

The Dynamics of Generalized Market Exchange

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The problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess.

F. A. Hayek (1945), p. 519.

Abstract

This paper develops a dynamical price and quantity adjustment process for a fully decentralized market economy, and shows for the first time, using agent-based modeling techniques, that in the long-run, prices and quantities approximate their equilibrium states. The resulting dynamical system is also highly resilient in the face of exogenous shocks.

The key design element is that each trader has a set of *private prices* that are updated through experience, less successful agents copying the strategies of more successful agents, as well as varying private prices in response to personal trading success.

The efficiency of the market is economy is, however, low when there are more than a few types of goods. By permitting agents to buy and sell goods that they do not consume, we show that a money good emerges from market exchange, and the resulting monetary system is extremely efficient in comparison with non-monetary exchange. Moreover, when an object is available for trade that is neither produced nor consumed and has low exchange and storage costs, this object will emerge as *fiat* money when it is scarce and in fixed supply.

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1 Introduction

Adam Smith (2000 [1759]), heavily influenced by the French Physiocrats (Quesnay 1972[1758]), gave us the vision of a decentralized economy that leads to an efficient allocation of resources through the “invisible hand” of market competition. This vision was given explicit analytical formulation by Léon Walras in 1874, and a rigorous proof of existence of equilibrium for a simplified version of the Walrasian economy was provided by Wald (1951 [1936]). Soon after, Debreu (1952), Arrow and Debreu (1954), Arrow and Hahn (1971) and others provided a fairly complete analysis of the existence of equilibrium in decentralized market economies.

One might reasonably have thought at the time that the problem of dynamic stability of Walrasian economies would have been relatively quickly solved, but surveying the state of the art some quarter century after Arrow and Debreu’s contributions, Franklin Fisher (1983) concluded that virtually no progress had been made in this direction. It is now more than another quarter century since Fisher’s assessment, and his conclusion remains. Despite the centrality of the general equilibrium model to economic theory, we know nothing systematic about market dynamics.

In Walras’ original description of general equilibrium (Walras 1954 [1874]), market clearing was effected by a public “auctioneer” possessing knowledge of the excess demand curves of all agents. The auctioneer would call out a price vector for the economy, measure excess demand in each sector, and gradually, through a process of groping around the price space (the famous “tâtonnement”) all excess demand would be squeezed from the system and equilibrium attained. It turns out, however, that only under quite implausible assumptions can the continuous ‘auctioneer’ dynamic be shown to be stable (Fisher 1983, Kirman 1992), and in a discrete period model, even these assumptions do not preclude chaos in price movements as the generic case (Saari 1985, Bala and Majumdar 1992).

An acceptable model of market dynamics may be based on two fundamental facts about market competition. The first, well known in the literature, is that real trades are bilateral with separate budget equations that must be satisfied for each transaction (Starr 1972). The second is conceptually clear but rarely stated in the general equilibrium literature. This is the fact that in a decentralized market economy out of equilibrium, *there is no price vector for the economy at all*. The assumption that there is a system of prices that are common knowledge to all participants (we may call these *public prices*) is reasonable in equilibrium, because all agents can, at least in principle, observe the same prices. However, out of equilibrium there is no single set of prices determined by market exchange. Rather, every agent has a subjective prior concerning prices, based on personal experience, that he uses to make and carry out trading plans. This is the essence of Hayek’s famous “The Uses of Knowledge in Society,” (1945) exemplified by the head quote of this

paper.

Hayek's pellucid yet profound observation suggests the futility of assuming a public price adjustment process. The analysis of market disequilibrium must start out with a vector of *private* prices, one for each agent in the economy, that is updated through the exchange experience. The only admissible forms of experience in a decentralized market economy involve producing, consuming, trading, and observing the corresponding behavior of other agents. Recall of one's own trading experience and knowledge of the trading strategies of exchange partners alone can be the basis for updating the system of private prices, and equilibrium can be achieved only if plausible models of inference and updating lead to a convergence of private prices to a system of public prices through market exchange (Howitt and Clower 2000).

It follows from the above reasoning that in a pure market system, all expectations are adaptive. The "rational expectations" notion that agents know the global structure of the economy and use macroeconomic information to form expectations is not plausible in this context.

My first foray into implementing such a model (Gintis 2006) assumed a simple barter economy with fixed-proportions utility functions and production functions constructed so that any public price vector is a market-clearing equilibrium. Shifts in production activity from lower to higher-profit sectors then led to what I term *market quasi-equilibrium* in the long run. A market quasi-equilibrium is a stationary state of a Markov chain representing repeated cycles of production, trade and consumption. Learning occurs in this model by imitation: an agent A from time to time learns the price vector of another agent B, and if B has had more trading success, A adopts B's price vector, perhaps with some random mutation. In addition, producers adapt by raising prices when they sell out quickly and lowering prices when they experience inventory accumulation. Similarly, consumers raise their offer prices when they failed to make trades in previous periods, and lower their offer prices when they have had previous success.

The main fact flowing from an analysis of this system using agent-based modeling techniques was that a system of private prices, randomly assigned in period one quickly converges to a set of *quasi-public* prices, in the sense that the variance across agents of relative prices moves from very large to fairly small, and this variance has no further secular trend. In the long run the quasi-public prices stabilize at levels entailing market quasi-equilibrium.

Quasi-public prices are conceptually distinct from public prices even when the variance of relative prices across individuals is very small. This is because with truly public prices, a small change in a relative price leads all agents to adjust in a synchronized manner. This synchronization leads to the instability of public prices Gintis (2007).

