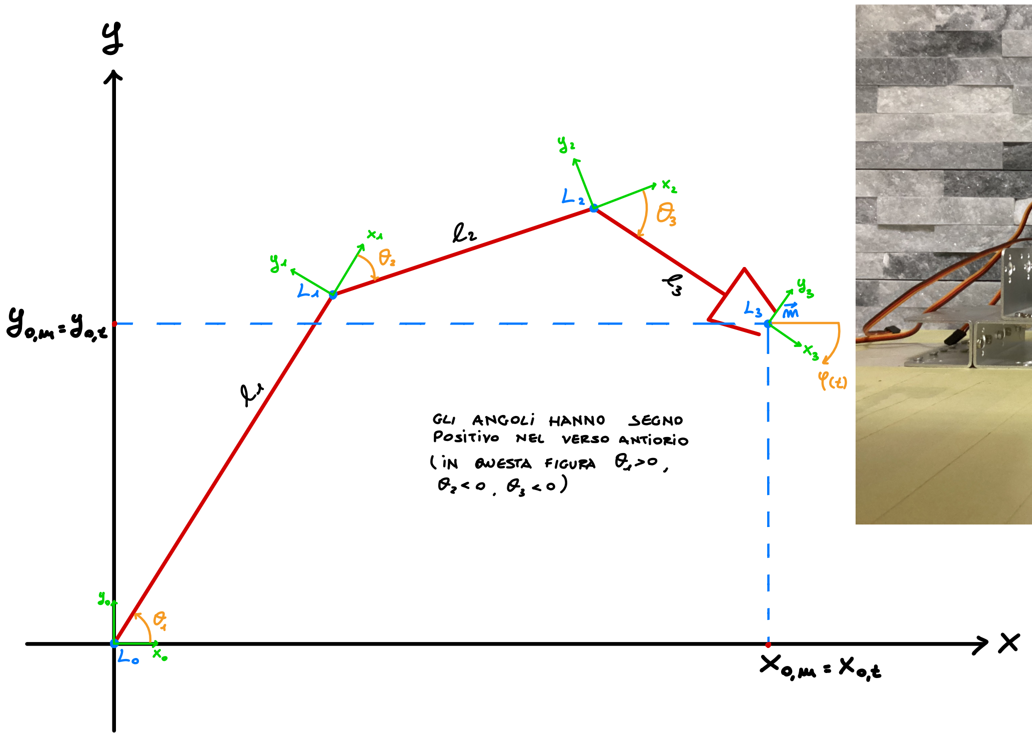
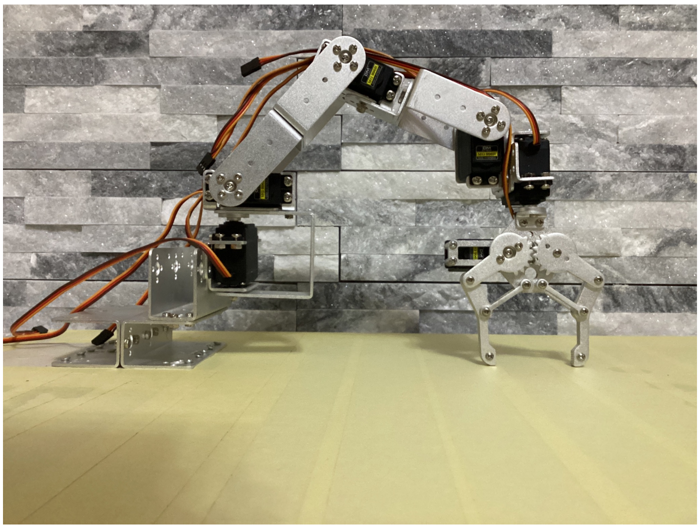
CINEMATICA DIRETTA (PLANARE)





- $$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = T_{23} \begin{pmatrix} x_3 \\ y_3 \\ 1 \end{pmatrix}$$
- $$\begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = T_{12} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = T_{12} T_{23} \begin{pmatrix} x_3 \\ y_3 \\ 1 \end{pmatrix}$$
- $$\begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix} = T_{01} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = T_{01} T_{12} T_{23} \begin{pmatrix} x_3 \\ y_3 \\ 1 \end{pmatrix}$$

- $$T_{23} = \begin{pmatrix} c\theta_3 & -s\theta_3 & \ell_3\, c\theta_3 \\ s\theta_3 & c\theta_3 & \ell_3\, s\theta_3 \\ 0 & 0 & 1 \end{pmatrix}$$
- $$T_{12} = \begin{pmatrix} c\theta_2 & -s\theta_2 & \ell_2\, c\theta_2 \\ s\theta_2 & c\theta_2 & \ell_2\, s\theta_2 \\ 0 & 0 & 1 \end{pmatrix}$$
- $$T_{01} = \begin{pmatrix} c\theta_1 & -s\theta_1 & \ell_1\, c\theta_1 \\ s\theta_1 & c\theta_1 & \ell_1\, s\theta_1 \\ 0 & 0 & 1 \end{pmatrix}$$

FORMULE ADD./SOT.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\bullet\ T_{12}T_{23} = \begin{pmatrix} c\theta_2 & -s\theta_2 & l_2\, c\theta_2 \\ s\theta_2 & c\theta_2 & l_2\, s\theta_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_3 & -s\theta_3 & l_3\, c\theta_3 \\ s\theta_3 & c\theta_3 & l_3\, s\theta_3 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_2c_3 - s_2s_3 & -c_2s_3 - s_2c_3 & l_3c_2c_3 - l_3s_2s_3 + l_2c_1 \\ c_2s_3 + s_2c_3 & c_2c_3 - s_2s_3 & l_3c_2s_3 - l_3s_2c_3 + l_2s_1 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(\overbrace{\theta_2+\theta_3}^{\theta_2+\theta_3=\theta'}) & -\sin(\theta_2+\theta_3) & l_3\cos(\theta_2+\theta_3) + l_2\cos\theta_2 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & l_3\sin(\theta_2+\theta_3) + l_2\sin\theta_2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\theta' & -\sin\theta' & l_3\cos\theta' + l_2\cos\theta_2 \\ \sin\theta' & \cos\theta' & l_3\sin\theta' + l_2\sin\theta_2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c' & -s' & l_3c' + l_2c_2 \\ s' & c' & l_3s' + l_2s_2 \\ 0 & 0 & 1 \end{pmatrix}$$

- $$T_{01}T_{12}T_{23} = T_{01}\begin{pmatrix} c' & -s' & l_3 c' + l_2 c_2 \\ s' & c' & l_3 s' + l_2 s_2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_1 & -s_1 & l_1 c_1 \\ s_1 & c_1 & l_1 s_1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c' & -s' & l_3 c' + l_2 c_2 \\ s' & c' & l_3 s' + l_2 s_2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_1c' - s_1s' & -c_1s' - s_1c' & l_3 c_1c' + l_2 c_1c_2 - l_3 s_1s' - l_2 s_1s_2 + l_1c_1 \\ c_1s' + s_1c' & c_1c' - s_1s' & l_3 s_1c' + l_2 s_1c_2 + l_3 c_1s' + l_2 c_1s_2 + l_1 s_1 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(\theta_1+\theta') & -\sin(\theta_1+\theta') & l_3\cos(\theta_1+\theta') + l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ \sin(\theta_1+\theta') & \cos(\theta_1+\theta') & l_3\sin(\theta_1+\theta') + l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 & 0 & 1 \end{pmatrix} =$$

SOTTOMATRICE DI ROTAZIONE $R(\theta_1,\theta_2,\theta_3)$

$$= \begin{pmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & l_3\cos(\theta_1+\theta_2+\theta_3) + l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ \sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & l_3\sin(\theta_1+\theta_2+\theta_3) + l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 & 0 & 1 \end{pmatrix} =$$

si può intuire che con n giunti la matrice risulta:

- $$T_{01}T_{12}\cdots T_{(n-2)(n-1)} = \begin{pmatrix} \cos(\theta_1+\cdots+\theta_{n-1}) & -\sin(\theta_1+\cdots+\theta_{n-1}) & l_{n-1}\cos(\theta_1+\cdots+\theta_{n-1}) + l_{n-2}\cos(\theta_1+\cdots+\theta_{n-2}) + \cdots + l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ \sin(\theta_1+\cdots+\theta_{n-1}) & \cos(\theta_1+\cdots+\theta_{n-1}) & l_{n-1}\sin(\theta_1+\cdots+\theta_{n-1}) + l_{n-2}\cos(\theta_1+\cdots+\theta_{n-2}) + \cdots + l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 & 0 & 1 \end{pmatrix}$$

possiamo concludere la trattazione della cinematica diretta:

$$\begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_{123} & -s_{123} & l_3 c_{123} + l_2 c_{12} + l_1 c_1 \\ s_{123} & c_{123} & l_3 s_{123} + l_2 s_{12} + l_1 s_1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{3,t} \\ y_{3,t} \\ 1 \end{pmatrix}$$

COORDINATE DI UN PUNTO RISPETTO AL SIS. DI RIF x_0, y_0

COORDINATE DI UN PUNTO RISPETTO AL SIS. DI RIF x_3, y_3

ricaviamo le coordinate della pinza rispetto a sistema L_0

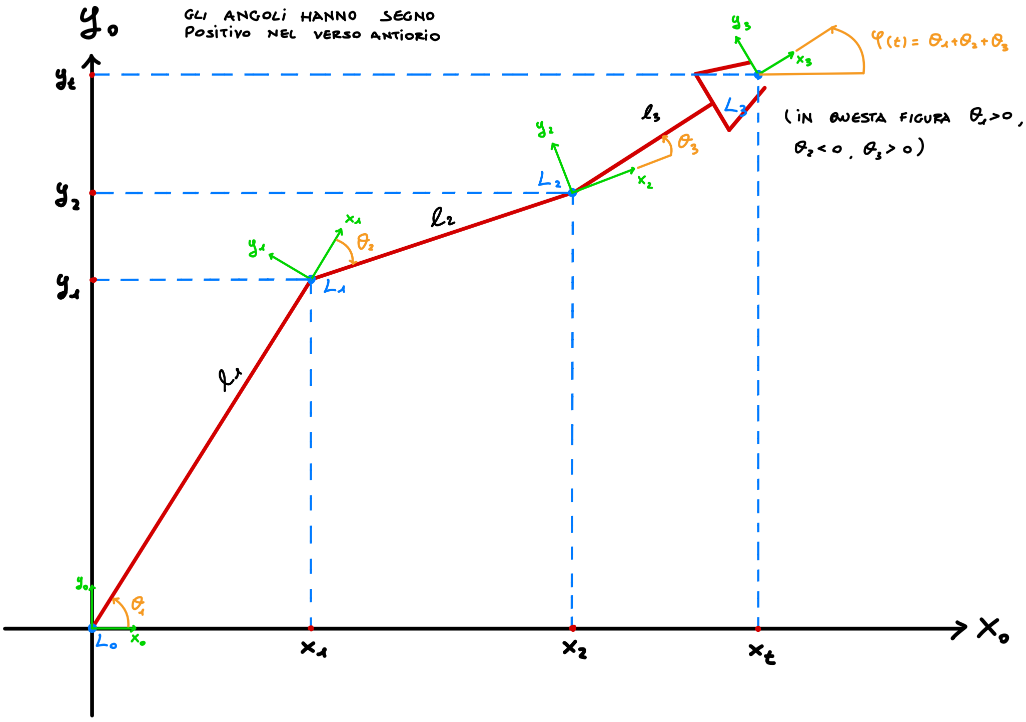
$$\begin{pmatrix} x_t \\ y_t \\ 1 \end{pmatrix} = \begin{pmatrix} c_{123} & -s_{123} & l_3 c_{123} + l_2 c_{12} + l_1 c_1 \\ s_{123} & c_{123} & l_3 s_{123} + l_2 s_{12} + l_1 s_1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{cases} x_t = x_{0,t} = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3) \\ y_t = y_{0,t} = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3) \\ \varphi(t) = \theta_1+\theta_2+\theta_3 \end{cases}$$

CINEMATICA INVERSA (PLANARE)

$$\begin{cases} x_1 = l_1 \cos\theta_1 \\ y_1 = l_1 \sin\theta_1 \end{cases}$$

$$\begin{cases} x_2 = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ y_2 = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{cases}$$



$$\begin{cases} x_t = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3) \\ y_t = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3) \end{cases}$$

INIZIAMO STUDIANDO SEPARATAMENTE I PRIMI 2 LINK:

- $x_2 = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2)$
- $y_2 = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)$

