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REGULAR ARTICLE

Evolutionary models in economics: a survey of methods and building blocks

Karolina Safarzyńska · Jeroen C. J. M. van den Bergh

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Abstract This paper assesses methods and components of formal evolutionary-economic modelling. Methods are broadly classified into evolutionary game theory and selection dynamics, evolutionary computation and multi-agent models, each with relevant subcategories. The components or building blocks are organized into diversity, innovation, selection, bounded rationality, diffusion, path dependency and lock-in, coevolution, multilevel and group selection, and mechanisms of growth. The number of alternatives

K. Safarzyńska (✉)
Institute for Environmental Studies, Vrije Universiteit,
De Boelelaan 1087, 1081 HV Amsterdam, The Netherlands
e-mail: karolina.safarzynska@ivm.vu.nl

J. C. J. M. van den Bergh
ICREA, Barcelona, Spain

J. C. J. M. van den Bergh
Institute for Environmental Science and Technology,
Department of Economics and Economic History,
Universitat Autonòma de Barcelona,
Edifici Cn—Campus UAB, 08193 Bellaterra, Spain
e-mail: jeroen.bergh@uab.es

J. C. J. M. van den Bergh
Faculty of Economics and Business Administration,
and Institute for Environmental Studies, Vrije Universiteit,
Amsterdam, The Netherlands

J. C. J. M. van den Bergh
Tinbergen Institute, Rotterdam, The Netherlands

J. C. J. M. van den Bergh
NAKE, Tilburg, The Netherlands

that has been proposed for each category is vast, making it difficult to comprehend the variety of assumptions and formalizations underlying existing evolutionary-economic models. Our survey aims to clarify for each model component the choice range, formal expressions, associated assumptions, and possible techniques for formalization. Our study is unique in that it provides more information about the formal details of specific model components and is considerably more inclusive than earlier reviews.

Keywords Agent based models · Diversity · Evolutionary computation · Evolutionary game theory · Growth mechanisms · Innovation · Multilevel evolution · Neo-Schumpeterian models · Selection dynamics

JEL Classification B52 · C60 · C73

1 Introduction

Many studies in evolutionary economics employ formal models of one type or another. They are characterized by a variety of approaches and elements. Perhaps this is a logical consequence of evolutionary economics allowing for many different assumptions regarding individual behavior, selection mechanisms and innovation processes. The current paper presents an overview of methods and components underlying formal economic models employing evolutionary approaches. This can assist inexperienced, and possibly also experienced, researchers in understanding the available variety of model components, their possible formalizations, and the importance of model design for deriving specific results. Some basic knowledge of evolutionary models may be relevant as well to evolutionary economists not working with models themselves, since models are intended to capture the essence of evolutionary processes and thus reflect the range of views and assumptions explored. Our survey is unique in three respects: it provides a comprehensive overview, it contains explicit representations of important formalizations of model components, and it clarifies the connection between components and general methods of modelling. Existing surveys are either old, considerably less exhaustive than ours, focus on one type of modelling (e.g., growth), or do not show many formal expressions of model components (Silverberg 1988, 1997; Saviotti and Metcalfe 1991; Andersen 1994; Kwasnicki 2001, 2007; van den Bergh 2004; Windrum 2004; Silverberg and Verspagen 2005a; Fagiolo et al. 2007).

In this survey, we consider as core building blocks: diversity, innovation and selection; and as additional components: bounded rationality, diffusion, path dependence and lock-in, coevolution, multilevel and group selection, and growth mechanisms. The relevance and potentially complementary role of these various components is supported by many studies in evolutionary economics, including a number of appreciative surveys (e.g., Hodgson 1993;

Nelson 1995; Dopfer 2005; Silva 2009; Witt 2008). We wish to clarify the variety of conceptualizations for each component, as well as determine which ones can be well addressed by particular methods. The combination of modelling methods and building blocks is important, as the particular method determines the specific assumptions as well as the possible hypotheses and questions that can be studied. Thus far, different evolutionary methods as applied to model economic phenomena have not been systematically compared. Nowak (2006) and Dercole and Rinaldi (2008) offer comparisons of methods and techniques but focus on applications in theoretical biology.

It is possible to identify distinct developments within evolutionary modelling, namely evolutionary game theory and selection dynamics (Friedman 1991; Weibull 1995; Samuelson 1997; Fudenberg and Levine 1997; Gintis 2000), evolutionary computation techniques (Fogel 2000; Eiben and Smith 2003), and multi-agent modelling (Weiss 1999; Wooldridge 2002; Tesfatsion and Judd 2006). They use various mathematical techniques, namely difference or differential equations, stochastic processes, graphs and evolutionary algorithms. Multi-agent models and evolutionary computation (genetic algorithms, learning classifier systems and genetic programming) are overlapping sets of techniques, which may create some confusion. Particularly, when agents do not learn or do not show changing behavior over time but a population of individuals evolves due to the processes of selective replication and stochastic variation, the distinction becomes diffuse. Generally, evolutionary computation offers techniques suitable for studying adaptive learning, search and optimization processes, which can be employed in multi-agent systems to model adaptive learning of the agents.

The organization of the reminder of the paper is as follows. Section 2 briefly discusses and compares the main evolutionary modelling approaches. Section 3 examines the various formalizations of the components of evolutionary-economic models, and relates these to the modelling approaches. Section 4 presents conclusions.

2 Evolutionary modelling techniques

2.1 Evolutionary game theory and selection dynamics

Evolutionary game theory originates from the work of Maynard Smith and Price (1973). It studies the strategic interactions of boundedly rational players. Individuals are drawn randomly from large populations and have little or no information about the game (Weibull 1998). A central concept in evolutionary game theory is the evolutionarily stable strategy, which denotes that strategies in an equilibrium are resistant to invading ‘mutant’ strategies.

To study dynamic paths to reach equilibrium underlying evolutionary games, various dynamic equations have been proposed, referred to as population or selection dynamics (Hofbauer and Sigmund 1998). A number of modelling alternatives can be distinguished, depending on whether time and payoffs are discrete or continuous, and whether population size and the number of strategies are finite or infinite. Dynamics underlying games can further involve deterministic or stochastic processes. In general, selection dynamics studies how the distribution of pure strategies changes over time. The distribution of such strategies defines a population state, which is mathematically equivalent to a notion of mixed strategy in the evolutionary game (Weibull 1998). Among deterministic selection dynamics, replicator dynamics predominates (Samuelson 1997), according to which frequencies of strategies in the population change over time according to their (relative) payoffs. Payoffs depend on the strategies of other players, and thus on the frequencies of these strategies within a population, which creates a feedback loop mechanism. Replicator dynamics focuses on selection processes and omits structural innovations, so that in effect it can be regarded as an incomplete representation of evolution.

Replicator dynamics describes one of many possible transmission mechanisms. Hofbauer and Sigmund (1998, 2003) suggest other deterministic selection dynamics, such as best response, Brown–von Neumann–Nash, imitation, and mutator dynamics (see also Nowak and Sigmund 2004). Best response dynamics requires agents to recognize a best reply to the mean population strategy, which in many situations may be beyond agents' computational and cognitive capabilities. Imitation of a rival's strategy in pairwise comparisons offers more realistic accounts for modelling social interactions, which can be captured with imitation dynamics. Among deterministic selection dynamics, only selection–mutation dynamics accounts for errors occurring during the replication process. Mutation can be interpreted here as agents switching between strategies already present in the population.

In general, deterministic selection dynamics are most relevant for studying dynamics over moderate time spans. To study interactions in the long-run, stochastic dynamics are more suitable (Sandholm 2007). Unlike in deterministic approaches, in stochastic models mutations are captured by random variables constantly perturbing model dynamics. Here, examining the impact of random mutations allows determining the stability of an equilibrium (Foster and Young 1990). A unique stationary distribution of strategies is found in the limit distribution as the mutation rate goes to zero, referred to as a stochastically stable strategy.

Alternatively, stochastic processes can be described in more detail at the individual level, involving birth, death and migration processes. This can be best expressed with Markov processes. Here, agents decide each period over which strategy to choose and occasionally employ a decision at random. The Markov process determines changes in individual states and consequently in the relative abundances of different strategies over time. Other stochastic processes describing changes in individual states, discussed here, are the Master equation and the Polya urn process.

2.1.1 Deterministic dynamics

1. Replicator dynamics

Replicator dynamics was first formalized by Taylor and Jonker (1978). It applies to any population divided into types E_1 to E_n, with corresponding frequencies x_1 to $x_n(\sum_i x_i = 1)$. According to the replicator model, individuals meet each other in random encounters. Whenever an individual of i-type meets individual of j–type, the payoff to i is a_{ij}. The motion for the frequency of type i is governed by (Hofbauer and Sigmund 1998):

$$\dot{x}_i = x_i \left((Ax)_i - x^T Ax\right).$$

where $(Ax)_i$ is the expected payoff for an individual of type i given by an $n \times n$ payoff matrix $A = (a_{ij})$, and $x^T Ax$ is the average payoff. The frequency of type i increases in the population if its payoff exceeds the average payoff in the population.

In the context of games with interactions occurring in groups with more than two members, fitness may be expressed as a nonlinear function of the frequencies (Nowak and Sigmund 2004). Replicator dynamics takes then the following generalized form:

$$\dot{x}_i = x_i \left(f_i(x) - \bar{f}(x)\right)$$

where $f_i(x)$ is a fitness function and $\bar{f}(x) = \sum_i x_i f_i(x)$ is the average fitness.

2. Best response dynamics

Best response dynamics may be applied to model myopic behavior of rational agents. It is derived under the assumption that, in large populations, a small fraction of individuals revise their strategies and choose the best reply to the population mean strategy x:

$$\dot{x} = \beta(x) - x$$

where $\beta(x)$ denotes the set of best replies b to strategy x such that $z^T Ax \leq b^T Ax$ for any $z, x, b \in S^n$. The best reply does not have to be unique.

3. Smoothed best replies

Best reply dynamics can be approximated by smooth dynamics such as the logit dynamics (in order to ensure a unique solution) for $\varepsilon > 0$:

$$\dot{x}_i = \frac{e^{a_i(x)/\varepsilon}}{\sum_j e^{a_j(x)/\varepsilon}} - x_i$$

for $\varepsilon \to 0$, this converges to best response dynamics.

4. Brown–von Neumann–Nash dynamics

The Brown–von Neumann–Nash dynamics is defined as:

$$\dot{x}_i = k_i(x) - x_i \sum_j k_j(x)$$

where $k_i(x) = max(0, a_i(x) - x^T a(x))$ denotes the positive part of excess payoff for strategy i. This equation ensures that, if there exists a strategy j with the excess payoff higher than i's, the frequency of strategy i will decrease in a population. The equation describes population dynamics resulting from myopic adjustment which, unlike replicator dynamics, allows for innovation but assumes less rationality than best response dynamics (Hofbauer et al. 2009).

