CHAPTER 11

Small-World Networks

Imagine that you are the supreme ruler of a number of islands within an archipelago. Your daily routine as dictator includes sipping on mojitos, getting a tan, jogging on the beach, and receiving wonderful massages. Occasionally, you do a few things that involve country management. Sometimes these things involve making hard choices. One recent choice you had to make concerns the allocation of boats on shipping routes.

Your country has only a limited number of boats. Either these boats can sail in and out of your country, docking at trading ports with the rest of the world, or the boats can be used to improve travel among the many islands that make up your country. Trading with foreign countries allows much needed resources to come in, such as food or other types of raw goods. In return, you export some things unique to your island, such as your very special coconuts. At the same time, if travel is made easier among your islands, it helps foster a sense of national identity and preserve your unique customs. Citizens within your country will now be able to travel to the island across the bay to visit their relatives, have a barbeque, and sing songs to your greatness with their extended families. This type of cohesion within your country is important—just in case a foreign country decides to invade you for your coconuts.

In the scenario presented above, the limited number of boats determines how you control physical space. When it comes to friendships and time, there are some parallels. Maintaining a good friendship may cost a considerable amount of investment and dedication on your part. It may be impossible to be good friends with everyone. Sometimes you have to spend quality one-on-one time with them to make them feel special, or invest money—another limited resource—on gifts or the cost of just hanging out.

In most countries, national borders are the result of politically drawn lines. Sometimes these lines are influenced by natural geographical features such as a river, a mountain range, or, in the case of your country, a sea. If we are to extend the analogy to friendships, the borders of your social circles are not as clear as a country's border. This is because the borders of your social circles are determined by whether or not your friends know each other. Suppose you met your main core group of friends from living on the same dorm floor as them. Then it is likely that some of your friends are friends with each other as well. Maintaining your social relations with this group of friends may be like investing in roads within your country. Investing in a road that leads outside your country may be like maintaining a relationship with a unique friend that is radically different from your core group of friends. This person also should not know any one from your core group of friends. So, in essence, your relationship with this person serves as a sort of bridge to a distant part of a social network.

Even this extended analogy is still not a clear-cut way to define the borders of your social circle. Sometimes you may have multiple groups or cliques of friends. Some friends may belong to more than one group. Even with these issues, the function of investing in relations within your group of friends versus relations with acquaintances on the fringe remains similar to building roads within and out of your country. You may receive unique social resources as a result of those fringe friendships. For example, your fringe friend is more likely to introduce you to something fresh and new than someone within your core group of friends, with whom you already share many of the same activities. This could be anything from a new restaurant recommendation to a new job offer. At the same time, you may enjoy spending time with your core group of friends, or at least find it easier, because of the redundant nature of within-group ties. Your core group of friends may all share a common reference point for gossip or have a common preference for music that prevents conflicts over what track is playing.

The bottom line is that our social connections function much like the roads in your imaginary country—they can be far-reaching or local. However, because of the interesting behavior of social network structure, our social connections can sometimes be both far-reaching and local. This is known as the small-world phenomenon.

SHORT NETWORK DISTANCES

In the previous chapter, we saw that random networks have very low distances between any two given vertices. The other network structures that were explored, such as lattices, did not have this basic property. As we will see, this property is important for social network models because evidence suggests that real-world networks tend to have very low distances as well. What this means is that our social connections function much like far-reaching roads—you don't have to keep changing highway routes to get where you want to go.

The Milgram Experiment

Social Psychologist Stanley Milgram was a classic social scientist who conducted a number of well-known experiments. You may have heard about his obedience experiments, where subjects were goaded into applying seemingly painful electric shocks to victims that were really actors. The grim black and white photos from the experiment would work as well in a horror documentary film as they would in a social psychology textbook.

In 1967, Milgram (Travers and Milgram, 1969) conducted another important experiment that was perhaps less well-known because it wasn't as shocking—excuse the pun. To put it in network terms, Milgram was interested in exactly how short distances were among individuals. Since he wanted results to be as surprising as possible, he chose subjects who were considered socially distant from each other. Thus instead of picking two random individuals in the same city, he choose a target individual in Boston, Massachusetts, a fairly urbanized East Coast city, and then selected subjects in Omaha, Nebraska, a more remote town in the Midwest. A subject chosen in this way would probably have different musical tastes than the target individual living in the city, or even a different accent. Keep in mind that that the Internet and cell phones didn't exist back then, so the social space between those two cities was a bit wider.

