosi (1988). Here, it suffices to summarise some of the major findings directly relevant to the diffusion of innovations concerning the nature of technology and the characteristics of firms and innovative environments.

- (a) Technology far from being a free good is characterised by varying degrees of appropriability, of uncertainty about the technical and, a fortiori, commercial outcomes of innovative efforts, of opportunity for achieving technical advance, of cumulativeness in the patterns of innovation and exploitation of technological know-how and hardware, and of tacitness of the knowledge and expertise on which innovative activities are based. Particular search and learning processes draw on technology-specific knowledge bases, related to both freely available information (e.g. scientific results) and more 'local' and tacit skills, experience and problem-solving heuristics embodied in people and organisations.
- (b) Technologies develop along relatively ordered paths (or 'trajectories') shaped by specific technical properties, search rules, 'technical imperatives' and cumulative expertise embodied in each 'technological paradigm' (cf. Dosi (1984); for similar arguments see Nelson and Winter (1977), Sahal (1981; 1985), Arthur (1985), Metcalfe (1985) and within somewhat different perspectives, Atkinson and Stiglitz (1969) and David (1975)). Relatedly, Winter (1984) defines different 'technological regimes' according to whether the knowledge base underpinning innovative search is primarily 'universal', and thus external to individual firms, or, alternatively, is primarily 'local' and firm-specific.
- (c) As a consequence of (a) and (b), diversity between firms is a fundamental and permanent characteristic of industrial environments undergoing technical change (see also Metcalfe (1985) on this point). Inter-firm diversity (even within an industry) can fall into three major categories.

First, there are technological gaps related to different technological capabilities to innovate, different degrees of success in adopting and efficiently using product and process innovations developed elsewhere, and different costs

of production of output. In Dosi (1984) we define these forms of diversity as technological asymmetries, meaning unequivocal gaps between firms which can be ranked as 'better' and 'worse' in terms of costs of production and product characteristics.

Second, diversity relates to differences between firms in their search procedures, input combinations and products, even with roughly similar production costs (on this point, see Nelson (1985)). Similarly, firms often search for their product innovations in different product-spaces and concentrate their effort on different sections of the market. Let us call this second set of sources of diversity technological variety, meaning all those technological differences which do not correspond to unequivocal hierarchies ('better' and 'worse' technologies and products).

Third, one generally observes within an industry (and even more so between industries) significant differences in the strategies of individual firms with respect to the level and composition of investment, scrapping, pricing, R & D, etc. Let us call these differences behavioural diversity.

Evolutionary processes in economic environments involving innovation and diffusion are governed to different degrees by selection mechanisms and learning mechanisms. Selection mechanisms tend to increase the economic dominance (e.g. profitability, market shares) of some firms with particular innovation characteristics at the expense of others. Learning mechanisms, on the other hand, may both spread innovative/imitative capabilities throughout the (possibly changing) set of potential adopters and reinforce existing disparities via cumulative mechanisms internal to the firm.

Learning processes generally occur via (a) the development of intra- and inter-industry 'externalities' (which include the diffusion of information and expertise, interfirm mobility of manpower, and growth of specialised services); (b) informal processes of technological accumulation within firms (of which learning-by-doing and learning-by-using are the best known examples of such 'internalised externalities'); and (c) processes of economically expensive search (R & D being, of course, the best example).

After a brief survey of the current state-of-the-art in the theory of innovation diffusion, we shall present a model which, in our view, makes a serious attempt to incorporate some of these features of innovative environments in a novel, yet consistent and realistic way.

II. DIFFUSION MODELS: RESULTS AND LIMITATIONS

Three basic approaches dominate current economic thought on innovation diffusion (cf. Stoneman (1983; 1986), Arcangeli (1986)). First, the line of enquiry pioneered by the seminal work of Mansfield (1961; 1968), and Griliches (1957) tries to identify the empirical regularities in diffusion paths, typically represented by S-shaped curves. In Mansfield's 'epidemic' approach, diffusion is generally found to be pushed by the expected profitability of the innovation and driven by the progressive dissemination of information about

its technical and economic characteristics. Thus, diffusion is interpreted as a process of adjustment to some long-term equilibrium contingent upon learning by potential adopters.¹

Empirical work on diffusion, however, whilst confirming the role of profitability in adoption decisions, has shown that differences in the characteristics of innovations, of product mixes, and of the potential adopters are also key factors in the diffusion process (see, for example, Nabseth and Ray (1974), Gold (1981), Davies (1979), David (1975)).

These findings, together with theoretical considerations about the crudely mechanical nature of epidemic diffusion models, lend support to a second approach, namely one based on 'equilibrium diffusion models'. Here, diffusion is seen as a sequence of equilibria determined by changes in the economic attributes of the innovation and the environment (see David (1969), Davies (1979), Stoneman and Ireland (1983), Ireland and Stoneman (1986), David and Olsen (1984), Reinganum (1981)). This approach has undoubtedly provided important insights into diffusion processes. Amongst other things, it has shown the importance of (i) differences (such as size) between potential adopters; (ii) the interactions between the supply decisions of the firms producing innovations and the pace of their adoption; (iii) the technological expectations of suppliers and adopters; (iv) the patterns of strategic interactions amongst both suppliers and adopters; (v) the market structure in both the supplying and using industries. However, these results are generally achieved at a high theoretical price. Radical uncertainty is de facto eliminated and maximising behaviour is assumed.² The analysis is often undertaken in terms of the existence and the properties of equilibria, while nothing is generally said about adjustment processes. Information about the techno-economic characteristics of the technologies is generally assumed to be freely available to all agents. The nature of 'technology' is radically simplified and assumed to be embodied in given technical features of production inputs.