My two previous papers on this topic have complementary weaknesses. The barter model (Gintis 2006) cannot illustrate the dynamics of price adjustment to equilibrium, because any public price system is a market equilibrium. The full Walrasian model (Gintis 2007) has so much institutional detail that it is difficult to separate basics from details and hence to infer the fundamental regularities of market dynamics in a fully decentralized system. The current paper overcomes these two weaknesses, thus facilitating future work towards a fully analytical model of market dynamics.

This paper endows agents with Cobb-Douglas rather than fixed proportion utility functions, the coefficients of which are drawn from a uniform distribution for each agent. The Cobb-Douglas assumption, together with simplifying assumptions concerning production described below, implies that there is a unique market equilibrium whose market-clearing prices can be calculated. This allows an unambiguous analysis of market dynamics, including the relationship between the long-run performance of the economy, which is the stationary distribution of the Markov process underlying the agent-based model, and the analytically-derived market equilibrium. I have also experimented with more general CES utility functions, finding no significant differences in the nature of market dynamics.

As in Gintis (2006, 2007), I find that over a wide range of parameter values (number of agents, number of goods, sector sizes, costs of production, transactions costs) the decentralized market system swiftly (within a few thousand trading periods) attains a set of quasi-public prices, but these are generally not market-clearing. In the same time period, the allocational efficiency of the economy improves considerably. In the stationary price distribution, approximated after several tens of thousands of periods, quasi-public prices closely approximate market clearing prices and the economy approximates the efficiency of market-clearing price equilibrium.

I then illustrate the value of the model by showing, for the first time, that money emerges dynamically from decentralized market exchange with fully endogenous out-of-equilibrium production, trade and price formation.

It is well-known that there is no role for money in the Walrasian general equilibrium mode because all adjustments of ownership are carried out simultaneously through the auspices of the auctioneer once the equilibrium prices are set. When there is actual exchange among individual agents in an economy without aggregated clearing mechanisms, two major conditions give rise to the demand for money, by which we mean a good that is accepted in exchange not for consumption or production, but rather for resale at a later date against other intrinsically desired goods. The first is the failure of the “double coincidence of wants,” (Jevons 1875), explored in recent years in this and other journals by Starr (1972) and Kiyotaki and Wright (1989,1991,1993). The second condition is the existence of transac-

tions costs in exchange, the money good being the lowest transactions-cost good (Foley 1970, Hahn 1971, Hahn 1973, Kurz 1974b, Kurz 1974a, Ostroy 1973, Ostroy and Starr 1974, Starrett 1974). We show that these conditions interact in giving rise to a monetary economy. When one traded good has very low transactions costs relative to other goods, this good may come to be widely accepted in trade even by agents who do not consume or produce it. Moreover, when an article that is neither produced nor consumed can be traded at very low transactions costs, this good, so-called *fiat* money, will emerge as a universal medium of exchange.

2 A Pure Market Economy

Our pure market economy, where agents buy a product only for consumption, has n sectors. Sector $k = 1, \dots, n$ produces good k in a “style” $s = 1, \dots, m$.¹ Each agent consumes a subset of his non-production goods, but only a single style of any good. In effect, then, there are nm distinct goods g_s^k , but only n production processes and correspondingly n prices, since goods g_s^k and g_t^k with styles s and t respectively, have the same production costs and price. We write $G = \{g_s^k | k = 1, \dots, n, s = 1, \dots, m\}$. We write $g = g_s^k$ when $g = g_s^k$ for some style s .

A producer of good g_s^k , termed a g_s^k -agent, produces with personal labor and no other inputs an amount $q_k(1 - v)$ of good g_s^k which depreciates to zero at the end of a trading period. The variable v represents the transactions cost of carrying a trade inventory over the previous period. In a non-monetary economy, only the production good is carried in inventory, but when individuals are permitted to acquire non-consumption goods, as in later sections of the paper, a trade inventory includes all goods that are not the agent’s consumption goods.

Each g_s^k -agent A is randomly assigned a set $H \subseteq G$, $g_s^k \notin H$ of consumption goods, at most one style of a given good, and is endowed with a utility function by drawing coefficients α_h , $h = 1, \dots, n$, $\alpha_h = 0$ if $h \notin H$ from a uniform distribution, and then normalizing the coefficients so their sum is unity. Agent A ’s utility function is thus of the form

$$u^A(x_1, \dots, x_n) = x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}, \quad (1)$$

where $\alpha_1 + \dots + \alpha_n = 1$ and $\alpha_g = 0$ if $g \notin H$.

At the start of the run of the Markov process, N agents are created and assigned as producers of a particular g_s^k good. Thus, in an economy with n goods in m styles, there are Nnm traders. Each of these traders is assigned a private price vector by choosing each price from a uniform distribution on $(0, 1)$.

¹We use styles to enrich the heterogeneity of goods without overly increasing the computational complexity of the model.