The goal of the study was for the participant in the Midwest to try to reach the target person through a chain of letters sent by mail. Each person in the chain could forward the letter only to a person whom they knew on a first-name basis. Of the 296 letter chains initiated by the Midwest residents, only 64 eventually reached their target resident in Boston. If you think about it in today's terms, this was an amazing result. Some of us get fairly annoyed at even moving our wrists and clicking our fingers to delete the spam in our email inbox. Imagine receiving one of these chain letters in 1964, and then having to write on paper, the name and address of someone you might not even know that well, and then having to get up to put this annoying packet in a mailbox. What a chore!

However, among the 64 letters that did reach their destination, it took an average of 5.5 steps. This result eventually was developed into the popular phrase “six degrees of separation”, or the idea that we are all somehow connected by six steps on average. Milgram's experiment not only suggested that real-world social networks had a low average distance but also demonstrated various methods that individuals used to search their networks. The subjects sent letters to contacts whom they knew who lived closer geographically to the target person. Of course, the path that the chain letter took to get to the target might not necessarily have been the shortest path. There might have been contacts whom the participants knew who might have been much closer to the target. It is also entirely possible that participants ended up forwarding the packet to an acquaintance who was even further from the target person than they were. Nonetheless, even without a map of the web of social relations or tempting incentives, Milgram demonstrated that people could reasonably navigate a packet through a large complex network.

Milgram's classic experiment was recently replicated on a worldwide scale by a research team led by Duncan J. Watts (Dodds et al., 2003) at Columbia University. Instead of document packets, participants used email. The results indicate that Milgram's initial ballpark of 5.5 was pretty close, as the average length of the completed email chains was about 5.

Kevin Bacon: Center of the Universe?

In April 1994, someone made a claim on an Internet newsgroup, which was the equivalent of the modern-day Internet forum back then, that a movie actor named Kevin Bacon was the center of the universe. The poster wasn't trolling. As proof, he proposed that any movie actor, no matter how obscure, could be linked back to Kevin Bacon in a very few number of steps. Movie actors were considered connected if they starred in the same movie together. The post quickly grew in length with other users putting the claim to the test. Eventually this turned into a popular game for those with an unhealthy knowledge of film actors. Someone nominates an actor, and another person tries to find the shortest distance between that actor and Kevin Bacon. This distance became known as the Bacon Number.

Today, there's a website called the Oracle of Kevin Bacon, where users can input any actor, and using the Internet Movie Database (IMDb), the website will find the shortest path between the given actor and Kevin Bacon. A bookmark to this website on your Internet phone is quite useful if you ever need to impress your friends at a party who happen to be playing the Kevin Bacon game—providing that you access the website discreetly.

Kevin Bacon isn't necessarily the center of the universe. He is, however, very well connected from starring in a wide variety of movies in different genres over a long period of time. And even though there are other actors who have an even shorter average distance to every other actor, the difference in distance between them and everyone else and some obscure actor and everyone else isn't that great either. The moral of the story is, however, that distances tend to be short in general, which is consistent with the results of Milgram's experiment.

SOCIAL CLUSTERING

At the beginning of this chapter, connections within social groups were used as an analogy for local roads. We now introduce a measure for quantifying how local or clustered a particular individual is in a network.

Density

As an abstract concept, density is simply how full or packed an object is. Suppose we convert this into a measure that is bounded by 0 and 1. You

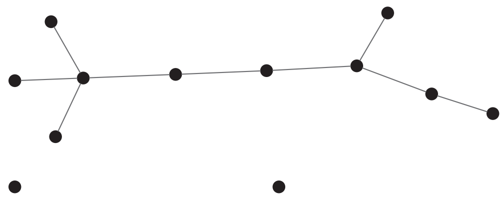

Figure 11.1. Network with two isolates.

could think of an empty glass of water being measured as 0, a glass half full (or half empty, for all you pessimists out there!) of water being measured as 0.5, and a glass filled to the brim being measured as 1. For networks, density describes how many edges exist over the total number of possible edges. If a graph is completely empty, it will have a density of 0, and if the graph is a complete graph, it will have a density of 1.