FACCIAMO I QUADRATI:

$$x_2^2 + y_2^2 = l_1^2 c_1^2 + l_2^2 c_{12}^2 + 2 l_1 l_2 c_1 c_{12} + l_1^2 s_1^2 + l_2^2 s_{12}^2 + 2 l_1 l_2 s_1 s_{12} =$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta_1 - (\theta_1+\theta_2)) =$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 \cos(-\theta_2) =$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2$$

$$\cos\theta_2 = \frac{x_2^2 + y_2^2 - l_1^2 - l_2^2}{2 l_1 l_2} \longrightarrow \bullet\ \theta_2 = \pm \arccos\left(\frac{x_2^2 + y_2^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$\longrightarrow$

- $x_t = x_2 + l_3 \cos(\theta_1+\theta_2+\theta_3) \longrightarrow x_2 = x_t - l_3 \cos(\underbrace{\theta_1+\theta_2+\theta_3}_{\varphi(t)})$
- $y_t = y_2 + l_3 \sin(\theta_1+\theta_2+\theta_3) \longrightarrow y_2 = y_t - l_3 \sin(\underbrace{\theta_1+\theta_2+\theta_3}_{\varphi(t)})$

$$\Longrightarrow \boxed{\theta_2 = \pm \arccos\left(\frac{(x_t - l_3 \cos\varphi(t))^2 + (y_t - l_3 \sin\varphi(t))^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)}$$

TROVIAMO θ_1.

$$x_2 = \ell_1 c_1 + \ell_2 c_{12} = \ell_1 c_1 + \ell_2 (c_1 c_2 - s_1 s_2) = (\ell_1 + \ell_2 c_2) c_1 - \ell_2 s_2 \cdot s_1$$

$$y_2 = \ell_1 s_1 + \ell_2 s_{12} = \ell_1 s_1 + \ell_2 (s_1 c_2 + c_1 s_2) = \ell_2 s_2 \cdot c_1 + (\ell_1 + \ell_2 c_2) s_1$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} \ell_1 + \ell_2 c_2 & -\ell_2 s_2 \\ \ell_2 s_2 & \ell_1 + \ell_2 c_2 \end{pmatrix} \begin{pmatrix} c_1 \\ s_1 \end{pmatrix} \longleftarrow \quad b = A x$$

$$x = A^{-1} b \longrightarrow \begin{pmatrix} c_1 \\ s_1 \end{pmatrix} = \frac{1}{\det(A)} \begin{pmatrix} \ell_1 + \ell_2 c_2 & +\ell_2 s_2 \\ -\ell_2 s_2 & \ell_1 + \ell_2 c_2 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

$$c_1 = \frac{1}{\det(A)} \cdot \left[(\ell_1 + \ell_2 c_2) x_2 + \ell_2 s_2 y_2 \right]$$

$$s_1 = \frac{1}{\det(A)} \left[-\ell_2 s_2 x_2 + (\ell_1 + \ell_2 c_2) y_2 \right]$$

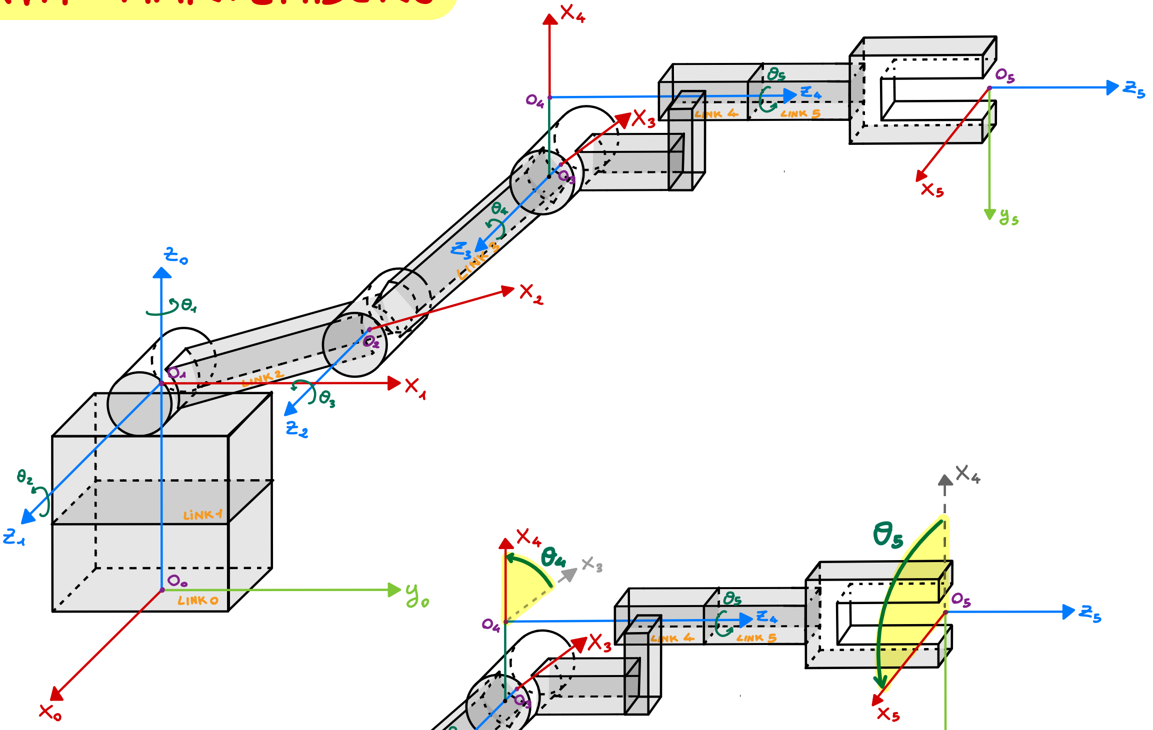
$$\theta_1 = \operatorname{arctg} \frac{s_1}{c_1} = \operatorname{arctg} \left(\frac{-\ell_2 s_2 x_2 + (\ell_1 + \ell_2 c_2) y_2}{(\ell_1 + \ell_2 c_2) x_2 + \ell_2 s_2 y_2} \right)$$

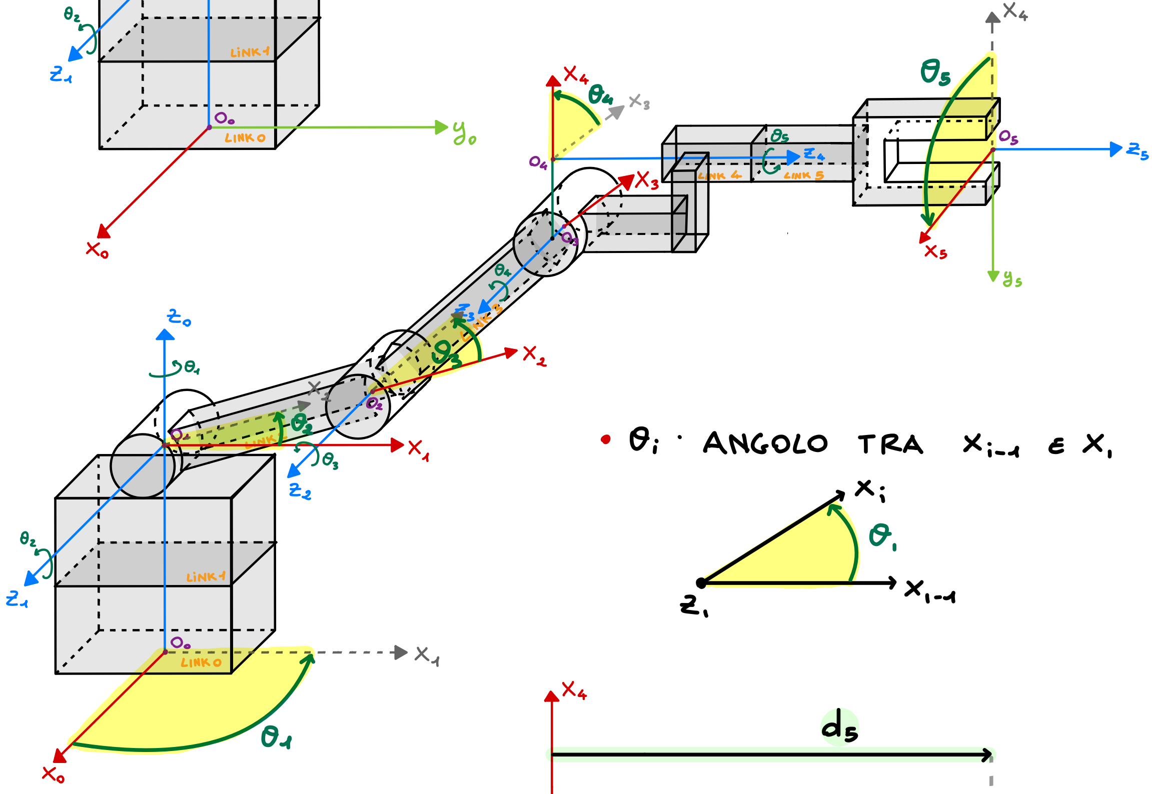
$$\longrightarrow \quad \theta_1 = \operatorname{arctg2} \left(-\ell_2 s_2 x_2 + (\ell_1 + \ell_2 c_2) y_2 \, , \, (\ell_1 + \ell_2 c_2) x_2 + \ell_2 s_2 y_2 \right)$$

$$\Longrightarrow \boxed{\theta_1 = \operatorname{arctg2} \Big(-\ell_2 s_2 (x_t - \ell_3 \cos\varphi(t)) + (\ell_1 + \ell_2 c_2)(y_t - \ell_3 \sin\varphi(t)) \, , \, (\ell_1 + \ell_2 c_2)(x_t - \ell_3 \cos\varphi(t)) + \ell_2 s_2 (y_t - \ell_3 \sin\varphi(t)) \Big)}$$

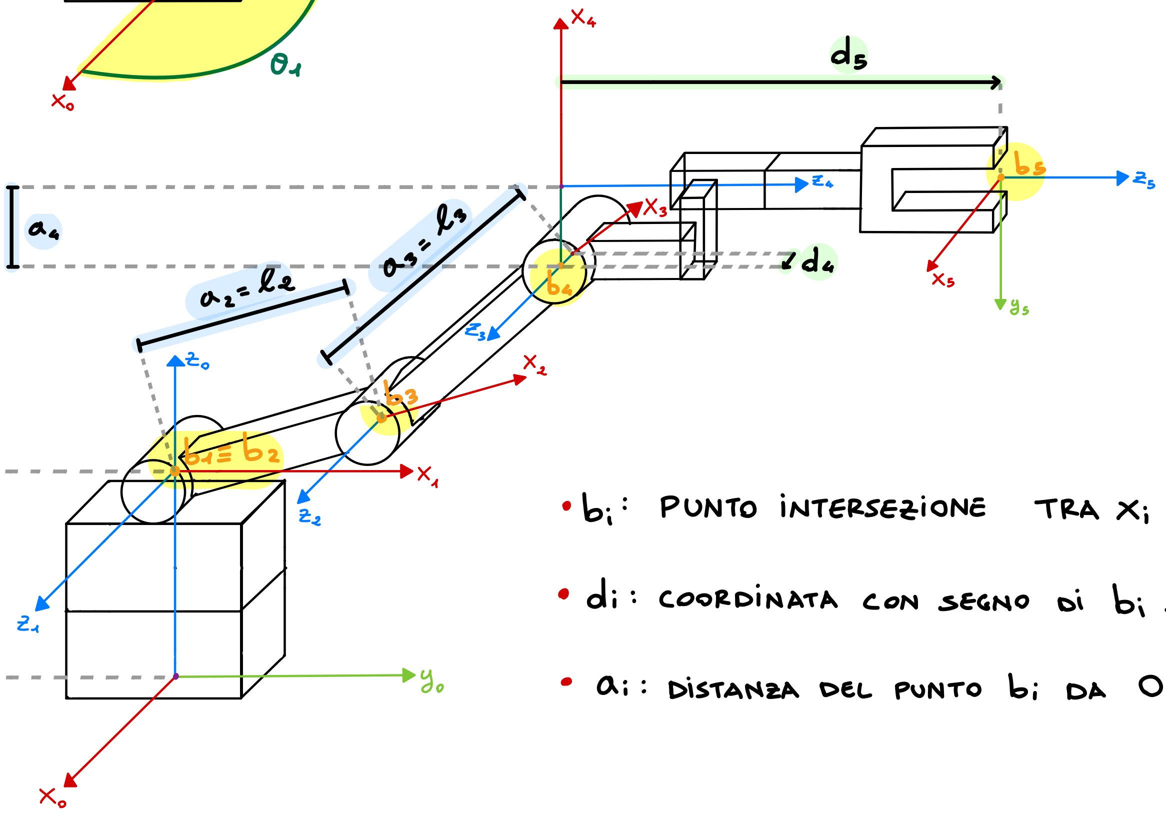
$$\Longrightarrow \boxed{\theta_3 = \varphi(t) - \theta_1 - \theta_2}$$

