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R.M.C. So
The Hong Kong Polytechnic University

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Ping Cheng
Mechanical Engineering Department
The Hong Kong University of Science and Technology
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NEAR-WALL MODELING OF TURBULENT HEAT TRANSFER WITH DIFFERENT PRANDTL NUMBERS

C. Y. Zhao and R. M. C. So
Mechanical Engineering Department
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
Fax: 852-2365-4703
E-mail: mmcyzhao@polyu.edu.hk

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ABSTRACT. Near-Wall modeling of the turbulent temperature field is much more complicated, because the boundary conditions are not as well defined as those for the velocity field. Up to now, most computational and theoretical investigations are still based on the hypothesis of a constant turbulent Prandtl number in order to remove the uncertainty of the turbulent temperature boundary condition. However, strictly speaking, the physical arguments for these assumptions are applicable only for fluids whose Prandtl number, Pr, is approximately 1. Near-wall asymptotes of the turbulence statistics show that these properties are Pr dependent. Another difficult in the modeling of near-wall heat transfer is the irregular geometries often encountered in heat transfer prblems. If the heat-flux models fail to reflect the Pr dependence and are geometry dependent, they would not be able to replicate the thermal asymptotes correctly as a wall is approached. The main objective of this paper is to develop a geometry independent near-wall two-equation heat-flux model for fluids with different Pr. In this paper, a geometry independent near-wall Reynolds stress turbulence model and the proposed two-equation heat-flux model are used to calculate heat transfer problems. As a first attempt, the proposed model is validated against fully developed turbulent channel flow with variable Pr. The mean temperature, turbulent kinetic energy, temperature variance, heat flux and the time scale ratio together with the near-wall characteristics are calculated and compared with direct numerical simulation (DNS) data. Good correlation with data is obtained.

INTRODUCTION

The wall boundary conditions of the thermal field are quite a bit more complicated than the velocity field in any turbulent heat transfer problem. For example, in the case of an incompressible flow, the fluid equation of state is automatically satisfied. Therefore, if the assumption is made that the temperature fluatuations vanish at the wall, that will lead to the conclusion that the fluctuating pressure also goes to zero at the wall [1]. Obviously, this is not true physically. The difficulty can be bypassed by invoking Reynolds analogy. In other words, the transport of heat is assumed to be similar to that of the transport of momentum [2]. As a result, a turbulent Prandtl number, Pr_t, can be defined and assumed to be given by a constant not too different from unity. This assumption is probably valid for simple heat transfer problems, such as fully-developed pipe and channel flow with a molecular Prandtl number, Pr, not too different from one [2]. If the Pr is much greater or smaller than one, the assumption of the heat and momentum transport process being similar is no longer valid [3, 4]. The heat transfer problem, even in the case of fully-developed flows, has to be investigated using higher level closure schemes [2].

The wall boundary condition is one of many difficulties encountered in complex heat transfer modeling. Another is the unavailability of models that are truly geometry independent. There are two components to the modeling of heat transfer problems; one is the velocity field model, the other is the thermal field model. Both models have to be geometry independent before the resultant model can be claimed to be one that is truly independent of geometry. Among the available velocity field models, the high Reynolds-number models are usually geometry independent while most near-wall models are not [5]. This is true for both two-equation and Reynolds-stress models. The reason is that wall unit normals and wall normal coordinate are frequently used to help devise near-wall damping functions to account for the effects of viscosity near a wall. Recently, a few geometry independent near-wall Reynolds-stress models have been put forward and they are shown to be quite valid for a range of flows with geometries that vary from the simple to the rather complex [6-8]. The other component is the heat flux model. As in the case of high Reynolds number velocity field models, the high Reynolds number heat flux models are also geometry independent. On the other hand, not many near-wall heat transfer models, be it two-equation or second order, are geometry independent [2]. In fact, even the more recently proposed near-wall models for flows with different Pr are either dependent on wall unit normals or wall normal coordinate [3, 4]. Therefore, if a truly geometry independent model for flows with different Pr is to be proposed for turbulent heat transfer problems, a new near-wall model has to be derived. The alternative is to modify one of the existing near-wall models to rid its dependence on wall unit normals and wall normal coordinate.