Suppose we have a network with only two individuals, and the relationship represented by the network is bidirectional friendship. Then these two individuals can be either connected or not, and therefore there are only two possible density values—0 and 1. If the relationship represented in this network were a directional relation such as respect, then there would be three possible density values: $\{0, 1/2, 1\}$. These individuals can both not respect each other, or one can respect the other without reciprocation, or they can both respect each other. Therefore, in directed graphs, the arcs are counted as the relationship in the network, and represented by m, whereas in undirected graphs, edges are represented by m. Because there are twice the total number of possible relationships in directed graphs, it is important to remember whether you are dealing with undirected or directed graphs when calculating density. We now generalize the formula for density in networks.

For undirected graphs, the density is given by,

$$d_{undirected}(G) = \frac{m}{\frac{n(n-1)}{2}} = \frac{2m}{n(n-1)} \tag{11.1}$$

where m is the number of edges and n is the number of vertices in the graph. The numerator is simply the number of relationships in the network, and the denominator represents the number of ways to select unordered subsets of 2 from a set of size n. In other words, the denominator represents all the ways to pick pairs of two people from the list of people in your network. For directed graphs, the formula simply becomes,

$$d_{directed}(G) = \frac{m}{n(n-1)} \tag{11.2}$$

Example 1. What is the density of the network given in Figure 11.1?

The network in Figure 11.1 has 12 vertices (including the 2 isolates) and 9 edges. Using the formula for the density of an undirected network, we get,

$$d_{undirected}(G) = \frac{2m}{n(n-1)} = \frac{2(9)}{12(11)} = 0.136 \tag{11.3}$$

Alternate Conceptualization

Another method of interpreting both the concept and the formula of density is to envision the adjacency matrix of a network. Recall that the adjacency matrix of a network is a $n \times n$ matrix where n is the number of people in the network. The entry in row i, column j in the matrix represents the value of the relation between person i and person j. In most types of networks, this is merely a binary value that reflects whether the tie is present (1) or not (0). Recall that the diagonal of an adjacency matrix is meaningless, as the people in the matrix cannot be connected with themselves. Therefore the number of possible valid cells in directed networks is $n^2 - n$, where n^2 represents the total number of cells in the matrix and $-n$ excludes the diagonal. Factoring out n, we obtain $n(n-1)$.

Example 2. Given the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, the density is 1.

In Example 2, all of the nondiagonal cells in the matrix are filled, so the network represented by the matrix has a density of 1.

Example 3. Given the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, the density is 4/6 or 2/3.

If the matrix given in Example 3 represented a digraph, then four out of the six possible nondiagonal cells are filled, so the network has a density of 4/6. Since the matrix is symmetric, with cell m_{ij} being equivalent to its reflection across the diagonal, cell m_{ji}, the matrix can also be seen as representing an undirected graph. In this case, 2 out of the 3 possible nondiagonal cells are filled.

Ego Networks

An ego network is sort of a mini network that consists of just an individual and the other individuals whom she is connected to. For example, your ego network would contain yourself and your immediate friends, and the relationships among you and your friends. However, it would not contain a friend of your friend whom you were not friends with. The diameter of an ego network can be at most two, for example, when you have two friends who do not know each other. A normal network of n individuals has n distinct ego networks.

Although an ego network is lacking in the rich structural features that normal, full-sized networks have, it is still popular and widely used in research because of its simplicity. One problem with collecting data on networks is defining the boundaries of the network. Suppose that you were not collecting ego network information, but instead you were mapping out the entire friendship network in a population. You ask one respondent who his friends are, and then you ask those friends who their friends are. Each new respondent refers you to her friends, and pretty soon the number of respondents in your study snowballs out of control. It is difficult to come up with a nonarbitrary criterion for when to stop collecting data. With an ego network, the boundary is determined by definition. You simply interview one respondent, then interview their friends, and stop.

Additionally, social networks that represent a subset of a larger population are likely to be unrepresentative of the population as a whole. Because of the principle of homophily, those in the same network are likely to share the same traits or attributes. Because ego networks are centered around a specific individual, as long as the person is representative, the set of ego networks may be somewhat representative. Some large-scale random surveys, such as the General Social Survey (GSS), have specific years when information on ego networks is collected.