A third approach is explicitly evolutionary and represents the diffusion of new techniques and new products under conditions of uncertainty, bounded rationality and endogeneity of market structures as a disequilibrium process (Nelson, 1968; Nelson and Winter, 1982; Metcalfe, 1985; Silverberg, 1984; Iwai, 1984a, b).³

The model that follows is in this evolutionary tradition, and thus allows for disequilibrium processes, endogeneity of market structures, etc. It also explicitly incorporates those assumptions of 'equilibrium' diffusion models which capture important empirical characteristics of innovative environments mentioned earlier, such as the relevance of expectations and differences between agents, as

¹ One may, for example, represent this diffusion process as the transition between two 'classical' long-term equilibrium positions: see Metcalfe and Gibbons (1988).

² To be precise, in Davies' original model adoption decisions are based on rules of thumb explicitly justified in terms of 'bounded rationality'. Yet, subsequent developments within this approach have been explicitly based on maximising behaviour of the agents.

³ See also Eliasson (1982; 1986). On the connection to empirical analysis see Gort and Klepper (1982), Gort and Konakayma (1982) and Levin et al. (1985).

well as some features implicit in Mansfield-type models, such as imperfect information and asymmetric technological knowledge.⁴

III. A SELF-ORGANISATION MODEL OF THE DIFFUSION OF INNOVATIONS AND THE TRANSITION BETWEEN TECHNOLOGICAL TRAJECTORIES

In two previous papers, one of the present authors (Silverberg, 1984; 1987) attempted to demonstrate the relevance to economic theory of the selforganisational approach to dynamic modelling pioneered by Eigen, Haken, Prigogine and others.⁵ In essence the argument proceeds from the observation that in complex interdependent dynamical systems unfolding in historical, i.e. irreversible time, economic agents, who have to make decisions today the correctness of which will only be revealed considerably later, are confronted with irreducible uncertainty and holistic interactions between each other and with aggregate variables. The a priori assumption of an 'equilibrium' solution to this problem to which all agents ex ante can subscribe and which makes their actions consistent and in some sense dynamically stable is a leap of methodological faith. Instead we proposed employing some of the recently developed methods of evolutionary modelling to show how the interaction of diverse capabilities, expectations and strategies with the thereby emerging selective pressures can drive a capitalistic economy along certain definite patterns of development.

Drawing on a dynamic model of market competition with embodied technical progress investigated in Silverberg (1987), we embed the question of diffusion into the larger one of the transition of an industry between two 'technological trajectories'. Choice of technique is no longer a choice between two pieces of equipment with given (but perhaps imperfectly known) characteristics, but now involves skills in using them which can be endogenously built up by learning by doing or by profiting from the experience of others, as well as expectations about future developments along the various competing trajectories. As we shall see, the diversity in firms' capabilities and expectations is an irreducible element driving the diffusion process.

In the sectoral approach taken here industry-level demand is taken as given and growing at some exponential rate. Firms command some market share of this demand at any given time, but market shares may change over time as a dynamic response with a characteristic time constant (reflecting the 'freeness' of competition and such factors as brand loyalty, information processing and search delays and costs, etc.) to disparities in the relative *competitiveness* of firms. This concept, so dear to close observers of the business scene, has to our knowledge evaded incorporation into a systematic economic theory until now.

⁴ A more detailed discussion of the empirical basis of the hypotheses entering into the model presented below can be found in Dosi et al. (1988).

⁵ For a multidisciplinary overview of self-organisational modelling and its methodological philosophy see Haken (1983), Nicolis and Prigogine (1977) and Prigogine (1976).

The evolution of market structure is governed in our approach by an equation relating the rate of change of a firm's market share to the difference between its competitiveness (defined below) and average industry competitiveness (averaged over all competing firms in an industry, weighted by their market shares). This equation is formally identical to the equation first introduced into mathematical biology by R. A. Fisher in 1930 and more recently applied in a variety of contexts and studied in considerable mathematical detail by Eigen (1971), Eigen and Schuster (1979), Ebeling and Feistel (1982), Hofbauer and Sigmund (1984), and Sigmund (1986). Our use of this equation differs from most biological applications, however, in that the competitiveness parameters, rather than being constants or simple functions of the other variables, themselves change over time in complex ways in response to the strategies pursued by firms and feedbacks from the rest of the system. In a systems theoretic sense this equation may be regarded as the fundamental mathematical description of competitive processes. It is worth emphasising the difference between our approach and standard theoretical conceptualisations of competition. The latter generally identify the circumstances under which no relative competitive shifts or profits can be realised (impossibility of arbitrage, uniform rate of profit, etc.) and then assume that the system must always be in or near this state.

