Mini project 1 i Matrix Theory, ht 2021

Mini project is best to do in groups of two two persons, but you can do it alone as well.

The task Write first a Matlab-program which decides if a given (integer) matrix A has integer eigenvalues and if this is the case calculates eigenvalues and their multiplicities.
The first line in program should be

```
function [ev,mult] = heltalsev(A,tol)
if nargin < 2, tol = favorittolerans; end
```

where ev is a column matrix which contains different eigenvalues and mult is a column matrix which contains the corresponding multiplicities. This construction gives that in general it is sufficient to call the program with heltalsev(A) where the other row above sets tol to a preliminary decided tolerance, but there is a possibility without changing the program to change the tolerance with call as heltalsev(A,nytol).
For calculations of eigenvalues may Matlabs own routines applies, e.g. eig and to decide if they are integers may round, norm, error and some other be useful. Besides that some of functions sort, unique, find, ismember may be useful. Write after that a Matlab-program which for matrix A with integer eigenvalues calculates Jordans normal form. The result should be a Jordan matrix of the same size as A. The first lines should be

```
function J = jordanmatris(A,tol)
if nargin < 2, tol = favorittolerans; end
[ev,mult] = heltalsev(A,tol);
```

Use method in Chapter 7.3, 7.5 (looks at example in the end of 7.6 as well). Here you can use functions as rank, null and diff. To construct later the jordanmatrix help diag, eye, zeros, ones and many other functions.
Test your program on many different matrices, part of them are in the book but it is easy to find own.

To think about When you calculate eigenvalues with eig note that this is a complex command. Matlab calculates with about 16 digits accuracy but the the error with eigenvalues may be much larger. So accept if obtained eigenvalues are not integers if the differers from integer is less than tolerance (and use integers instead). To choose a reasonable favorite tolerance (*favorittolerans*) for tol is a part of the problem.

(To estimate how large can be error try to calculate eigenvalues of

$$\begin{pmatrix} -9 & 11 & -21 & 63 & -252 \\ 70 & -69 & 141 & -421 & 1684 \\ -575 & 575 & -1149 & 3451 & -13801 \\ 3891 & -3891 & 7782 & -23345 & 93365 \\ 1024 & -1024 & 2048 & -6144 & 24572 \end{pmatrix}$$

which has size only 5×5 and has 0 as the only eigenvalue (with multiplicity). This matrix is can be found on home page inside the function testmatris.m.) But such a matrix is an exception and you should not chose your tolerance to fit it *favorittolerans* otherwise the risk is to accept the matrix which have no integer eigenvalue as in matrix

$$\begin{pmatrix} 1000 & 1 \\ 1 & 0 \end{pmatrix}.$$

Thus there is no universal tolerance and that is why we have an option to use the tolerance as a parameter.

Final test When you feel that the program is working on small examples check it on 10×10 matrix

```
A = compan(poly(1:10));
```

(companion matrix to polynomial with roots in i 1,2,...,10 which we know has eigenvalues exactly 1,2,...,10). Either it works and it is time to sen the program or crashes the program. Probably this depends on unnecessary matrix multiplication (high powers of large matrices as A above contains the numbers that are larger than 10^{16} which results to approximation errors. Thus you need to reduce the number of multiplications, especially when the multiplicity is low. Often you need only small modifications.
If you did not used that the eigenvalues are real it is possible that your program works with Gauss numbers (complex integers) as in matrix as

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Sending Put both your functions after each other i a file with jordanmatris first and heltalsev last, give the name jordanm.m (nothing else!) and send (not later than 2/11) the file as attachment by e-mail to victor.ufnarovski@math.lth.se Use miniprojekt-1 as subject. Check that the program is commented and easy to follow. Do not forget to write who have written it!

Good luck!