At the start of a trading period, given his private prices p_1^A, \dots, p_n^A , his inventory a_1^A, \dots, a_n^A of consumption goods (left over from the previous period, provided he failed to consume in , the previous period) and his stock $q_k(1 - v)$ of his production good, agent A maximizes utility by setting his demand for consumption good h to

$$x_h^{A*} = \frac{\alpha_h(p_k^A q_k(1 - v) + \sum_k p_k^A a_k^A)}{p_h^A} - a_h^A. \quad (2)$$

For each good $g_s^k \in G$ there is a market $m[k, s]$ of traders who sell good g_s^k . In each period, the traders in the economy are randomly ordered and are permitted one-by-one to engage in active trading. When the g_t^h -agent A is the current active trader, for each good g_t^h for which A has positive demand (i.e., $x_h^{A*} > 0$), A is assigned a random member $B \in m[h, t]$ who consumes g_s^k . A then offers B the maximum quantity y_k of g_s^k , subject to the constraints $y_k \leq \mathbf{i}_k^A$, where \mathbf{i}_k^A represents A's current inventory of good g_s^k , and $y_k \leq p_h^A x_h^A / p_k^A$, where x_h^A is A's current demand for g_t^h . This means that if A's offer is accepted, A will receive in value at least as much as he gives up, according to A's private prices. A then offers to exchange y_k for an amount $y_h = p_k^A y_k / p_h^A$ of good g_t^h ; i.e., he offers B an equivalent value of good g_t^h , the valuation being at A's prices. B accepts this offer provided the exchange is weakly profitable at B's private prices; i.e., provided $p_k^B y_k \geq p_h^B y_h$. However, B adjusts the amount of each good traded downward if necessary, while preserving their ratio, to ensure that what he receives does not exceed his demand, and what he gives is compatible with his inventory of g_t^h . If A fails to trade with this agent, he still might secure a trade giving him g_s^k , because $A \in m[k, s]$ may also be on the receiving-end of trade offers from g_t^h -agents at some point during the period. If a g_s^k -agent exhausts his supply of g_s^k , he leaves the market for the remainder of the period.

Several aspects of the trading process deserve comment. First, the assumption that each trading encounter is between agents each of whom produces a good that the other consumes is rather unrealistic. A more plausible assumption is that each g_s^k -producer A can locate the producers of his consumption goods, but that finding such a producer who also consumes g_s^k will require a separate search. We simply collapse these two stages, noting that when a second search is required and its outcome costly or subject to failure, the relative inefficiency of the non-monetary economy, by comparison with the monetary economies described below, is considerable magnified. However, while A's partner does consume g_s^k , he may have already fulfilled his demand for g_s^k for this period by the time A makes his offer, in which case no trade will take place. Second, the trade algorithm involves only one substantive design choice, that of allowing A to make a single take-it-or-

leave-it relative price offer, while obliging A to accept quantity terms that are set by B. I have experimented with alternatives, such as allowing B to make the take-it-or-leave-it offer, and choosing the mean of the two offers provided that each is acceptable to the other. These alternatives do not alter the market dynamics in observable ways.

After each trading period, agents consume their consumption inventories provided they have a positive amount of each consumption good, and agents replenish their inventories of their production goods. Moreover, each trader updates his private price vector on the basis of his trading experience over the period, raising the price of a consumption or production good by 0.02% if his inventory is empty (i.e., if he failed to purchase any of the consumption good or sell all of his production good), and lowering price by 0.02% otherwise (i.e., if he succeeded in obtaining his consumption good or sold all his production inventory). In general, it would be desirable to allow this adjustment strategy to evolve endogenously according to learning or imitation processes, but I have not investigated this possibility.

After every ten trading periods, the population of traders is updated using the following process. For each market $m[k, s]$ and for each g_s^k -trader A, let f^A be the accumulated utility of agent A since the last updating period (or since the beginning of the Markov process if this is the first updating period). Let f_* and f^* be the minimum and maximum, respectively, over f^A for all g_s^k -agents A. For each g_s^k -agent A, let $p^A = (f^A - f_*) / (f^* - f_*)$, so p^A is a probability for each A. If r agents are to be updated, we repeat the following process r times. First, choose an agent for reproducing as follows. Identify a random agent in $m[k, s]$ and choose this agent for reproduction with probability p^A . If A is not chosen, repeat the process until one agent is eventually chosen. Note that a relatively successful trader is more likely to be chosen to reproduce than an unsuccessful trader. Next, choose an agent B to copy A's private prices as follows. Identify a random agent B in $m[k, s]$ and choose this agent with probability $1 - p^B$. If B is not chosen, this process is repeated until B is chosen. Clearly, a less successful trader is likely to be chosen by this criterion. Repeat until an agent B is chosen. Finally, endow B with A's private price vector, except for each such price, with a small probability $\mu =$ randomly increase or decrease its value by a small percentage ϵ .

The resulting updating process is a discrete approximation of a monotonic dynamic in evolutionary game theory, and all monotonic dynamics have the same dynamical properties (Taylor and Jonker 1978, Samuelson and Zhang 1992). Other monotonic approximations, including the simplest, which is repeatedly to choose a pair of agents in $m[k, s]$ and let the lower-scoring agent copy the higher-scoring agent, produce similar dynamical results.

There is nothing intrinsically desirable about using utility as the fitness criterion. Because utility functions are heterogeneous and individuals who prefer goods

with low prices do better than agents who prefer high-priced goods independent of the trading prowess, there is significant noise in the imitation dynamic. I have used other criteria, including frequency and/or volume of trading success, with results similar to those reported herein.

The result of the dynamic specified by the above conditions is the change over time in the distribution of private prices. The general result is that the system of private prices, which at the outset are randomly generated, in rather short time evolves to a set of *quasi-public* prices with very low inter-agent variance. In this same time frame, the economy attains a fairly high level of efficiency, even though the quasi-public prices remain unequal to their market-clearing values. Over the long term, these quasi-public prices move toward their equilibrium, market-clearing levels, and near constrained Pareto-optimal allocational efficiency is achieved (constrained, that is, by the informational asymmetries of the economy). In effect, the quasi-public prices represent a set of common conventions for trade, and they are sufficiently close to their market-clearing values that price changes after the establishment of quasi-public prices and the attainment of the associated quasi-equilibrium, have relatively minor effects of allocative efficiency.

