Probability construction (general setting)
Operatively, there are two ways of furnishing a (measure of) probability on a measurable space : it may be claimed
The first is common in applications such as Bayesian Statistics, where you make an hypothesis on the distribution a
In both cases some check are in order, as we need to ensure that the probability is *coherent*. Note that in the secon
Discrete setting
Some results of this kind were produced already: we have dealt with the case of discrete partitions of the sample sp
[Existence and Uniqueness] Let be a discrete partition of Ω and let the function : $E_k \in \mathcal{E} \to p_k \in R$ be such that $\sum$
$p_k \geq 0 \quad$ *for all* $k \in I$. Then, there exist a unique probability P on $\sigma(\mathcal{E})$ such that P and agree on $\mathcal{E}$. A function with t
Morally, the discrete probability density on a partition describes a unique and consistent probability on the generat
Let be a discrete partition of Ω. Then

Now suppose that : $E_k \in \mathcal{E} \to p_k \in R$ is a discrete probability density. Then, if P is the probability defined in Theorem

A special case worth mentioning is that of the atomic partition on a discrete Ω. Here, the generated is $\mathcal{P}(\Omega)$ and th

General setting
For more general settings the short message is that Theorem holds, provided some conditions, while no explicit cha
We present a powerful theorem of measure theory, that does just that. It will allow us to extend a *pre-probability*, a
Let be an algebra defined on Ω. Then, $:\to R$ is a *pre-probability* if
$(\Omega) = 1$
$(\bigcup_{i=1}^n A_i) = \sum_i^n (A_i)$ for $A_i \in$
if $\bigcup_{i \in N} A_i \in$, then $(\bigcup_{i \in N} A_i) = \sum_{i \in N} (A_i)$.
The latter is the condition that ensure coherence with respect to the probability, and can be read as a *need-based* .
[Carathéodory's Theorem] Let be an algebra defined on Ω, and suppose that $:\to R$ is a pre-probability. Then, ther
We can refine the result for practical purposes by exploiting the equivalence between σ-additivity and continuity[1].
[Carathéodory's Theorem, continuity characterization] Let be an algebra defined on Ω, and suppose that $:\to R$ sat
$(\Omega) = 1$,
$(\bigcup_{i=1}^n A_i) = \sum_i^n (A_i)$,
if $A_i \downarrow \emptyset$, then $(A_i) \downarrow 0$,
where $A_n \in$. Then, there exists a unique probability $P : \sigma() \to R$ such that and P agree on . The usefulness of this is
Another refinement is in order: the minimal information condition could be weaker. This is an application of the M
[π-system] A collection $\mathcal{C}$ is said to be a π-system if stable under finite intersection. Explicitly, if $A_1, \ldots, A_n \in \mathcal{C}$ the
This theorem is not just about comparisons. It furnishes sharp[2] conditions for the uniqueness in Theorem : it requi
*Borel's σ-algebra
*σ-algebra on Bernoulli Space