If we denote by f_i the market share in percentage of real orders of the ith firm, by E_i its competitiveness and by $\langle E \rangle$ the average competitiveness of all firms in the industry $(=\sum f_i E_i)$, then the evolution of market shares is governed by the following equation:

$$\dot{f_i} = A_9(E_i - \langle E \rangle) f_i. \tag{1}$$

We define the competitiveness parameter as a linear combination of terms reflecting relative price and delivery delay differentials:

$$E_i = -\ln p_i - A_{10} \, dd_i, \tag{2}$$

where p_i is the market price of the *i*th firm and dd_i its current delivery delay.⁶

Silverberg (1987) presents a basic dynamic structure for dealing with strategic investment in the face of uncertainty with respect to the future course of embodied technical progress, overall demand and changes in relative competitiveness. In this framework, entrepreneurs are seen as being fully conscious of the ongoing, process nature of economic growth and technological change, so that their decisions, particularly concerning fixed investment, take account of and try to anticipate these developments. Decision-making is incorporated on the one hand in certain robust rules of thumb (for the most part feedback rules dealing with oligopolistic pricing and production policies) and 'animal spirits' in the form of decision rules governing replacement policy (the payback period method) and expansion of capacity ('estimates' or 'guesses' of future demand growth corrected by experience). Technical change

⁶ Product quality factors could also be included in this expression, but for simplicity we restrict the analysis here to markets with fully standardised commodities.

is embodied in vintages, and the resulting capital stocks are not assumed to start in, and in general need not converge to steady-state distributions.

The capital stock (measured in units of productive capacity) of each firm is represented as an aggregation over nondecaying vintages between the current period t and the scrapping date $T_i(t)$:

$$K_i(t) = \int_{T_i}^t K_i(t, t') dt',$$
 (3)

where $K_i(t, t)$ is gross investment at time t (in capacity units),

$$K_i(t, t') = K_i(t', t')$$
 if $T_i(t) < t' < t$ and
= 0 otherwise.

This aggregate capital stock may be a composite of different technologies as well as different vintages of a single technological trajectory. A payback calculation is performed by each firm with its desired payback period (which may differ between firms) to determine a desired scrapping date for its capital stock $T_{di}(t)$ by solving:

$$P(t)/[c(T_{di})-c(t)] = b_i,$$
 (4)

where P(t) is the price of new capital equipment per unit capacity, c(...) is the unit operating cost at time t of the vintage in question, and b_i is the target payback period of the ith firm.⁷

The actual scrapping date adjusts to this desired date via a first-order catch up procedure: $\dot{T}_i = z_i \max \left[A_{11} (T_{di} - T_i), o \right]$ (5)

where z_i is a rationing parameter between o and I (the ratio of current cash flow to desired gross investment) which may arise if the *i*th firm, due to financial constraints, is not able to finance its desired investment programme fully (otherwise it is I). The amount of capacity scrapped as a result of this decision (as well as a possible desire to reduce overall capacity) is

$$S_i = K_i(t, T_i) \dot{T}_i. \tag{6}$$

Net expansion (or contraction) of capacity is governed by a desired expansion rate r_i for each firm:

$$N_i = r_i K_i. (7)$$

⁷ In the economics literature a number of seemingly 'self-evident' rules have been applied to decide when technologically obsolete equipment should be replaced by new equipment. One calls for an old vintage to be replaced when its unit variable costs exceed total unit costs of current best practice. Another indicates replacement when unit variable costs exceed the price attained per unit of output. A substantial specialised literature exists, however, dealing with optimal replacement beginning with Terborgh (Terborgh, 1949; see also Smith, 1961). Under suitable assumptions about the rate of future technical progress this leads to the so-called square root rule.

Terborgh shows that the payback criterion is a reasonable approximation to the square root rule. Given that uncertain technological expectations (which are an extrapolation from past experience in this rule) play a major role, and that surveys of industrial practice consistently reveal rate of return or payback period calculations to be widely entrenched, we have opted for this simple criterion in our treatment of replacement. For a discussion of optimal replacement in the evolutionary framework employed here see Silverberg (1987).

The capital stock changes over time due to additions from gross investment and removals due to scrapping:

$$\dot{K}_i = N_i = K_i(t, t) - S_i. \tag{8}$$

The desired rate of capacity expansion may be set initially at any level ('animal spirits') but is revised over time using first-order feedback from the deviation of the rate of capacity utilisation u from its desired level u_0 :

$$\dot{r}_i = A_{13}(u_i - u_0). \tag{9}$$

Labour is assumed to be the only current cost of production and can be decomposed into prime and overhead components.⁸ The prime unit labour coefficient is an average over the historical technological labour/output coefficients a(t) weighted by vintage (in the following the firm subscript i has been suppressed for simplicity):

$$\langle a \rangle = \int_{T}^{t} a(t') K(t, t') dt' / K(t). \tag{10}$$

It changes over time due to additions of more productive new equipment through investment and removal of marginal equipment through scrapping according to the following equation derived from (10) by differentiation:

$$\langle \dot{a} \rangle = \{ K(t, t) \left[a(t) - \langle a \rangle \right] + S[\langle a \rangle - a(T)] \} / K. \tag{11}$$

If net investment is taking place, i.e. N > 0, then all scrapping serves the purpose of replacement investment R, so that K(t,t) = N + R, S = R and

$$\langle \dot{a} \rangle = \{ N[a(t) - \langle a \rangle] + R[a(t) - a(T)] \} / K, \tag{12}$$

which shows that replacement investment contributes more to lowering unit costs per unit of investment outlay than does expansion investment. Thus unit costs are determined by the age structure of the capital stock and the history of technological change it represents. They will vary over time as a result of the scrapping and expansion strategies of the firm under the constraint of its ability to finance its investment plans, itself a function of cost and profitability.⁹

Overhead labour per unit output at full operating capacity is assumed proportional to prime unit labour. Total overhead labour is then this value multiplied by total productive capacity K (and thus is independent of the rate of capacity utilisation, contrary to total prime labour, which is directly proportional to it).