3 A Baseline Example

I will illustrate this dynamic with a particular agent-based simulation of the above model. The model assume $n = 6$, $m = 5$, and $N = 300$, so there are 30 distinct goods which we write as g_1^1, \dots, g_5^6 , and 9000 traders in the economy. There are thus six distinct prices p_1^A, \dots, p_6^A for each agent A . We treat g^6 as the numeraire good for each trader, so $p_6^A = 1$ for all traders A . A unit of a g^k -agent's labor produces $q_k = k$ units of g^k , so the unit labor cost of g^k is $1/k$. We assume that there are equal numbers of producers of each good, so the economy produces goods g^k and g^h in proportion k/h . Agent utility functions are Cobb-Douglas as described above, where H has three elements (i.e., a trader consumes three of his five non-production goods). We can calculate equilibrium prices rather easily with Cobb-Douglas utility functions, using the fact that an individual with utility function $u(x_1, \dots, x_n) = x_1^{\alpha_1} \dots x_n^{\alpha_n}$ spends a fraction α_i of income on good x_i . Because the average α_i for the economy is $1/3$ for each good, we calculate that the approximate equilibrium price of g^k is $6/(7 - k)$. This result is accurate to within a fraction of 1%, as can be ascertained by actual measurement during a simulation. Population updating occurs every ten periods, and the number of encounters per sector is 10% of the number of agents in the sector. The mutation rate is $\mu = 0.01$ and the error correction is $\epsilon = 0.01$.

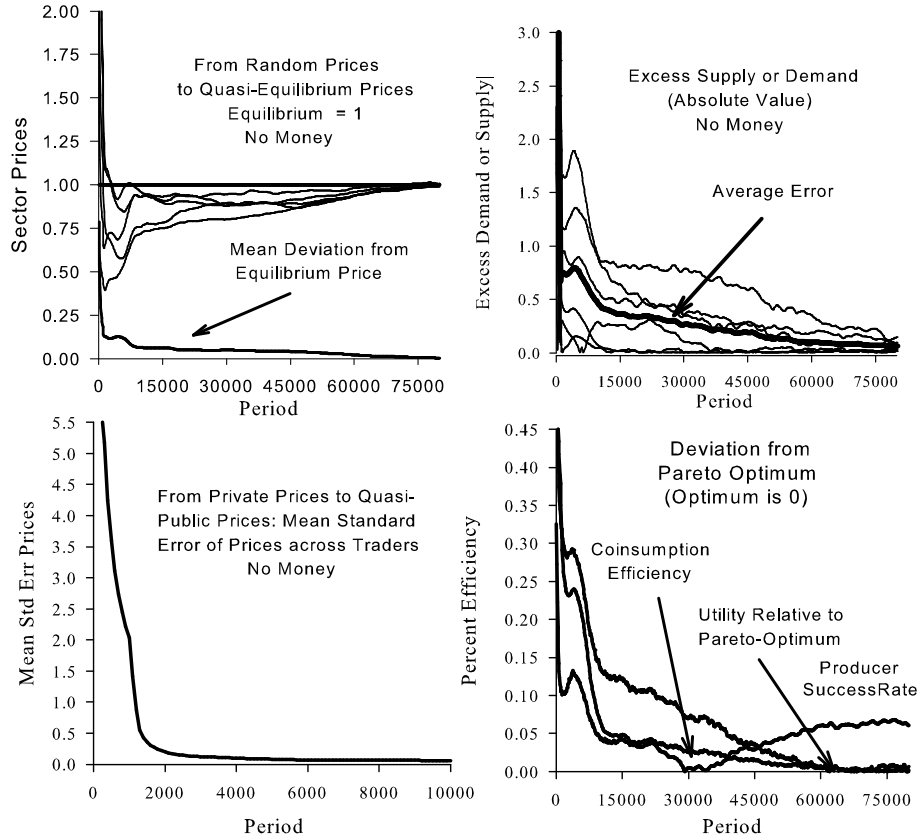


Figure 1: The Market Economy

The results of a typical run of this model is illustrated in Figure 1. The south-west pane of the figure shows the passage from private to quasi-public prices over the first 10,000 trading periods. The mean standard error of prices is computed as follows. For each good g we measure the standard deviation of the price of g across all g -agents, where for each agent, the price of the numeraire good g_6 is set to unity. Figure 1 shows the average of the standard deviations divided by the means for all six goods. The passage from private to quasi-public prices is quite dramatic, the standard error of prices across individuals falling by an order of magnitude within 1500 periods, and falling another order of magnitude over the next 8500 periods. The final value of this standard error is 0.0223, as compared with its initial value of 6.3992.

The distinction between low-variance private prices and true public prices is significant, even when the standard error of prices across agents is extremely small, because stochastic events such as technical changes propagate very slowly when prices are highly correlated private prices, but very rapidly when all agents react in parallel to price movement. In effect, with private prices, a large part of the reaction to a shock is a temporary reduction in the correlation among prices, a reaction that is impossible with public prices, as the latter are always perfectly correlated.