In heat transfer modeling, it is advisable to use a heat flux model that is at least one level lower than that of the velocity field model [9]. If the geometry independent near-wall Reynolds-stress models are used to calculate the velocity field, this means that the heat flux model should be limited to either a two-equation model or a constant Pr_t assumption. For incompressible heat transfer problems, the velocity field affects the thermal field but not the other way around. In isothermal flows, there is only one turbulence time scale,. This is given by the ratio of the turbulent kinetic energy, k, over its dissipation rate, ε. For non-isothermal incompressible flows, another time scale given by the ratio of the temperature variance, $\overline{\theta^2}$, and its dissipation, ε_θ, is also important. As a result, a time scale ratio, $R = \overline{\theta^2}\varepsilon/(2\,k\,\varepsilon_\theta)$, can be defined. Strictly speaking, if R is assumed constant, the model is essentially a constant Pr_t model because it can be shown that $Pr_t = \sqrt{R}$ [2]. On the other hand, if R is to be determined, two equations governing the transport of $\overline{\theta^2}$ and ε_θ have to be solved. Consequently, there is no one-equation model for the heat flux [2]. There are many different types of near-wall two-equation models. Some are formulated for a particular Pr, while others can account for different Pr. However, none is geometry independent. Even the more recent proposals [3, 4] have damping functions in the heat flux model that are dependent on the wall unit normal and the wall normal coordinate. In the work of So and Sommer [3], a Reynolds-stress and a two-equation model were used to calculate the velocity field and both models have near-wall corrections that rendered them geometry dependent. On the other hand, Nagano and Shimada [4] only used a two-equation model to resolve the velocity field. However, their model is not geometry independent either. The present approach is to attempt to modify one of the two heat flux models to make it geometry independent and to use it in conjunction with a near-wall Reynolds-stress model that is also geometry independent. That way, a truly geometry independent heat transfer model that can account for fluids with different Pr is available.

The choice of models for both the velocity and temperature fields is dictated by the desire to have relatively simple models, the least modifications need be made to the heat flux model and a fairly accurate prediction of Pr effects. Among the near-wall Reynolds-stress models that are geometry independent, the model by Craft and Launder [7] is very complex, however, their predicted results of simple flows are not too much different from those given by Durbin [6] and So and Yuan [8]. The model proposed by Durbin [6] relies on the solution of an additional second order tensor equation. Since the So and Yuan [8] model is fairly simple, the choice for the present study is quite straightforward. As for the heat flux, the models of So and Sommer [3] and Nagano and Shimada [4] give equally good results for the prediction of Pr effects. The selection of the So and Sommer [3] model is based on the fact that it only involves the modification of a single damping function, while the model of Nagano and Shimada [4] requires the revision of several damping functions in order to rid its dependence on wall unit normals and wall normal coordinate. Therefore, the present approach to formulate a geometry independent model for heat transfer calculations is to adopt the So and Yuan [8] model for the velocity field and to modify the So and Sommer [3] model for the thermal field. Thus formulated, the model will be suitable for complex heat transfer problems with variable Pr. The present paper only attempts to verify the model calculations against the direct numerical simulation (DNS) data of Abe *et al.* [10] where fully-developed channel flows with different Pr have been reported. A full validation of the model against complex flows with different Pr will be reported in a subsequent paper.

VELOCITY FIELD MODEL

The detailed derivation of the geometry independent near-wall Reynolds-stress model has already been reported by So and Yuan [8]. Therefore, for the sake of brevity, only the modeled equations and the associated constants are given here. The interested readers should go to [8] for details. For incompressible flows, the exact Reynolds-stress equations can be written as

$$\frac{D\overline{u_iu_j}}{Dt} = \frac{\partial}{\partial x_k}\left(\nu \frac{\partial \overline{u_iu_j}}{\partial x_k}\right) + D^T_{ij} + P_{ij} + \Pi^*_{ij} - \varepsilon_{ij} \quad , \tag{1}$$

where U_i, u_i are the mean and fluctuating velocity vectors, x_i is the coordinate vector, t is time, ν is the fluid kinematic viscosity and the models proposed for the turbulent diffusion tensor, the production tensor, the velocity pressure-gradient correlation tensor and the dissipation rate tensor, D^T_{ij}, P_{ij}, Π^*_{ij}, ε_{ij}, respectively, are given by