Clustering Coefficient: Local Density

Now we combine the concepts of both network density and ego networks and arrive at a clustering coefficient. The clustering coefficient of an individual is just the density of that individual's ego network *with the individual removed*. In other words, it's the density of the network formed by the individual's friends and the relations among them. The last part about excluding the original individual is important because clustering coefficients are not simply the density of the ego network. The individual and ties from the individual to her friends are excluded because that information is trivial and redundant by definition. Everyone has a friendship relation with his friend. If he does not have a friendship relation with the friend, then that person is not a friend and is excluded from the ego network.

Example 4. Suppose Adam has three friends: Bob, Conrad, and Dan. Of Adam's three friends, only Conrad and Dan are friends with each other. What is Adam's clustering coefficient?

The correct way to calculate the clustering coefficient is to consider the network of only Adam's friends, which consists of Bob, Conrad, and Dan ($n = 3$). Since only one friendship exists in this network, the tie between

Conrad and Dan, $m = 1$. Using the formula for the density of an undirected network, we get,

$$d_{undirected}(G) = \frac{2m}{n(n-1)} = \frac{2(1)}{3(2)} = 1/3 \tag{11.4}$$

Thus, the clustering coefficient for Adam is 1/3, which is correct.

Now, suppose we forgot to exclude Adam and the ties to his friends in the calculation and computed the density of Adam's ego network. This network would have four people ($n = 4$), and it would have four edges: Adam's three ties to his three friends and the extra tie between Conrad and Dan ($m = 4$). If we used the formula for the density of an undirected network in this situation, we would get,

$$d_{undirected}(G) = \frac{2m}{n(n-1)} = \frac{2(4)}{4(3)} = 2/3 \tag{11.5}$$

The above answer of 2/3 is not the correct clustering coefficient for Adam, but it represents the raw density of Adam's ego network. Thus, we see that although the clustering coefficient can conceptually be considered the local density, or the density of an actor's ego network, it is not exactly the same thing in practice.

It is important to note that the clustering coefficient maps an individual in the network to a number between 0 and 1. It is not a measure associated with the entire network. However, when we talk about the clustering of a network, we are talking about the average of the valid clustering coefficients for everyone in the network. Individuals who are disconnected and isolated are not included in the calculations because their friendship network does not exist. Individuals with only one connection are also excluded because their clustering coefficient is undefined. If they have only one connection, then the denominator of their clustering coefficient is $n(n-1) = 1(0) = 0$, giving you a divide by zero scenario. Therefore, the clustering of a network measure is not a real average of clustering coefficients for all individuals in the network, but only for those with two or more connections.

Sometimes the terms "clustering coefficient" and "clustering" are used interchangeably for individuals and networks. For the purposes of this text, we will refer to clustering coefficients as a measure of the local density of a specific individual, while clustering will refer to the average of all valid clustering coefficient values in a network.

Another way to interpret the clustering coefficient of an individual in a friendship network is to think of it as the probability that the individual's friends are friends with each other. Someone with a high clustering coefficient probably spends a lot of time in group activities with her friends. For example, they could all be students who live in the same dorm or frat house or play on the same sports team. The group activity

Table 11.1.
How density decreases in large networks

n	$m = 15n$	$d_{example}(G) = \frac{2m}{n(n-1)}$
31	465	1
100	1,500	0.303030
500	7,500	0.060120
1,000	15,000	0.030030
10,000	150,000	0.003000
1,000,000	15,000,000	0.000030
6,700,000,000	100,500,000,000	0.000000

increases the likelihood that everyone knows each other. Someone with a low clustering coefficient might be someone who has a lot of Internet friends from different places who may not necessarily know each other.

Clustering in Random Networks

In random networks, the clustering of the network is approximately equal to the density of the network. If on a macro level, the entire random network is dense, then on a micro level, when you examine the local network around a single individual, that network is likely to be dense. Conversely, if the random network is sparse overall, then the local networks around individuals will be sparse as well.