DENAVIT-HARTENBERG



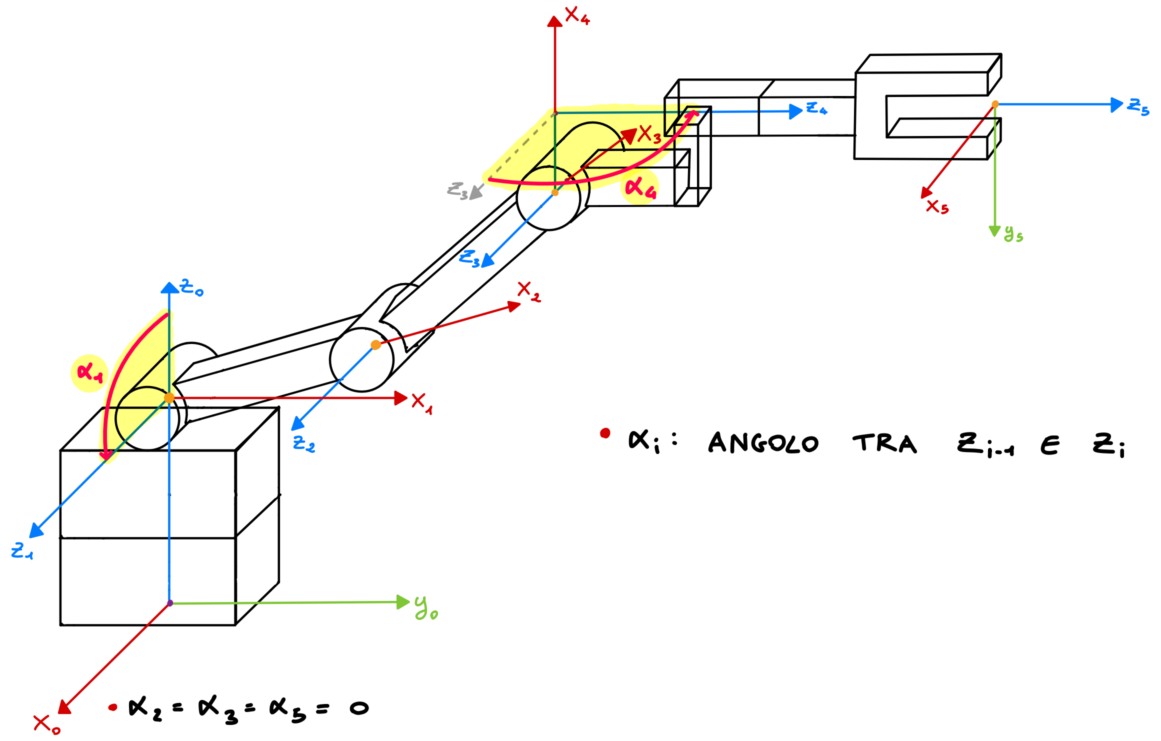


- θ_i : ANGOLO TRA x_{i-1} E x_i VISTO DA z_{i-1}



- b_i : PUNTO INTERSEZIONE TRA x_i E z_{i-1}
- d_i : COORDINATA CON SEGNO DI b_i SU z_{i-1}
- a_i : DISTANZA DEL PUNTO b_i DA O_i

- $d_2 = d_3 = 0$
- $a_1 = a_5 = 0$
- $a_2 = l_2$
- $a_3 = l_3$



- α_i : ANGOLO TRA z_{i-1} E z_i VISTO DA x_i

- $\alpha_2 = \alpha_3 = \alpha_5 = 0$
- $\alpha_1 = \alpha_4 = \frac{\pi}{2}$

quindi possiamo riassumere :

i	θ_i	d_i	a_i	α_i	
1	θ_1^*	d_1	$\emptyset$	$\frac{\pi}{2}$	$\rightarrow T_0^1$
2	θ_2^*	$\emptyset$	ℓ_2	$\emptyset$	$\rightarrow T_1^2$
3	θ_3^*	$\emptyset$	ℓ_3	$\emptyset$	$\rightarrow T_2^3$
4	θ_4^*	d_4	a_4	$\frac{\pi}{2}$	$\rightarrow T_3^4$
5	θ_5^*	d_5	$\emptyset$	$\emptyset$	$\rightarrow T_4^5$

PER PASSARE DA L_{i-1} A L_i LE OPERAZIONI DA ESEGUIRE SONO:

1) ROTAZIONE DI UN ANGOLO θ_i DELLA TERNA L_i INTORNO A z_{i-1}
2) TRASLAZIONE DI d_i DELLA TERNA L_i LUNGO z_{i-1}

$\Big\}$ SU ASSE z
- $T_z(\theta_i, d_i)$

3) ROTAZIONE DI UN ANGOLO α_i DELLA TERNA L_i INTORNO A x_i
4) TRASLAZIONE DI a_i DELLA TERNA L_i LUNGO x_i

$\Big\}$ SU ASSE x
- $T_x(\alpha_i, a_i)$

PER QUESTO ROBOT SARÁ:

ROTAZIONE DI UN ANGOLO θ_1 DELLA TERNA L_1 INTORNO A z_0
TRASLAZIONE DI d_1 DELLA TERNA L_1 LUNGO z_0
ROTAZIONE DI $\frac{\pi}{2}$ DELLA TERNA L_1 INTORNO A x_1
$\Big\} \; T_0^1$

ROTAZIONE DI UN ANGOLO θ_2 DELLA TERNA L_2 INTORNO A z_1
TRASLAZIONE DI ℓ_2 DELLA TERNA L_2 LUNGO x_2
$\Big\} \; T_1^2$

ROTAZIONE DI UN ANGOLO θ_3 DELLA TERNA L_3 INTORNO A z_2
TRASLAZIONE DI ℓ_3 DELLA TERNA L_3 LUNGO x_3
$\Big\} \; T_2^3$

ROTAZIONE DI UN ANGOLO θ_4 DELLA TERNA L_4 INTORNO A z_3
TRASLAZIONE DI d_4 DELLA TERNA L_4 LUNGO z_3
TRASLAZIONE DI a_4 DELLA TERNA L_4 LUNGO x_4
ROTAZIONE DI $\frac{\pi}{2}$ DELLA TERNA L_4 INTORNO A x_4
$\Big\} \; T_3^4$

ROTAZIONE DI UN ANGOLO θ_5 DELLA TERNA L_5 INTORNO A z_4
TRASLAZIONE DI d_5 DELLA TERNA L_5 LUNGO z_4
$\Big\} \; T_4^5$

IN GENERALE LA MATRICE DEL CAMBIAMENTO DELLE COORDINATE T_{i-1}^i RISULTA:

$$\longrightarrow T_{i-1}^i = T_z(\theta_i, d_i)\, T_x(\alpha_i, a_i)$$

$$\longrightarrow T_z(\theta_i, d_i) = \left(\begin{array}{c|c} R_z(\theta_i) & \vec{t} \\ \hline 0\;0\;0 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow T_x(\alpha_i, a_i) = \left(\begin{array}{c|c} R_x(\alpha_i) & \vec{p} \\ \hline 0\;0\;0 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow T_{i-1}^i = \left(\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$= \begin{pmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $T_0^1 = \left(\begin{array}{ccc|c} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$

- $T_1^2 = \left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$

- $T_2^3 = \left(\begin{array}{ccc|c} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$

- $T_3^4 = \left(\begin{array}{ccc|c} c\theta_4 & 0 & s\theta_4 & a_4 c\theta_4 \\ s\theta_4 & 0 & -c\theta_4 & a_4 s\theta_4 \\ 0 & 1 & 0 & d_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$

- $T_4^5 = \left(\begin{array}{ccc|c} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$

i	θ_i	d_i	a_i	α_i	
1	θ_1^*	d_1	$\emptyset$	$\frac{\pi}{2}$	$\rightarrow T_0^1$
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3	θ_3^*	$\emptyset$	ℓ_3	$\emptyset$	$\rightarrow T_2^3$
4	θ_4^*	$\emptyset$	$\emptyset$	$\frac{\pi}{2}$	$\rightarrow T_3^4$
5	θ_5^*	d_5	$\emptyset$	$\emptyset$	$\rightarrow T_4^5$

$\longrightarrow T_0^5 = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 = \dots$

- $T_0^2 = T_0^1 T_1^2 = \left(\begin{array}{ccc|c} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) =$

$$= \begin{pmatrix} c_1 c_2 & -c_1 s_2 & s_1 & \ell_2 c_1 c_2 \\ s_1 c_2 & -s_1 s_2 & -c_1 & \ell_2 s_1 c_2 \\ s_2 & c_2 & 0 & d_1 + \ell_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $T_1^3 = T_1^2 T_2^3 = \left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) =$

$$= \begin{pmatrix} c_2 c_3 - s_2 s_3 & -c_2 s_3 - s_2 c_3 & 0 & \ell_3 c_2 c_3 - \ell_3 s_2 s_3 + \ell_2 c_2 \\ s_2 c_3 + c_2 s_3 & -s_2 s_3 + c_2 c_3 & 0 & \ell_3 s_2 c_3 + \ell_3 c_2 s_3 + \ell_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_{23} & -S_{23} & 0 & \ell_3 C_{23} + \ell_2 C_2 \\ S_{23} & C_{23} & 0 & \ell_3 S_{23} + \ell_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\bullet\ T_0^3 = T_0^1 T_1^2 T_2^3 = T_0^1 T_1^3 =$$

$$= \begin{pmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{23} & -s_{23} & 0 & \ell_3 c_{23} + \ell_2 c_2 \\ s_{23} & c_{23} & 0 & \ell_3 s_{23} + \ell_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & \ell_3 c_1 c_{23} + \ell_2 c_1 c_2 \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & \ell_3 s_1 c_{23} + \ell_2 s_1 c_2 \\ s_{23} & c_{23} & 0 & d_1 + \ell_3 s_{23} + \ell_2 s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\bullet\ T_2^4 = T_2^3 T_3^4 = \left(\begin{array}{ccc|c} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} c\theta_4 & 0 & s\theta_4 & a_4 c\theta_4 \\ s\theta_4 & 0 & -c\theta_4 & a_4 s\theta_4 \\ 0 & 1 & 0 & d_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) =$$

$$= \begin{pmatrix} c_3 c_4 - s_3 s_4 & 0 & c_3 s_4 + s_3 c_4 & a_4 c_3 c_4 - a_4 s_3 s_4 + \ell_3 c_3 \\ s_3 c_4 + c_3 s_4 & 0 & s_3 s_4 - c_3 c_4 & a_4 s_3 c_4 + a_4 c_3 s_4 + \ell_3 s_3 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{34} & 0 & s_{34} & a_4 c_{34} + \ell_3 c_3 \\ s_{34} & 0 & -c_{34} & a_4 s_{34} + \ell_3 s_3 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\bullet\ T_1^4 = T_1^2 T_2^4 = \left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} c_{34} & 0 & s_{34} & a_4 c_{34} + \ell_3 c_3 \\ s_{34} & 0 & -c_{34} & a_4 s_{34} + \ell_3 s_3 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_2 c_{34} - s_2 s_{34} & 0 & c_2 s_{34} + s_2 c_{34} & a_4 c_2 c_{34} - a_4 s_2 s_{34} + \ell_3 c_2 c_3 - \ell_3 s_2 s_3 + \ell_2 c_2 \\ s_2 c_{34} + c_2 s_{34} & 0 & s_2 s_{34} - c_2 c_{34} & a_4 s_2 c_{34} + a_4 c_2 s_{34} + \ell_3 s_2 c_3 + \ell_3 c_2 s_3 + \ell_2 s_2 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_{234} & 0 & s_{234} & a_4 c_{234} + \ell_3 c_{23} + \ell_2 c_2 \\ s_{234} & 0 & -c_{234} & a_4 s_{234} + \ell_3 s_{23} + \ell_2 s_2 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\bullet\ T_0^4 = T_0^1 T_1^4 = \begin{pmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{234} & 0 & s_{234} & a_4 c_{234} + \ell_3 c_{23} + \ell_2 c_2 \\ s_{234} & 0 & -c_{234} & a_4 s_{234} + \ell_3 s_{23} + \ell_2 s_2 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_1C_{234} & S_1 & C_1S_{234} & a_4C_1C_{234} + \ell_3C_1C_{23} + \ell_2C_1C_2 + d_4S_1 \\ S_1C_{234} & -C_1 & S_1S_{234} & a_4S_1C_{234} + \ell_3S_1C_{23} + \ell_2S_1C_2 - d_4C_1 \\ S_{234} & 0 & -C_{234} & a_4S_{234} + \ell_3S_{23} + \ell_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $T_3^5 = T_3^4 T_4^5 = \begin{pmatrix} c\theta_4 & 0 & s\theta_4 & a_4c\theta_4 \\ s\theta_4 & 0 & -c\theta_4 & a_4s\theta_4 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} C_4C_5 & -C_4S_5 & S_4 & a_4c\theta_4 + d_5S_4 \\ S_4C_5 & -S_4S_5 & -C_4 & a_4s\theta_4 - d_5C_4 \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $T_2^5 = T_2^4 T_4^5 = \begin{pmatrix} C_{34} & 0 & S_{34} & a_4C_{34} + \ell_3C_3 \\ S_{34} & 0 & -C_{34} & a_4S_{34} + \ell_3S_3 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} C_{34}C_5 & -C_{34}S_5 & S_{34} & a_4C_{34} + \ell_3C_3 + d_5S_{34} \\ S_{34}C_5 & -S_{34}S_5 & -C_{34} & a_4S_{34} + \ell_3S_3 - d_5C_{34} \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $T_1^5 = T_1^4 T_4^5 = \begin{pmatrix} C_{234} & 0 & S_{234} & a_4C_{234} + \ell_3C_{23} + \ell_2C_2 \\ S_{234} & 0 & -C_{234} & a_4S_{234} + \ell_3S_{23} + \ell_2S_2 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} C_{234}C_5 & -C_{234}S_5 & S_{234} & a_4C_{234} + \ell_3C_{23} + \ell_2C_2 + d_5S_{234} \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & a_4S_{234} + \ell_3S_{23} + \ell_2S_2 - d_5C_{234} \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $T_0^5 = T_0^4 \, T_4^5 =$