$$D^T_{ij} = \frac{\partial}{\partial x_m}\left[C_s \frac{k}{\varepsilon}\left(\overline{u_iu_l}\frac{\partial \overline{u_ju_m}}{\partial x_l} + \overline{u_ju_l}\frac{\partial \overline{u_mu_i}}{\partial x_l} + \overline{u_mu_l}\frac{\partial \overline{u_iu_j}}{\partial x_l}\right)\right] \quad , \tag{2}$$

$$P_{ij} = -\left[\overline{u_iu_m}\,\partial U_j/\partial x_m + \overline{u_ju_m}\,\partial U_i/\partial x_m\right] \quad , \tag{3}$$

$$\Pi^*_{ij} = \Pi_{ij} + \Pi^w_{ij} + D^p_{ij} \quad , \tag{4}$$

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\,\delta_{ij} + \varepsilon^w_{ij} \quad . \tag{5}$$

Here, C_s is the model constant. In (4), the tensor has been split into three parts, a pressure strain part, Π_{ij}, a wall correction part, Π^w_{ij}, and a pressure diffusion part, D^p_{ij}. As for ε_{ij}, near the wall, a correction, ε^w_{ij}, is added to account for the anisotropic behavior of the dissipation rate tensor. The pressure strain part is given by the high Reynolds number model of Speziale *et al.* [11], or

$$\Pi_{ij} = -\left(C_1\varepsilon + C^*_1\widetilde{P}\right)b_{ij} + C_2\varepsilon\left(b_{im}b_{mj} - \frac{1}{3}II_b\,\delta_{ij}\right) - \alpha_1\left(P_{ij} - \frac{2}{3}\widetilde{P}\,\delta_{ij}\right)$$
$$- \beta_1\left(D_{ij} - \frac{2}{3}\widetilde{P}\,\delta_{ij}\right) - 2\left(\gamma_1 + \frac{C^*_3}{2}II_b^{1/2}\right)k\,S_{ij} \quad . \tag{6}$$

The near-wall correction for Π^w_{ij} derived by So and Yuan [8] is given by

$$\Pi^w_{ij} = f_{w1}\left[\left(C_1\varepsilon + C^*_1\widetilde{P}\right)b_{ij} - C_2\varepsilon\left(b_{im}b_{mj} - \frac{1}{3}II_b\,\delta_{ij}\right) + \alpha^*\left(P_{ij} - \frac{2}{3}\widetilde{P}\,\delta_{ij}\right) + 2\gamma^* k\,S_{ij}\right] \quad . \tag{7}$$

Only one near-wall correction is proposed for the combined term, $D^p_{ij} + \varepsilon^w_{ij}$. This term is again written as ε^w_{ij} and is given by

$$\varepsilon^w_{ij} = -(2/3)\,f_{w1}\delta_{ij}\varepsilon + f_{w1}(\varepsilon/k)\,\overline{u_iu_j} + \varepsilon^*_{ij} \quad , \tag{8}$$

where ε^*_{ij} is modeled by the function

$$\varepsilon^*_{ij} = \frac{1}{2}\left[\frac{\partial}{\partial x_m}\left(\nu\frac{\partial \overline{u_iu_j}}{\partial x_m}\right) - \frac{\overline{u_iu_j}}{k}\frac{\partial}{\partial x_m}\left(\nu\frac{\partial k}{\partial x_m}\right)\right] \quad . \tag{9}$$

One damping function has been introduced and this is $f_{w1} = exp\left[-(Re_t/200)^2\right]$ where $Re_t = k^2/\varepsilon\nu$. The other unknown functions are defined by $D_{ij} = -\left[\overline{u_iu_m}\,\partial U_m/\partial x_j + \overline{u_ju_m}\,\partial U_m/\partial x_i\right]$, $b_{ij} = \left\{\overline{u_iu_j} - (2/3)\,k\,\delta_{ij}\right\}/2k$, $II_b = b_{ij}b_{ij}$ and $2\widetilde{P} = P_{ii}$, and the model constants are given by $C_1 = 3.4$, $C_2 = 4.2$, $C^*_1 = 1.8$, $C^*_3 = 1.3$, $\alpha_1 = 0.4125$, $\beta_1 = 0.2125$, $\gamma_1 = 0.01667$, $\alpha^* = -0.29$ and $\gamma^* = 0.065$.