If we use random networks as our model for real-world networks and assume that people could maintain only a finite number of relationships, then in very large populations, the density of such a random network approaches zero. For example, let's make the reasonable assumption that, on average, someone maintains 30 friendships. Since each friendship tie consists of two people, then in this network, each person will contribute 15 to the number of edges ($m = 15n$). Thus the formula for the density of this network is,

$$d_{example}(G) = \frac{2m}{n(n-1)} = \frac{(2)(15n)}{n(n-1)} \tag{11.6}$$

Table 11.1 gives the density of the network with different numbers of people. When this network has 31 people, the density of the network is 1 because each person is connected to all the remaining 30 people in the network, and thus all the possible edges exist. Notice that as the number of people in the network increases, the density of the network decreases. If we used this random network to model the approximately 6.7 billion people on earth, the density would be practically zero.

Since the network is random, this means the clustering is practically zero as well. This is because we know in random networks, the density of local networks on the individual resembles the entire network. This means that if friendship networks were random, it would be almost statistically impossible for your friends to be friends with each other. However, we know this is not the case. Although random networks exhibit the low path lengths that we observe in real-world networks, they do not exhibit the clustering property. We now introduce the small-world network model, which combines both low path lengths and relatively high clustering.

THE SMALL-WORLD NETWORK MODEL

Watts and Strogatz (1998) presented a network model that was both far-reaching and local. The idea is elegant in its simplicity: start with a localized graph with high clustering and high average distances between vertices, and then randomly rewire edges. Rewiring is defined as deleting an existing edge and creating a new edge between two previously unconnected vertices. With each rewired edge, the expected distance between any two vertices decreases. The clustering coefficients, which were initially high, also decrease. However, an interesting side effect of the rewiring is that the average distance between vertices drops much faster relative to clustering. The average distance between vertices in the rewired networks approaches that of a random graph with only a few rewires, while clustering can still remain fairly high. When a network is rewired to a point where it exhibits these properties, it is considered a small-world network. However, with additional rewiring, the clustering will continue to fall until both distances and clustering are low. The resulting network at this point is no longer a small-world network but a random one.

In addition to introducing the small-world network model, Watts and Strogatz suggested that this small-world network structure was natural. While it may have been created due to random chance, there was something unique and special about it. In addition to the social example of the movie actor network, Watts and Strogatz showed that networks with high clustering and low average path length also occurred in the neural network of an earthworm and also in power grid networks. Later on, this structure was discovered in many more natural phenomena by other researchers. The small-world network therefore has broad interdisciplinary appeal.

We now explore a few examples of the generation of a small-world network.

Clustered Circle Graph

The clustered circle graph was one of the first models that Watts and Strogatz used. They called it the α model. Figure 11.2 shows a clustered circle graph with 10 vertices. With this small graph, you can notice that

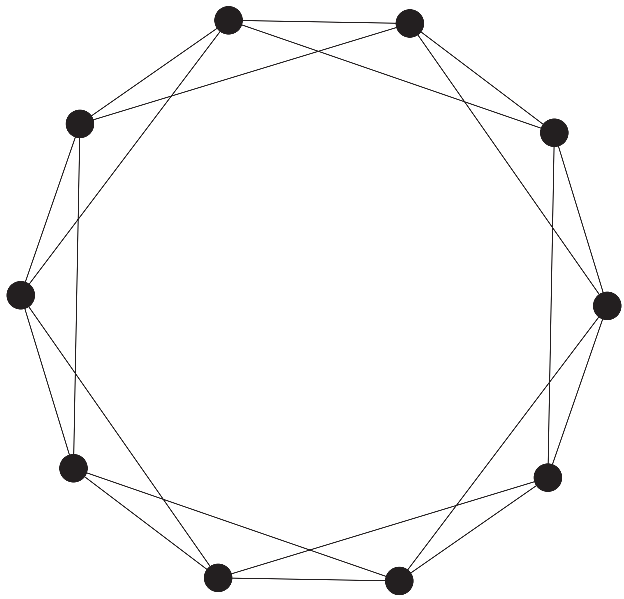

Figure 11.2. Clustered circle graph, $n = 10$.

there is a pattern to how each vertex is connected: each vertex in this graph is connected to four other vertices, the two closest neighbors in both the clockwise and counterclockwise direction. Suppose we had a clustered circle graph with 50 vertices. First, let's find the density of this graph. We know that $n = 50$, and we want to obtain the number of edges in this graph, m. We know that each vertex has 4 edges, so the average degree is 4. Using the formula for average degree, we deduce that the network has 100 edges.