The level of production is set such as to compensate for deviations of the current delivery delay (dd) from some industry-wide standard level (dd_0) :

$$\dot{u} = A_5(dd - dd_0) u(1 \cdot 1 - u^2), u < 1,$$

= 0, $u = 1$ and rhs above > 0. (13)

⁸ Other current costs of production could be incorporated by making the prime unit labour coefficient and nominal wage rate vectors.

⁹ The exact functional relationship is reminiscent of Kaldor's technical progress function, but shows that the rate of change of *average* productivity is a function of the gaps between best practice, average and marginal vintage productivities *and* the division of gross investment between modernisation and expansion.

The quadratic saturation term is introduced to represent bottlenecks in the production process near the full capacity limit. Delivery delay dd is the ratio of order backlog L to current production y (=uK), and the order backlog is governed by the rate equation

 $\dot{L} = d - y, \tag{14}$

where d is incoming orders $(=f_i \times \text{total market demand})$.

Firms' prices are determined as a dynamic compromise between the desired mark-up on unit costs and relative competitiveness. Since only relative prices are of importance here, we take the logarithm of price variables throughout. Let p_i be the log of the *i*th firm's market price and p_{ci} its desired markup price based on its unit prime costs. Then

$$\dot{p}_i = A_7(p_{ci} - p_i) + A_8(E_i - \langle E \rangle). \tag{15}$$

Pricing policy is regarded as a compromise (depending on the 'degree of monopoly' characteristic of an industry) between strict cost-plus pricing and a concession to the 'prevailing' market price (the geometric mean of all prices weighted by market shares) via relative competitiveness. This structure of pricing allows the changing relative cost structure of firms to be transmitted through the market and makes intelligible such phenomena as price leadership or being under price pressure. Firms at a competitive (in general mostly cost) disadvantage are thus forced to lower their prices somewhat to prevent excessive losses of market share, while firms enjoying a competitive advantage are free to realise short-term profits by raising their prices. The ratio of A_8 to A_7 determines to what extent competitive pressures overrule the markup principle (which remains valid, however, at the aggregate level) and enables the model to span the entire range of market structures between pure monopoly and pure competition. 10

As the model now stands, with a single vintage structure for each firm, it already accounts for the diffusion of new technology in the case in which a unique best practice technology is apparent to all agents (this perspective on diffusion was first introduced by Salter, 1962). The process of investment under the assumption of some long-term rate of technical progress implicit in the payback method ensures that advances in productivity will be continually incorporated into the capital stock, even if entrepreneurs differ in their assessment of the appropriate payback to use. Thus diffusion of technical progress is already guaranteed by the standard methods of investment policy at this first level of analysis.

However, in order to capture the collective dynamic of advance along different technological trajectories we propose the following additional structure. We compare two technological trajectories representing at any time the maximum productivities attainable in best practice vintages of the respective technologies. We assume that these are both changing at some rate, and that the second technology is always absolutely superior in productivity. Moreover, the relative price/capacity unit of the two technologies may also be

¹⁰ For a more detailed discussion of the price interactions to which this system leads see Silverberg (1987).

changing. The actual productivities realised by firms are a product of this underlying value and the specific efficiency or skill with which firms master each technology (between o and 100%). For simplicity we assume all firms begin with technology 1, and technology 2 first becomes available at time t^* . Furthermore, technology 1 is already mature, i.e. skill levels are saturated at 100 %. The firms initially possess lower (and possibly varying) efficiencies with technology 2, but the margin for further development is not knowable with any precision. Firms only know the product of this efficiency and the underlying potential. They may (and in fact must) make guesses about the rate at which further improvements in efficiency (equally applicable to previously installed vintages) and further embodied technical progress (only applicable to current investment) will be achieved. This formulation reflects the fact that the productivity of a technology realised in practice is not just a function of the presence of the requisite machines, but conjointly requires certain levels of specific expertise and experience (from specialised scientific and engineering training to shop floor apprenticeship and work discipline) both internal and external to the firm. Hence investment decisions are not merely a question of determining the best practice technology at a given time, but one of weighing the prospects for further development either by acquiring experience with it now to gain a jump on competitors or waiting for a more opportune moment and avoiding possible development costs.11

We identify the evolution of the efficiency parameter with movement down the well-known learning or experience curve using a logistic dynamic and a variable rate of change equal to the rate of growth of cumulative production with the technology (this corresponds to the classic power law learning curves on cumulative production reported in the literature for values well below saturation). This represents internal learning and is only achieved if the firm actually produces with the new technology. Writing s_i for the internal skill level of the ith firm using the new technology, P_i for its current production and CP_i for its cumulated production with the new technology, we have

$$\dot{s}_i = A_{15}[P_i/(CP_i + C)] \, s_i (\mathbf{I} - s_i), \qquad \text{if } s_i > s_p, \tag{16}$$

where C is a constant proportional to the capital stock and s_p is the level of skill generally available in the industry even to those firms not yet producing on the new technological trajectory.