Finally, the modeled ε-equation can be written as

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \varepsilon}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left[C_\varepsilon\frac{k}{\varepsilon}\overline{u_iu_j}\frac{\partial \varepsilon}{\partial x_i}\right] + C_{\varepsilon 1}f_1\frac{\varepsilon}{k}P_k - C_{\varepsilon 2}f_2\frac{\varepsilon\widetilde{\varepsilon}}{k} \quad , \tag{10}$$

where $\widetilde{\varepsilon} = \varepsilon - (C_{\varepsilon 3}/f_2C_{\varepsilon 2})\,\nu\left(\partial k^{1/2}/\partial x_j\right)^2$ and the damping functions are defined as $f_1 = (1 - \alpha f_{w2})$ and $f_2 = 1 - (2/9)\exp\left[-(Re_t/6)^2\right]$. The constants are specified as $C_\varepsilon = 0.12$, $C_{\varepsilon 1} = 1.50$, $C_{\varepsilon 2} = 1.9$, $C_{\varepsilon 3} = 2.9556$ and $\alpha = 1$. It can be seen that all modeled equations are not dependent on any wall unit normals or wall normal coordinate. As such, the near-wall corrections only depend on local variables, hence the model is geometry independent.

THERMAL FIELD MODEL

The near-wall heat flux model selected for improvement is the two-equation model proposed by So and Sommer [3]. This model could re-produce the near-wall heat transfer effects correctly for fluids with different Pr. It is also versatile enough to handle different wall thermal boundary conditions, whether they are given by a constant wall heat flux or a constant wall temperature or a mixed of both. The important thing is that the model could even replicate the thermal field fairly correctly under the assumption of a vanishing wall fluctuating temperature in spite of the fact that its prediction of ε_θ in the near-wall region is in error [1]. The only drawback of this model is its geometry dependence.

So and Sommer [3] assumed the turbulent thermal diffusivity to be given by $\alpha_T = C_\lambda f_\lambda k\left[k\,\overline{\theta^2}/\varepsilon\,\varepsilon_\theta\right]^{1/2}$, where a mixed time scale is invoked and f_λ is the near-wall correction proposed to mimick the viscous damping effects near a wall. If the model has to reproduce the *Pr* effects correctly, f_λ has to be parametric in Pr. Their proposal not only reflects the dependence of f_λ on Pr, but is also parametric in y, the wall normal coordinate. In addition, some terms in their proposed near-wall correction to the modeled ε_θ-equation also depends on y. As a result, their model is not coordinate independent and modifications are needed in order to render the modeled equations to be completely dependent on local properties.

The present approach is to modify the damping function and the near wall correction to the ε_θ-equation, so that their dependence on *y* could be eliminated. An asymptotic analysis similar to that used by Lai and So [12] to derive near-wall corrections to the modeled Reynolds-stress equations is used to deduce an f_λ and a near-wall correction for the ε_θ-equation that are independent of y. Instead of using the wall coordinate as a length scale, a turbulence length scale given by $k^{3/2}/\varepsilon$ is taken to be the characteristic length scale near a wall. Without going into details, the equations for the modified heat flux model can be written as

$$\frac{\partial\overline{\theta^2}}{\partial t} + U_j\frac{\partial\overline{\theta^2}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\alpha\frac{\partial\overline{\theta^2}}{\partial x_j}\right) + \frac{\partial}{\partial x_i}\left(C_{\theta^2}\,\overline{u_iu_j}\,\frac{k}{\varepsilon}\frac{\partial\overline{\theta^2}}{\partial x_j}\right) - 2\overline{u_j\theta}\frac{\partial\Theta}{\partial x_j} - 2\varepsilon_\theta \quad , \tag{11}$$

$$\frac{\partial\varepsilon_\theta}{\partial t} + U_j\frac{\partial\varepsilon_\theta}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\alpha\frac{\partial\varepsilon_\theta}{\partial x_j}\right) + \frac{\partial}{\partial x_i}\left(C_{\varepsilon\theta}\,\overline{u_iu_j}\,\frac{k}{\varepsilon}\frac{\partial\varepsilon_\theta}{\partial x_j}\right) + C_{d1}\frac{\varepsilon_\theta}{\overline{\theta^2}}P_\theta + C_{d2}\frac{\varepsilon}{k}P_\theta$$
$$+ C_{d3}\frac{\varepsilon_\theta}{k}\tilde{P} - C_{d4}\frac{\tilde{\varepsilon}_\theta}{\overline{\theta^2}}\varepsilon_\theta - C_{d5}\frac{\tilde{\varepsilon}}{k}\varepsilon_\theta + \xi_{\varepsilon\theta} \quad , \tag{12}$$