$$= \begin{pmatrix} C_1C_{234} & S_1 & C_1S_{234} & a_4C_1C_{234} + \ell_3C_1C_{23} + \ell_2C_1C_2 + d_4S_1 \\ S_1C_{234} & -C_1 & S_1S_{234} & a_4S_1C_{234} + \ell_3S_1C_{23} + \ell_2S_1C_2 - d_4C_1 \\ S_{234} & 0 & -C_{234} & a_4S_{234} + \ell_3S_{23} + \ell_2S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & C_1S_{234} & a_4C_1C_{234} + \ell_3C_1C_{23} + \ell_2C_1C_2 + d_4S_1 + d_5C_1S_{234} \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & S_1S_{234} & a_4S_1C_{234} + \ell_3S_1C_{23} + \ell_2S_1C_2 - d_4C_1 + d_5S_1S_{234} \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & a_4S_{234} + \ell_3S_{23} + \ell_2S_2 + d_1 - d_5C_{234} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

VERIFICA:

- $T_0^5 = T_0^1 T_1^5 =$

$$= \begin{pmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{234}C_5 & -C_{234}S_5 & S_{234} & a_4C_{234} + l_3C_{23} + l_2C_2 + d_5S_{234} \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & a_4S_{234} + l_3S_{23} + l_2S_2 - d_5C_{234} \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & C_1S_{234} & a_4C_1C_{234} + l_3C_1C_{23} + l_2C_1C_2 + d_4S_1 + d_5C_1S_{234} \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & S_1S_{234} & a_4S_1C_{234} + l_3S_1C_{23} + l_2S_1C_2 - d_4C_1 + d_5S_1S_{234} \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & a_4S_{234} + l_3S_{23} + l_2S_2 + d_1 - d_5C_{234} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

è uguale alla matrice calcolata precedentemente

- $T_0^5 = T_0^2 T_2^5 =$

$$= \begin{pmatrix} C_1C_2 & -C_1S_2 & S_1 & l_2C_1C_2 \\ S_1C_2 & -S_1S_2 & -C_1 & l_2S_1C_2 \\ S_2 & C_2 & 0 & d_1 + l_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{34}C_5 & -C_{34}S_5 & S_{34} & a_4C_{34} + l_3C_3 + d_5S_{34} \\ S_{34}C_5 & -S_{34}S_5 & -C_{34} & a_4S_{34} + l_3S_3 - d_5C_{34} \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & C_1S_{234} & a_4C_1C_{234} + l_3C_1C_{23} + l_2C_1C_2 + d_4S_1 + d_5C_1S_{234} \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & S_1S_{234} & a_4S_1C_{234} + l_3S_1C_{23} + l_2S_1C_2 - d_4C_1 + d_5S_1S_{234} \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & a_4S_{234} + l_3S_{23} + l_2S_2 + d_1 - d_5C_{234} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

COLCOLO LE COORDINATE DEI VARI SISTEMI DI RIFERIMENTO L_i RISPETTO AI SISTEMI DI RIFERIMENTO PRECEDENTI AD L_i STESSO

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{0,1}} \\ \vdots \\ 1 \end{pmatrix} = T_0^1 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ d_1 \\ 1 \end{pmatrix}$$

COORDINATE DI O_1 RISPETTO A L_0: $\overrightarrow{O_{0,1}} = \begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{0,2}} \\ \vdots \\ 1 \end{pmatrix} = T_0^2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1c_2 & -c_1s_2 & s_1 & \ell_2c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & \ell_2s_1c_2 \\ s_2 & c_2 & 0 & d_1+\ell_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \ell_2c_1c_2 \\ \ell_2s_1c_2 \\ d_1+\ell_2 s\theta_2 \\ 1 \end{pmatrix}$$

COORDINATE DI O_2 RISPETTO A L_0: $\overrightarrow{O_{0,2}} = \begin{pmatrix} \ell_2c_1c_2 \\ \ell_2s_1c_2 \\ d_1+\ell_2 s\theta_2 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{1,2}} \\ \vdots \\ 1 \end{pmatrix} = T_1^2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \ell_2c_2 \\ \ell_2s_2 \\ 0 \\ 1 \end{pmatrix}$$

COORDINATE DI O_2 RISPETTO A L_1: $\overrightarrow{O_{1,2}} = \begin{pmatrix} \ell_2c_2 \\ \ell_2s_2 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{0,3}} \\ \vdots \\ 1 \end{pmatrix} = T_0^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1c_{23} & -c_1s_{23} & s_1 & \ell_3c_1c_{23}+\ell_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & \ell_3s_1c_{23}+\ell_2s_1c_2 \\ s_{23} & c_{23} & 0 & d_1+\ell_3s_{23}+\ell_2s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \ell_3c_1c_{23}+\ell_2c_1c_2 \\ \ell_3s_1c_{23}+\ell_2s_1c_2 \\ d_1+\ell_3s_{23}+\ell_2s_2 \\ 1 \end{pmatrix}$$

COORDINATE DI O_3 RISPETTO A L_0: $\overrightarrow{O_{0,3}} = \begin{pmatrix} \ell_3c_1c_{23}+\ell_2c_1c_2 \\ \ell_3s_1c_{23}+\ell_2s_1c_2 \\ d_1+\ell_3s_{23}+\ell_2s_2 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{1,3}} \\ \vdots \\ 1 \end{pmatrix} = T_1^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_{23} & -s_{23} & 0 & \ell_3 c_{23} + \ell_2 c_2 \\ s_{23} & c_{23} & 0 & \ell_3 s_{23} + \ell_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \ell_3 c_{23} + \ell_2 c_2 \\ \ell_3 s_{23} + \ell_2 s_2 \\ 0 \\ 1 \end{pmatrix}$$

COORDINATE DI O_3 RISPETTO A L_1 :

$$\overrightarrow{O_{1,3}} = \begin{pmatrix} \ell_3 c_{23} + \ell_2 c_2 \\ \ell_3 s_{23} + \ell_2 s_2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{2,3}} \\ \vdots \\ 1 \end{pmatrix} = T_2^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \ell_3 c\theta_3 \\ \ell_3 s\theta_3 \\ 0 \\ 1 \end{pmatrix}$$

COORDINATE DI O_3 RISPETTO A L_2 :

$$\overrightarrow{O_{2,3}} = \begin{pmatrix} \ell_3 c\theta_3 \\ \ell_3 s\theta_3 \\ 0 \end{pmatrix}$$

$$\begin{matrix} \ell_3 C_1 C_{23} + \ell_2 C_1 C_2 \\ \ell_3 S_1 C_{23} + \ell_2 S_1 C_2 \\ d_1 + \ell_3 s_{23} + \ell_2 s_2 \\ 1 \end{matrix}$$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{0,4}} \\ \vdots \\ 1 \end{pmatrix} = T_0^4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_1 C_{234} & S_1 & C_1 S_{234} & a_4 C_1 C_{234} + \ell_3 C_1 C_{23} + \ell_2 c_1 c_2 + d_4 s_1 \\ S_1 C_{234} & -C_1 & S_1 S_{234} & a_4 S_1 C_{234} + \ell_3 S_1 C_{23} + \ell_2 S_1 C_2 - d_4 C_1 \\ S_{234} & 0 & -C_{234} & a_4 S_{234} + \ell_3 S_{23} + \ell_2 S_2 + d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_4 C_1 C_{234} + \ell_3 C_1 C_{23} + \ell_2 c_1 c_2 + d_4 s_1 \\ a_4 S_1 C_{234} + \ell_3 S_1 C_{23} + \ell_2 S_1 C_2 - d_4 C_1 \\ a_4 S_{234} + \ell_3 S_{23} + \ell_2 S_2 + d_1 \\ 1 \end{pmatrix}$$

COORDINATE DI O_4 RISPETTO A L_0 :

$$\overrightarrow{O_{0,4}} = \begin{pmatrix} a_4 C_1 C_{234} + \ell_3 C_1 C_{23} + \ell_2 c_1 c_2 + d_4 s_1 \\ a_4 S_1 C_{234} + \ell_3 S_1 C_{23} + \ell_2 S_1 C_2 - d_4 C_1 \\ a_4 S_{234} + \ell_3 S_{23} + \ell_2 S_2 + d_1 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \overrightarrow{O_{1,4}} \\ \vdots \\ 1 \end{pmatrix} = T_1^4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} C_{234} & 0 & S_{234} & a_4 C_{234} + \ell_3 C_{23} + \ell_2 C_2 \\ S_{234} & 0 & -C_{234} & a_4 S_{234} + \ell_3 S_{23} + \ell_2 S_2 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_4 C_{234} + \ell_3 C_{23} + \ell_2 C_2 \\ a_4 S_{234} + \ell_3 S_{23} + \ell_2 S_2 \\ d_4 \\ 1 \end{pmatrix}$$

COORDINATE DI O_4 RISPETTO A L_1 :

$$\overrightarrow{O_{1,4}} = \begin{pmatrix} a_4 C_{234} + \ell_3 C_{23} + \ell_2 C_2 \\ a_4 S_{234} + \ell_3 S_{23} + \ell_2 S_2 \\ d_4 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \vec{O}_{2,4} \\ \vdots \\ 1 \end{pmatrix} = T_2^4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_{34} & 0 & s_{34} & a_4 c_{34} + \ell_3 c_3 \\ s_{34} & 0 & -c_{34} & a_4 s_{34} + \ell_3 s_3 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_4 c_{34} + \ell_3 c_3 \\ a_4 s_{34} + \ell_3 s_3 \\ d_4 \\ 1 \end{pmatrix}$$

COORDINATE DI O_4 RISPETTO A L_2 : $\vec{O}_{2,4} = \begin{pmatrix} a_4 c_{34} + \ell_3 c_3 \\ a_4 s_{34} + \ell_3 s_3 \\ d_4 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \vec{O}_{3,4} \\ \vdots \\ 1 \end{pmatrix} = T_3^4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} c\theta_4 & 0 & s\theta_4 & a_4 c\theta_4 \\ s\theta_4 & 0 & -c\theta_4 & a_4 s\theta_4 \\ 0 & 1 & 0 & d_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_4 c\theta_4 \\ a_4 s\theta_4 \\ d_4 \\ 1 \end{pmatrix}$$