5. Imitation dynamics

The frequency of certain strategies can increase in a population through imitation. Imitation dynamics is derived under the assumption that an individual selects randomly another player in the population and decides whether to adopt his strategy. It takes the form:

$$\dot{x}_i = x_i \sum_j \left[f_{ij}(x) - f_{ji}(x) \right] x_j$$

where f_{ij} is the rate at which a player of type j adopts type's i strategy.

The simplest rule, proposed by Hofbauer and Sigmund (2003), is 'imitate the better'. In this case, the rate depends only on the payoffs achieved by the two players:

$$\begin{aligned} f_{ij}(x) = f\left(a_i(x), a_j(x)\right) &= 0 \text{ for } a_i(x) < a_j(x) \\ &= 1 \text{ otherwise} \end{aligned}$$

The frequency of strategy i increases if i's payoff exceeds j's (the term $\left[f_{ij}(x) = f_{ji}(x) \right]$ is in this case equal to 1). Alternatively, the switching rate may depend on the payoff difference, i.e. $f_{ij}(x) = f\left(a_i(x), a_j(x)\right) = \varphi\left[a_i(x) - a_j(x)\right]$ with φ a monotonically increasing function. The dynamic process then becomes:

$$\dot{x}_i = x_i \sum_i \psi\left[a_i(x) - a_j(x)\right] x_j$$

where $\psi(.)$ is an increasing and odd function $-\psi(x) = \psi(-x)$, (i.e. the graph of an odd function has 180° rotational symmetry with respect to the origin). The equation may be interpreted as players imitating strategies of other agents with a probability proportional to the expected gain from switching.

6. Selection–mutation dynamics

The selection mechanisms discussed under 1–5 describe selection without any drift or mutation. To allow for errors to occur during the process, models combining selection and mutation can be employed, such as mutator and replicator–mutator dynamics. According to mutator dynamics, the processes of replication and mutation are sequential (Helbing 1995; Brenner 1998):

$$\dot{x}_i = x_i\left((Ax_i) - x^T Ax\right) + \sum_j \left[x_j q_{ji} - x_i q_{ij}\right]$$

where q_{ij} is a mutation probability from strategy i to j, and q_{ji} from j to i. The first term on the right-hand side depicts replicator dynamics, and the second term describes the flow towards and away from strategy x_i.

In population genetics, biochemistry, and models of language learning, the replicator–mutator equation is used: $\dot{x}_i = \sum_j x_j f_j(x)\, q_{ij} - \bar{f}(x)\, x_i$ (Bürger 1998; Komarowa 2004; Nowak and Sigmund 2004). Here, mutation occurs during the replication process. The mutation matrix $Q = [q_{ij}]$ is a stochastic matrix, where each entry is a probability that replication of i will result in j, with $\Sigma_j q_{ij} = 1$. The replicator–mutator contains both replicator dynamics and quasi-species equations as special cases. If the matrix Q is an identity matrix, the equation reduces to replicator dynamics (perfect learning). The quasi-species equation describes deterministic mutation–selection dynamics on a constant fitness landscape. The fitness values are independent of the frequencies of other strategies in a population. Formally, the quasi-species equation takes the form: $\dot{x}_i = \sum_j x_j f_j q_{ij} - \bar{f} x_i$, where f_i is a reproductive rate (fitness) of strategy i and $\bar{f} = \sum_i x_i f_i$ is the average fitness.

7. Adaptive dynamics

The term adaptive dynamics was first used by Hofbauer and Sigmund (1990) and Nowak and Sigmund (1990). The approach allows investigating a number of theoretical issues, including genetic variation, coevolution and speciation. Nevertheless, its theoretical and mathematical properties are not well understood (Waxman and Gavrilets 2004). Formally, adaptive dynamics requires a population in which almost all individuals use a strategy p. The population can be invaded by a strategy q if a payoff for an individual playing the strategy q, while all others play p, exceeds the payoff he would receive from playing the strategy p. Adaptive dynamics takes the form:

$$\dot{p} = \frac{\partial f(q, p)}{\partial q}\Big|_{q=p}$$

The function $f(q,p)$ denotes the payoff for an individual playing strategy q in a homogenous population with strategy p. The derivative of this function determines the direction of the mutant's advantage. Adaptive dynamics captures adaptive learning and the process of myopic search in a homogenous population where most individuals use the same strategy and only a small number of agents ('mutants') use alternative strategies.

2.1.2 Stochastic dynamics

Foster and Young (1990) were the first to introduce a stochastic term into replicator dynamics. They claim that the biological model on which replicator dynamics is based is inherently stochastic in nature, so that not every encounter between *i*-type and *j*-type individuals must result in exactly the same change in fitness. Under the assumption of a large population size and frequent interactions, Foster and Young approximate any source of variability in the payoffs by a continuous-time Wiener process:

$$\dot{x}_i\left(t\right)=x_i\left(t\right)\left[\left(Ax\left(t\right)\right)_i\Delta t-x\left(t\right)^T Ax\left(t\right)\Delta t+\sigma\left(\Gamma\left(x\right)\Delta W\left(t\right)\right)_i\right]$$

where $x\left(t\right)=\left[x_1\left(t\right),..x_n\left(t\right)\right]^T$ is the proportion of different strategies; $W(t)$ is a continuous, white-noise process with a zero mean and an unit rate covariance matrix; $\Gamma(x)$ is continuous in x and has the property $x^T\Gamma(x)=[0,0,..,0]^T$. The stochastic version of replicator dynamics is suitable for models where random perturbations σ constantly affect the selection process and thus system dynamics.

Other stochastic approaches to the study of dynamics in evolutionary games include Markov processes, the master equation, and the Polya urn. These techniques describe changes in the population state based on individual stochastic processes (e.g., birth, death and migration of individuals). According to a Markov process, a probability of transition from state x to y at time t is conditional on all past states, but it can be reduced to a probability that is conditional only on the state visited in the previous time $t-1$:

$$\Pr\left(X_t=y\,|X_{t-1}=x,..,X_0=x_0\right)=\Pr\left(X_t=y\,|X_{t-1}=x\right)$$

Economic variables modelled as Markov processes are ‘memory-less’: their values depend solely on the values in the previous period. Wheeler et al. (2006) offer an application of a discrete-time Markov chain to model adaptive learning in the context of the Cobweb model.

The so-called master equation is a special case of a Markov chain (in finite time space). It may be employed to model agents’ discrete choices. The equation describes a transition probability based on probabilities of flows into and out of the set of states. Formally, it can be written down, using vector notation, as (Aoki 1996; 117):

$$\partial P\left(x',t\right)/\partial t=\sum_{x\neq x'}P\left(x,t\right)\omega\left(x'\,|x,t\right)-\sum_{x\neq x'}P\left(x',t\right)\omega\left(x\,|x',t\right)$$

where $P(x,t)$ denotes a probability of being in state x at time t, while $\omega(x'|x,t)$ is a transition rate from state x to x'. The first term is the sum of probability of flows into state x', while the second is the probability of flows out of state x'. Weidlich and Braun (1992) discuss application of such Markov chains to economics. Cantner and Pyka (1998b) use the master equation to model technological progress emerging from technological interactions involving spillover effects.

Alternatively, the Polya urn describes a dynamic process with reference to an urn that is filled with balls of two colors. Each time, one ball is drawn randomly: the selected ball is returned to the urn, while a ball of the same color is added. The probability of adding a ball of a particular color equals exactly the proportion of balls of this color in the urn. Arthur et al. (1987) propose a more general model in which the probability of adding a ball of type j is an arbitrary function (q) of the color frequencies. It augments the standard Polya urn with a perturbation component. Formally, the urn consists of n balls of N colors, where a vector $X_n = \left\{X_n^1,\ X_n^2, .., \ X_n^N\right\}$ describes the proportions of balls of colors 1 to N respectively. At each time period, one ball is added; the probability that it is a ball of a color i is equal to $q_n^i\left(X_n\right)$. The frequency of the i-color ball is:

$$X_{n+1}^i = X_n^i + \frac{1}{w+n}\left[q_n^i\left(X_n\right) - X_n^i\right] + \frac{1}{w+n}\mu_n^i\left(X_n\right)$$

Here, $\mu_n^i\left(X_n\right) = \beta_n^i\left(X_n\right) - q_n^i\left(X_n\right)$, while $\beta_n^i\left(X_n\right)$ equals 1 with a probability $q_n^i\left(X_n\right)$ and 0 otherwise.

The Polya urn mechanism as described refers to a non-linear Polya process (Arthur et al. 1987). Dosi et al. (1994a) apply the general urn scheme to modelling technology choice, and Fagiolo (2005) to coordination games.

2.2 Evolutionary computation

Evolutionary computation offers algorithms based on the mechanisms of natural selection and genetics, such as genetic algorithms (Back 1996; Mitchell 1996; Goldberg 1989), genetic programming (Banzhalf et al. 1989), evolutionary programming (Back 1996), learning classifier systems (Lazi et al. 1998; Bull 2004) and evolutionary strategies (Beyer 1998).[1] These techniques are increasingly applied to evolutionary-economic modelling (see Arifovic 2000; Dawid 1999). In evolutionary computation models, individuals do not change over time, but a population evolves due to selective replication and variation processes. Riechmann (1999) has proposed to interpret these evolutionary operators in terms of socio-economic interactions, namely as learning by imitation (selective replication), learning by communication (crossover) and learning by experimentation (mutation).

Central to all techniques in evolutionary computation is the search process for better solutions. It involves generating new options with mutation and recombination operators. A mutation operator is always stochastic. It acts by changing a value of a random characteristic of an individual with some positive probability. Recombination (crossover) merges information (characteristics) from two parent codes into an offspring code. The important difference between mutation and recombination is that mutation is a unary operator; it

[1] Since economic applications of evolutionary programming and evolutionary strategies are rare, we do not discuss them further here.

requires one object as an input, while crossover is typically a binary operator applied to two objects (parents). In addition, the possibility of recombination with more than two parents is also possible in a socioeconomic or technological context (see Eiben 2000). This creates a very wide spectrum of innovation outcomes. Notably, the role of mutation and recombination differs among alternative ‘evolutionary dialects’ from being the only variation operator for creating diversity, the only search operator for scanning the parameter space in search for a better solution, a mixture of these two, to not being used at all (Eiben and Smith 2003).