$$\begin{aligned} avgdeg(v) &= \frac{\sum deg(v)}{n} = \frac{2m}{n} \\ 4 &= \frac{2m}{50} \\ 200 &= 2m \\ 100 &= m \end{aligned} \tag{11.7}$$

Recall that the local density or clustering for a random network is approximately equivalent to its overall density. So, if this were a random graph, we would expect a clustering coefficient of 0.166667.

$$\frac{2m}{n(n-1)} = \frac{2(100)}{50(49)} = 0.082 \tag{11.8}$$

However, because the graph isn't random, let's calculate the initial clustering coefficient for our clustered circle graph. Since all vertices are in an identical position, we need to calculate the clustering coefficient for only one vertex, and assume that value to be the average of all positions. Let's pick a random vertex, and call its neighbors A, B, C, and D. Among these four vertices there are three connections: {A–B, B–C, C–D}. Thus $n = 4$ and $m = 3$. Using the density formula, we get,

$$\frac{2m}{n(n-1)} = \frac{2(3)}{4(3)} = 0.5 \tag{11.9}$$

As 0.5 is much greater than 0.082, we see that this network has a much higher than expected value for clustering in a random graph.

Another thing to note is that distances in this clustered circle graph are relatively high. The diameter of this circle graph is 13. If you picked a pair of vertices that were spatially the furthest from each other, it would take 13 hops to reach the other. You'll have to trust me on the next calculation, but the average distance between all 1,225 pairs of vertices is 6.63. This is a huge distance considering there are only 50 vertices in the network. Remember that according to Stanley Milgram, and later on Duncan Watts, in a world with more than 6 billion people, the proposed average distance between any two random individuals is only about 5 or 6!

Figure 11.3 shows the network after one rewiring and then five rewirings. Notice that dramatic changes can already be seen in the network after just one edge is randomly deleted, then readded. The new edge forms a sort of bridge in an area that is fairly close to the center of the network. If you had to bet some cash on it, you would probably bet that the average path length in the network has decreased dramatically from this new connection. To be exact, the average distance has decreased from 6.63 to 5.87. Because the rewiring is a random procedure, if we reset the network and randomly rewired another edge, the change could be larger or smaller depending where the new bridge forms. The best decrease in distance would be if the new edge appeared exactly in the center of the circle graph, dividing it evenly into two smaller ovals. After five rewirings, the average distance has dropped to 4.83. On the other hand, after five rewirings, the clustering of the network is still around 0.39.

While the small-world model may at first appear abstract and mechanical, without analogues in the real world, it does offer some sociological insights that may be worthy of discussion for those who like to sit at cafes with their goatees, shades, and Frappuccinos. Duncan Watts compared the early clustered start of a small-world network model to a primitive caveman society, where humans lived as isolated tribes. Watts also argued that the random network, which is the ultimate product of the prolonged rewiring process, could be representative of a futuristic society, where someone's set of friends may include random people spread across the entire planet because of communication and travel technologies. Our real world could be thought of as somewhere in the middle.

The idea is not entirely new. Social theorists have often pondered the effects of modernization and the role of increasing distant connections.