COORDINATE DI O_4 RISPETTO A L_3 : $\vec{O}_{3,4} = \begin{pmatrix} a_4 c\theta_4 \\ a_4 s\theta_4 \\ d_4 \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \vec{O}_{0,5} \\ \vdots \\ 1 \end{pmatrix} = T_0^5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_1 c_{234} c_5 + s_1 s_5 & -c_1 c_{234} s_5 + s_1 c_5 & c_1 s_{234} & a_4 c_1 c_{234} + \ell_3 c_1 c_{23} + \ell_2 c_1 c_2 + d_4 s_1 + d_5 c_1 s_{234} \\ s_1 c_{234} c_5 - c_1 s_5 & -s_1 c_{234} s_5 - c_1 c_5 & s_1 s_{234} & a_4 s_1 c_{234} + \ell_3 s_1 c_{23} + \ell_2 s_1 c_2 - d_4 c_1 + d_5 s_1 s_{234} \\ s_{234} c_5 & -s_{234} s_5 & -c_{234} & a_4 s_{234} + \ell_3 s_{23} + \ell_2 s_2 + d_1 - d_5 c_{234} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} a_4 c_1 c_{234} + \ell_3 c_1 c_{23} + \ell_2 c_1 c_2 + d_4 s_1 + d_5 c_1 s_{234} \\ a_4 s_1 c_{234} + \ell_3 s_1 c_{23} + \ell_2 s_1 c_2 - d_4 c_1 + d_5 s_1 s_{234} \\ a_4 s_{234} + \ell_3 s_{23} + \ell_2 s_2 + d_1 - d_5 c_{234} \\ 1 \end{pmatrix}$$

COORDINATE DI O_5 RISPETTO A L_0 : $\vec{O}_{0,5} = \begin{pmatrix} a_4 c_1 c_{234} + \ell_3 c_1 c_{23} + \ell_2 c_1 c_2 + d_4 s_1 + d_5 c_1 s_{234} \\ a_4 s_1 c_{234} + \ell_3 s_1 c_{23} + \ell_2 s_1 c_2 - d_4 c_1 + d_5 s_1 s_{234} \\ a_4 s_{234} + \ell_3 s_{23} + \ell_2 s_2 + d_1 - d_5 c_{234} \end{pmatrix}$

$$\begin{pmatrix} \vdots \\ \vec{O}_{1,5} \\ \vdots \\ 1 \end{pmatrix} = T_1^5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} C_{234}C_5 & -C_{234}S_5 & S_{234} & a_4C_{234}+\ell_3C_{23}+\ell_2C_2+d_5S_{234} \\ S_{234}C_5 & -S_{234}S_5 & -C_{234} & a_4S_{234}+\ell_3S_{23}+\ell_2S_2-d_5C_{234} \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} a_4C_{234}+\ell_3C_{23}+\ell_2C_2+d_5S_{234} \\ a_4S_{234}+\ell_3S_{23}+\ell_2S_2-d_5C_{234} \\ d_4 \\ 1 \end{pmatrix}$$

COORDINATE DI O_5 RISPETTO A L_1: $$\vec{O}_{1,5} = \begin{pmatrix} a_4C_{234}+\ell_3C_{23}+\ell_2C_2+d_5S_{234} \\ a_4S_{234}+\ell_3S_{23}+\ell_2S_2-d_5C_{234} \\ d_4 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \vec{O}_{2,5} \\ \vdots \\ 1 \end{pmatrix} = T_2^5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} C_{34}C_5 & -C_{34}S_5 & S_{34} & a_4C_{34}+\ell_3C_3+d_5S_{34} \\ S_{34}C_5 & -S_{34}S_5 & -C_{34} & a_4S_{34}+\ell_3S_3-d_5C_{34} \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} a_4C_{34}+\ell_3C_3+d_5S_{34} \\ a_4S_{34}+\ell_3S_3-d_5C_{34} \\ d_4 \\ 1 \end{pmatrix}$$

COORDINATE DI O_5 RISPETTO A L_2: $$\vec{O}_{2,5} = \begin{pmatrix} a_4C_{34}+\ell_3C_3+d_5S_{34} \\ a_4S_{34}+\ell_3S_3-d_5C_{34} \\ d_4 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \vec{O}_{3,5} \\ \vdots \\ 1 \end{pmatrix} = T_3^5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} C_4C_5 & -C_4S_5 & S_4 & a_4c\theta_4+d_5S_4 \\ S_4C_5 & -S_4S_5 & -C_4 & a_4s\theta_4-d_5C_4 \\ S_5 & C_5 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_4c\theta_4+d_5S_4 \\ a_4s\theta_4-d_5C_4 \\ d_4 \\ 1 \end{pmatrix}$$

COORDINATE DI O_5 RISPETTO A L_3: $$\vec{O}_{3,5} = \begin{pmatrix} a_4c\theta_4+d_5S_4 \\ a_4s\theta_4-d_5C_4 \\ d_4 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \vec{O}_{4,5} \\ \vdots \\ 1 \end{pmatrix} = T_4^5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ d_5 \\ 1 \end{pmatrix}$$

COORDINATE DI O_5 RISPETTO A L_4: $$\vec{O}_{4,5} = \begin{pmatrix} 0 \\ 0 \\ d_5 \end{pmatrix}$$

CINEMATICA DIRETTA:

IN CONCLUSIONE POSSIAMO CALCOLARE LE COORDINATE FINALI DEL ROBOT (PINZA) OVVERO $\vec{O}_{0,5}$:

- $x_d = x_{pinza} = a_4 C_1 C_{234} + \underline{l_3 C_1 C_{23} + l_2 C_1 C_2} + d_4 S_1 + d_5 C_1 S_{234}$
- $y_d = y_{pinza} = a_4 S_1 C_{234} + \underline{l_3 S_1 C_{23} + l_2 S_1 C_2} - d_4 C_1 + d_5 S_1 S_{234}$
- $z_d = z_{pinza} = a_4 S_{234} + \underline{l_3 S_{23} + l_2 S_2 + d_1} - d_5 C_{234}$

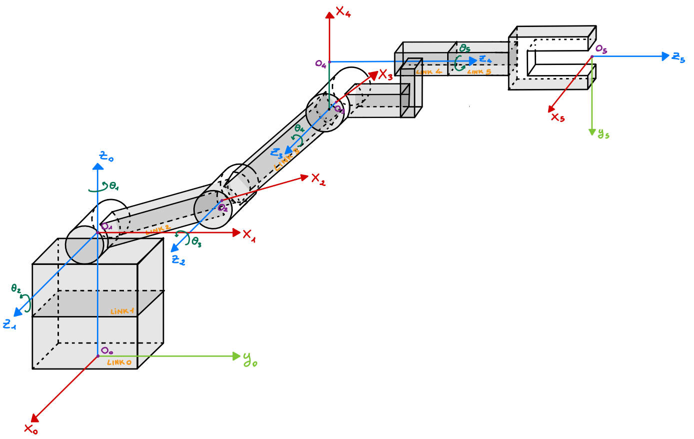
$$= a_4 C_1 C_{234} + d_4 S_1 + d_5 C_1 S_{234} + x_{polso}$$

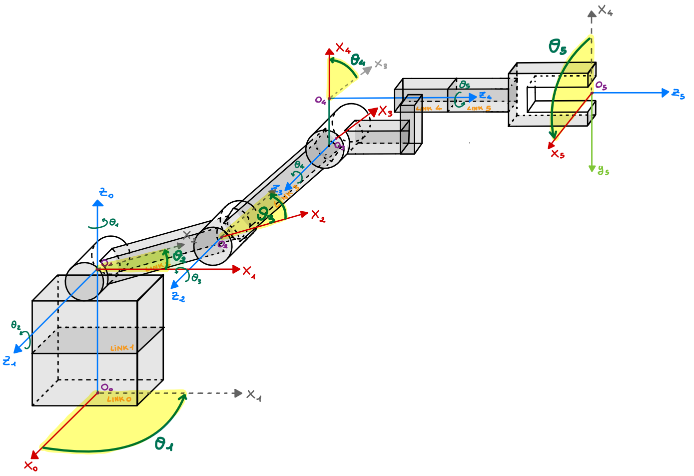
$$= a_4 S_1 C_{234} - d_4 C_1 + d_5 S_1 S_{234} + y_{polso}$$

$$= a_4 S_{234} - d_5 C_{234} + z_{polso}$$

INOLTRE:

- $\omega_d = \theta_5 \longleftarrow$ ANGOLO DI ROLL
- $\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 \longleftarrow$ ANGOLO DI PITCH





CINEMATICA INVERSA:

DATE $\vec{P}_e = (x_d, y_d, z_d)^T$ E β_d, ω_d RICAVARE $\theta_1, \theta_2, \cdots, \theta_5$:

SENZA CONSIDERARE d_4: $\theta_1^* = \text{atan2}(y_d, x_d)$

SI RICAVA DA CONSIDERAZIONI GEOMETRICHE: $\boxed{\theta_1 = \text{atan2}\left(y_d + d_4 \cos(\theta_1^*),\ x_d - d_4 \sin(\theta_1^*)\right)}$

RIPRENDIAMO:

- $x_d = x_{pinza} = a_4 c_1 c_{234} + \underline{l_3 c_1 c_{23} + l_2 c_1 c_2} + d_4 s_1 + d_5 c_1 s_{234}$
- $y_d = y_{pinza} = a_4 s_1 c_{234} + \underline{l_3 s_1 c_{23} + l_2 s_1 c_2} - d_4 c_1 + d_5 s_1 s_{234}$

ED ORA SVILUPPIAMO:

$$x_{polso} c_1 + y_{polso} s_1 = (\underline{l_3 c_1 c_{23} + l_2 c_1 c_2}) c_1 + (\underline{l_3 s_1 c_{23} + l_2 s_1 c_2}) s_1 =$$

$$= (c_1^2 + s_1^2)(l_3 c_{23} + l_2 c_2) = l_3 c_{23} + l_2 c_2$$

INTRODUCIAMO DUE VARIABILI AUSILIARIE:

$$A_1 = l_3 c_{23} + l_2 c_2$$

$$A_2 = z_{polso} - d_1 = l_3 s_{23} + l_2 s_2$$

→ LE RICAVEREMO DOPO

SVILUPPIAMO i QUADRATI:

$$A_1^2 + A_2^2 = (l_3 c_{23} + l_2 c_2)^2 + (l_3 s_{23} + l_2 s_2)^2 =$$

$$= l_3^2 c_{23}^2 + l_2 c_2^2 + 2 l_3 c_{23} l_2 c_2 + l_3^2 s_{23}^2 + l_2^2 s_2^2 + 2 l_3 s_{23} l_2 s_2 =$$

$$= l_3^2 + l_2^2 + 4 l_3 l_2 c_2 c_{23} = l_3^2 + l_2^2 + 2 l_3 l_2 (c_2 c_{23} + s_2 s_{23}) =$$

$$= l_3^2 + l_2^2 + 2 l_3 l_2 \cos(\theta_2 + \theta_3 - \theta_2) = l_3^2 + l_2^2 + 2 l_3 l_2 c_3$$

$$\longrightarrow A_1^2 + A_2^2 = l_3^2 + l_2^2 + 2 l_3 l_2 c_3 \longrightarrow c_3 = \frac{A_1^2 + A_2^2 - l_2^2 - l_3^2}{2 l_2 l_3}$$

$$\boxed{\theta_3 = \pm \arccos\left(\frac{A_1^2 + A_2^2 - l_2^2 - l_3^2}{2 l_2 l_3}\right)}$$

RIPRENDIAMO :

$$\begin{cases} A_1 = \ell_3 c_{23} + \ell_2 c_2 \\ A_2 = \ell_3 s_{23} + \ell_2 s_2 \end{cases} \qquad \begin{cases} A_1 = \ell_3 (c_2 c_3 - s_2 s_3) + \ell_2 c_2 = (\ell_3 c_3 + \ell_2)\cdot c_2 - \ell_3 s_3 \cdot s_2 \\ A_2 = \ell_3 (c_2 s_3 + s_2 c_3) + \ell_2 s_2 = \ell_3 s_3 \cdot c_2 + (\ell_2 + \ell_3 c_3)\cdot s_2 \end{cases}$$

$$\begin{pmatrix} \ell_3 c_3 + \ell_2 & -\ell_3 s_3 \\ \ell_3 s_3 & \ell_2 + \ell_3 c_3 \end{pmatrix} \begin{pmatrix} c_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

RISOLVIAMO IL SISTEMA CON IL METODO DI CRAMER:

$$\Delta = \begin{vmatrix} \ell_3 c_3 + \ell_2 & -\ell_3 s_3 \\ \ell_3 s_3 & \ell_2 + \ell_3 c_3 \end{vmatrix} = (\ell_3 c_3 + \ell_2)^2 + \ell_3^2 s_3^2 = \ell_3^2 + \ell_2^2 + 2\ell_3 \ell_3 c_3$$

$$\Delta_1 = \begin{vmatrix} A_1 & -\ell_3 s_3 \\ A_2 & \ell_2 + \ell_3 c_3 \end{vmatrix} = (\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2$$

$$\Delta_2 = \begin{vmatrix} \ell_3 c_3 + \ell_2 & A_1 \\ \ell_3 s_3 & A_2 \end{vmatrix} = (\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1$$

- $c_2 = \dfrac{\Delta_1}{\Delta} = \dfrac{(\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2}{\ell_3^2 + \ell_2^2 + 2\ell_3 \ell_3 c_3}$

