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Compressed Sensing: Hope or Hype?

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Compressed sensing is often motivated by a conundrum present in modern digital imagers: significant effort is expended to fabricate imaging devices with increasingly greater resolution, yet, for efficient storage on a mobile device, the captured images are digitally compressed, with little perceptible information loss. From so very many pixels captured, relatively few bits are stored. The paradox of this acquire-then-compress strategy is well captured by an oft-repeated sentiment by Brady [1]: "One can regard the possibility of digital compression as a failure of sensor design. If it is possible to compress measured data, one might argue that too many measurements were taken. "

The key contribution of compressed sensing to sensor design is a significant reduction in the number of measurements required to faithfully reproduce sparse or compressible signals. Implications of sample reduction are domain-dependent, but may include faster acquisition time, greater instantaneous bandwidth, or lower size, weight, power, and/or cost.

Nevertheless, the potential benefits of compressed sensing do not come without cost. The price to pay for sample reduction may include reduced SNR, and often a dramatic increase in the computational burden of signal recovery. The net utility of a compressed sensing device greatly depends how its design or purpose exploits opportunities in this cost-benefit curve. While CS may not have broad-sweeping implications for all sensor domains, it does offer a unique design paradigm that may provide key benefits in some domains.

Compressed Sensing Theory

For the purpose of exposition, digital imaging is presented as an application to introduce key terminology, but it should be understood that compressed sensing theory relates broadly to sampling.

Compressed sensing is the study of how to sample and reconstruct a compressible image from a small number M of non-adaptive linear "measurements"—many fewer than the total number of pixels N. Intuitively, this is possible because a compressible image contains only $K < M \ll N$ significant coefficients when represented in an appropriate compression basis.

A measurement vector y is acquired according to the linear model

$$\boldsymbol{y} = \Phi\boldsymbol{x} + \boldsymbol{n},$$

where Φ is an $M \times N$ sampling matrix that represents the measurements employed in the system, e.g., M measurements of an N pixel image $\boldsymbol{x}$ (here, represented as column-scanned vector of intensities). Each row of Φ encodes how a single measurement of $\boldsymbol{x}$ is collected. Often, each measurement is comprised of a global, multiplexed measurement of the scene, plus measurement noise.

Key to compressed sensing is the assumption that the image $\boldsymbol{x}$ is compressible, meaning that it can be decomposed as $\boldsymbol{x} = \Psi\boldsymbol{\alpha}$, where Ψ is a user-specified compression basis or dictionary suitable to the domain, and the vector $\boldsymbol{\alpha}$ has only $K \ll N$ significant coefficients. For example, $\boldsymbol{\alpha}$ could represent the wavelet coefficients of an image with the wavelets captured in the basis Ψ. Or, $\boldsymbol{x}$ could, itself, represent a sparse image (e.g., a "star-field" image) without any compression, so that $\Psi = I$. A vector $\boldsymbol{x}$ is called K-*sparse* if, under a suitable representation $\boldsymbol{x} = \Psi\boldsymbol{\alpha}$, the coefficient vector $\boldsymbol{\alpha}$ has at most K nonzero elements. Alternatively, $\boldsymbol{x}$ is *compressible to* K *coefficients* if all but K coefficients in $\boldsymbol{\alpha}$ have near-zero magnitude.

Given that the image one wishes to capture (but hasn't yet) is compressible, a hardware system is designed to implement the inner product $\Phi\boldsymbol{x}$. In order to guarantee that the image can be acquired in $M \ll N$ measurements, not just any Φ will do. Indeed, Φ must be chosen to satisfy properties that guarantee recoverability of $\boldsymbol{x}$ from "undersampled" measurements $\boldsymbol{y}$.

Designing Φ to acquire samples of $\boldsymbol{x}$ that is compressible by some decomposition $\boldsymbol{x} = \Psi\boldsymbol{\alpha}$ is equivalent to specifying $A = \Phi\Psi$ for recovering $\boldsymbol{\alpha}$ from $y = A\boldsymbol{\alpha}$. Since $M \ll N$, $y = A\boldsymbol{\alpha}$ is a dimensionality reduction or compression, that in general, loses information. However, if only K elements of $\boldsymbol{\alpha}$ are nonzero, recovering $\boldsymbol{\alpha}$ from (noiseless) measurements $\boldsymbol{y}$ might be achieved through an NP-hard combinatoric search for the support of $\boldsymbol{\alpha}$. Although alternative methods have been successful, basis pursuit—an exact and computationally tractable ℓ_1 inverse problem that is a convex relaxation of the combinatoric recovery problem—is a broadly-used approach that enjoys rich theoretical guarantees. Basis pursuit for noiseless measurement solves

$$\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_1 \quad \text{subject to} \quad A\boldsymbol{\alpha} = \boldsymbol{y},$$

where $\|\boldsymbol{\alpha}\|_1 = \sum_{i=1}^{N} |\alpha_i|$, and the constraint may be replaced by $\|A\boldsymbol{\alpha} - \boldsymbol{y}\|_2^2 \leq \sigma^2$ for noisy measurements. A myriad of efficient methods exist to solve basis pursuit in polynomial time [2] [3] [4] [5]. Once $\boldsymbol{\alpha}$ is recovered, the image of interest is easily reconstructed via $\boldsymbol{x} = \Psi\boldsymbol{\alpha}$.