The process of selective replication transfers a set of individuals hosting distinct strategies from one generation to the next. In evolutionary algorithms, selection consists of two processes: parent and survival selection (Eiben and Smith 2003). The role of parent selection is to stimulate better individuals to become parents of the next generation. Parent selection is typically probabilistic: better quality individuals have a higher chance to reproduce. For instance, parents may be selected in proportion to their relative fitness (a quality measure assigned to each solution). The approach is also known as roulette wheel selection: the chance of selecting a particular parent may be envisaged as spinning a roulette wheel, where the size of each pocket is proportional to the parent’s fitness. Other types of selection mechanisms are linear sorting and tournament selection. According to the first, an algorithm sorts all individuals based on their fitness and then assigns a selection probability to each individual according to its rank. Alternatively, in tournament selection, an algorithm chooses randomly two parents and creates an offspring of the fitter parent. Subsequently, parents are returned to the initial population. The process is repeated n times to create a succeeding population of n offsprings. The second type of selection is survival selection (often deterministic). Here, offspring compete for a place in the next generation based on their fitness. A new population can be constructed from a set of parents and offspring, referred to as fitness bias selection, or solely from the offspring population, known as age bias selection.

Evolutionary algorithms may be employed to model individual learning in multi-agent systems. In such models, each agent observes a representation of the current state and undertakes an action according to a selected decision rule (from a finite set of rules). After all agents undertake their decisions, payoffs are revealed, and the effectiveness of rules is evaluated. The most effective rules have a higher chance to be selected in the future. Over time, an evolutionary algorithm evolves the optimal rule or set of rules in response to a changing environment.

2.2.1 Genetic algorithms

Holland (1980, 1992), inspired by genetic processes, developed the Genetic Algorithm (GA) method to study adaptive behavior. A simple genetic algorithm is characterized by a population of strings of equal length, representing sequences of binary or real values. A GA operates as follows: from an initial parent population, some strings are chosen with a probability proportionate to their fitness. Offspring are created by applying variation operators to the selected parents: mutation 'flips' the value of any bit-string with some positive probability, while recombination (crossover) switches sequences of consecutive bits between two parents' strings. A new generation is then created from parent and offspring populations (or only from the offspring population). The process is repeated a finite number of times until convergence occurs or some (other) stopping rule is satisfied.

Genetic algorithms are widely employed in evolutionary modelling. The string representation offers a convenient way to code consumer preferences (Aversi et al. 1997), production designs (Windrum and Birchenhall 1998, 2005), firm routines (Kwasnicki and Kwasnicka 1992), production rules in cobweb models (Arifovic 1994, 1995; Dawid and Kopel 1998; Frenke 1998), production functions (Birchenhall 1995; Birchenhall et al. 1997), pricing strategies (Curzon Price 1997), or strategies in a Prisoners Dilemma (Axelrod 1987; Miller 1996).

2.2.2 Learning classifier systems

A classifier system was designed by Holland (1992) as an adaptive system where rules are activated depending on the state of the environment. Each rule consists of a condition–action part (for example 'if X appears then do Y'). Classifier conditions are strings of symbols {0, 1, #}, while actions are expressed as binary strings. Classifier systems work as follows. First, the state of the environment is coded on a binary string and transmitted to the system. If a condition part of a rule matches the message from the environment, the rule enters a competition with other rules that have satisfied this condition. The outcome of the process depends on strengths of rules describing a rule's past performance. The strengths are updated over time with a particular learning algorithm (e.g., bucket-bridge or Q-learning). In a second stage, a genetic algorithm is run on the population of rules to generate new and delete poorly performing rules with the use of one-point crossover and bitwise mutation. The purpose of employing classifier systems is to create a cooperative set of rules that together solve the problem (Bull 2004). Classifier systems are typically employed to model agent's adaptive behavior (Marimon et al. 1990; Arthur 1991; Arthur et al. 1996; Vriend 1995; Kirman and Vriend 2001).

2.2.3 Genetic programming

Genetic programming (GP) represents the youngest technique in the artificial intelligence and computational literature. It was developed by Koza (1992, 1994) and builds on the concept of functions applied to arguments; these functions are organized into trees, the nodes of which are described with a set of basic functions (e.g., the arithmetic, Boolean, relation, if–then operators) plus some variables and constants {+, −, *, /,....., OR, AND, NOT, >, <, =, ...v_1, v_2, v_3...c_1, c_2, c_3...}. Operators have connections with other operators or variables. Variables, which have no further connection, constitute 'leaves' of the tree.

GP proceeds by evaluating each solution according to its fitness and selecting the best solutions for 'reproduction'. In order to generate new solutions, the fittest among the existing ones are modified and recombined. For example, crossover operates by selecting randomly two nodes in the parents' trees and swapping the sub-trees, which have such nodes as roots. The idea of generating new, possibly better functions or trees in GP is similar to the way in which genetic algorithms (GA) operate.

GP as a member of the evolutionary algorithm family shares some properties with GA. Formally, GP is a variant of GA characterized by a different data structure. The two approaches differ with respect to the application area: GP is used to seek models with maximum fit to the environment, while GA aims to find an optimal solution (Eiben and Smith 2003). Working with GP allows for more flexibility: trees take the form of complex structures with nested components, while the size of trees may vary within a population. By contrast, a GA population consists of fixed-length binary strings. However, the complex structures of GP may hinder their usefulness, in particular making the interpretation of results difficult (Arifovic 2000). Genetic programming has been employed in a number of economic applications, for instance, to evolve an optimal price-setting rule (Dosi et al. 1999) or an optimal trading rule (Neely et al. 1997; Allen and Karjalainen 1999), and to model speculators' adaptive behavior (Chen and Yeh 2000).

2.2.4 Fitness landscape and NK models

In evolutionary algorithms, there is no assumption being made about the fitness landscape. Hill and ORiordan (2001) suggest employing the *NK* model to represent solutions in the search space and subsequently to evaluate the performance of variation operators in evolutionary algorithms. In general, the fitness landscape can be constructed by assigning to each solution the height corresponding to its fitness value in the search space. The *NK* model has been proposed by Kauffman (1993) as a stochastic method for constructing an adaptive fitness landscape that can be gradually tuned from smooth to rugged. Here, N stands for the number of elements, while K denotes the complexity of the system (interdependence of dimensions). Each element has its own sub-function(s) within the system. It is assigned a fitness value w_n drawn randomly from the uniform distribution [0, 1]. Elements in an *NK* system are interdependent; these dependencies are often referred to as 'epistatic relations'. If a value of a particular element changes, the change affects both the fitness (and functioning) of this element and the fitness (and functioning) of elements that are interlinked with it. The total fitness of the system changes according to the average fitness of its elements:

$$W(s) = \frac{1}{N} \sum\nolimits_{n=1}^{N} w_n(s)$$

where w_n denotes the fitness of the n-th element. In this context, search is modelled as a trail-and-error process. Each time step the value of one element is mutated and the fitness of the system before and after mutation is compared. If the average fitness has increased, mutation continues, otherwise the state of the system is restored to the previous configuration. The process is repeated until a (local or global) optimum is reached. The *NK* framework can be employed to model myopic search for better solutions (Altenberg 1997; Auerswald et al. 2000; Frenken and Nuvolari 2004).

2.3 Multi-agent models

Multi-agent models (sometimes referred to as agent-based models, multi-agent systems, multi-agent simulations, or multi-agent based simulations) enable the study of coordination processes, self-organization, distributed processing, micro diversity and innovation through recombination, all in a way that is far beyond the capabilities of any representative agent model (Potts 2000). In early studies, the approach was employed to model social interaction processes (Schelling 1978; Axelrod 1984). The most ambitious in this sense has been Epstein and Axtell's (1996) multi-agent 'Sugarscape' model, which integrates elements of demography, sociology, psychology and economics. Exercises with this model show the way in which spatial-temporal interactions of agents can generate a variety of social phenomena, for example, the transmission of culture, the rise of conflicts, the spread of a disease, the diffusion of price information, and migration. In economics the method of multi-agent simulations became more the work of Andersen et al. (1988) and Holland and Miller (1991). These authors proposed to view widely known through the economy as a complex, dynamic, and adaptive system with a large number of autonomous agents. Multi-agent simulations offer a powerful tool for addressing interactions of heterogeneous, boundedly rational agents characterized by learning, increasing returns and path dependence.

The basic structure of a multi-agent system involves specifying a large number of parameters and variables: time, the number of agents, micro states (actions) that can be endogenously modified by agents, micro parameters containing information about agents' behavioral and technological characteristics, time independent variables governing the fixed technological and institutional setup, the structure of interactions and information flows among agents, and aggregate macro variables (Pyka and Fagiolo 2007).

Formally, agents can be defined as computational entities, usually showing some form of bounded rationality (myopia, local search), situated in some environment, capable of undertaking flexible autonomous actions with the objective of meeting their goals (Wooldridge 1999). Intelligent agents are characterized as capable of perceiving the environment and responding to it; of exhibiting goal-oriented behavior, and of interacting with other agents. These interactions can take place indirectly through the environment in which agents are embedded, or in direct communication among agents (Weiss 1999).

Agents' interactions as well as feedback from aggregate (macro) to disaggregate (micro) phenomena are the sources of nonlinear dynamics.

Multi-agent models have been applied to a wide range of topics. This includes: agent learning (Arthur 1991; Ishibuchi et al. 2001; Klos and Nooteboom 2001), the evolution of norms, and conventions (Axelrod 1997; Thebaud and Locatelli 2001; Hodgson and Knudsen 2004), financial markets (Arthur et al. 1996; Caldarelli et al. 1998; LeBaron 2001; Levy et al. 2000), diffusion of innovations and industry dynamics (Aversi et al. 1997; Cantner and Pyka 1998a; Cantner et al. 1998, 2000; Gilbert et al. 2001; Windrum and Birchenhall 1998, 2005; Saint-Jean 2006; Schwoon 2006), land use and environmental management (Paker et al. 2003), labor economics (Tassier and Menczer 2001; Gabriele 2002; Fagiolo et al. 2004), and environmental policies (Janssen and Jager 2002; Carrillo-Hermosilla 2006). Multi-agent models have been also applied to various specific markets, including the textile market (Brannon et al. 1997), fish market (Kirman and Vriend 2001), wholesale electricity market (Bower and Bunn 2001), and agricultural practices in a developing country (Lansing and Miller 2004).

Despite numerous contributions, a common protocol for the design and validation of multi-agent models has not yet emerged (Matteo et al. 2006; Fagiolo et al. 2007; Pyka and Fagiolo 2007). Behavioral rules and different heuristics are often introduced *ad hoc,* as there are no or little data available to validate model assumptions and to calibrate the parameters. The diversity of approaches follows from the fact that an array of assumptions can be justified with reference to stylized facts (David and Fagiolo 2008). Existing approaches for the validation of multi-agent models include the indirect calibration approach, the Werker–Brenner approach, and the construction of history friendly models. However, each method has its shortcoming (Fagiolo et al. 2007). For these reasons, the use of multi-agent models for the evaluation and especially design of economic policy should be approached with care (see also a special issue on "Agent-based models for economic policy," David and Fagiolo 2008).