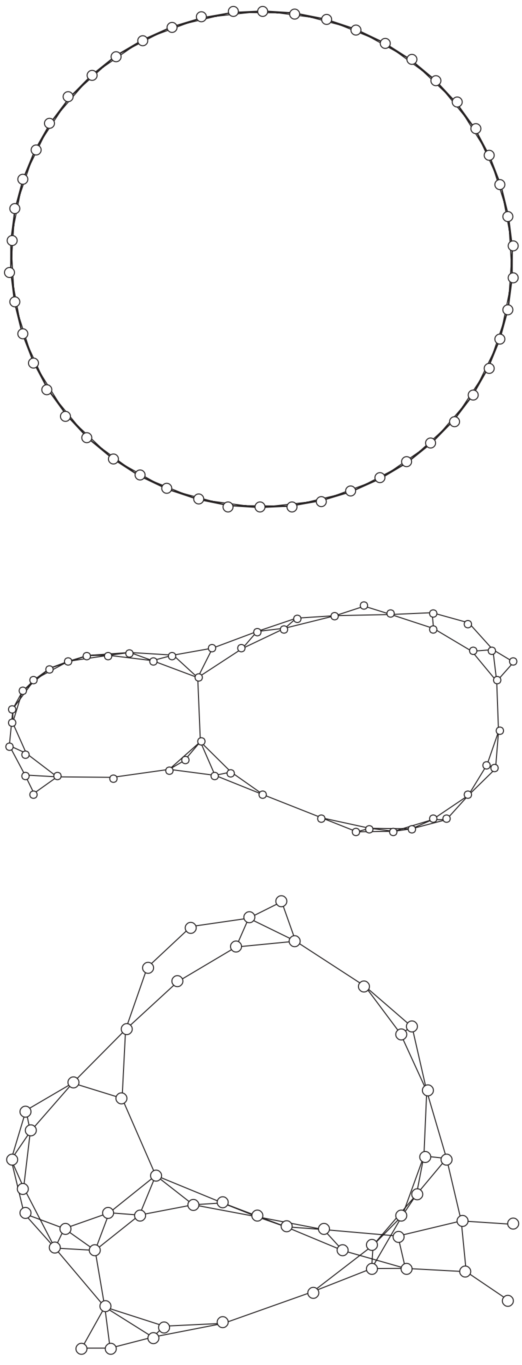

Figure 11.3. Clustered circle graph with no rewiring, one rewiring, and five rewirings, $n = 50$.

Anthony Giddens, one of the most influential sociologists of the 20th century, argued in his theory of structuration that as societies become modernized, the concept of time becomes increasingly independent from space, and subsequently the concept of space becomes independent from place. In a society without wheels, lightbulbs, or a sundial, an individual's daily routine and environment are very dependent on each other. Consequently, there will be a lot of overlap with other individuals in the same community, which could lead to what we call high clustering. Today, clustering still exists in our social networks but is just as often caused by similarity of interests or hobbies, as the space in which someone inhabits. Humans have increasingly become more mobile over the ages, through horses, the wheel, the boat, and the plane. The tools for communication have increasingly defied space through the telephone, the cell phone, the Internet, and now Internet cell phones. Communication tools have also become independent from time through writing, the phonograph, the cassette, the compact disc, and now Internet streaming videos. These have increasingly allowed the people greater opportunity to form random, far-reaching connections. The impact is that someone's local social space is increasingly "penetrated by and shaped in terms of social influences quite distance from them" (Giddens, 1990, 19). Giddens presents a number of consequences for such a phenomenon. For example, the property for social structures to be both local and global facilitates the growth of rationalized bureaucratic organizations. Thus, the small-world model could be considered an abstraction for the effects of modernity on social structure.

Even if you don't buy the story above, there is also the idea that the small-world concept through social network analysis has turned something that might have previously been thought of as abstract, qualitative, and unmeasurable into something concrete and quantitative. Network diagrams essentially show social structure, typically an abstract concept before social network analysis. We can implement the abstract terms coined by Giddens, such as "phantasmagoric" or "distanciation" as the average path length. Social cohesion could be operationalized as clustering coefficients, although there are many other ways to define and measure the concept of cohesion. We can collect network data on two separate societies, compare a network statistic such as clustering, and not only say which one has more clustering but also calculate how much more. Being able to say that the speed of light is 299,792,458 meters per second is much more precise than saying that the speed of light is very, very fast. Of course, many abstract sociological concepts may lose a lot of color when reduced to mere numbers, but measures can always be refined.

Finally, remember that even though models may not be empirical, they are at least theoretical. The random network model was a point of comparison, something to which we can compare our empirical networks and point out differences. At the very least, models may serve as our null hypothesis or controls. With the small-network model, we have noted the discrepancy in clustering between random and many real-world social networks, and have adjusted for it. In the next chapter, we explore another discrepancy.

EXERCISES

1. Given a clustered circle graph with 10 vertices and 4 connections per vertex, calculate the expected clustering given a random graph with the same number of vertices and edges, and the actual clustering of the circle graph. What is the diameter of this graph?
2. Given a 10×8 two dimensional lattice with 80 vertices, with each vertex being connected to its immediate orthogonal neighbors, calculate the expected clustering given a random graph with the same number of vertices. Next, calculate the actual clustering of the lattice. What is the diameter of this graph?