The northwest pane in Figure 1 shows the passage of average quasi-public prices for the six goods, normalized as above, and divided by the calculated market-clearing equilibrium prices of the goods, which are $1/6$, $1/3$, $1/2$, $2/3$, $5/6$, and 1 , so that market clearing occurs when each value is unity. As can be seen, the six prices are within one percent of their market equilibrium values after 80,000 periods. This shows dramatically the snail pace of convergence of quasi-public prices to market equilibrium. If we think of a trading period as a day, quasi-public prices are fully formed after three years, but market equilibrium prices emerges only after more than two hundred years. The weakness of the forces leading prices towards equilibrium is perhaps surprising, and is quite a contrast with the rapid convergence of prices towards market clearing in a full Walrasian model with firms, as well as product, labor, and financial markets(Gintis 2007).

The reason for the slow convergence of prices is quite clear. In a non-monetary economy with many goods, most agents spend most of their time attempting to find a trading match. It is impossible to tell whether success and failure are related to the agent's private prices vector, or to simple luck or the lack thereof. In our model, there are separate markets for each good of each quality, so a trader can direct his efforts to others whom he knows have a good he consumes and who consume his production good. But he does not know whether that agent is still in the market of his production good. Stochasticity is thus inherent in the market economy.

The northeast pane in Figure 1 shows the absolute value of excess supply in the six sectors. This pane shows that the dynamics of trade dramatically lower excess demand, falling from an initial value of 16.76 to a final value of 0.0598.

The southwest pane in Figure 1 shows several measures of the efficiency of the economy. All measures exhibit the same quite unexpected phenomenon: each efficiency measure is within 80% of its equilibrium value after about 5000 periods, but comes within 95% of its equilibrium value only after some 45,000 periods. The three efficiency measures are as follows. The consumption efficiency curve shows the average amount of his desired consumption the consumer consumes when he does have a positive amount of all his consumption goods. The producer efficiency curve shows the fraction of an agent's stock of his production good he sells in each period. Finally, the relative utility curve shows an average agent's mean utility as a fraction of utility in full market-clearing equilibrium. To estimate the equilibrium

value of these statistics, I first run the economy forcing each agent to start out with market equilibrium prices. Within a few periods, I measure each of the three statistics. The southeast pane of Figure 1 is the absolute value of the difference between the current statistic and its estimated equilibrium value.

There is nothing special about the parameters used in the above simulation. Of course adding more goods or styles increases the length of time until quasi-public prices become established, as well as the length of time until market quasi-equilibrium is attained. Increasing the number of agents speeds up both of these time intervals.

4 Learning, Imitation, and Market Dynamics

The above simulation assumed change in the distribution of private prices are the twin results of individual adjustment of prices in the face of trading experience and global updating using an imitation mechanism approximating the replicator dynamic of evolutionary game theory. It is useful to separate out the contribution of each of these mechanisms, as well as to estimate the strength of their interaction. Figure 2 show the effect of dropping the imitation/updating process, thus relying on individual learning and price adjustment alone.

These results are truly dramatic and unexpected, although they appear in every run of the model under conditions of pure individual adjustment. The north-west pane of Figure 2 show that there is some initial movement towards equilibrium prices, but after 40,000 periods prices begin to move systematically against market-clearing levels. The southwest pane shows that the movement to quasi-public prices is strong and unambiguous, but the rate of convergence is at least ten times slower than the benchmark in Figure 1. In the long run, the mean standard error of prices is about 0.4, which is two orders of magnitude greater than the benchmark. Finally, the right-hand panes of the figure show that the model moves strongly away from efficiency after 40,000 periods, and individual markets move away from rather than towards equilibrium.

Market dynamics with the imitation/mutation mechanism present but individual adjustment suppressed is depicted in Figure 3. As might be expected, market dynamics in this case market dynamics is much more favorable to allocational efficiency, as illustrated in the southeast pane of the figure. On the other hand, the