For a more extensive discussion of multi-agent modelling, see Tesfatsion (2001a, b), Axelrod (2003), Windrum (2004), Dawid (2006), Vriend (2006), and Epstein (2007). We discuss aspects of some of the aforementioned models in greater detail later on in the sections dealing with particular building blocks of evolutionary-economic models.

2.3.1 Spatial and network structures

Agent interactions in multi-agent models can occur through spatial structures or networks taking the form of a graph, grid or lattice (Antonelli 1996; Solomon et al. 2000; Conlisk et al. 2001; Nowak 2006, Chapters 8 and 9; Noailly et al. 2007, 2009). Graphs compromise Ising models, small world models and random graphs (Watts and Strogatz 1998; Cowan 2004; Frenken 2006). In Ising models, agents are located at fixed points in a regular integer space, and they are connected to their *n*-nearest neighbors only. In small world models, agents can interact with some distant (i.e. not direct neighboring) sites. The network structure in small world models is characterized by high cliquishness, i.e. a high density of agents' interactions, and short average path lengths between agents (Cowan and Jonard 2000). Alternatively, in random graphs, agents are connected with some positive probability regardless of their location; the networks do not reflect explicit geographical space. Watts and Strogatz (1998) proposed a one-parameter random graph model compromising these three approaches (Ising and small world models, and random graphs). A parameter p, reflecting a probability of connecting a random agent to each link within the network, is used to scale between the regular and random graph (e.g., $p = 0$ gives Ising model, $p = 1$ random graph).

Recently, percolation models have achieved some attention in modelling technology diffusion and spillovers in innovations (Silverberg and Verspagen 2005b; Cantono and Silverberg 2008; Hohnisch et al. 2008). Here, interactions occur among neighboring cells on a lattice. Cells are active (e.g., adoption of a particular good) or inactive. An algorithm defines conditions under which a cell can change its state; for instance, an inactive cell becomes activated. Percolation is said to occur if one or more clusters of active cells emerge (Hohnisch et al. 2008).

2.4 Comparing methods

Little attention has been devoted to comparing the different methods of evolutionary modelling in evolutionary economics. In fact, there is not much communication between researchers using distinct techniques (Witt 2008). Important differences between the methods relate to mathematical representation, the possibility of deriving analytical solutions, dynamics being stochastic or deterministic, the way selection and innovation mechanisms are or can be formalized, and the level of aggregation. As a result, the choice of method will influence model design.

Evolutionary game theory describes interactions between randomly drawn individuals from a population, which indeed can be interpreted as a micro-level and population approach (e.g., Friedman 1991). However, with the aim to derive analytical solutions, applications usually reduce heterogeneity in a population to a few strategies or subgroups, which often means a kind of aggregation of information, certainly in comparison with multi-agent approaches that distinguish between sometimes hundreds of individual agents. In evolutionary computation and multi-agent models, individuals within a population are described in detail, resulting in evolutionary dynamics that are analytically intractable. The multi-agent model is unique in the sense of allowing for interactions between many heterogeneous agents, who can moreover exhibit changing behavior and learning over their life time. By contrast, in models developed with evolutionary computation techniques, individuals do not change over time, but the population evolves due to selection and variation processes. Evolutionary computation has been mainly used to study adaptive learning or to perform optimization in complex, nonlinear systems. Each of the evolutionary algorithms (i.e. genetic algorithms, learning classifier systems and evolutionary programming) is associated with specific formal representations of individuals (string, tree of functions etc.). Multi-agent modelling is much more flexible in this respect.

As already mentioned in Section 1, the classification of different methods employed in evolutionary-economics is not straightforward, especially if different methods are combined. For instance, evolutionary games can underlie interactions of individuals in multi-agent models, while evolutionary algorithms can be used to model agent learning and search in multi-agent settings. This certainly allows for more flexibility in the model designs, but may create a difficulty in classifying, and comparing results of, different models.

3 Building blocks of evolutionary-economic models

In this section, we present an overview of the various formalizations of components of evolutionary-economic models that have been proposed in the literature. The following categorization is employed (as motivated in Section 1): (1) diversity, (2) innovation, (3) selection, (4) bounded rationality, (5) diffusion, (6) path dependence and lock-in, (7) coevolution, (8) multi-level and group selection, and (9) mechanisms of evolutionary growth. Table 1 summarizes the manner in which these components can be formally conceptualized using the various methods. Statements in the cells of the table are elaborated and explained in relevant subsequent subsections.

3.1 Diversity

Central to any evolutionary model is a heterogeneous population, i.e. a population consisting of diverse elements or members. Diversity relates to progress through Fisher's principle (Fisher 1930), which says that the greater the variability upon which selection for fitness acts, the greater the expected improvement in fitness. In evolutionary-economic models, diversity is formalized in a number of different ways. In evolutionary game theory, diversity is limited to a very small number (most commonly two) of strategies. However, individuals may do different things on different occasions, formally captured by the notion of mixed strategies. This has been referred to as 'individual behavior mixing' as opposed to the situation in which individuals demonstrate constant behavior over time while different individuals can show diverse behaviors, referred to as 'developmental coin flipping' (Bergstrom and Godfrey-Smith 1998). The latter is characteristic of model design in evolutionary computation (Section 2.2) where the number of different strategies in the population is typically large. Finally, in multi-agent systems, variety of agent characteristics and time variation of strategies can be combined. Agents can differ here with respect to behavioral rules, knowledge, goals, physiological features (e.g., vision and energetic efficiency in the Sugarscape model;

Table 1 Relation between components of evolutionary-economic models and evolutionary methods

Method	Model component								
	Diversity	Innovation	Selection	Bounded rationality	Diffusion	Path dependence and lock-in	Coevolution	Multi-level and group selection	Growth mechanisms
Evolutionary game theory and selection dynamics	Limited	Absent (replicator dynamics) or captured by mutation (adaptive dynamics, selection-mutation dynamics), possibly stochastic (stochastic replicator dynamics)	Frequency dependent (replicator) or constant (quasi-species)	Myopic search, errors during replication captured by mutation	By local (imitation dynamics) or global imitation (replicator, best response, Brown-von Neumann-Nash dynamics)	Depends on the type of selection dynamics, stochastic factors, and the fitness function. (constant versus frequency dependent fitness)	In principle possible, but no economic examples available	Group selection modelled using Price equation	Has been capital addressed by combining selection dynamics and accumulation of capital
Evolutionary computation	Large	New solutions generated with variation operators: mutation and recombination	Fitness dependent, tournament selection, or linear sorting	Learning through reinforcement, myopic search (NK model)	Replication (biological analogue)	Present Possible lock-in to suboptimal state(local optimum)	Feasible but no economic applications	Emergent properties, social learning	Not present
Multi-agent models	Medium to large	Can address a wide range of innovation processes: innovation in consumers' preferences, firms' production techniques, routines, products etc.	Replication or imitation of certain strategies (frequency dependent)	Agent behavior covers imitation, myopic search and imperfect learning (e.g., through networks)	Imitation (possibly on a grid or lattice)	Present Many feedback mechanisms can be included	Coevolution of supply and demand, of industries, and of socio-economic groups	Emergent properties, group competition and conflict, status-peer groups	Capital accumulation, population growth, resource dynamics

Epstein and Axtell 1996) or signals. This creates a wide spectrum of opportunities to realize heterogeneity.

The concept of diversity can be elaborated as having three properties: variety, balance, and disparity (Stirling 2004, 2007). Variety is defined as the number of categories into which a population can be partitioned; the larger this number the larger the diversity. Balance relates to the distribution of shares of each category in the population; the more equal the shares, the more even the distribution and the larger the diversity. Finally, disparity refers to the degree to which options differ; it captures the distance between categories. Disparity is a qualitative property, which represents a rather subjective and context-dependent aspect of diversity.

Stirling suggests a simple diversity measure that combines these components. It takes the form of a multiplicative function, representing an integrated diversity heuristic measure D (Stirling 2007):

$$D = \sum_{i,\, j(i \neq j)} d_{ij}^{\alpha} \left(p_i p_j\right)^{\beta}$$

Here d_{ij} is the distance in a Euclidean disparity space between options i and j, and p_k is the frequency of element k in the population. The parameters α and β may take values 0 or 1. In the reference case, α and β both equal 1 and the measure captures balance- and disparity-weighted variety. If $\alpha = 0$ and $\beta = 1$, the index reduces to balance-weighted variety, while if $\beta = 0$ and $\alpha = 1$, it reduces to disparity-weighted variety. For $\alpha = 0$ and $\beta = 0$, the measure depicts scaled variety.

For the purpose of statistical analysis, a number of other diversity measures have been proposed (Theil 1967; Weitzman 1992, 1998a; Önal 1997; Frenken et al. 1999; Saviotti 2001). However, Stirling (2007) shows that most of these are not very well balanced, as they ignore some aspects of diversity. For instance, an entropy-based index is a dual measure combining diversity and balance, while the Weitzman index is limited to disparity. The entropy-based Shannon index is defined as $-H = -\sum_{i=1}^{n} p_i \ln(p_i)$, where n is the number of species, and p_i is the share of the ith species. $H = 0$ indicates the lowest diversity. The Simpson index takes the form of the sum of the squared shares of each option in the portfolio: $H = \sum\limits_{i} p_i^2$. A related entropy measure is that of Önal (1997), proposed for the purpose of creating a more operationally and computationally convenient index. It defines the structural diversity index as: $V(x) = 1 - \frac{1}{2(n-1)} \sum\limits_{i,j} \left|s_i - s_j\right|$ (n is the number of species, and s_i, s_j are shares of i and j species respectively). For a given pair of groups i and j, $\left|s_i - s_j\right|$ measures the relative diversity between the two groups. Maximum diversity occurs when all groups in an assembly have equal numbers of elements, while a minimum value is realized if one group contains all of the elements.

Alternatively, Weitzman's index (1992, 1998a) emphasizes the distance between entities. The measure can be applied to both discrete and continuous variables. It classifies entities in groups on their dissimilarity through a distance measure d. Formally, diversity $V(S)$ is the solution of the recursion: $V(S) = \max_{y\in S}(V(S\backslash y) + d(S\backslash y, y))$, where $S\backslash y$ stands for a set S without a member y and $d(S\backslash y, y)$ captures the distance between this set and y. The Weitzman's index addresses disparity alone; it does not account for the relative abundance of different options within a population.

