Task for lecture 13

Consider the elliptic partial differential equation

$$-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 1 + x + y \qquad \text{FOR} \qquad (x, y) \in \Omega \qquad (1)$$

where $\Omega = \{(x, y)|0 < x < 1, 0 < y < 1\}$ and $u(x, y) = 0$ for $(x, y) \in \partial\Omega$

- Set up the system of linear equations for $N = 4, 8, 16, ...$

- Solve the system for $u(0.5, 0.5)$.

- Perform Richardson extrapolation and error estimation for $u(0.5, 0.5)$.

Tips

- Consider Dirichlet boundary conditions.

- Use header files 'nr3.h' and 'banded.h' from the recipe library.

- See slide 4 in 'uge13pres.pdf' for incrementing x and y where j are columns and k are rows.

- Use the index function on slide 6 in 'uge13pres.pdf' and setup A matrix and ϕ vector.

- Use the band matrix function 'Bandec' from the 'banded.h' header file on matrix A (it is a numerical crime not to).