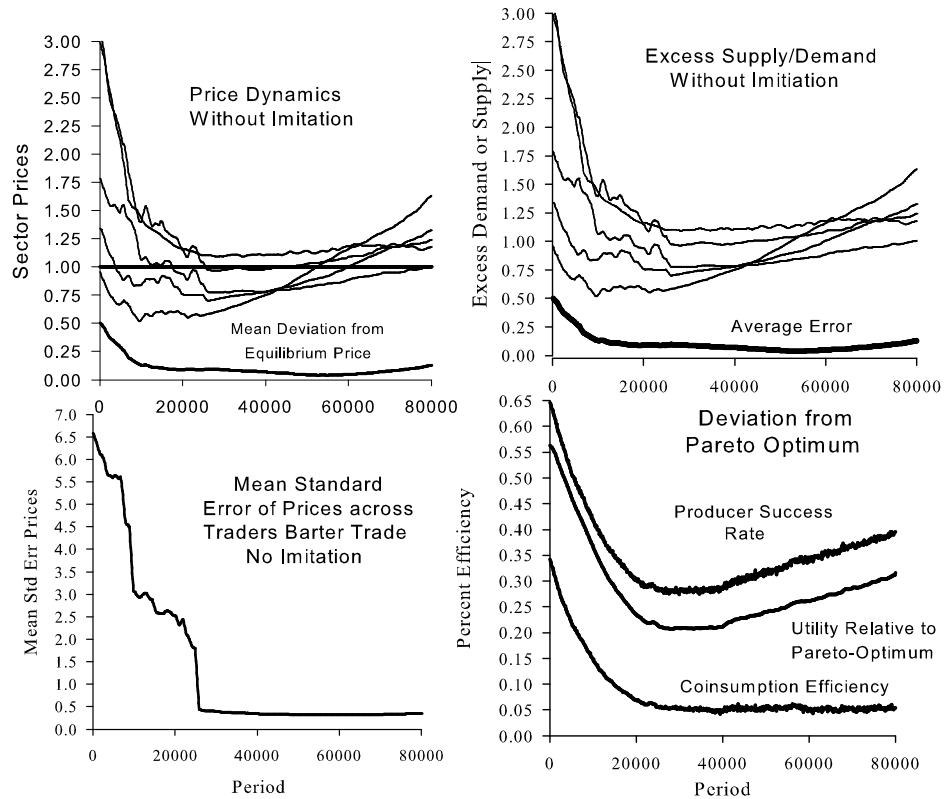


Figure 2: The Dynamics of the Market Economy when the Imitation Mechanism is Suppressed

rapid movement to quasi-public prices, shown in the southwest pane, is not followed by a further reduction in the standard error of private prices, and in the long run, this standard error is about 0.67, as compared to 0.022 in the baseline case. The movement of prices towards equilibrium is also extremely weak, the mean deviation from equilibrium being 0.18 in the no learning case, as compare with 0.001 in the baseline case.

The straightforward conclusion is that imitation and mutation alone are sufficient to bring the market economy to a point where allocation is quite efficient, but there is little tendency to approximate market-clearing prices even in the very long run. Private price adjustment strongly reinforces the capacity of imitation to bring about market clearing.

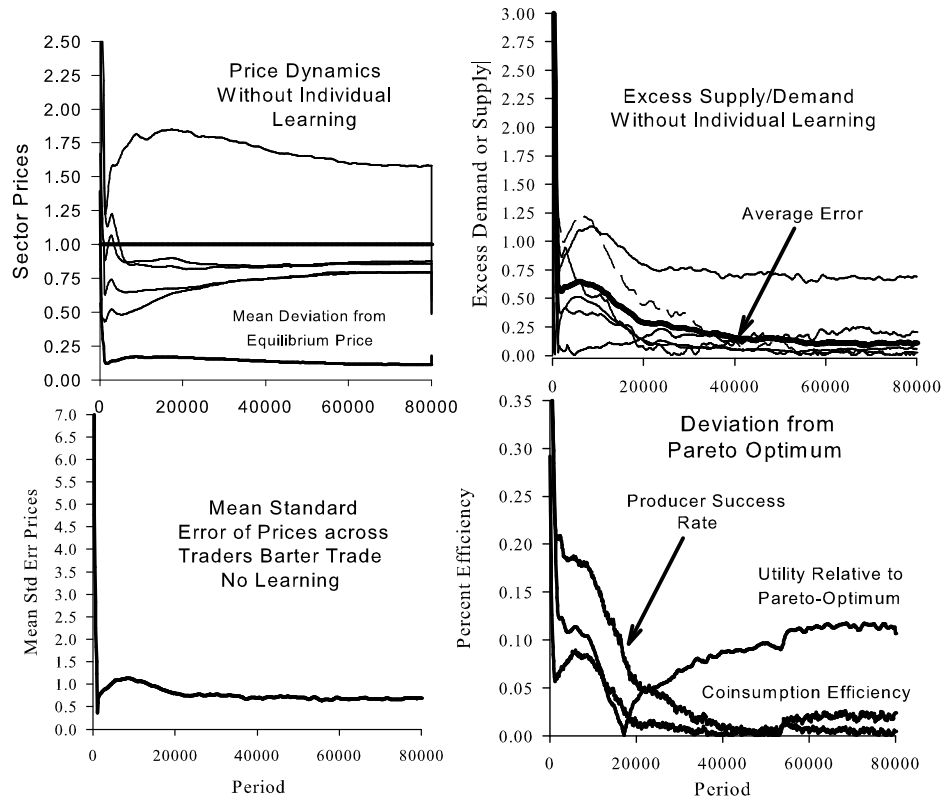


Figure 3: The Dynamics of the Market Economy when Individual Learning and Adjustment is Suppressed

5 The Emergence of Money

We now permit traders to buy and sell at will any good that they neither consumer nor produce. We call such a good a *money good*, and if there is a high frequency of trade in one or more money goods, we say the market economy is a *money economy*. We assume that traders accept all styles of a money good indifferently. We first investigate the emergence of money from market exchange by assuming zero inventory costs, so the sole value of money is to facilitate trade between agents even though the direct exchange of consumption and production goods between a pair of agents might fail because one of the parties is not currently interested in buying the other's production good. The trade algorithm in case agents accept a

good that they do not consume is as follows. At the beginning of each period, each agent calculates how much of each consumption good he want to acquire during that period, as follows. The agent calculate the market value of his inventory of production and money goods he holds in inventory, valued at his private prices. This total is the agent's income constraint. The agent then choose an amount of each consumption good to purchase by maximizing utility subject to this income constraint. The trade algorithm is similar to the case of pure market, except that either party to a trade may choose to offer and/or accept a money good in the place of his production good.