where Θ and θ are the mean and fluctuating temperature, respectively, and $P_\theta = -\overline{u_j\theta}\left(\partial\Theta/\partial x_j\right)$ is the production of $\overline{\theta^2}$. Similar to $\tilde{\varepsilon}$, the reduced ε_θ is defined as $\tilde{\varepsilon}_\theta = \varepsilon_\theta - \alpha\left[\partial\left(\overline{\theta^2}\right)^{1/2}/\partial x_j\right]^2$. This definition still gives the correct asymptotic behavior for $\tilde{\varepsilon}_\theta$ near a wall because the derivatives along the stream and transverse directions are of higher order than that along the normal direction. Therefore, the two leading terms in $\tilde{\varepsilon}_\theta$ are contributed by the normal derivative only. Finally, the near-wall correction function is given by

$$\xi_{\varepsilon\theta} = f_{w,\varepsilon\theta}\left[\left(C_{d4} - 4\right)\frac{\tilde{\varepsilon}_\theta}{\overline{\theta^2}}\varepsilon_\theta + C_{d5}\frac{\tilde{\varepsilon}}{k}\varepsilon_\theta - \frac{\tilde{\varepsilon}_\theta^2}{\overline{\theta^2}} + \left(2 - C_{d1} - C_{d2}Pr\right)\frac{\varepsilon_\theta}{\overline{\theta^2}}P_\theta^*\right] \quad . \tag{13}$$

The C's in these equations are model constants to be specified. Here, P_θ^* is P_θ due to a streamwise mean temperature gradient alone and $f_{w,\varepsilon\theta} = \exp\left[-\left(R_t/80\right)^2\right]$ is a damping function proposed to render (11) zero far away from the wall. The P_θ^* term is a consequence of the constant wall heat flux boundary condition, where $\partial\Theta/\partial x$ is finite. Once $\overline{\theta^2}$ and ε_θ are known, α_T can be evaluated provided f_λ is properly specified, and the heat flux vector is then given by $-\overline{u_i\theta} = \alpha_T\left(\partial\Theta/\partial x_i\right)$.

The heat flux model is complete by specifying f_λ and the model constants. Following So and Sommer [3], the damping function $f_\lambda = C_{\lambda 1}\left(1 - f_{\lambda 1}\right)/Re_t^{1/4} + f_{\lambda 1}$ is assumed. In this expression, $C_{\lambda 1}$ is assumed to be parametric in Pr, while $f_{\lambda 1}$ is taken to be a function of the dimensionless characteristic length $\eta = k^{3/2}u_\tau/\varepsilon\nu$ and Pr. Here, u_τ is the friction velocity. Again, using asymptotic analysis, $f_{\lambda 1}$ can be shown to be given by $f_{\lambda 1} = \left[1 - \exp\{-g(\eta)/A^+\}\right]$. The function g is determined to be

$$g(\eta) = 1.313 + 0.249\,\eta - 0.0045\,\eta^2 + 7.643 \times 10^{-5}\,\eta^3 \quad , \tag{14a}$$

for $\eta < 65$ and

$$g(\eta) = 0.991\,\eta - 45.0 \quad , \tag{14b}$$

for $\eta \geq 65$. The quantities A^+ and $C_{\lambda 1}$ are parametric in *Pr* and are defined as $A^+ = 10/Pr$ for $Pr < 0.25$ and $A^+ = 39/Pr^{1/16}$ for $Pr \geq 0.25$; $C_{\lambda 1} = 0.4/Pr^{1/4}$ for $Pr < 0.1$ and $C_{\lambda 1} = 0.07/Pr$ for $Pr \geq 0.1$. Other constants are given by $C_{d1} = 1.80$, $C_{d2} = 0$, $C_{d3} = 0.72$, $C_{d4} = 2.20$, $C_{d5} = 0.80$, $C_\lambda = 0.10$, $C_{\theta 2} = 0.11$ and $C_{\varepsilon\theta} = 0.11$. Again, it can be seen that, after modifications, the modeled equations are not dependent on the wall unit normal or the wall normal coordinate. Hence, this modified two-equation heat flux model is geometry independent.