In addition, the experience acquired by individual firms can 'leak' out and become available to the rest of the industry. In practice this can take the form of skilled labour and management moving between firms (or setting up their own companies), manufacturers diffusing the results of experience gained with

¹¹ Thus in a very suggestive study of the diffusion of numerically controlled machine tools in German industry, Kleine (1983) reports that some firms invested in the new technology even though it did not yet satisfy their normal investment criteria because they hoped to build up superior skills specific to a technology which they anticipated would play a decisive role in the future. Others took a more conservative attitude, by no means irrational prima facie, and waited for the smoke to clear before buying into a more mature technology. The spread of knowledge about the availability and purported superiority of NC equipment played almost no role since the firms surveyed were well informed from the start by suppliers and trade publications. Similar observations on computer adoption decisions have been made by Stoneman (1976).

their equipment to other users in the form of operating instructions and the like, trade organisations and publications, educational institutions, or even industrial espionage. We represent this by having the level of generally available skill (public skill) lag behind the average of internal skill levels with an exponential delay:

 $\dot{s}_p = A_4(\langle s \rangle - s_p), \tag{17}$ $\langle s \rangle = \sum f_s s_s.$

where

Firms profit from this learning externality because they 'float' on the rising general skill level even if they are not yet employing the new technology:

$$\dot{s}_i = \dot{s}_n \quad \text{if} \quad s_i = s_n. \tag{18}$$

In deciding on whether to switch to the new technology firms may want to abandon their normal investment criteria to take into consideration the gains in productivity they may be able to realise even after new equipment is installed as well as their desire to attain early proficiency in its use and thereby get on a possible virtuous circle. These will depend on how optimistic they are about the future development potential of the new trajectory and the extent to which temporary advantages can be appropriated (which is related to the relative rates of internal and external learning) as well as what their competitors are planning. To this end firms select an 'anticipation bonus' they award to the new technology in making their choice of technique. They multiply the current realisable productivity by their bonus for the new technology and compare it with the best practice productivity of the old in a payback calculation. This means that the new technology is preferred if its adjusted productivity is higher than that of the old and (i) it is cheaper per unit of capacity at the time of comparison or (ii) it is more expensive but the difference in price can be recouped within the desired pay-back period by the savings in labour cost. If c_1 , P_1 and c_2 , P_2 are the unit cost and price per efficiency unit of the old and the new technique, respectively, then the calculation is

$$(P_2 - P_1)/(c_1 - c_2/s_i X_i) \le b_i, \tag{19}$$

where X_i is the anticipation bonus of the *i*th firm (cf. equation (4)). The last case in which its (adjusted) productivity is lower but its price is also lower is excluded here as being of limited empirical interest.

It remains to decide what changes this introduces into the replacement rule. The reference value entering into the payback calculation for replacement uses the maximum of the old best practice productivity and the currently realisable new best practice productivity. This ensures that scrapping does not fall below the rate that would have prevailed if investment had continued in the old technology, and that it only accelerates when the new technology actually proves its superior performance on the shop floor.

The above model represents a dynamical system which, due to the vintage structure, should be categorised as a set of differential-difference equations with age-dependent effects. This is a class of systems whose mathematical properties, even in the most simple cases, are still only poorly understood. Many of the

mathematical elements going into the model, however, have a well-known pedigree, such as the replicator dynamics governing market shares (see, e.g. Sigmund, 1986). Consequently, we are forced to resort to 'experimental mathematics' in the form of a computer implementation to uncover some of the economic properties of the model.

IV. MARKET DYNAMICS, DIFFUSION, AND THE COLLECTIVE RATIONALITY OF THE ADOPTION DECISION

The system as described above admits several dimensions of structural and behavioural variability over time: firms can be of different sizes, characterised by different unit costs, delivery delays, rates of capacity utilisation, skill levels, age profiles of capital stock, etc. Of course, in this model as in a large part of the diffusion literature (e.g. David, 1969; Stoneman and Ireland, 1983), a distribution of initial characteristics of firms, with uniform expectations across firms, other things being equal, will lead to a distribution of adoption dates. In the general case of diverse firm characteristics and diverse technological expectations, the distribution of adoption decisions will both result from these initial distributions and contribute to their endogenous transformation. Thus firms' sizes, skill levels, and the like cannot be regarded as fixed characteristics to which the diffusion process can be referred, but rather must be seen themselves as in part products of that process. However, to focus more clearly on the strategic aspects of the diffusion process and the problem of the interdependence of behaviour even in the absence of diverse firm characteristics, we will neglect this dimension of the problem. Instead, we will single out the role of the anticipation bonus (reflecting expectations about the future course of the new trajectory) in relation to what we term dynamic appropriability and set all other characteristics identical across firms.

In the three runs we will now consider, technology 2 is potentially 100% more productive than technology 1 and both are advancing in the embodied sense at 4% p.a., as are nominal wages. Overall demand is growing at 5%. Technology 2 starts out being priced higher per capacity unit but this price declines at the rate of 1% p.a. All 10 firms employed in the first case (Figs. 1-4) start out identical in every respect except in their propensity to innovate, i.e. their innovation bonuses. The initial efficiency level on technology 2 is 30% for all firms. The anticipation bonuses range from 3.33 to 1.0 with a clustering around 1.33 (i.e. the firm evaluates the productivity of technology 2 in its choice of technique decision 33% higher than its actual present value). The vertical dotted lines indicate the date of adoption of the firm with the corresponding number.