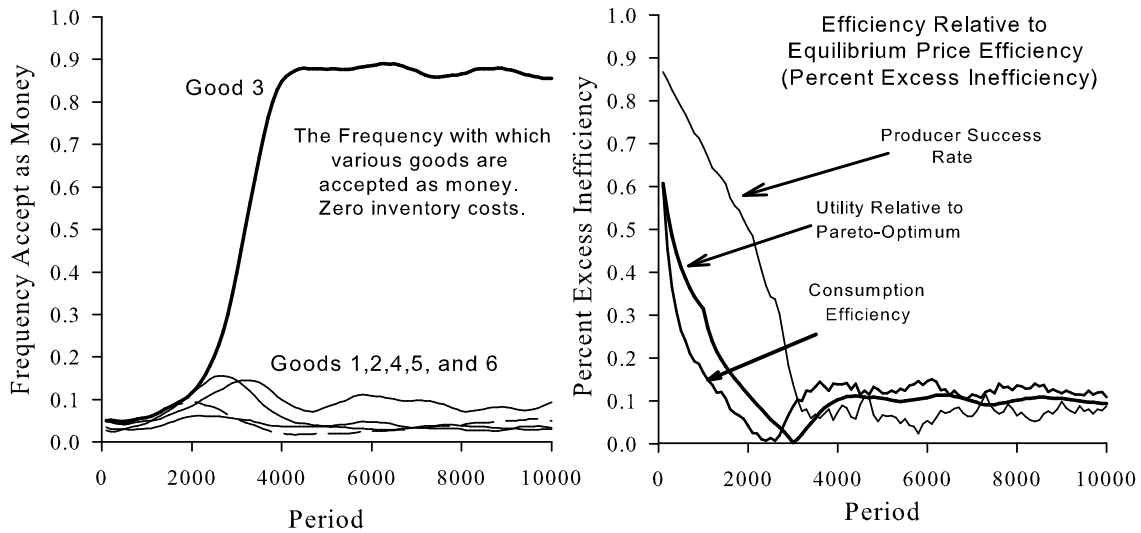


Figure 4: The Emergence of Money in a Market Economy. The parameters of the model are the same as in the baseline case treated previously. Inventory costs are assume absent.

We simulate this economy using the same parameters as in our previous simulations, including zero inventory costs. The left-hand pane of Figure 4 shows that the use of money increases monotonically over the first 2000 periods, spread almost equally among the six goods. The right-hand pane shows that efficiency increased according to all of the various efficiency measures over this period. From period 2000 to 4000, one good becomes a virtually universal currency, driving the

use of the others to low levels. It is purely random which good becomes the universal medium of exchange, but one does invariably emerge as such after several thousand periods. If we add inventory costs with g^1 being lower cost than the others, g^1 invariably emerges as the medium of exchange after 1000 periods, and the other goods are not used as money at all. I did not include graphs of the passage to quasi-public prices or other aspects of market dynamics because they differ little from the baseline economy described above.

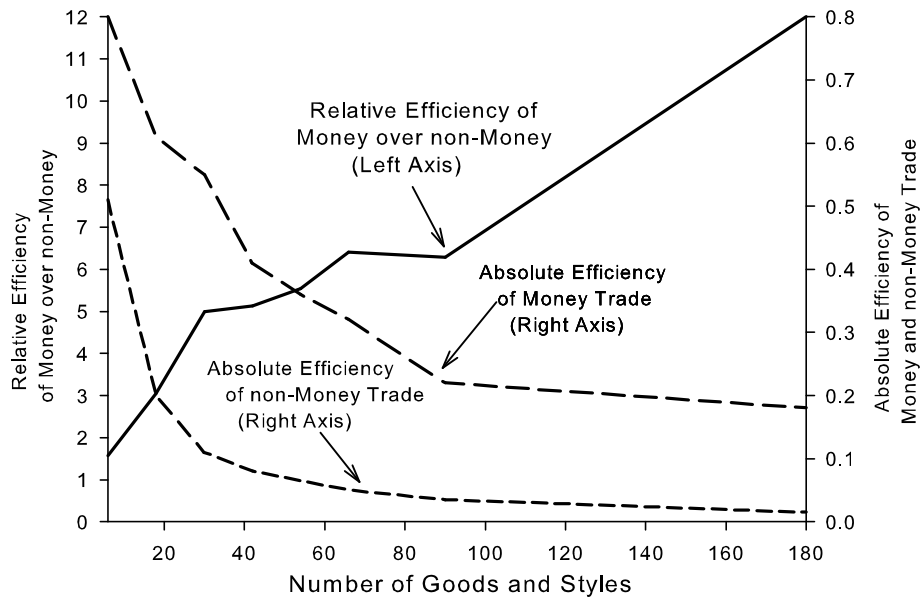


Figure 5: The Relative Efficiency of Money in a Market Economy

As in traditional monetary theory (Menger 1892, Wicksell 1911, Kiyotaki and Wright 1989,1991) money emerges from goods trade both because it is a low transactions cost good and it solves the problem of the “double coincidence of wants” that is required for market exchange (Jevons 1875). The relative efficiency of money over direct goods trade increases with the number of goods, as illustrated in Figure 5. While with six goods and one style the relative efficiency of money is only 150%, for nine goods and twenty styles (180 goods), the relative efficiency is 1200%.

6 The Robustness of the Decentralized Market Economy

Among the more interesting facts gleaned from an agent-based model of the dynamics of the Walrasian economy is its extreme fragility in the face of exogenous shocks (Gintis 2007). This fragility does not undermine the stability of equilibrium, but it leads to frequent large excursions from the mean ergodic values indicated by the stationary distribution of the economy. The market economy with or without money exhibits no such fragility.