- $s_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{(\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1}{\ell_3^2 + \ell_2^2 + 2\ell_3 \ell_3 c_3}$

- $\theta_2 = \text{atan2}(s_2, c_2) =$

$$= \text{atan}\left(\frac{\dfrac{(\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1}{\cancel{\ell_3^2 + \ell_2^2 + 2\ell_3 \ell_3 c_3}}}{\dfrac{(\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2}{\cancel{\ell_3^2 + \ell_2^2 + 2\ell_3 \ell_3 c_3}}} \right)$$

$$\boxed{\bullet\ \theta_2 = \text{atan2}\Big((\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1 \ ,\ (\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2 \Big)}$$

É IMMEDIATO RICAVARE θ_4 E θ_5 :

$$\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 \longrightarrow \boxed{\bullet\, \theta_4 = \frac{\pi}{2} - \theta_2 - \theta_3 - \beta_d}$$

$$\omega_d = \theta_5 \longrightarrow \boxed{\bullet\, \theta_5 = \omega_d}$$

TROVIAMO A_1 E A_2 :

- $x_d = a_4 C_1 C_{234} + \underline{\ell_3 C_1 C_{23} + \ell_2 C_1 C_2} + d_4 S_1 + d_5 C_1 S_{234}$
- $y_d = a_4 S_1 C_{234} + \underline{\ell_3 S_1 C_{23} + \ell_2 S_1 C_2} - d_4 C_1 + d_5 S_1 S_{234}$
- $z_d = a_4 S_{234} + \underline{\ell_3 S_{23} + \ell_2 S_2 + d_1} - d_5 C_{234}$

$$\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 \longrightarrow \theta_2 + \theta_3 + \theta_4 = \frac{\pi}{2} - \beta_d$$

$$x_{polso} = x_d - a_4 C_1 C_{234} - d_4 S_1 - d_5 C_1 S_{234} =$$

$$= x_d - a_4 C_1 \cos\left(\frac{\pi}{2} - \beta_d\right) - d_4 S_1 - d_5 C_1 \sin\left(\frac{\pi}{2} - \beta_d\right)$$

$$y_{polso} = y_d - a_4 S_1 C_{234} + d_4 C_1 - d_5 S_1 S_{234} =$$

$$= y_d - a_4 S_1 \cos\left(\frac{\pi}{2} - \beta_d\right) + d_4 C_1 - d_5 S_1 \sin\left(\frac{\pi}{2} - \beta_d\right)$$

$$z_{polso} = z_d - a_4 S_{234} + d_5 C_{234} = z_d - a_4 \sin\left(\frac{\pi}{2} - \beta_d\right) + d_5 \cos\left(\frac{\pi}{2} - \beta_d\right)$$

$$\bullet\ A_1 = x_{polso} c_1 + y_{polso} s_1 = \left(x_d - a_4 c_1 \cos\left(\tfrac{\pi}{2} - \beta_d\right) - d_4 s_1 - d_5 c_1 \sin\left(\tfrac{\pi}{2} - \beta_d\right) \right) c_1 +$$

$$+ \left(y_d - a_4 s_1 \cos\left(\tfrac{\pi}{2} - \beta_d\right) + d_4 c_1 - d_5 s_1 \sin\left(\tfrac{\pi}{2} - \beta_d\right) \right) s_1 =$$

$$= x_d c_1 + y_d s_1 - a_4 c_1^2 \cos\left(\tfrac{\pi}{2} - \beta_d\right) - a_4 s_1^2 \cos\left(\tfrac{\pi}{2} - \beta_d\right) - \cancel{d_4 s_1 c_1} + \cancel{d_4 c_1 s_1}$$

$$- d_5 c_1^2 \sin\left(\tfrac{\pi}{2} - \beta_d\right) - d_5 s_1^2 \sin\left(\tfrac{\pi}{2} - \beta_d\right) =$$

$$= x_d c_1 + y_d s_1 - a_4 \cos\left(\tfrac{\pi}{2} - \beta_d\right) - d_5 \sin\left(\tfrac{\pi}{2} - \beta_d\right) =$$

$$= x_d c_1 + y_d s_1 - a_4 \sin(\beta_d) - d_5 \cos(\beta_d)$$

$$\bullet\ A_2 = z_{polso} - d_1 = z_d - a_4 \sin\left(\tfrac{\pi}{2} - \beta_d\right) + d_5 \cos\left(\tfrac{\pi}{2} - \beta_d\right) - d_1 =$$

$$= z_d - a_4 \cos(\beta_d) + d_5 \sin(\beta_d) - d_1$$

IN CONCLUSIONE I PASSI DA SEGUIRE SONO

1. $$\boxed{\theta_1 = \operatorname{atan2}\left(y_d + d_4 \cos(\theta_1^*),\ x_d - d_4 \sin(\theta_1^*) \right)}$$ CON $\theta_1^* = \operatorname{atan2}(y_d, x_d)$

2. $$\boxed{\begin{array}{l} \bullet\ A_1 = x_d c_1 + y_d s_1 - a_4 \sin(\beta_d) - d_5 \cos(\beta_d) \\ \bullet\ A_2 = z_d - a_4 \cos(\beta_d) + d_5 \sin(\beta_d) - d_1 \end{array}}$$

3. $$\boxed{\bullet\ \theta_3 = \pm \arccos\left(\frac{A_1^2 + A_2^2 - \ell_2^2 - \ell_3^2}{2 \ell_2 \ell_3} \right)}$$

4. $$\boxed{\bullet\ \theta_2 = \operatorname{atan2}\left((\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1,\ (\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2 \right)}$$

5. $$\boxed{\bullet\ \theta_4 = \frac{\pi}{2} - \theta_2 - \theta_3 - \beta_d}$$

6. $$\boxed{\bullet\ \theta_5 = \omega_d}$$

YERIFICA NUMERICA:

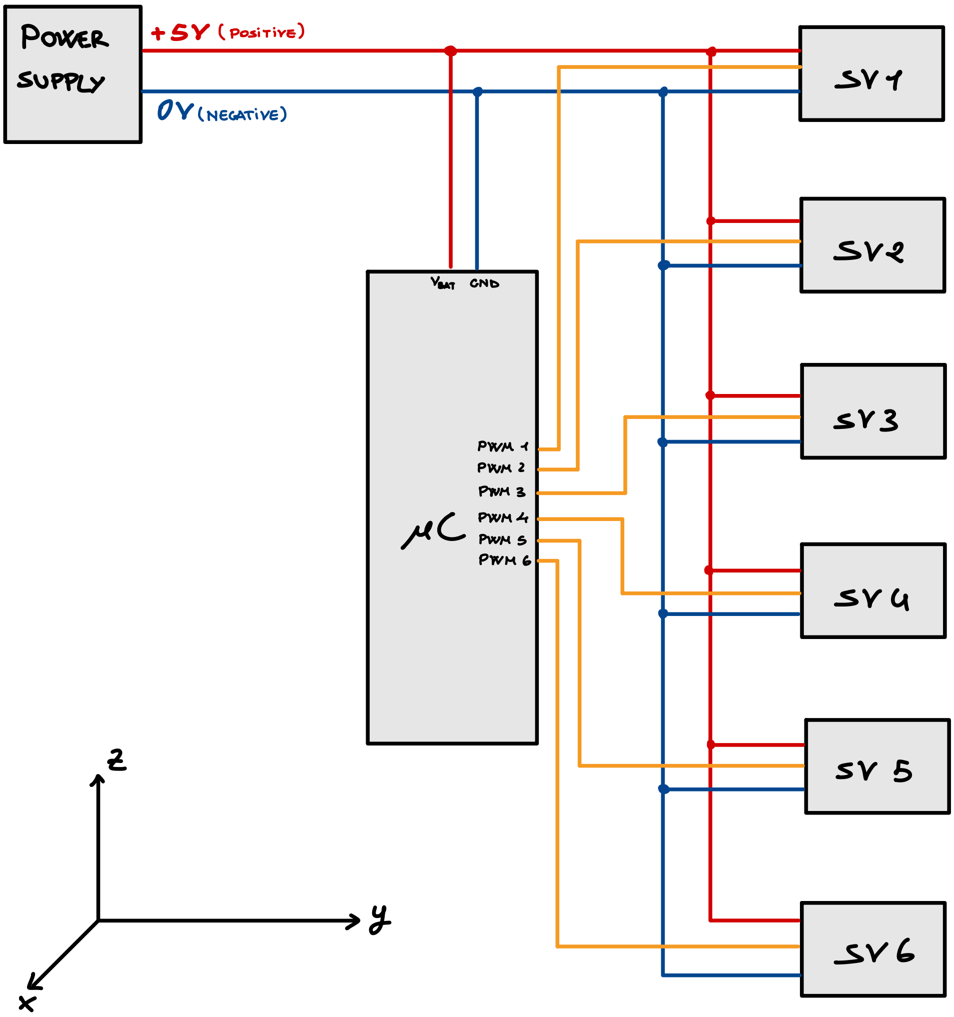
$\vec{P}_e = (-20,\ 30,\ 20)^T \qquad \beta_d = 40° \qquad \omega_d = 10°$

$d_1 = 30\ cm \qquad \ell_2 = 20\ cm \qquad \ell_3 = 20\ cm \qquad d_5 = 10\ cm \qquad d_4 = 35\ cm \qquad a_4 = 30\ cm$

- $\theta_1^* = atan2(y_d, x_d) = atan2(30, -20) = 123.7°$ $\qquad + \qquad -$
- $\theta_1 = atan2(y_d + d_4\cos(\theta_1^*),\ x_d - d_4\sin(\theta_1^*)) = atan2(30 + 35\cos(123.7°),\ -20 - 35\sin(123.7°)) = 167.84°$
- $A_1 = x_d c_1 + y_d s_1 - a_4\sin(\beta_d) - d_5\cos(\beta_d) = -1.073$
- $A_2 = z_d - a_4\cos(\beta_d) + d_5\sin(\beta_d) - d_1 = -26.553$
- $\theta_3 = \pm\arccos\left(\dfrac{A_1^2 + A_2^2 - \ell_2^2 - \ell_3^2}{2\ell_2\ell_3}\right) = \pm 96.73°$
- $\theta_2 = atan2\left(\underbrace{(\ell_3 c_3 + \ell_2)A_2 - \ell_3 s_3 A_1}_{\substack{-447.51 \\ -490.14}},\ \underbrace{(\ell_2 + \ell_3 c_3)A_1 + \ell_3 s_3 A_2}_{\substack{-546.293 \\ +508.508}}\right) = -140.68°$
  $(-43.95°)$
- $\theta_4 = \frac{\pi}{2} - \theta_2 - \theta_3 - \beta_d = 93.95°$
  $(190.68°)$
- $\theta_5 = \omega_d = 10°$

VERIFICA:

- $x_d = x_{pinza} = a_4 c_1 c_{234} + \underline{\ell_3 c_1 c_{23} + \ell_2 c_1 c_2} + d_4 s_1 + d_5 c_1 s_{234} =$
- $y_d = y_{pinza} = a_4 s_1 c_{234} + \underline{\ell_3 s_1 c_{23} + \ell_2 s_1 c_2} - d_4 c_1 + d_5 s_1 s_{234}$
- $z_d = z_{pinza} = a_4 s_{234} + \underline{\ell_3 s_{23} + \ell_2 s_2 + d_1} - d_5 c_{234}$
- $\omega_d = \theta_5 = 10°$ ok
- $\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 = 40°$ ok



SV1: (PIANO xy)

180° ≡ 2200 ms

90° ≡ 1645 ms

0° ≡ 490 ms

SV2: (PIANO VERTICALE)

90° ≡ 1800 ms

180° ≡ 3000 ms — 600 ms ≡ 0°

SV3 (PIANO VERTICALE)

90° ≡ 1800 ms

400 ms ≡ 0° — 270° ≡ 3300 ms

SV 4 (PIANO VERTICALE)

90° ≡ 1900 ms

750 ms ≡ 0° — 180° ≡ 3100 ms

SV 5 (ROT. PINZA)