Fig. 1 graphs three measures of diffusion. The curve marked with squares shows the classic measure of inter-firm diffusion discussed in the literature: the percentage of potential adopters already employing some quantity of the new technology. It shows the typical S-shape familiar to students of diffusion. Just near it (marked with a diamond) is a curve depicting the current market share of adopters. If this curve lies above/below the previous one, adopters as a whole

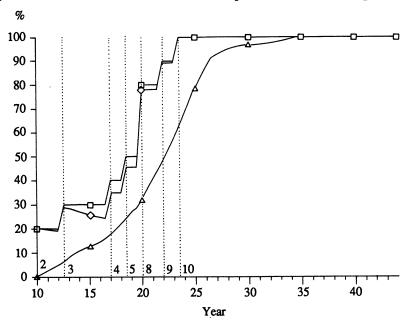
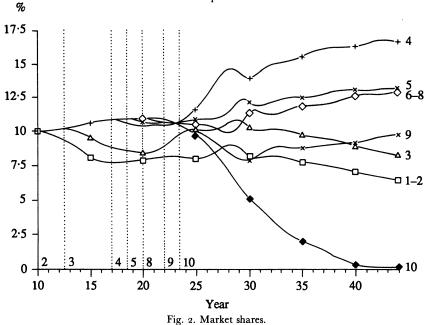


Fig. 1. Diffusion curves. □, Percentage of firms; △, percentage capacity; ⋄, market share of adopters.



have gained/lost market shares over time. The aggregation hides the fact that the vicissitudes of individual adopters can vary quite widely. The last curve represents the percentage of overall productive capacity embodied in the new technology. This results from both inter and intrafirm diffusion as well as shifts in the relative sizes of firms and is the key variable in analysing the impact of

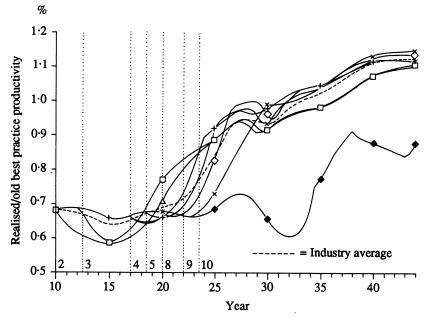


Fig. 3. Ratio of realised average productivity to best practice productivity of technique 1.

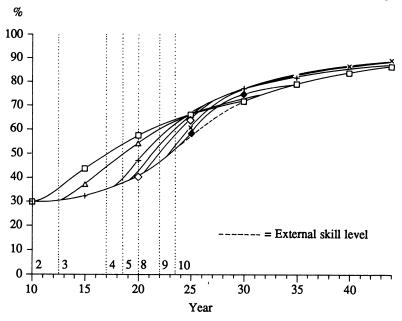


Fig. 4. Skill levels: firm-specific and external.

the innovation at the industry and the economy-wide levels. It displays the classic smooth S-shaped form Fisher and Pry (1971) found in measuring diffusion in capacity terms.

Fig. 2 plots market shares and reveals the microeconomic drama going on beneath the aggregation surface. Firms 1 and 2, which adopt as soon as the

innovation appears on the market (year 10), just manage to maintain respectable market shares. Firm 3 innovates $2\frac{1}{2}$ years later and does around 2 percentage points better in holding on to market share. Firm 4 is the clear winner in this saga, benefitting from the mistakes of firms 1–3 but still getting in on the ground floor to increase its market share by over 50%. Firms 5–8 are also net profiters from the market reshuffle to a small extent. Even firm 9, one of the laggards, manages to recover its initial market share after taking something of a beating. Firms 10 demonstrates the pitfalls of missing the boat by not providing for an anticipation bonus. It has evidently been pushed into a vicious downward spiral which completely eliminates it from the market.

Fig. 3 depicts the realised productivity of the entire capital stock of each firm, corrected for the rate of capacity utilisation, and divided by the old best practice productivity to eliminate the underlying exponential trend. The early adopters suffer a loss as they first go down the learning curve and then pull ahead. The middle adopters suffer only minor losses and soon overtake the early group, while the late adopters manage to get on the 'track' but consistently remain below the industry average (the dashed curve). Firm 10, finally, is thrown completely off the track and never comes close to closing the gap.

The evolution of the firm-specific and external skill levels is shown in Fig. 4. The early adopters do indeed build up a lead in their internal efficiency, but the middle and late adopters start from a higher initial level due to external learning and eventually overtake them. Even firm 10 manages to rise above the public skill level for a while after it adopts.

If we now naively rerun history (in a run there is no point in plotting) by giving all 10 firms the anticipation bonus used by the winner of the first round (firm 4), something surprising occurs. The new technology is not adopted at all because no firm is willing to incur the development costs associated with bringing it to commercial maturity. This makes it clear that technological innovation and diffusion are characterised by collective effects and an inextricable tension between private and social gain.