Several studies have applied these diversity measures: Saviotti and Trickett (1992) in a study of helicopters, Bourgeois et al. (2005) for refinery processing, Frenken and Nuvolari (2004) for the steam engine, Frenken and Windrum (2005) for microcomputers and laptops, van den Heuvel and van den Bergh (2009) for the solar photovoltaic industry, and van den Bergh (2008) in an abstract model of optimal diversity in investment. Frenken et al. (1999) use both the entropy and Weitzman's diversity measure to analyze the evolution of technology in four industries: aircrafts, helicopters, motorcycles and microcomputers. They define a population of products in terms of the distribution of product characteristics. Changes of variety in each particular industry are investigated as changes in the composition of the population structure over time (measured with diversity indexes). The results reveal a tendency for decreasing variety towards product standardization for helicopters and microcomputers and increasing variety for aircrafts and motorcycles.

3.2 Innovation

Innovation is an inherent feature of any evolutionary system. It is essential for diversity creation. Technological evolution may take the form of a series of incremental improvements in already existing designs or the introduction of a design radically different from the latest technological achievement. Mokyr (1990) distinguishes in this respect between micro and macro inventions, following Schumpeter (1939). Although innovations are intrinsically uncertain, and for this reason in most evolutionary-economic models treated as stochastic, it would be incorrect to consider the process of innovation as totally random. Innovations may be expected to occur in a systematic manner, namely preceded by the cumulativeness of relevant technical advances. The innovative process is often depicted as following relatively ordered technological pathways, as is reflected by notions such as natural trajectories (Nelson and Winter 1977), technological guide points (Sahal 1985), technological paradigms (Dosi 1982), and socio-technological regimes (Geels 2002, 2005).

Innovations are conceptualized in formal models in a number of ways: as a stochastic process (e.g., Poisson) that can result in structural discontinuity, variation and recombination of existing technological options, or random or myopic search on a fitness (technology) landscape. Innovations may be associated with a new vintage of capital (e.g., Iwai 1984a, b; Silverberg and Lehnert 1993; Silverberg and Verspagen 1994a, b, 1995). In vintage models developed in an evolutionary game setting, innovations tend to transform a firm as whole. For instance, Iwai (1984a) develops a capital vintage model to examine the way in which dynamic interactions between the equilibrating force of imitation and the disequilibrating force of innovation shape the evolutionary pattern of an industry. The market consists of M firms (active and potential producers) and n production methods with corresponding unit costs c_i ($c_n>$ .. $>c_1$). Firms face two alternatives, namely innovate or imitate the technology exhibiting a lower than current cost of production. If innovation occurs, it creates a new cumulative frequency $F_t(C_N) = 1/M$, where C_N denotes the unit cost of the best production method that is technologically possible at time t. The relative frequency of firms with the unit cost equal to c or lower than c changes according to:

$$\Delta F_t(c) = \{\mu F_t(c)(1 - F_t(c)) + \nu M (1 - F_t(c))(1/M)\} \Delta t$$

where μ and ν are indices of the effectiveness of firm imitation and innovation activities, respectively; $\nu \Delta t M$ denotes the probability that an innovation is carried out successfully by one of the firms over a small time period Δt.

In micro-simulation models of industry dynamics, each firm is engaged in the search process for better solutions. In Nelson and Winter's (1982) pioneering model, search is modeled as a two-stage random process. In the first stage, imitation and innovation draws determine the firm's probability of undertaking R&D activities (0 or 1). If a firm i gets an imitation draw, then in the second stage it copies the industry's best practice. If it gets an innovation draw, it samples productivity A from a distribution of technological opportunities $F(A;\, t,\, A_{i,t})$, where A_{it} is firm i's current productivity level. Finally, if a firm obtains a combination of imitation and innovation draws, its new productivity level is determined by $A_{i,t+1} = Max(A_{i,t}, \overline{A}_t, \overline{A}_{i,t})$, where $A_{i,t}$ is firm i's current productivity level, $\overline{A}_t$ is the best practice productivity level at time t, and $\overline{A}_{i,t}$ is a random variable resulting from the innovation draw.

In Nelson and Winter's model, firms are treated as a single unit of selection. Alternatively, a firm can be treated as a multi-operation unit (e.g., Kwasnicki and Kwasnicka 1992; Chiaromonte and Dosi 1993; Dosi et al. 1994b, 2006). For instance, in Kwasnicki and Kwasnicka's (1992) model of industry dynamics, each firm is characterized by two types of routines: active ones employed in everyday practice, and latent ones stored but not actually applied. Routines here are modelled with genetic algorithms. Each set of routines is divided into separate segments, consisting of similar routines employed by firms in different domains of their activities. New routines evolve due to recombination, mutation, transition or transposition. With a certain probability, the lth routine in the kth sector changes (mutation) or the segment k of a firm-unit i is recombined with the segment k of a firm-unit j (recombination). Alternatively, a single routine may be transmitted from another firm (transition) or within a single firm a latent routine can be transposed from a latent into an active state (transposition).

Modelling innovations on the supply side is well established in the evolutionary economics' literature. By contrast, conceptualizing innovations on the demand side has not led to a common approach, especially in the context of modelling endogenous preferences of consumers (see Section 3.4). An interesting attempt to formalize evolving preferences in an abstract model has been undertaken by Potts (2000).[2] The author sketches eight ways in which the schematic preferences, coded on a string, may evolve with the use of a genetic algorithm. In the context of an agent choosing a set of goods from the available set {a, b, c, d,...}, the change in his preferences may be captured with (# implies indifference):

1. Point mutation: <aaab>→<aaaa>
2. Cross over: <aabc><bbcc>→<aacc>
3. Inversion <abca>→<acba>
4. Slide <##aabbcc##>→<aaaabb####>
5. Reclustering <abcabcaabc>→<aaabbbccc>
6. Emergence/Closure <aaaaa###>→<aaaaa>
7. Higher or lower specification: <aabb##>→<aabbc#>; <aabb##>→<aab###>
8. Birth or death: <...>→<aabbc#>; <aabb##>→<...>

This list can be augmented with other mechanisms corresponding to genetic processes. In addition to 'point mutation' and 'recombination', 'insertion' and 'deletion' are distinguished (in genetics). Insertion implies adding a string to the existing sequence of code. Deletion characterizes the reverse process, i.e. the loss of a string of code (Nowak 2006). New solutions may also result from hybridization of more than two existing ideas, a process known as multi-parent recombination in evolutionary computation, or modular evolution in biology. In particular, modular evolution is the source of radical innovations in both natural and social–technological history. Watson (2006) theoretically supports this by formally showing that modular evolution can realize more complex systems, or similarly complex systems in a shorter time, than gradual evolution.

A number of models address the notion of recombinant innovation in an economic context (Weitzman 1998b; Olsson and Frey 2002; Tsur and Zemel 2007; van den Bergh 2008). Weitzman presents a formal model in which the number of new combinations is a function of the number of existing ideas. He shows that if this number is the only limiting factor in knowledge production, super-exponential growth may result. Tsur and Zemel extend this model with endogenous growth elements. Olsson and Frey (2002) connect Weitzman's recombinant growth with Schumpeter's view of the entrepreneur, who innovates by combining existing ideas or technologies in a convex way. They demonstrate that the resulting combinatory process is constrained by following factors: convexity implies the exhaustion of technological opportunities; the cost of combining ideas increases with distance (disparity) between them so that profit maximization requires combining ideas that are technologically sufficiently close; social acceptance constrains or prohibits certain combinations; and a

[2]For examples of formalization of endogenous preference change, see Aversi et al. (1997) and the coevolutionary models described in Section 3.7.

ruling technological paradigm limits the scope for recombinant growth. Van den Bergh (2008) develops a model to derive optimal diversity resulting from the trade-off between increasing returns to scale and benefits of recombinant innovation.

3.3 Selection

Certain aspects of selection models were already discussed in Section 2. Here we adopt a broader approach. Selection in the simplest form can be understood in terms of picking a subset from a certain set of elements according to a criterion of preference, referred to as subset selection (Price 1995). Alternatively, selection can be seen by analogy with natural selection as the outcome of two independent processes, namely replication of an encoded instruction set, and interaction of entities with their environment, causing differential replication (Knudsen 2002). If the second process applies, a population of offspring is not a subset of parents but consists of new entities. Similar to Price (1995), we can describe a general selection process that unifies subset and natural selection as follows. Formally, a set P includes n_i units of entities i with value x_i for some property x. A set P' is composed of new entities corresponding to entities of P. Selection on the set P in relation to the property x can then be defined as a process of producing the corresponding set P' such that n_i' is a function of x_i. According to subset selection, $n_i' \leq n_i$, while $x_i = x_i'$. These assumptions are not required in the case of natural selection.

An early discussion in evolutionary economics focused on firms being selected by the market, in the sense of surviving competition, with possible effects on profit seeking or even maximizing behavior (Alchian 1950; Friedman 1953; Winter 1964). In later models of industry dynamics, selection was formalized with replicator type of dynamics by analogy with natural selection. Accordingly, market shares of firms generating above-average profits increase over time. In this context, technology diffusion is treated as an outcome of selective competition between rival technologies, where selection covers both traditional types of competitiveness e.g., price competition and product differentiation (e.g., Nelson and Winter 1982; Iwai 1984a, b; Soete and Turner 1984; Silverberg et al. 1988; Metcalfe 1988). The system of firms competing by offering new, improved product characteristics or services, which enable them to capture some temporary monopoly rents, has been referred to as Schumpeterian competition (Saviotti and Pyka 2004).

Formally, in evolutionary models, frequency-dependent selection predominates. The most commonly used model, replicator dynamics, ignores the possibility of mistakes, imperfect learning, and costly experimentation during selection and replication processes. Alternative models of selection dynamics exist (as already discussed in Section 2.1), but these have seen little application to economic phenomena (an exception is Safarzynska and van den Bergh 2008). Important exceptions are Foster and Young (1990), Canning (1992), Young (1993), and Kandori et al. (1993), who propose models of adaptive learning in the context of repeated 2 × 2 games. Here, mistakes by players constantly disturb the process of learning and thus the selection dynamics.

Note that, although selection environments are often modelled as being constant, this does not need to be the case. For example, the dynamics of consumer preferences may alter the selection environment for firms, leading to demand–supply coevolution (see Section 3.7). The latter can be best expressed with multi-agent modelling. Alternatively, selection may be modelled as a two-stage or a multi-level process: internal and external to the firm. Internal selection concerns selection of routines at the level of a firm, while external selection is typically understood in terms of market selection (Kwasnicki and Kwasnicka 1992; Lazaric and Raybaut 2005). For instance, in Kwasnicki and Kwasnicka (1992) each firm searches for new routines (or new combinations of existing routines) to increase its overall competitiveness. After a firm has made decisions concerning production, its performance is subject to external (market) selection. As a result, a firm's market share depends on relative prices, relative values of products, and the market saturation level. Although in most models external selection takes the form of market selection, the possibility of non-market selection—for example, due to institutional pressure—is at least theoretically possible. For more general discussion on multi-level evolution, see Section 3.8.