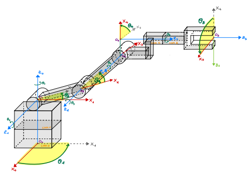
90° ≡ 1900 ms

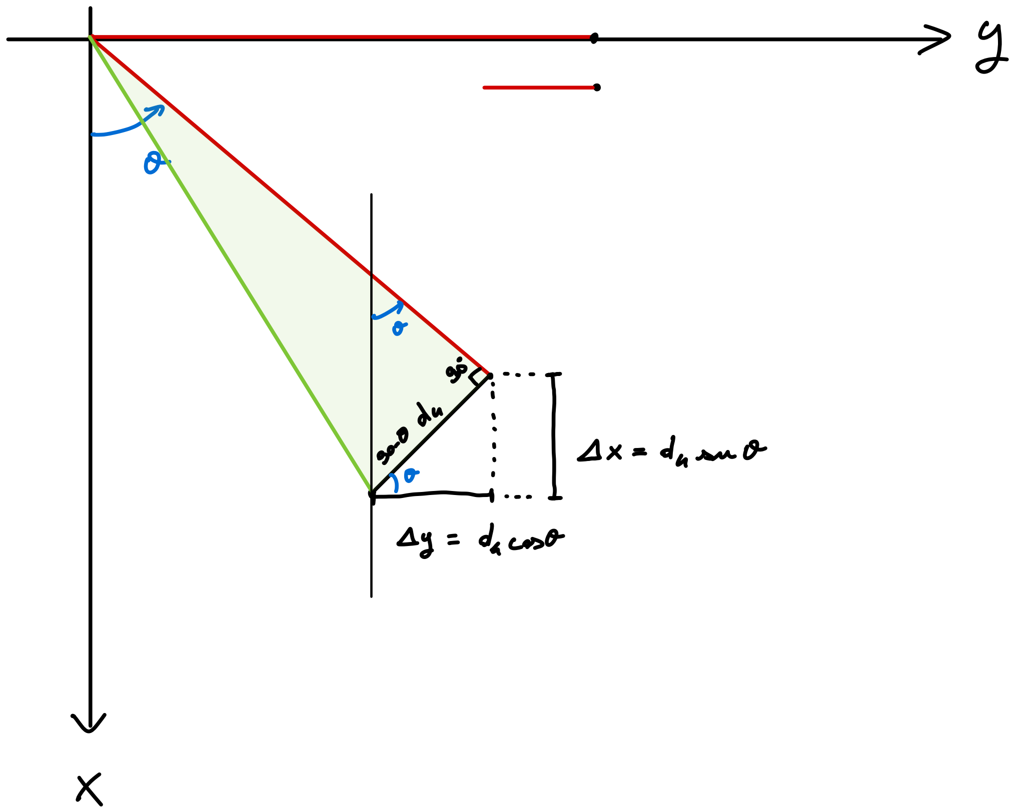
600 ms ≡ 0° — 180° ≡ 3050 ms

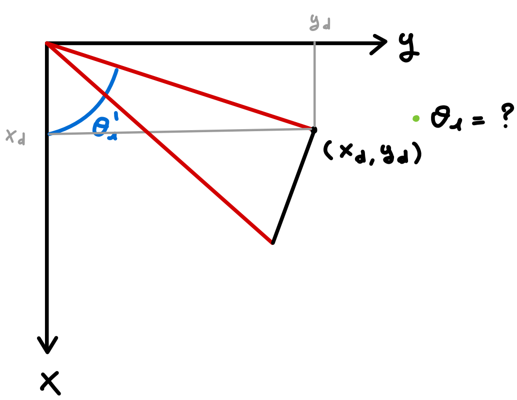
SV6 (PINZA)

min: 1200 ms PINZA APERTA

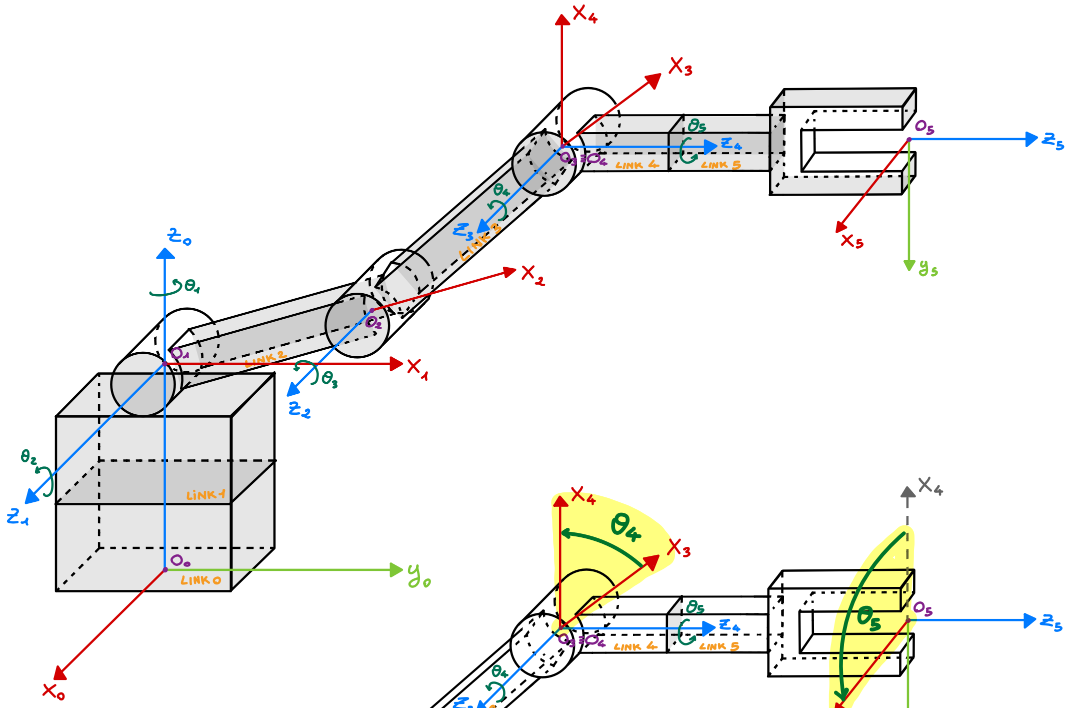
max: 2100 ms PINZA CHIUSA

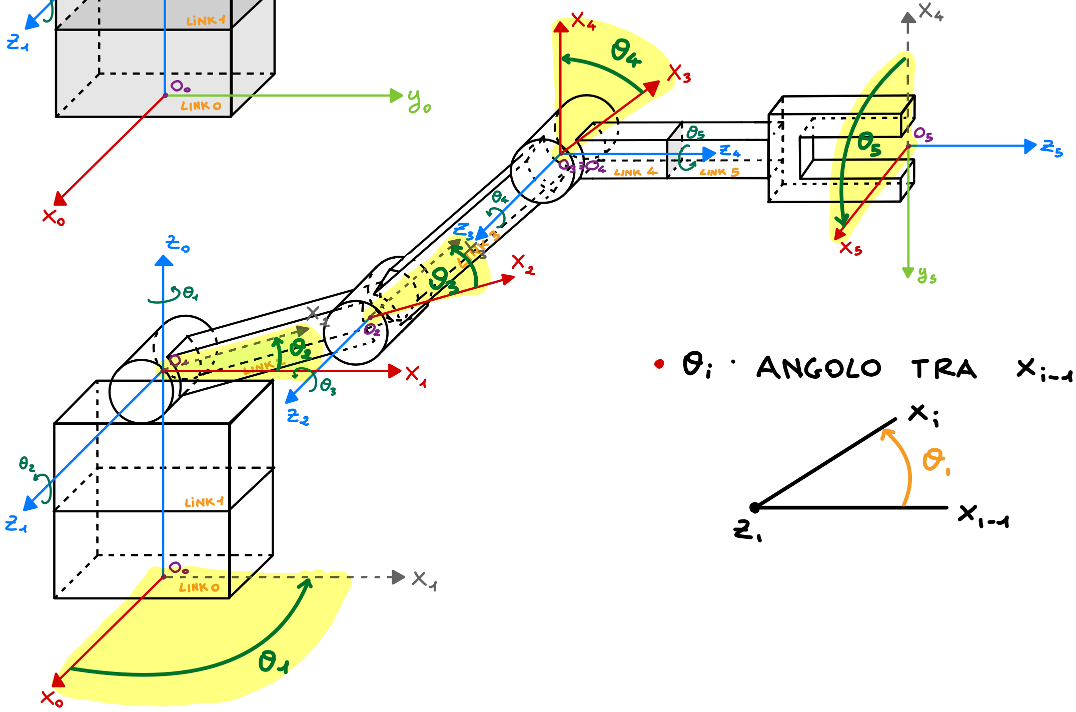



y
θ
θ
90°
90-θ
d4
θ
Δx = d4 sin θ
Δy = d4 cos θ
X

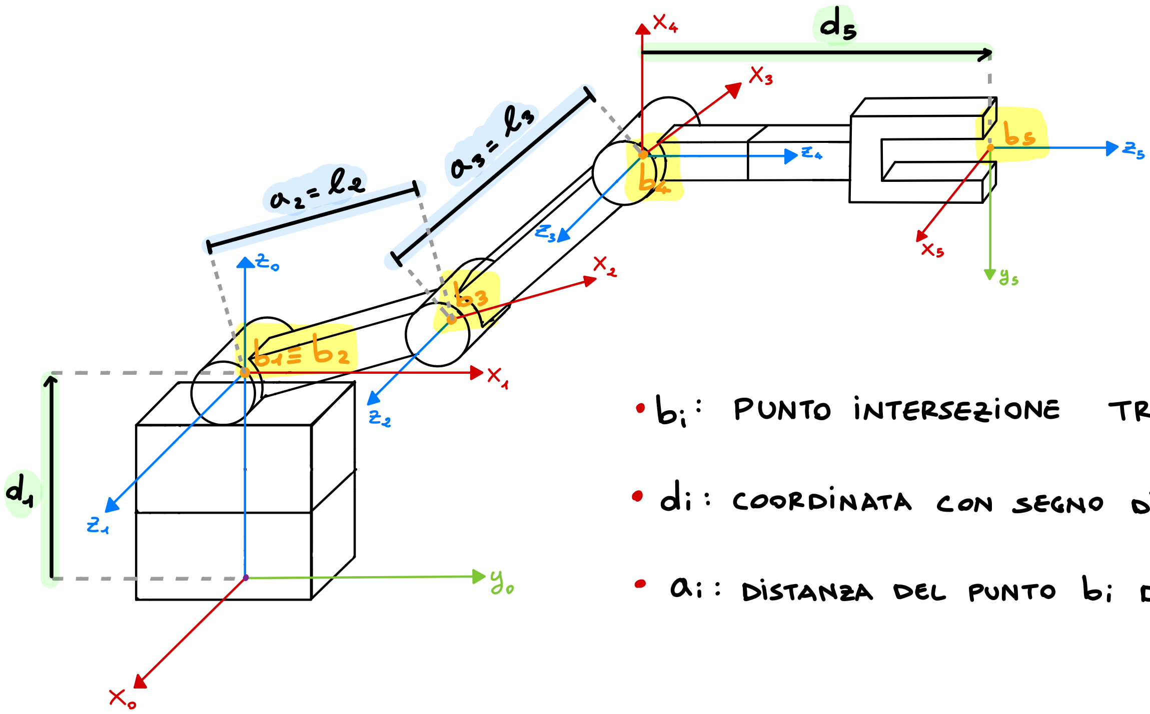

yd
y
xd
θ'd
(xd, yd)
θd = ?
X

DENAVIT - HARTENBERG





- θ_i · ANGOLO TRA x_{i-1} E x_i VISTO DA z_i



- b_i: PUNTO INTERSEZIONE TRA x_i E z_{i-1}
- d_i: COORDINATA CON SEGNO DI b_i SU z_{i-1}
- a_i: DISTANZA DEL PUNTO b_i DA O_i

- $d_2 = d_3 = d_4 = 0$
- $a_1 = a_4 = a_5 = 0$
- $a_2 = \ell_2$
- $a_3 = \ell_3$

i	θ_i	d_i	a_i	α_i	
1	θ_1^*	d_1	0	$\frac{\pi}{2}$	$\rightarrow T_0^1$
2	θ_2^*	0	ℓ_2	0	$\rightarrow T_1^2$
3	θ_3^*	0	ℓ_3	0	$\rightarrow T_2^3$
4	θ_4^*	0	0	$\frac{\pi}{2}$	$\rightarrow T_3^4$
5	θ_5^*	d_5	0	0	$\rightarrow T_4^5$

- $$T_0^1 = \left(\begin{array}{ccc|c} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

- $$T_1^2 = \left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

- $$T_2^3 = \left(\begin{array}{ccc|c} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

- $$T_3^4 = \left(\begin{array}{ccc|c} c\theta_4 & 0 & s\theta_4 & 0 \\ s\theta_4 & 0 & -c\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

- $$T_4^5 = \left(\begin{array}{ccc|c} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow T_0^5 = T_0^1 \, T_1^2 \, T_2^3 \, T_3^4 \, T_4^5 = \dots$$