RESULTS AND DISCUSSION

The second order velocity field model and the two-equation heat flux model are used to calculate different kinds of heat trtansfer problems. Since the velocity field model has been verified to give good results for a variety of complex flows without heat transfer [8], its credibility needs no further validation. However, the performance of the heat transfer model needs verification. Two steps are taken to verify the model; the first is its ability to predict flows with different Pr after the modifications, and the second is its performance in the prediction of heat transfer problems with complex geometry. This paper reports on the first attempt using DNS data derived from Abe *et al.* [10], while leaving the second to a more detailed study using reliable data.

The available DNS heat transfer data is usually derived from fully-developed channel and pipe flows calculations at different *Pr* and either constant wall heat flux or constant wall temperature boundary conditions. Under these assumptions, the governing mean flow equations for channel/pipe flows can be written in a combined form as

$$\frac{1}{y^j}\frac{d}{dy}\left[y^j\left(\nu\frac{dU}{dy} - \overline{uv}\right)\right] - \frac{1}{\rho}\frac{dP}{dx} = 0 \quad , \tag{15}$$

$$\frac{1}{y^j}\frac{d}{dy}\left[y^j\left(\alpha\frac{d\Theta}{dy} - \overline{v\theta}\right)\right] - S = 0 \quad , \tag{16}$$

where $S = 2(2^j)\,Uq_w/(\rho C_p U_m D)$ is the source term for (16) with constant wall heat flux, q_w, specified, $S = -q''/(\rho C_p)$ is the source term for (16) with constant wall temperature specified, j = 0 denotes channel flows, j = 1 denotes pipe flows and dP/dx is related to the wall shear. Here, q" is the heat source per unit volume, C_p is the specific heat at constant pressure, U_m is the mean velocity and D is either the pipe diameter or the channel height. Similarly, the modeled turbulence equations are also reducible to ordinary differential equations. As a result, the governing equations can be solved by standard numerical techniques and the numerical errors are essentially the same for all calculated cases. The results to be compared [10] covered a range of Pr, from 0.025 to 5.0 at a Reynolds number based on the friction velocity, u_τ, and D of $Re_\tau = 180$.

The results are reported in wall variables, which are defined as $U^+ = U/u_\tau$, $k^+ = k/u_\tau^2$, $y^+ = yu_\tau/\nu$, $\Theta^+ = \Theta/\Theta_\tau$, $\theta^+ = \overline{\theta^2}/\Theta_\tau^2$, $-v\theta^+ = -\overline{v\theta}/u_\tau\Theta_\tau$, and the time scale ratio R. Here, Θ_τ is the friction temperature. In order to show that the seond order velocity field model is indeed giving a correct prediction of the velocity and turbulence field, the calculated U^+ and k^+ are also compared with the DNS data [13] which was reported at an $Re_\tau = 180$. Some sample plots of the results are shown in Figs. 1 through 5. In these figures, besides the present model predictions, the results of the calculations using the model of So and Sommer [3] and the second order velocity field model of So and Yuan [8] are also shown for comparison. The present calculations are designated as "present model" while those using the model of So and Sommer [3] and the DNS data are simply indicated by the reference numbers. This comparison is intended to show that the modifications made to the So and Sommer [3] model do not cause any deterioration in the model performance as far as its ability to replicate Pr effects.

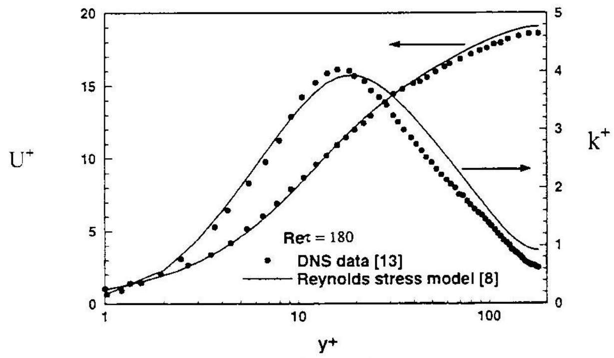

Fig. 1 Comparisons of U^+ and k^+ with DNS data.