This is further brought out in the third run (Figs. 5–8), which is an example of 'early adopters receive their just deserts'. All parameters of this run are identical to those of the first one except for a doubling of coefficient A_{15} in equation (14). This accelerates the rate of internal learning and thereby raises the dynamic appropriability of the innovation for the early adopters. Although the actual times of adoption have hardly changed, the relative fortunes of the competing firms change considerably. The first adopters (firms 1 and 2) are clear net beneficiaries, followed by firm 3. All of the middle adopters are huddled closely together with little change in their market shares, while the straggler firm 10 is once again catapulted from the market.

What story does this one sequence of runs out of many possible ones have to tell us about strategic behaviour in innovative environments? In some respects

¹² It should be borne in mind that productivity is only part of competitiveness. The response of delivery delays to production and capacity expansion decisions also contributes to changes in market shares and realised price margins.

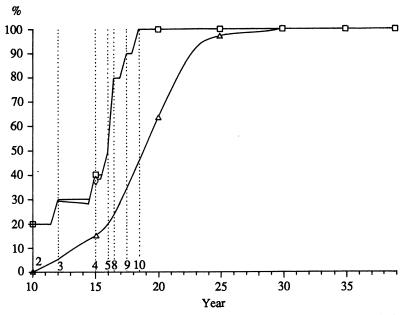


Fig. 5. Diffusion curves: ☐, Percentage of firms; △, capacity; ♦, market share of adopters.

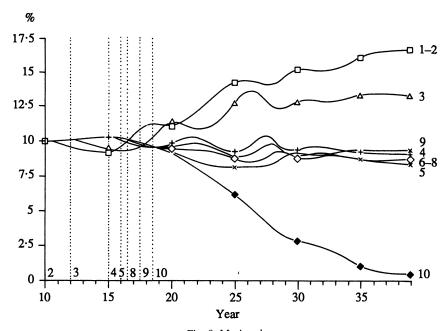


Fig. 6. Market shares.

the prospective shift of technological paradigm creates a Prisoner's Dilemma situation: conservative entrepreneurs would all prefer to avoid accelerated capital replacement and costly development expenditures. Yet profits may eventually be reaped and irreversible market share gains realised by adopting

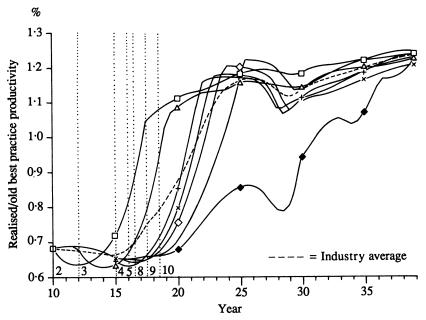


Fig. 7. Ratio of realised average productivity to best practice productivity of technique 1.

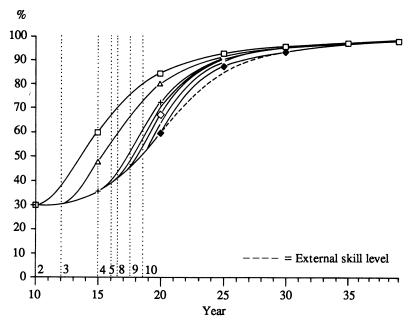


Fig. 8. Skill levels: firm-specific and external.

early. This threat thus forces entrepreneurs to take an anticipatory position and can be ultimately self-justifying, if the innovation is indeed potentially superior. This latter fact, of course, remains uncertain until the diffusion process is well underway. Moreover, the adoption decision is complicated by the learning

externality. The dynamic appropriability of the new trajectory, as we have seen, serves as a bifurcation parameter. For high enough values a first-in strategy is preferable, for lower values a second-in one is. But a second-in strategy is only possible if there is a sacrificial lamb in the form of a first adopter. And first adopters exist because the precise value of the dynamic appropriability is unknown.

One might inquire whether some distribution of adoption times may not exist satisfying a Nash equilibrium, i.e. given that it is clear that adoption will actually take place in a certain sequence, no single entrepreneur has an incentive to deviate locally from his adoption decision. This is precisely what Reinganum (1981) has studied in a static context. In comparison with our model, however, it should first be noted that the payoffs to adoption cannot simply be expressed as a function of the percentage of the industry already adopting. This is because nonlinear cumulative causation (a) makes the form of the interaction between agents exceedingly complex and subject to bifurcations, and (b) in evolutionary games such as our own, outcomes are in terms of expansion, survival or extinction and not of one-time monetary payments.

In the biological literature the evolutionary stable strategy (ESS) concept corresponds to that of a Nash equilibrium.¹³ Instead of the rationality postulate, the concept of noninvadability is used. The justification for this procedure is that interactions between strategies are microevents repeated sufficiently often against unchanging boundary conditions to ensure convergence. To what extent can such an argument be invoked to explain market dynamics? Routine rules such as the payback period investment criterion may become established historically through some such process, as is argued in Silverberg (1987). A shift between technological paradigms confronts us with an altogether different situation which is in some ways comparable to the lockin and standards phenomena discussed by Arthur (1985; 1988), David (1085), and Katz and Shapiro (1985; 1986). Because major innovations entailing a new endogenous skill regime occur infrequently, there can be no 'learning' process to ensure a convergence of strategies before the strategies have been irreversibly implemented. It is the diversity of positions adopted by firms that allows the potential superiority of a new technological regime to be developed and exploited. In that process losses and gains will almost invariably be made before routine procedures can reassert themselves. Rationality cannot be invoked to guarantee equilibrium because the system is not sufficiently transparent and there is no ex ante coordination mechanism. But if diversity may have inevitably negative consequences for some participants, at the system-level it is necessary to probe the development potential and trigger the collective development process. Lockin to an inferior technology of course is an associated danger. By the time another such decision arises, crucial parameters such as the dynamic appropriability will have almost certainly changed, so that

¹³ For a discussion of the specific features of evolutionary games in various contexts see Axelrod (1984), Hofbauer and Sigmund (1984), Thomas (1984), and Zeeman (1979 and 1981).