Fundamental to CS theory is that if A is properly designed with $M \geq O(K \log\frac{N}{K})$ rows (measurements), stable recovery of $\boldsymbol{\alpha}$ is guaranteed through basis pursuit. The tightest guarantee to date is provided by the restricted isometry property (RIP)—which essentially guarantees that A is nearly distance-preserving for any sparse vector $\boldsymbol{\alpha}$. To certify that RIP holds for an arbitrary A is a challenging task. An alternative guarantee is based on the mutual coherence property, which states that the sensing kernels employed in Φ are incoherent with (cannot be efficiently compressed by) the compression basis Ψ.

Popular design strategies of Φ that provide guarantees for recovering $\boldsymbol{x}$ include:

- randomly draw the entries of Φ from a sub-Gaussian distribution (a family that includes the normal distribution or balanced -1/+1 Bernoulli distribution);
- choose Φ to by randomly selecting M rows of an $N \times N$ Fourier matrix;
- more generally, select M rows at random from any $N \times N$ orthogonal matrix, such as a Hadamard matrix;
- design each row of Φ to be a sparse binary (0/1) pattern [6], with only a few nonzero elements per row; or,

- for streaming data, choose a compressive sampling vector that is repeated at each time-shift, forming a structured Toeplitz and circulant matrix Φ [7] [8] (this allows for efficient matrix multiplication, useful both in signal acquisition and in efficient signal recovery).

In each case, each of the M compressed measurements is obtained through a kind multiplexed sensing: combinations of many parts of the signal or image are mixed together in a pre-determined, but randomly-generated way. In some cases, such as drawing entries of Φ from a sub-Gaussian distribution, any choice of Ψ will guarantee that $A = \Phi\Psi$ satisifies properties requisite for guaranteed recovery of $\boldsymbol{\alpha}$, and hence, $\boldsymbol{x}$. This provides a very attractive universality condition, in which the compression basis Φ can be selected (or changed) even after the measurements $\boldsymbol{y}$ have already been collected!

An instructive example of a compressive imaging sensor is the Rice single-pixel camera [9] [10]. Like a conventional camera, the scene is illuminated by a light source. However, the reflected light from the illuminated object is not directly focused onto a detector array. Rather, an image is formed on a digitial micromirror device (DMD) that acts as a spatial light modulator (SLM). Each micromirror on the DMD can be individually switched into a focusing or a non-focusing position. A configuration of mirrors results in a single compressive measurement of the imaging system: for the ith measurement, the mirror array encodes the ith row $\boldsymbol{\phi}_i^T$ of the measurement matrix Φ. Pixels corresponding to mirror elements in the "on" position (a 1 in $\boldsymbol{\phi}_i$) are focused onto a single photodetector that performs in analog the vector inner-product between the scene x and sensing function $\boldsymbol{\phi}_i$. The process of reconfiguring the DMD and digitizing the result at the photodetector is repeated M times. An algorithm then recovers the image from the compressed measurements and explicit knowledge of the DMD mirror encoding scheme.

Compressed Sensing Hardware Examples

Although the Rice single-pixel camera, as a visible light camera, is not likely to replace consumer camera devices based on focal plan arrays (FPAs), its basic design has led to a host of other compressed sensing image devices that can reduce cost. InView Corporation (Austin, TX) markets a direct descendent of the early Rice single-pixel camera: a low-cost, high resolution shortwave infrared (SWIR) camera workstation that provides images at a fraction of the cost of FPA approaches. Indeed, the competitive cost of a traditional SWIR cameras is about $40K *per megapixel*, since Indium Gallium Arsenide (InGaAs) semiconductors and associated manufacturing expenses must be employed to capture images at SWIR wavelengths. The InView camera utilizes a DMD, thereby mitigating the need for an expensive FPA. By reducing the cost of the detector, InView Corporation achieves good SWIR imaging performance at a fraction of the cost. Many other imaging devices employing a DMD with a single detector have been proposed or demonstrated as prototypes, including hyperspectral imaging [11], a low SWaP laser radar [12], and reduced exposure time in fluorescence miscroscopy [13], to name a few.