3.4 Bounded rationality

The notion of bounded rationality originated in the 1950s from Herbert Simon's critique of 'economic man'. Simon (1955, 1956) proposed the concept of bounded rationality, which involves considerations of extensiveness, complexity and uncertainty (Hodgson 1997). Under extensiveness, information may be readily accessible and comprehensible, even though time and other resources are required to obtain it. Complexity stipulates the existence of a gap between the computational capacity of an agent and the complexity of his environment. Under uncertainty, agents have difficulties in assessing probabilities of future events. In these cases, individuals are likely to exhibit habits and rule-driven behavior.

In models of firm and organizational behavior, bounded rationality has taken the form of rules and routines. Nelson and Winter (1982) claim that firms operate to a large extent according to decisions rules that are not consistent with profit maximization but instead take the form of complex patterns of routinized behavior. Heuristics, cognitive and learning processes are crucial for decision-making. In particular, imitation is an important mechanism underlying firm behavior in models of technology diffusion. It allows saving on costs of individual learning, experimentation or searching by exploiting information already acquired by others (horizontal and vertical transmission). In the context of social interactions, imitation can take the form of either copying the' the most successful' or 'the majority' strategy. Copying 'the most successful' is also known as prestigious-bias transmission; it occurs when individuals seek to copy the most influential, knowledgeable or skillful behavior (Henrich et al. 1999). Copying the majority strategy has been termed by Boyd and Richardson (1985) as conformist transmission. It refers to a propensity of an individual to adopt cultural traits that appear most frequently in the population, which can be formalized with frequency-dependent selection (see Section 2.1).

Bounded rationality is implicit in many evolutionary economics models or results from specific choices made with regard to the other model components (e.g., selection models), and for this reason the opportunities to review explicit formalisations are limited. Conlisk (1996) offers an extensive (appreciative) overview of different types of bounded rationality. In evolutionary game settings, boundedly rational agents are incapable of anticipating actions of other agents or consequences of their own decisions. They may engage in myopic search for better solutions or imitate the most frequent behavior. Various forms of selection dynamics have been proposed to model boundedly rational behavior, as described in Section 2.1. Notably, imitating the most successful or majority strategy in a population requires the assumption of common knowledge. One way to deal with this rather unrealistic setting is to limit the environment in which agents operate (Kirman 1997). This can be achieved by assuming that agents interact with a limited number of other agents, for instance, through networks (Axelrod 1997; Janssen and Jager 2002; Silverberg and Verspagen 2003, 2005a; Morone and Taylor 2004; Cowan and Jonard 2004; Cowan et al. 2006). The latter predominates in evolutionary multi-agent models, which allow for explicit modelling of interactions within and between heterogeneous groups, and within networks of consumers (Section 2.3). In addition, networks play an important role in facilitating communication, specialization of competences, standardization of complementary technologies, and flows of knowledge between firms. A number of studies have analyzed behavior of firms and strategic arrangements within specific networks (Malerba 2006).

In general, a variety of assumptions regarding boundedly rational behavior can be encountered in evolutionary-economic models. In many cases, they are introduced *ad hoc* without clear empirical, experimental or theoretical support. Evidence and theories in behavioral economics can help to provide a better foundation of behavioral assumptions of evolutionary-economic models. In particular, relevant insights are offered by prospect theory (Kahneman and Tversky 1979), quasi-hyperbolic instead of exponential discounting (Thaler 1981; Prelec and Loewenstein 1991; Frederick et al. 2002), various social preferences (Guth et al. 1982), regret theory (Bell 1985; Loomes and Sugden 1986), and case-based theory (Gilboa and Schmeidler 1995). Prospect theory, which describes decision making under uncertainty, has received much attention. It builds upon the premise that individuals differently evaluate losses and gains relative to a situation-specific reference point. The theory of social preferences is inspired by experimental evidence that players tend to sacrifice own benefits to reduce inequality of payoffs, while they are likely to reciprocate behaviors that have benefited them. Regret theory assumes that, whenever the outcome of the prospect is worse than expected, a sense of disappointment is generated, while in case the outcome of the prospect is good, a person experiences elation. Finally, case based theory suggests that people choose acts based on their performance in similar problems in the past. It provides insight into habit formation. Although theories in behavioral economics offer useful examples of bounded rationality of individuals, still more research is needed on the conditions under which they apply (Fudenberg 2006; Pesendorfer 2006).

3.5 Diffusion

Diffusion is closely intertwined with the selection mechanisms. It determines the pace of adoption of particular technologies, goods and behaviors that have been already adopted (selected) by a fraction of the population. Diffusion typically follows a logistic or sigmoid (S) curve over time: the diffusion rate first rises at initially low but increasing adoption rates, leading to a period of relatively rapid adoption. At some later stage, the diffusion rate starts to decline, until finally a regime of satiation is reached. In general, models of technology diffusion aim at explaining the logistic pattern of diffusion process. For overviews, see Metcalfe (1988), Silverberg et al. (1988), Geroski (2000), and Manfredi et al. (2004).

The diffusion process relies on the progressive dissemination of information about technical and economic characteristics of products within a population of potential adopters (Silverberg et al. 1988). The minimal structure of such a diffusion model requires distinguishing between mutually exclusive sub-groups of users and non-users, while the analysis of model dynamics focuses on the spread of information from adopters to non-users. Several related frameworks can be distinguished. They are typically described with difference equations.

According to the epidemic model (the seminal work is by Mansfield 1961), technology spreads like a disease. An individual adopts a particular technology after having had contact with the 'infected population' i.e. individuals who already have adopted the innovation. The framework explains patterns of innovation diffusion from the date of its first implementation (not invention) by some percentage of users. The evolution of the number of adopters follows the pattern given by: $y(t) = N(1 - exp[-\alpha t])$, where N is the number of potential adopters, and α denotes the percentage of the population that has learned about a new technology. The model applies to a situation in which information spreads from a central source.

Alternatively, 'word of mouth models' account for direct communication: users independently contact non-users with a positive probability β. The process of diffusion follows an S-curve over time: the rate of infection increases as a population of users gradually rises (increasing the aggregate source of information) until it reaches the maximum. Then it starts declining, as non-users become more hard to find and therefore to infect.

Mixed information source models combine the epidemic and the word of mouth approaches. The information spreads with a probability equal to a sum of a constant rate at which an individual learns about new technology from the central source, plus a flexible rate at which an individual learns about novelty from other users: $\alpha + \beta y(t)$ (see Bass 1969).

Finally, the probit model was developed for the analysis of individual adoptions. A simplified version of this approach assumes that individuals differ in some characteristic x, which is randomly distributed in a population according to a function $f(x)$. Only individuals whose characteristic value exceeds a threshold level x^* adopt the innovation. Over time, technology gets cheaper and the threshold value falls. As a consequence, more people have a chance to adopt it. If the distribution underlying $f(x)$ is normal, the gradual movement of the threshold level across the distribution generates the $S-$shaped diffusion curve.

The aforementioned models have been criticized for lacking a description of individual decision-making. They do not provide insight into how the possible saturation level is reached or determined. Multi-agent models can better explain micro foundations of diffusion patterns as described in aggregate models (discussed above). They allow for a description of individual imitating behavior of earlier adopters (e.g., information cascades), of neighboring sites in case of a game with a spatial dimension (agents are located on a grid), or of individuals who belong to the relevant social network (e.g., Janssen and Jager 2002; Alkemade and Castaldi 2005; Delre et al. 2007, see also Section 2.3). For instance, Delre et al. (2007) develop a multi-agent model, where adoption decisions depend on agents' personal networks and external marketing efforts. The results suggested that the speed of diffusion is highly sensitive to the network structure and the degree of consumer heterogeneity.

Evolutionary graph theory may provide interesting insights for studying the effect of population structure on diffusion. Individuals are placed here on the vertices of the graph and connected by edges. Edges denote reproductive rates at which individuals place offspring into adjacent vertices. The analysis of the fixation probability indicates how likely it is that a single mutant, placed randomly within the network, takes over a whole population (Nowak 2006, Chapter 8). In this context, some graphs act as suppressors or amplifiers of selection. In particular, amplifying structures increase the probability of fixation of advantageous mutants (with high relative fitness) and reduce the probability of fixation of disadvantageous mutants. The superstar, funnel and metafunnel are examples of such amplifier structures (Lieberman et al. 2005). Evolutionary dynamics on graphs have been applied to study social games (e.g., Prisoner Dilemma, Dove and Hawk) in spatially structured populations.

3.6 Path-dependence and lock-in

Economic systems are characterized by various reinforcement and feedback mechanisms that explain why, after a system follows a particular path of development, it may be difficult to reverse or change the direction of system change. Feedback mechanisms associated with increasing returns may arise from economies of scale, learning-by-doing, technological interrelatedness, the accumulation of knowledge and experience, and agglomeration or spillover effects (see Arrow 1962; Arthur 1988; Metcalfe 1994). These are typically mechanisms associated with supply-side dynamics. In addition, increasing returns on the demand side play a role, in particular network externalities, informational increasing returns, imitation and bandwagon effects, learning-by-interacting, and external influences such as advertising, education (Katz and Shapiro 1986; Lundvall 1988).

Increasing returns are the sources of lock-in and path dependence. A simple model illustrating dynamics in the presence of increasing returns was developed by Arthur (1989). This model considers two technologies, A and B, competing for adoption by two types of economic agents: an agent R, who has a natural or intrinsic preference for technology A, and an agent S, having a natural inclination to chose technology B. Choices are made sequentially; at each point in time, a randomly drawn type of agent (either R or S) decides which technology to adopt by comparing payoffs from two technology variants. The matrix of payoffs is described as:

	Technology A	Technology B
R-agent	$a_R + rn_A$	$b_R + rn_B$
S-agent	$a_S + sn_A$	$b_S r + sn_B$

where a_R, a_S denotes returns to technology A exhibited by agent R and agent S, respectively ($a_R > a_S$); analogously b_R, b_S, ($b_R < b_S$), r, s are agent R's and S's returns to adoption and n_A, n_B are the number of previous adopters of technology A and B, respectively. These payoff functions reflect the notion that returns from adoption of a particular technology depend on the number of its previous adopters. This dependence causes increasing returns to scale: the more it is adopted, the more attractive is a technology. It is a self-reinforcing mechanism, which may be the source of lock-in: once a certain technology becomes dominant; subsequent adoptions will most likely be of the same type enhancing its leading position.