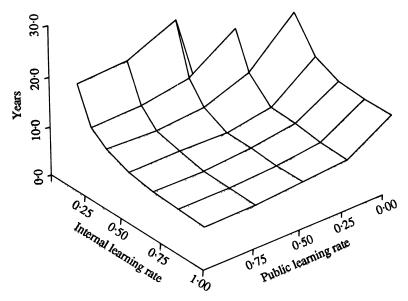


Fig. 9. Time to diffuse from 10-90%.

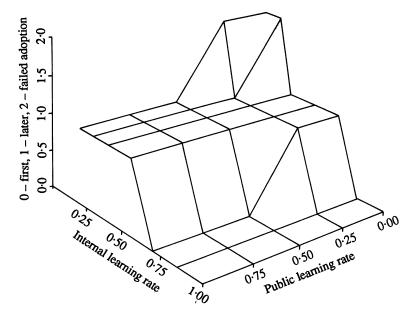


Fig. 10. Diffusion winner: first vs. later adopters.

the successful strategies of the last round may no longer be valid. Or they invalidate themselves because now they are being copied.

Figs. 9 and 10 and Tables 1 and 2 summarise the results of 35 runs conducted for the same distribution of anticipation bonuses as in Figs. 1–8 but for a range of values of the internal and public learning rates $(A_{15}$ and $A_4)$. Fig. 9 plots the time for the new technology to increase its share in total capacity from 10–90%

		Table	I			
Time for capacity	share of neu	technology to	diffuse from	10 to	90 % for	different
		of parameters				-

_			$A_{\underline{4}}$		•
A_{15}	0.1	0.322	0.52	0.775	1.00
0.5	fd	fd	29.51	25.23	21.20
0.3	fd	27'02	19.01	16.02	15.52
0.4	29.22	19.03	14.21	12.54	12.04
0.55	22.04	14.04	11.54	10.99	10.01
0.4	19.00	13.03	10.99	9.52	9.01
0.85	17.53	11.53	10.01	9.02	8.53
I.O	16.20	10.56	9.52	8.53	8.04

fd = failed diffusion.

Table 2
Asymptotic results of diffusion process in terms of relative market shares

_	A_4				
A_{15}	0,1	0.322	0.55	0.775	1.00
0.5	fd	fd	l	l	l
0.3	fd	l	l	l	l
0.4	l	ı	l	l	l
0.55	l	l.	l	l	l
0.4	l	l	f	f	f
0.85	\overline{f}	\overline{f}	f	f	f
1.0	f	f	f	f	f

fd = failed diffusion and relative decline of adopting firms, l = later adopters attain largest market shares, f = first adopters attain largest market shares.

and is a measure of the speed of diffusion. A very regular pattern emerges, with the speed of diffusion increasing, but at a declining rate, as a function of both parameters. A threshold can be discerned in the region of low learning rates. Below it only a few pioneer firms adopt the new technology, but are then driven off the market. Non-adopters manage to dominate the industry, the diffusion process reverses, and the technology ultimately disappears, even though it is potentially superior.

In Fig. 10 the bifurcations in the qualitative nature of the market reshuffle become visible. As we have seen, for low values of the learning parameters, non-adopters eventually increase their market shares at the expense of adopters (the peak of the cliff). For a middle range of values some (but of course not all) of the later adopters profit most in terms of markets share gains from introducing the new technology (the plateau). As we suspected, a threshold exists in the A_4-A_{15} plane beyond which first adopters emerge with the largest gains in market share (the foot of the hill). This threshold is primarily a function of the internal learning rate but, surprisingly, declines somewhat with higher public learning rates. Tables 1 and 2 summarise the original data.

V. CONCLUSIONS

In the first part of this paper we underscored the role of technological expectations, cumulativeness, internal and public knowledge, and strategic competition in any discussion of the dynamics of innovation-induced technological change. We then went on to formulate a model based on a number of behavioural assumptions on the one hand, and a structure of feedback loops on the other. This removed the question of diffusion from the largely static framework in which it has traditionally been placed and led to a dynamic coupling between the behaviour of individual agents and the environment in which they are operating. Although, as we have argued, a considerable range of microeconomic diversity and disequilibrium must remain an irreducible feature of such a system, the diffusion process itself shows a rather stable and invariant structure. Thus our simulations show that while some firms may be incurring short-term losses for long-term gains in market share, and others are driven onto a vicious spiral towards backruptcy, nevertheless the S-shaped form of the diffusion curve (which, however, is not necessarily a logistic and may even decline during the early phases) stands out. It is this superposition of microeconomic drama and system-level logic which makes the Schumpeterian entrepreneur a crucial element in the innovation process.

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