- $$T_0^2 = T_0^1 \, T_1^2 = \left(\begin{array}{ccc|c} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) =$$

$$= \begin{pmatrix} c_1 c_2 & -c_1 s_2 & s_1 & \ell_2 c_1 c_2 \\ s_1 c_2 & -s_1 s_2 & -c_1 & \ell_2 s_1 c_2 \\ s_2 & c_2 & 0 & d_1 + \ell_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $$T_1^3 = T_1^2 \, T_2^3 = \left(\begin{array}{ccc|c} c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & \ell_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) =$$

$$= \begin{pmatrix} c_2 c_3 - s_2 s_3 & -c_2 s_3 - s_2 c_3 & 0 & \ell_3 c_2 c_3 - \ell_3 s_2 s_3 + \ell_2 c_2 \\ s_2 c_3 + c_2 s_3 & -s_2 s_3 + c_2 c_3 & 0 & \ell_3 s_2 c_3 + \ell_3 c_2 s_3 + \ell_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_{23} & -s_{23} & 0 & \ell_3 c_{23} + \ell_2 c_2 \\ s_{23} & c_{23} & 0 & \ell_3 s_{23} + \ell_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

CINEMATICA INVERSA:

DATE $\vec{P}_e = (x_d, y_d, z_d)^T$ E β_d, ω_d RICAVARE $\theta_1, \theta_2, \cdots, \theta_5$:

É IMMEDIATO RICAVARE: $\boxed{\theta_1 = \text{atan2}(y_d, x_d)}$

RIPRENDIAMO:

- $x_d = x_{pinza} = d_5 c_1 s_{234} + \ell_3 c_1 c_{23} + \ell_2 c_1 c_2$
- $y_d = y_{pinza} = d_5 s_1 s_{234} + \ell_3 s_1 c_{23} + \ell_2 s_1 c_2$

ED ORA SVILUPPIAMO:

$$x_{polso} c_1 + y_{polso} s_1 = (\ell_3 c_1 c_{23} + \ell_2 c_1 c_2) c_1 + (\ell_3 s_1 c_{23} + \ell_2 s_1 c_2) s_1 =$$

$$= (c_1^2 + s_1^2)(\ell_3 c_{23} + \ell_2 c_2) = \ell_3 c_{23} + \ell_2 c_2$$

INTRODUCIAMO DUE VARIABILI AUSILIARI:

$$A_1 = \ell_3 c_{23} + \ell_2 c_2$$

$$A_2 = z_{polso} - d_1 = \ell_3 s_{23} + \ell_2 s_2$$

$\rightarrow$ LE RICAVEREMO DOPO

SVILUPPIAMO I QUADRATI:

$$A_1^2 + A_2^2 = (\ell_3 c_{23} + \ell_2 c_2)^2 + (\ell_3 s_{23} + \ell_2 s_2)^2 =$$

$$= \ell_3^2 c_{23}^2 + \ell_2 c_2^2 + 2\ell_3 c_{23} \ell_2 c_2 + \ell_3^2 s_{23}^2 + \ell_2^2 s_2^2 + 2\ell_3 s_{23} \ell_2 s_2 =$$

$$= \ell_3^2 + \ell_2^2 + 4\ell_3 \ell_2 c_2 c_{23} = \ell_3^2 + \ell_2^2 + 2\ell_3 \ell_2 (c_2 c_{23} + s_2 s_{23}) =$$

$$= \ell_3^2 + \ell_2^2 + 2\ell_3 \ell_2 \cos(\theta_2 + \theta_3 - \theta_2) = \ell_3^2 + \ell_2^2 + 2\ell_3 \ell_2 c_3$$

$$\rightarrow A_1^2 + A_2^2 = \ell_3^2 + \ell_2^2 + 2\ell_3 \ell_2 c_3 \rightarrow c_3 = \frac{A_1^2 + A_2^2 - \ell_2^2 - \ell_3^2}{2\ell_2 \ell_3}$$

$$\boxed{\bullet\ \theta_3 = \pm \arccos\left(\frac{A_1^2 + A_2^2 - \ell_2^2 - \ell_3^2}{2\ell_2 \ell_3}\right)}$$

RIPRENDIAMO :

$$\begin{cases} A_1 = \ell_3 c_{23} + \ell_2 c_2 \\ A_2 = \ell_3 s_{23} + \ell_2 s_2 \end{cases} \qquad \begin{cases} A_1 = \ell_3 (c_2 c_3 - s_2 s_3) + \ell_2 c_2 = (\ell_3 c_3 + \ell_2) \cdot c_2 - \ell_3 s_3 \cdot s_2 \\ A_2 = \ell_3 (c_2 s_3 + s_2 c_3) + \ell_2 s_2 = \ell_3 s_3 \cdot c_2 + (\ell_2 + \ell_3 c_3) \cdot s_2 \end{cases}$$

$$\begin{pmatrix} \ell_3 c_3 + \ell_2 & -\ell_3 s_3 \\ \ell_3 s_3 & \ell_2 + \ell_3 c_3 \end{pmatrix} \begin{pmatrix} c_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

RISOLVIAMO IL SISTEMA CON IL METODO DI CRAMER:

$$\Delta = \begin{vmatrix} \ell_3 c_3 + \ell_2 & -\ell_3 s_3 \\ \ell_3 s_3 & \ell_2 + \ell_3 c_3 \end{vmatrix} = (\ell_3 c_3 + \ell_2)^2 + \ell_3^2 s_3^2 = \ell_3^2 + \ell_2^2 + 2 \ell_3 \ell_3 c_3$$

$$\Delta_1 = \begin{vmatrix} A_1 & -\ell_3 s_3 \\ A_2 & \ell_2 + \ell_3 c_3 \end{vmatrix} = (\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2$$

$$\Delta_2 = \begin{vmatrix} \ell_3 c_3 + \ell_2 & A_1 \\ \ell_3 s_3 & A_2 \end{vmatrix} = (\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1$$

- $c_2 = \dfrac{\Delta_1}{\Delta} = \dfrac{(\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2}{\ell_3^2 + \ell_2^2 + 2 \ell_3 \ell_3 c_3}$

- $s_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{(\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1}{\ell_3^2 + \ell_2^2 + 2 \ell_3 \ell_3 c_3}$

- $\theta_2 = \text{atan2}(s_2, c_2) = \text{atan}\left(\dfrac{\dfrac{(\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1}{\cancel{\ell_3^2 + \ell_2^2 + 2 \ell_3 \ell_3 c_3}}}{\dfrac{(\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2}{\cancel{\ell_3^2 + \ell_2^2 + 2 \ell_3 \ell_3 c_3}}} \right)$

$$\boxed{\theta_2 = \text{atan2}\big((\ell_3 c_3 + \ell_2) A_2 - \ell_3 s_3 A_1 \,,\; (\ell_2 + \ell_3 c_3) A_1 + \ell_3 s_3 A_2 \big)}$$

É IMMEDIATO RICAVARE θ_4 E θ_5 :

$$\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 \longrightarrow \boxed{\bullet\ \theta_4 = \frac{\pi}{2} - \theta_2 - \theta_3 - \beta_d}$$

$$\omega_d = \theta_5 \longrightarrow \boxed{\bullet\ \theta_5 = \omega_d}$$

TROVIAMO A_1 E A_2 :

- $x_d = d_5 C_1 S_{234} + \underline{l_3 C_1 C_{23} + l_2 C_1 C_2} = d_5 C_1 S_{234} + x_{polso}$
- $y_d = d_5 S_1 S_{234} + \underline{l_3 S_1 C_{23} + l_2 S_1 C_2} = d_5 S_1 S_{234} + y_{polso}$
- $z_d = -d_5 C_{234} + \underline{d_1 + l_3 S_{23} + l_2 S_2} = -d_5 C_{234} + z_{polso}$

$$\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 \longrightarrow \theta_2 + \theta_3 + \theta_4 = \frac{\pi}{2} - \beta_d$$

$$x_{polso} = x_d - d_5 C_1 S_{234} = x_d - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) c_1$$

$$y_{polso} = y_d - d_5 S_1 S_{234} = y_d - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) S_1$$

$$z_{polso} = z_d + d_5 C_{234} = z_d + d_5 \cos\left(\frac{\pi}{2} - \beta_d\right)$$

- $A_1 = x_{polso} C_1 + y_{polso} S_1 = \dots$
- $A_2 = z_{polso} - d_1 = z_d + d_5 \cos\left(\frac{\pi}{2} - \beta_d\right) - d_1 = z_d + d_5 \sin\beta_d - d_1$

$$A_1 = \left(x_d - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) c_1\right) c_1 + \left(y_d - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) s_1\right) s_1 =$$

$$= x_d C_1 - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) C_1^2 + y_d S_1 - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) s_1^2 =$$

$$= x_d C_1 + y_d S_1 - d_5 \sin\left(\frac{\pi}{2} - \beta_d\right) = x_d C_1 + y_d S_1 - d_5 \cos\beta_d$$

$$\boxed{\begin{array}{l} \bullet\ A_1 = x_d C_1 + y_d S_1 - d_5 \cos\beta_d \\ \bullet\ A_2 = z_d + d_5 \sin\beta_d - d_1 \end{array}}$$

IN CONCLUSIONE i PASSI DA SEGUIRE SONO

1. $\theta_1 = \text{atan2}(y_d, x_d)$

2. $A_1 = x_d C_1 + y_d S_1 - d_5 \cos\beta_d$

   $A_2 = z_d + d_5 \sin\beta_d - d_1$

3. $\theta_3 = \pm \arccos\left(\dfrac{A_1^2 + A_2^2 - \ell_2^2 - \ell_3^2}{2\ell_2\ell_3}\right)$

4. $\theta_2 = \text{atan2}\left((\ell_3 C_3 + \ell_2)A_2 - \ell_3 S_3 A_1 \,,\; (\ell_2 + \ell_3 C_3)A_1 + \ell_3 S_3 A_2\right)$

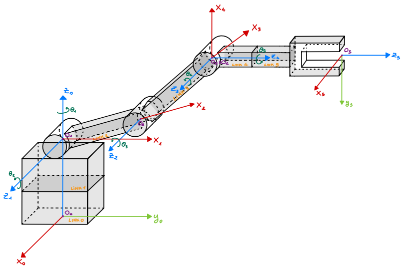
5. $\theta_4 = \frac{\pi}{2} - \theta_2 - \theta_3 - \beta_d$

6. $\theta_5 = \omega_d$

VERIFICA NUMERICA:

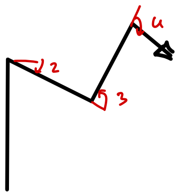
$\vec{P}_e = (-20,\ 30,\ 20)^T \qquad \beta_d = 40° \qquad \omega_d = 10°$

$d_1 = 30\ cm \qquad \ell_2 = 20\ cm \qquad \ell_3 = 20\ cm \qquad d_5 = 10\ cm$

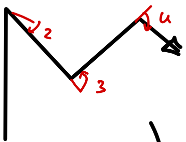


- $\theta_1 = atan2(y_d, x_d) = atan2(30, -20) = 123.7°$

- $A_1 = x_d C_1 + y_d S_1 - d_5 \sin(\frac{\pi}{2} - \beta_d) = 28.39$
- $A_2 = z_d + d_5 \cos(\frac{\pi}{2} - \beta_d) - d_1 = -3.57$



- $\theta_3 = \pm \arccos\left(\dfrac{A_1^2 + A_2^2 - \ell_2^2 - \ell_3^2}{2\ell_2\ell_3}\right) = \pm 88,66°$



- $\theta_2 = atan2\left(\underbrace{(\ell_3 C_3 + \ell_2)A_2 - \ell_3 S_3 A_1}_{\substack{-640.71 \\ +494.57}},\ \underbrace{(\ell_2 + \ell_3 S_3)A_1 + \ell_3 S_3 A_2}_{\substack{+1064.06 \\ -476.26}}\right) = -31°$

  $(+133, 91°)$

- $\theta_4 = \frac{\pi}{2} - \theta_2 - \theta_3 - \beta_d = 12.48°$

  $+4.75$

- $\theta_5 = \omega_d = 10°$

VERIFICA:

- $x_d = d_5 C_1 S_{234} + \ell_3 C_1 C_{23} + \ell_2 C_1 C_2 = -19,89 \cong -20.01$ ok
- $y_d = d_5 S_1 S_{234} + \ell_3 S_1 C_{23} + \ell_2 S_1 C_2 = 29,82 \cong 30$ ok
- $z_d = d_1 + \ell_2 S_2 + \ell_3 S_{23} - d_5 C_{234} + = 20.17$ ok
- $\omega_d = \theta_5 = 10°$ ok
- $\beta_d = \frac{\pi}{2} - \theta_2 - \theta_3 - \theta_4 = 40°$ ok