I shall illustrate this behavior in the context of the same baseline economy except we now admit of a single new good, *fiat* money, that is neither produced nor consumed, and enjoys zero inventory storage costs. When such a good is available, it quickly becomes a universal medium of exchange for the economy, accepted by almost 100% of market traders. The nature of market dynamics in this context is not visibly different from the baseline case of non-monetary exchange, and the efficiency of the economy is heightened in the same manner as depicted in Figure 5. Suppose, however, that every 5000 periods, we impose a shock on the economy consisting of a reduction in the *fiat* money holdings of each trader to 20% of its current level. The reduced holding are maintained for 100 periods, after which the money holdings of each trader is multiplied by five, restoring the money stock for the economy to its initial level. The southwest pane in Figure 6 shows the course of the money supply. The southeast pane of the figure shows that relative efficiency suffers during the shock period, but is perfectly restored upon the resumption of the normal money supply. The northeast and northwest panes show that prices and excess demand are virtually unaffected by the sequence of 100 period shocks.

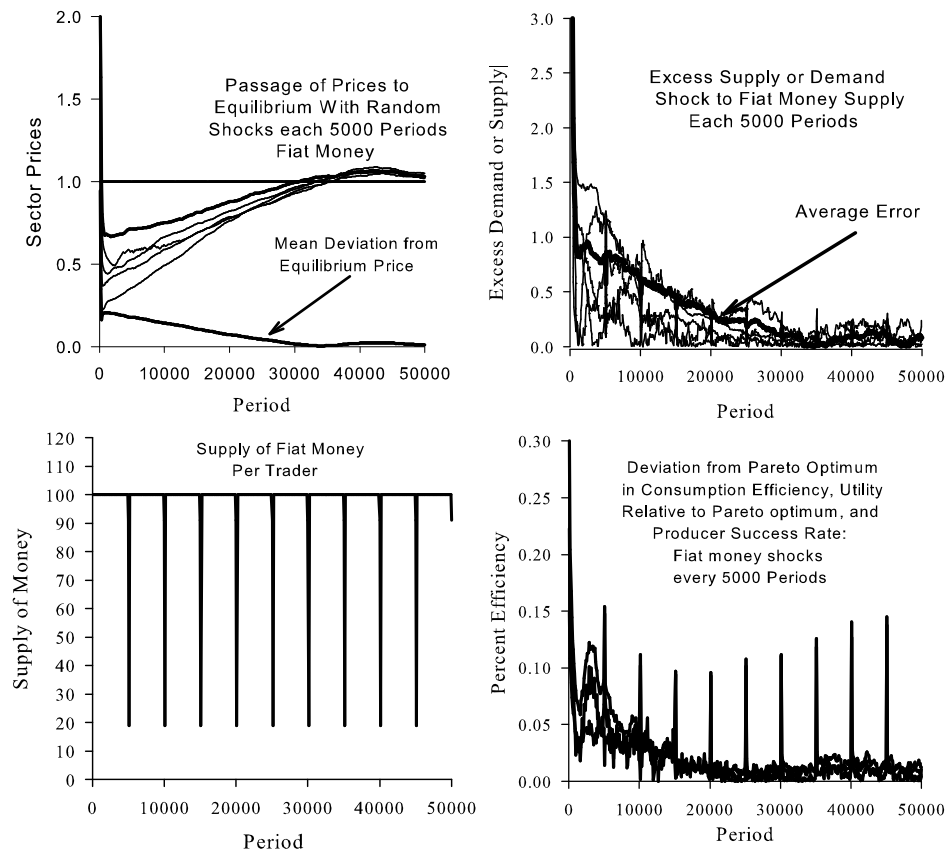


Figure 6: The Resilience of a Market Economy with *Fiat* Money

7 Conclusion

Attempting to explain market dynamics assuming public prices has been a fruitless enterprise. This is hardly surprising, because out of equilibrium there is no reasonable sense in which public prices exist in a market economy. By modeling market exchange assuming each agent has a set of *private prices* that is updated through learning and imitation, we have seen that market economies behave in extremely stable and robust ways. Such economies are nevertheless complex dynamical systems that to date cannot be properly modeled using standard analytical techniques, but reveal their basic properties through agent-based models, which treat the system of prices and quantities as a finite Markov chain with a stationary distribution whose properties we can analyze computationally.

What are these properties? First, starting from a state of pure randomness, under the twin influence of learning and imitation, private prices rapidly converge to *quasi-public prices*, which have the property of differing across individuals, but with a very small standard error. Quasi-public prices have much of the virtue of public prices in that they support a relatively high level of economic efficiency, while at the same time acting as shock absorbers in the face of random exogenous perturbations to the economy.

Second, quasi-public prices adjust to their equilibrium levels in the long run, leading to a *quasi-market equilibrium* with approximately constrained Pareto-optimal allocations, at least in the simple case of market exchange in which each agent produces a single good using only personal labor.

Third, individual learning is insufficient to produce market equilibrium. The imitation process is much more powerful than individual learning, but the two in combination are quite powerful even in the case of many goods. The economist's faith in the general equilibrium model is in this sense completely vindicated.

Finally, the resulting economy is extremely robust in the face of exogenous shocks. This contrasts with the extreme fragility of the full Walrasian economy analyzed in Gintis (2007). Apparently there is some point in the passage from simple market to a full Walrasian system at which fragility enters. Identifying this point through agent-based modeling would be a valuable enterprise.

Given the clarity, uniformity, and simplicity of our findings for a market economy, it is perhaps surprising that an analytical model exhibiting the above three characteristics remains to be developed. This should prove a worthy task for contemporary mathematical economists, and might serve as a prolegomena to the analytical dynamics of a full Walrasian economy.

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