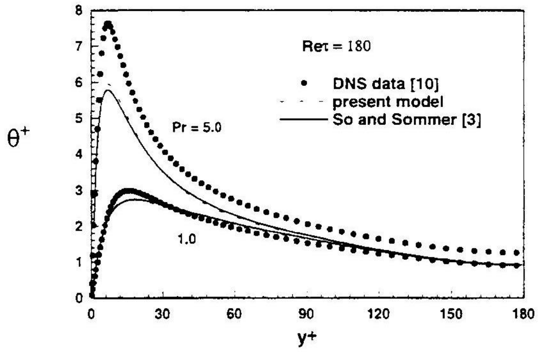

Fig. 3b Comparison of θ^+ with DNS data at Pr = 1 and 5.

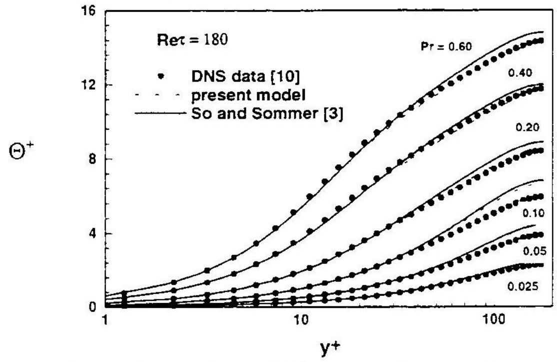

Fig. 2a Comparison of Θ^+ with DNS data at Pr varying from 0.025 to 0.60.

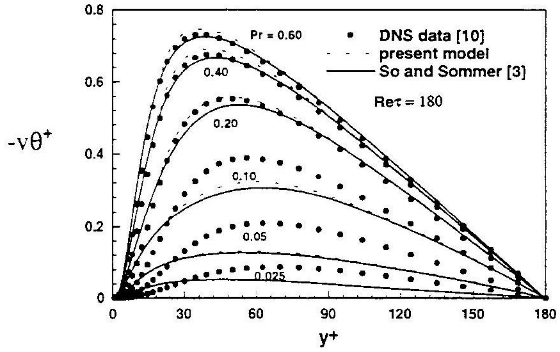

Fig. 4a Comparison of $-v\theta^+$ with DNS data at Pr varying from 0.025 to 0.60.

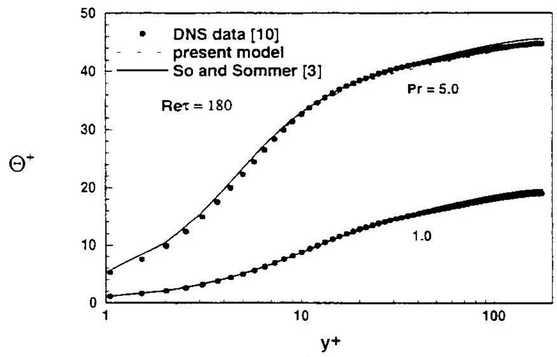

Fig. 2b Comparison of Θ^+ with DNS data at Pr = 1and 5.

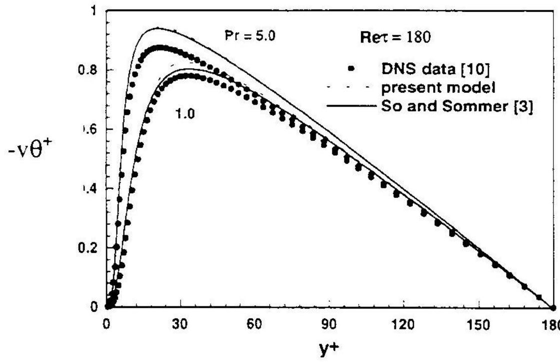

Fig. 4b Comparison of $-v\theta^+$ with DNS data at Pr =1 and 5.