Witt (1997) notes that lock-in critically depends on the assumption of an infinitely growing population of adopters. This, together with the presence of only two types of agents and specific interactions between adopters (imitation), prevents model dynamics from exhibiting cyclic or more complex behavior. If a finite or constant population is assumed, an unstable fixed point rather than an inescapable state of lock-in results. Arthur and Lane (1993), Kirman (1993) and Dosi et al. (1994a) show that lock-in is not a necessary outcome if interactions between agents take a different form than in the basic Arthur model. For instance, Dosi et al. (1994a) reformulate Arthur's model with the generalized Polya urn schemes approach. Here, new adopters choose the technology used by the majority of a sample m of other adopters with probability α, while with probability $1 - \alpha$ they adopt the technology used by the minority. Due to the presence of a stochastic factor, technology shares never converge to either 0 or 1, ensuring co-existence of variety. In addition, Leydesdorff and van den Besselaar (1998) use Arthur's model to demonstrate that, under the assumption of limited cognitive capabilities of individuals, i.e. agents being unable to perceive small differences in the adoption rate below a certain threshold, lock-in disappears.

Path dependence and lock-in are important features of technological change in the context of environmental regulation. Problems of lock-in and unlocking policy are closely related to the difficulty of making a transition to sustainable systems in energy, transport and agriculture (Unruh 2000; van den Bergh et al. 2006; van den Bergh 2007). Lock-in does not need to be permanent. Assuming that everyone switches, the change from an inferior state is possible (Arthur 1994). For instance, actors might coordinate their decisions to adopt a new technology when they recognize that coordinated action yields special benefits (Foray 1997). In line with the above remarks, Witt (1997) argues that the capacity to pass a "critical mass threshold" in terms of the number of potential adopters of a market alternative is the key to the success of unlocking the market. He notes that, in fact, governments and innovating firms take account of the critical mass phenomenon. For instance, with promotion campaigns firms undertake efforts to convince potential adopters that others are already about to adopt the new variant in order to stimulate coordinated adoption decisions.

Since the seminal work by David (1985) and Arthur (1988, 1989), lock-in and path dependence have received increasing attention in the context of policy studies in multi-agent models (Janssen and Jager 2002; Carrillo-Hermosilla 2006; Schwoon 2006). For instance, Carrillo-Hermosilla (2006) develops a framework in which a public authority representing the collective interest of society tries to guide the market (individual decisions) by supporting the socially preferable technology with a subsidy. The conditions are investigated under which escaping a lock-in of environmentally unstable practices is possible. It is further examined whether a system can move between equilibria (i.e. be un-locked) without a need for public intervention, and if the timing and the direction of these spontaneous transitions would be socially optimal.

3.7 Coevolution

The term coevolution refers to a situation in which two or more evolutionary systems or populations are linked together in such a way that each influences the evolutionary trajectory of the others. It is achieved through reciprocal selective pressures among evolving populations. Linking an evolutionary to a non-evolutionary system does not produce strict coevolutionary dynamics but co-dynamics of sub-systems (van den Bergh and Stagl 2004; Winder et al. 2005).

Coevolutionary dynamics underlie many economic processes. In an early contribution, Norgaard (1984) discusses coevolution as the interaction between knowledge, values, organization, technology and environment. However, without explicitly referring to population dynamics, this should better be regarded as system dynamics due to co-dynamics of subsystems. Nevertheless, different sub-systems (market, technology, institutions, scientific knowledge, etc.) can be seen as consisting of heterogeneous, changing populations (producers, consumers, policymakers, universities, etc.). Their interactions may give rise to coevolution and, over time, render irreversible changes in socio-technological trajectories. In spite of this, there are relatively few contributions to coevolutionary modelling available. Most formal applications focus on demand–supply coevolution (Janssen and Jager 2002; Windrum and Birchenhall 1998, 2005; Saint-Jean 2006; Schwoon 2006; Safarzynska and van den Bergh 2007; Windrum and Birchenhall 2009a, b). Models of other types of coevolutionary dynamics exist, but are rare. For instance, Noailly (2008) develop a formal coevolutionary framework to analyze the effect of human activity (total pesticide use) on the size and the composition of pests, while Malerba et al. (2005) propose a history friendly model that captures coevolution of computer and semiconductor industries.

In economic models developed with evolutionary game theory and selection dynamics, coevolution seems not to have been explored much. Nevertheless, the method in principle seems to allow for describing coevolution of two or more interdependent populations, for instance, by interlinking fitness functions of different populations (McGill and Brown 2007). This is illustrated by the model of Noailly (2008), in which two replicator dynamics representing separate populations (harvesters using pesticide strategies and pests) are coupled to give rise to coevolution. In addition, evolutionary algorithms can also be used to model coevolution, but this does not seem to have been used in economic applications.

In fact, all existing coevolutionary models of demand and supply are developed with the multi-agent method, which easily accommodates feedback mechanisms between multiple populations. In a coevolutionary model developed by Saint-Jean (2006), the probability that a consumer adopts a particular good depends on the distinct product characteristics and the relative weights a consumer assigns to each of them. If product characteristics receive relatively high weights from consumers, they are considered as their priorities. During every period, firms invest in quality improvements. Each firm reallocates R&D budget towards characteristics that are priorities for consumers and in which a firm has reached a sufficiently high performance level. On the other hand, consumers' preferences evolve over time in response to technological advances and changes in the industry structure. These mechanisms create strong feedbacks between supply and demand.

In a coevolutionary model by Windrum and Birchenhall (1998, 2005), firms aim to offer product designs maximizing the average utility of a randomly selected consumer class. Consumers can move between classes, depending on how well they are served by the incumbent firms. In order to improve its competitiveness, each firm engages in product innovation. It implements a new design only if it yields a higher utility of consumers in its target class than the current design. Evolving consumer preferences influence the direction of such product innovations. Formally, firms compete by offering distinct designs or different points in a multi-dimensional (service characteristic, price) space.

Their success depends on realizing a utility of the target consumer class i above the average level:

$$\varphi_{i,t+1} = \varphi_{i,t} \frac{w_{it}}{W_t}$$

where $\varphi_{i,t+1} = \frac{G_{it}}{G}$; G is the total number of consumers; G_{it} is the number of consumers in class i at time t; w_{it} denotes the average utility in the i class in time t; and W_t is the average level of utility across classes. Consequently, technological change (product succession) is modelled here as an outcome of a coevolutionary process involving interactions between consumers and producers. Recently, Windrum and Birchenhall (2009a, b) applied the earlier approach to address the substitution of more by less polluting firms

Building upon Windrum and Birchenhall (1998, 2005), Safarzynska and van den Bergh (2007) propose a multi-agent model of demand–supply coevolution to assess the probability of market lock-in which results from dynamic interactions of the most important types of increasing returns on the demand and supply side. They further consider the effect of different demand side specifications. On the supply side, a technological trajectory arises from the interplay of incremental innovation, search for a new product design and marketing activities. On the demand side, two disequilibrating forces underlie consumer choices, namely a desire for distinction (status) and imitation of other consumers within the social network (peer group).

To conclude, coevolution of demand and supply is an important theme in economics. However, no canon for designing coevolutionary demand–supply dynamics has emerged so far. Researchers derive conclusions based not only on differently formalized behavioral rules but also on different technical model specifications, including: number of consumers and consumer classes, number of firms, the length of a single simulation run, and the number of overall simulations conducted. Results from Safarzynska and van den Bergh (2007) suggest that the technical specification of the models, e.g. the number of firms, is important for coevolutionary dynamics. Consequently, detailed and specific guidelines for modelling may be useful and allow systematic comparison and validation of different coevolutionary models.

3.8 Multi-level evolution and group selection

The economy can be seen as a complex, hierarchical structure comprising various levels and subsystems linked together through strong feedback mechanisms. The micro-interactions among heterogeneous elements lead to the emergence of a higher structure, while variation and selection processes occurring in any of the subsystems affect changes in the total environment. In this context, Potts (2000) has called for a new evolutionary microeconomics based on discrete, combinatorial mathematics, and in a practical sense graph theory and multi-agent modelling. A standard graph theory model is described by the elements $S = (V, E)$ S-system, V-elements, E-connections. According to Potts, connections are crucial for the analysis of dynamics, complexity and system change. Due to the introduction of connections, the notions of emergence and hierarchy can be combined into a single construct, termed a hyperstructure. Formally, this requires recognizing that a system itself can be an element of a higher-level system, while an element may itself be a system at a lower level $\left(S^n = V^{n+1}\right)$.

Gunderson and Holling (2001) develop an alternative complexity model build upon the notion of resilience: panarchy. The idea of panarchy combines the concept of space–time hierarchies with the context of adaptive structures. Elements of a complex adaptive system, which emerge through local interactions among various components, are recursively nested to form a hierarchy. The framework may be applied to evolving systems: economic, ecological or social. For instance, nature (forests, lakes) and humans (cultures, governance structures) can be interlinked through the panarchy in never-ending adaptive cycles of growth, accumulation, restructuring, and renewal. The approach has seen formalization through multi-agent evolutionary models (e.g., Janssen and Carpenter 1999).

A multilevel theory of evolution that is receiving much attention presently is built on the combination of individual and group selection (Wilson and Sober 1994; Wilson 2002, 2006; Henrich 2004; van den Bergh and Gowdy 2009). Group selection theory tries to elucidate emerging phenomena by taking into account individual and group level processes framed in a multi-level model. There are many relevant models available now (see Bergstrom 2002; Garcia and van den Bergh 2007). The minimal structure of a group selection model requires defining a reproducing population composed of groups characterized by more intense or regular interactions among members than with outsiders. Two main approaches can be identified to attain a group formation for the next generation. In a haystack or migration pool type model, after reproducing, groups are pooled together and then randomly sampled. Alternatively, in propagule types of models, groups are formed solely on the basis of a single parent group; in this case, offspring are continuously added to the parent group that splits into two after reaching a certain size (Bowles et al. 2004; Traulsen and Nowak 2006). The second approach makes selection more effective. To further increase the effectiveness of group selection, non-random assortment typical of cultural and economic systems may be included (Bergstrom 2003).