Other imaging devices employ SLM designs to generate compressive measurements of a scene. Coded Aperture Snapshot Spectral Imager (CASSI) technology offers video-rate spectral imaging for dynamic scenes [14] [15] [16]. In CASSI, one or more dispersive elements are used to spectrally and/or spatially "mix" pixels in the scene's reflectivity. CASSI is cited as an underlying technology by startup company Centice Corp, where co-inventor David Brady is a member of the executive team. An evaluation of the CASSI hyperspectral imaging framework is provided in [17].

Rather than reducing the cost of the detector, some have proposed reducing the power consumption of FPA detectors through compressed sensing. Unlike the aforementioned compressed sensing imaging embodiments, some solutions in the space have proposed that the compressive mixing of pixels occur digitally, rather than in the optical domain. For example, a prototype 256x256 CMOS image sensor has been prototyped that acquires each entry of the measurement vector $\boldsymbol{y}$ simultaneously in a single image capture by shifting CMOS pixel values sequentially to a multiplexer controlled by the values in Φ [18]. Measured results from this embodiment showed no loss in SNR or sensitivity relative to normal image capture, and close to linear reduction in energy consumption per frame, as a function of the compressed sensing undersampling ratio M/N. The implication is that images can be captured in the CMOS device with reduced power requirements, or alternatively, high frame rates can be achieved at nominal power consumption. However, reconstructing the images from measurements prevents immediate image display.

Magnetic resonance imaging (MRI) was the first, and to date, is perhaps the most mature technology to leverage compressed sensing [19]. MRI imaging consists of taking a series of measurements in spatial frequency (k-space), then inverting the data. Traditionally, high-quality images required a rather dense sampling of k-space, despite the fact that the reconstructed image is quite compressible. Thus, compressed sensing is a natural fit for MRI: by reducing the number of k-space measurements (and thereby dramatically reducing MRI scan times), high-quality images can still be recovered. Despite its maturity as a research field, with several sparse MRI prototypes, manufacturers have yet to release a product that incorporates an appropriate pulse sequence generator with embedded image reconstruction software. This is largely due to the need for clinical validation, constrained by relatively few sites/tools to perform that validation. Nevertheless, it is anticipated that some elements of CS will make its way into MRI products by 2016, enabling higher spatial and temporal resolution.

Like MRI, synthetic aperture radars (SARs) collect samples in k-space, and images are obtained via Fourier inversion. Since transmit power can account for much of the energy budget on a mobile platform, compressed sensing may provide a way to reduce SWaP for SARs on small platforms. In [20], a simple method was proposed in which few radar pulses are used to illuminate the scene of interest, those pulses also being transmitted at randomly-spaced intervals, to enable low-power imaging of the scene. Compressed sensing for SAR and other radar applications is a maturing field, but as highlighted in a 2012 JASON report, still requires significant investment for government and military applications [21].

Analog-to-information converters (AIC) have been proposed as an enabling technology to capture extremely high bandwidth signals—out of reach for current high-speed analog to digital (ADC) technology—in which the instantaneous bandwidth (occupancy) is very low. Cognitive radio is a motivating application, in which the radio spectrum is monitored for unallocated use and subsequently filled. A useful critique of AIC designs showed that if one accounts for jitter and aperture errors in the AIC design, a 2-10× reduction in power consumption is feasible, but only when low signal gain and moderate resolutions are acceptable [22]. In all other cases, traditional ADCs provide more favorable bit error rates or power constraints.

Summary

There are many subfields of compressed sensing that employ CS theory to reduce SWaP, cost, or increase sampling efficiency for a particular class of sensors. Often, as with the Rice camera, this requires redesigning a sensor from the ground up in order to properly encode the sampling matrix Φ. In some cases, such as compressed MRI or synthetic aperture radar (SAR), the sensor naturally provides measurements that are compressive in nature, and one needs only to refit the system to provide a pseudorandom sampling strategy. In any case, software must be written to recover the signal of interest from compressed measurements.

Part of the difficulty in building compressive sensing hardware comes from the gap between hardware domain specialists and compressive sensing theoreticians. Indeed, ground-up redesign of a CS sensor necessitates a tightly integrated production cycle of CS design in which theoreticians adapt to physical constraints, and provide feedback to hardware designers [23]. Currently, the learning curve for both theoreticians and domain experts to meet in the middle is not insignificant. Dealing with physically realizable sampling kernels which, for example, do not satisfy theoretical "sufficient, but not necessary" conditions, remains a problem. Despite recent advances in theory and bounds, measurement noise and sensor calibration are also limiting factors [23].

Compressed sensing is one of the most trending topics in engineering. But presently, most advances are theoretical in nature, albeit with a burgeoning collection of proposed and prototype sensor paradigms. Undoubtedly, like other fields that enjoyed early enthusiasm, compressed sensing will find a role in many application domains. However, some theoreticians warn that hype may curtail innovation. In the meantime, research that critically evaluates hopeful applications of compressed sensing are important.

About the Author

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