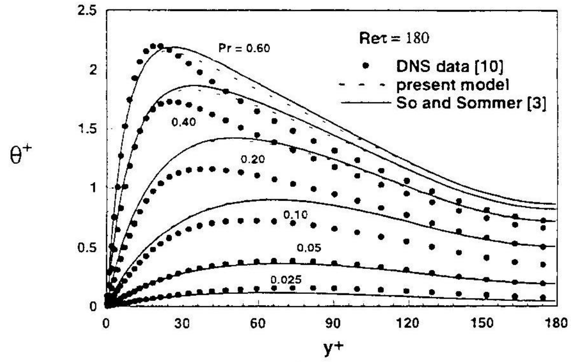

Fig. 3a Comparison of θ^+ with DNS data at Pr varying from 0.025 to 0.60.

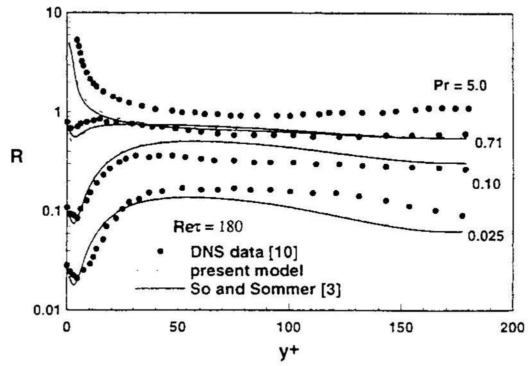

Fig. 5 Comparison of R with DNS data at four different Pr.

The comparisons of U^+ and k^+ are shown in Fig. 1 and they simply verify that the second order model is giving good results for this fully developed channel flow. Mean temperature Θ^+ is compared in Fig. 2 for eight different Pr. In this and subsequent figures, the comparisons with *Pr* less than one are shown in part (a) of the figure while the comparisons with Pr greater or equal to one are given in part (b). Both the present model and that of So and Sommer [3] yield the same result for the whole range of Pr calculated and the Pr effects are predicted correctly. In fact, the discrepancy is so small that it is not noticeable in the plots. The temperature variance predictions for the range of Pr considered are compared in Fig. 3. Here, differences between predictions and the DNS data show up at large Pr. At Pr = 5.0, a subatantial under-prediction of the θ^+ peak is noticed, while differences between predictions and DNS data also occur at other Pr in the range, 0.1 to 1.0. In spite of this, there is still not much discrepancies between the calculations of the present model and that of [3]. The normal heat flux is compared next in Fig. 4. In general, the discrepancies between predictions and DNS data occur either at Pr < 0.20 or at Pr = 5.0, while there is little difference between the two model calculations. Finally, the time scale ratio is compared in Fig. 5. The trend predicted by the models is essentially correct, even though there are minor differences between calculations and DNS data. For Pr < 1.0, the models yield a fairly correct prediction of R in the near-wall region. On the other hand, the predicted R is lower than the DNS data for Pr = 5.0. These results, therefore, verify that the modifications made to render the heat flux model geometry independent do not unnecessarily affect the ability of the model to replicate the Pr effects in an incompressible flow with heat transfer.

CONCLUSIONS

The present study attepts to formulate a geometry independent turbulence model for incompressible heat transfer problems that involve fluids with different Pr. Two different components are required; they are the velocity field model and the thermal field model. Both models have to be independent of the flow geometry before the combined model can be claimed to be truly geometry independent. Furthermore, it is advisable to have a thermal field model that is at least one level lower than the velocity field model. In view of these constraints, it is decided to formulate a heat transfer model that is based on a second order velocity field model and a two-equation thermal field model. The alternative is to have a two-equation model for the velocity field and to invoke a constant Pr_t assumption for the thermal field, which is not a viable alternative for complex heat transfer problems. Since the second order model of So and Yuan [8] is geometry independent, the proposed approach is to adopt this model for the velocity field and to modify the two-equation model of So and Sommer [3] for the heat flux. This latter model has been formulated for different Pr, but it is not geometry independent because of the presence of wall unit normals and wall normal coordinate in the near-wall correction functions. Asymptotic analysis is used to derive alternative near-wall correction functions that are capable of mimicking the Pr effects and yet are truly geometry independent. Validation of the combined model has been carried out in two steps, the first is against simple channel/pipe flows with different Pr and the second is against complex flows at a fixed Pr. The calculations are compared with DNS data over a wide range of Pr. Good correlation with mean temperature is obtained. However, for the turbulence statistics, minor differences exist at either Pr = 5.0 or Pr < 0.2. The comparison with flows having complex geometries will be carried out later.

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