A wide range of techniques can be used to build a group selection model, such as difference and differential equations, deterministic and stochastic models, spatial models and multi-agent frameworks. The Price equation is often used to decompose evolutionary change into effects of within- and between-group components (Price 1970). Formally, it takes the form of:

$$\overline{w}\Delta\overline{z} = Cov\left(w_i, z_i\right) + E\left(w_i, \Delta z_i\right)$$

Here, $\Delta\overline{z}$ depicts a change in the average characteristic (trait) over generations according to $\Delta\bar{z} = \sum_i q_i{}' z_i' - \sum_i q_i z_i$, where q_i is the frequency of the type i with the characteristic z_i in the parent population, q_i' the frequency of the type i with the characteristic z_i' in a descendant (offspring) population, and Δz_i measures the change in the trait value for the type i as $\Delta z_i = z_i' - z_i$. In addition, the frequency of type i in the offspring population is proportional to the relative fitness of the type i in the parent population: $q_i' = q_i {w_i}/_{\overline{w}}$, where w_i stands for the fitness of i type and $\overline{w}$ denotes the average fitness of the population. The components of the Price equation are open to a wide variety of interpretations (Frank 1995; Andersen 2004). For instance, the equation may decompose the evolutionary process into selection and transmission. In the context of group selection models, the covariance and expectation terms can be construed as effects of between- and within-group selection on the average trait frequency in the population (Henrich 2004). The Price equation is often mistaken for being a generally applicable analytical tool, while its role is solely to decompose evolutionary change. Ultimately, the equation is an identity or mathematical tautology (Grafen 2000). Van Veelen (2005) suggests distinguishing clearly between statistical and probability (stochastic) analysis. He claims that the Price equation can be employed to address two types of questions. First, it can be used to assess a possibility (likelihood) of certain modelling assumptions being correct. Alternatively, one may employ the equation to make interferences given a set of assumptions and mechanisms leading to a theoretical (evolutionary) model.

Group selection has not been employed in many economic applications, but has the potential to provide a theoretical explanation for the emergence and evolution of all sorts of institutions. For instance, selection on the group level may contribute to a better understanding of the processes of replication of successful and extinction of ineffective institutions, the evolutions of power relations and firm organizational structures, and the dynamics of conflicts over economic distributions (van den Bergh and Gowdy 2009).

3.9 Mechanisms of growth

Endogenous growth theory has tried to explain the rate of technological progress by endogenizing human capital or R&D research (e.g., Romer 1986, 1990; Lucas 1988; Grossman and Helpman 1991). In addition, new endogenous growth theories devote more attention to the importance of creativity and innovations in the process. For instance, Aghion and Howitt (1992) develop a model embedding Schumpeter's idea of creative destruction, where the expected growth rate of the economy depends upon the economy-wide amount of research. Each innovation is regarded here as an act of creation aimed at capturing monopoly rents, while it simultaneously destroys rents that motivated the previous discovery. The model relies on a temporal equilibrium, a representative agent and rational expectations, so that it cannot be categorized as an evolutionary-economic approach. Silverberg and Yildizoglu (2002) indeed show numerically that the behavior of Aghion and Howitt's model critically depends on the rational agent assumption.

With the seminal work by Nelson and Winter (1982), evolutionary economics contributed to opening the 'black box' of growth theories. Models developed in an evolutionary spirit describe diversity of production techniques at the level of individual firms characterized by bounded rationality, i.e.

production routines. Opportunities of innovation can arise any time, as entities (agents, firms) are constantly involved in search activities. The analysis focuses on structural change and differential growth of a population of firms. In the classic evolutionary model of growth by Nelson and Winter (1982, Chapter 12), heterogeneous firms produce the same homogenous product but with different techniques. Model dynamics are driven by investment rules and search processes relating to each individual firm. Firm i's desired expansion or contraction (of the capital stock K) at time $t$is determined by gross investment $I(.)$, the output per unit capital A_{it}, price P_t, profit on capital Π_{it}, the depreciation rate of the capital δ, the production cost c, and the market share Q_{it}/Q_t:

$$K_i\left(t+1\right)=I\left(\frac{P_t A_{i,t+1}}{c},\frac{Q_{i,t}}{Q_t},\Pi_{i,t},\delta\right)K_{i,t}+\left(1-\delta\right)K_{i,t}$$

Industry output results from aggregating over individual firms' production levels: $Q_t=\sum_i Q_{i,t}$.

Nelson and Winter built their evolutionary growth model from the bottom-up. They carried out simulations of micro data, which generated patterns consistent with observed macro aggregates. The model initiated a new phase in evolutionary growth theorizing. Later contributions to evolutionary growth theory can be categorized into models following Nelson and Winter's perspective of micro foundations and evolutionary growth theories formulated at the macro level (Silverberg and Verspagen 2005a). Within the first type, two distinct approaches can be identified (Kwasnicki 2007): (1) capital-vintage type of models (e.g., Silverberg and Verspagen 1994a, b, 1995; Iwai 2000); and (2) two-sector type of models (Chiaromonte and Dosi 1993; Dosi et al. 1994b; Fagiolo and Dosi 2003), where the single economy is divided into an industry fabricating inputs for production and an industry manufacturing final goods. In these models, dynamics at the firm level underlie the growth rate of aggregate output. The common modelling technique is computer simulation. Models differ in the degree of complexity, technology representation, and firm behavior rules. In addition, an extension to a multi-country framework is possible. For instance, Silverberg and Verspagen (1995) propose an evolutionary model of endogenous growth to explain the convergence between countries' productivity levels. In each country, there are q firms producing homogenous goods from different types of capitals. Formally, the accumulation of capital j in firm i is governed by replicator dynamics:

$$\dot{k}_{ij}=r_{ij}+\alpha\left(r_{ij}-r_i\right)-\sigma$$

where k_{ij} is capital, r_i is the average profit over all types of capital employed in firm i, r_{ij} is profit of firm j realized with capital i, σ is a depreciation rate, and α a parameter.

Contributions to the macro approach to evolutionary growth do not include micro foundations explicitly. Here, dynamics are analyzed at the sector or industry level directly. Different techniques are employed, namely analytical methods and computer simulations (Silverberg and Verspagen 2005a). The aggregate growth rate of output may be driven by an increase in labor or output productivity (Conlisk 1989; Silverberg and Lehnert 1993; Metcalfe et al. 2006) or by a growing variety of the economic system (Saviotti and Pyka 2004, 2008). In the growth model by Metcalfe et al. (2006), output growth depends on the growth rates in different interdependent economic sectors. The interdependence arises due to income and expenditure flows through market interactions. The (average) output growth in the economy is described as: $\widehat{q}=\frac{\overline{\alpha}_e+\overline{\beta}_e g_z}{(\sum e_j\Psi_j)(1-\overline{\beta}_u)+\overline{\beta}_e}$, where $\overline{\beta}_u$ and $\overline{\beta}_e$ are the average elasticities of technological progress constructed with weights corresponding to the income elasticities and employment shares in each sector, respectively. Furthermore, $\overline{\alpha}_e$ is the average rate of residual progress (due to investments unrelated to the current capacity), e_j refers to the share of employment in sector j, and Ψ_j is elasticity of capital income in sector j.

Recently, growth through variety has achieved more attention, indicating an interesting direction for further research. For instance, Saviotti and Pyka (2004, 2008) develop a model in which the emergence of new products and services allows for a continuation of economic development. Here, an industry is defined as a collection of firms producing variants of goods with different characteristics along the same dimensions of the characteristics space. The growth rate of the number of firms in each industry depends on firms' entry and exit, and thus on the size of the potential market, financial availability, the intensity of competition, and a number of mergers and acquisitions, formally expressed as:

$$N_i^{t+1} - N_i^t = k_1 \times FA_i^t \times AG_i^t - IC_i^t - MA_i^t$$

where N_i^t is the number of firms in industry i at time t; FA_i^t is financial availability in industry i at time t; IC_i^t is intensity of competition in industry i at time t; and MA_i^t are mergers and acquisitions in industry i at time t. For each industry there exists a saturation level; once it is reached, firms innovate radically by offering a new product in the characteristic space. As a result, new sectors emerge and old ones disappear.

4 Conclusions

This paper has reviewed methods underlying, and components of, evolutionary models in economics. The main methods, namely evolutionary game theory and selection dynamics, evolutionary computation, and multi-agent models, were described in some detail. In addition, an overview was given of components or theoretical building blocks of evolutionary economic models. We discussed the various ways in which these components have been conceptualized in models developed using a range of modelling techniques.

In evolutionary-economic models, replicator dynamics is the most popular variant of deterministic selection dynamics. According to this method, repeated selection can cause convergence to a single strategy, which makes sense given that no mechanism generating diversity—i.e. the emergence of new strategies—is required. This simplifies resulting models considerably and in turn may allow for analytical solutions. Other selection dynamics, such as selection-mutation and stochastic dynamics, allow for errors to occur during the process of replication. On the other hand, in evolutionary algorithms innovations are essential. New solutions are generated with the variation operators: mutation and crossover. Evolutionary algorithms can be employed to study adaptive learning and optimization processes, also within the setting of a multi-agent system. The number of evolutionary contributions to multi-agent modelling has increased drastically in recent years. However, so far no common rule for model specification, conducting simulations and validating results has been established.

The main goal of the paper was to clarify the variety of specific choices made with regard to formal conceptualization of evolutionary system (and resulting model) components, namely core ones—diversity, innovation and selection—and additional ones—bounded rationality, diffusion, path dependency and lock-in, coevolutionary dynamics, multilevel and group selection, and growth mechanisms. The review shows that there is much variety and little agreement on how to conceptualize many of these building blocks in formal models. This is perhaps in the nature of evolutionary economics, which steps away from rational, representative agents as the decision units, and market processes as the main driving force of economic dynamics. The variety of choices available for each model component translates into an even larger variety of possible combinations of these components, that is, particular evolutionary-economic models.

Nevertheless, some building blocks have converged to a certain modelling standard. On the supply side, mechanisms underlying innovation, diffusion and evolutionary growth are well established in evolutionary economics. On the other hand, there is still no consensus on how to model consumer behavior on the demand side, in particular, how to conceptualize consumer diversity, social interaction, and bounded rationality. Concepts are often tailored to the application context and vary depending on the method used. However, a full understanding of the economy as a complex evolving system requires accounting for interdependencies among various groups and entities, including consumers. This can be only achieved if consumers and producers attain equal balance, resulting in coevolutionary demand–supply models. These are, however, still very uncommon. In addition, a (cultural) group selection approach has been rarely employed in modelling economic phenomena, although it potentially provides a concrete formal theory of selection at multiple levels (individual and group). Its application could enhance our understanding of the emergence and evolution of human organizations and institutions.

Multi-agent modelling is definitely the most flexible in addressing the nine building blocks, as it allows for a variety of assumptions. However, the main difficulties associated with this method are validation of model results and communication with other researchers, in the absence of a protocol for design of such models. Evolutionary game theory is much more limited, especially in addressing diversity, innovation and coevolution, but it has the advantage that—under certain conditions—analytical solutions can be obtained. Finally, the potential of evolutionary computation techniques for modelling economic dynamics has not been exhausted. Evolutionary algorithms are mostly employed to address population (social) learning. All in all, it seems as though one should not expect a general convergence on the specification of building blocks and choice of modelling techniques in the near future. Nevertheless, a good understanding of the properties of modelling methods and their implications for designing model components is essential for further progress of evolutionary economics.

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