Basic ERGM estimation

Simone Santoni

2024-11-19

Synopsis This notebook shows how to fit a basic ERGM on a one-mode, directed network dataset

Notebook setup

Load libraries

We need to load three libraries:

- `car`, which stands for 'Companion to Applied Regression,' provides utility functions regarding regression models
- `sna`, a Statnet's library, includes many network descriptives
- `ergm`, another Statnet's library, implements ERGMs.

```
library(car)
library(sna)
library(ergm)
```

Load data

ERGM 101

What is the objective of ERGMs?

ERGMs test how and to what extent an observed network exhibits certain tie formation mechanisms. Example of tie formation mechanisms include (but they are not limited to):

- In-degree centrality, the tendency of a node i to receive ties
- Out-degree centrality, the tendency of a node i to send ties
- Reciprocity, a *dyadic* tendency such that $i \rightarrow j$ & $j \rightarrow i$
- Transitive closure, a *triadic* tendency such that i.e., $i \rightarrow j$ & $j \rightarrow w$ & $i \rightarrow w$
- Balance, the tendency for two nodes i and j to share alters $a = a_1, a_2, ..., a_n$
- Node attributes, i.e., node i's qualities

- Dyadic attributes, i.e., the similarity (or differences) in nodes i and j qualities

What is the intuition behind ERGMs?

ERGMs consider observed networks as mixtures of network effects

The General Form of ERGM

ERGMs estimate the probabilities that the observed network y comes from the class Y based on a set of endogenous and exogenous tie formation explanations (aka 'model effects'). ERGM's general form is the following:

where θ^T is the vector of regression coefficients regarding the model effects $g(y, X)$, and $k(\theta)$ is the summation of the numerator's value over the set of all possible networks y.

To better understand ERGM's general form, we can dispense the numerator of the previous equation as follows:

This equation highlights that the probability of observing a particular network in a set of networks—e.g., ten-node networks exhibiting significant in-degree popularity and reciprocity—as a function of many $g(y)$. We can also dispense the equation in terms of the log odds of an edge:

where $\delta[(y, X)]$ is the 'change' statistic, that is, the change in $g(y, X)$ when the value of only the ij dyad is changed from 0 to 1.

Examples of Model Effects

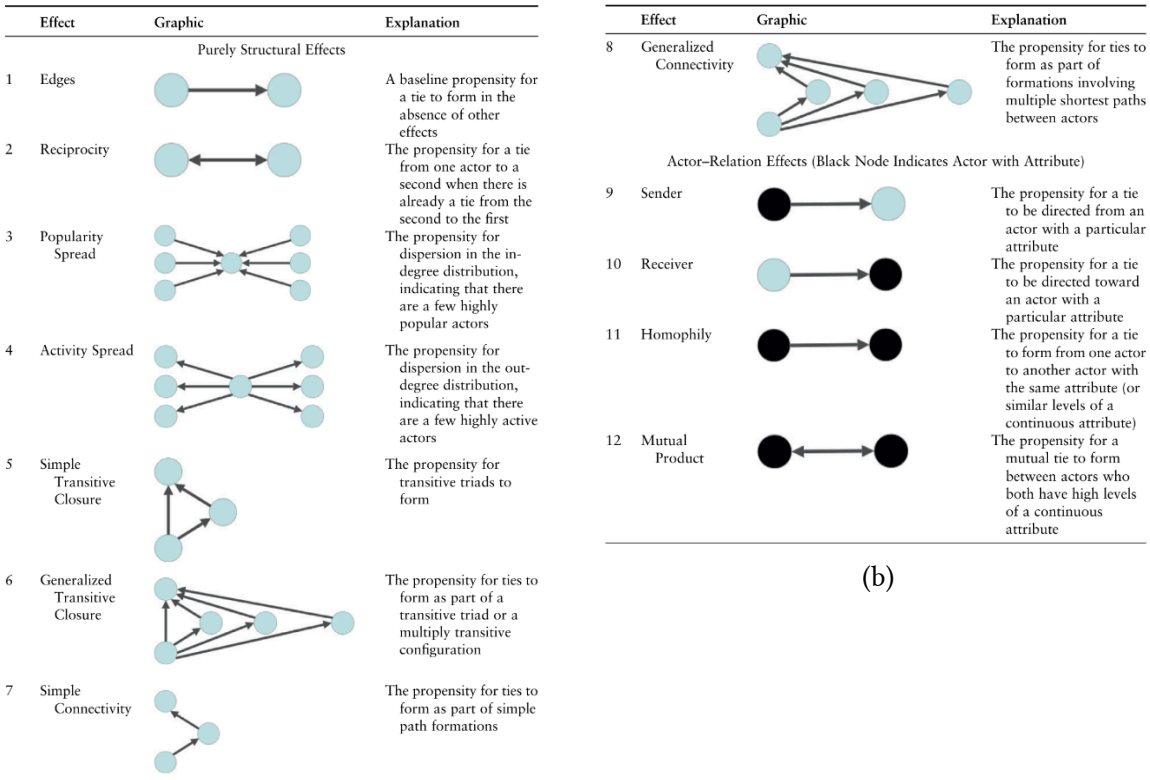


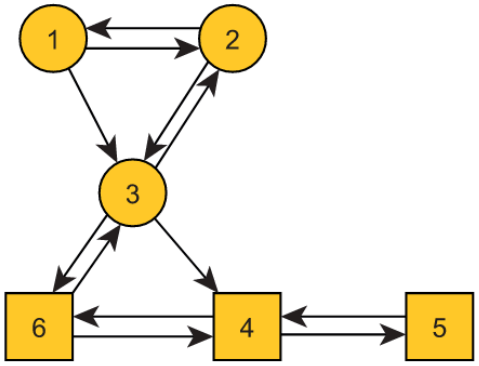
Figure 1: Model Effects reported in Rawlings et al. (2023, pp. 322-323)

How Do I Compute the Change Statistic $\delta[g(y, X)]$?

ERGM libraries, like R's `ergm`, do that for you. However, it is important that you familiarize yourself with computing the change statistic $\delta[g(y, X)]$. Here are two key premises:

- Mainly, the procedure aims to create the regressors for the above-displayed Logit model. For example, one may want to regress the likelihood to observe a tie from i to j against i and/or j's degree, the existence of the tie from j to i, the fact that i and j are involved in a triad to which a third node w, and so on and so forth
- Overall, the procedure consists of a 'thought experiment.' For each tie involving a pair of nodes $\{i, j\}$, we ask ourselves:
  - "Does adding the tie from i to j make the relationship 'reciprocal', that is, $i \rightarrow j$ & $j \rightarrow i$?"
  - "Does adding the tie from i to j make the triad involving i, j, and w transitive?"
  - "Is the tie from i to j involving two similar (equivalent) or dissimilar (different) nodes"

The below-displayed figure illustrates this kind of thought experiment visually. The algorithm will replicate the thought experiment for us, iterating over all possible pairs of node ${i, j} creating the input for the Logit regression. The final dataset will have $N \cdot (N - 1)$ rows (aka ties) and $K + 1$ columns, where K is the number of selected model effects. The $+1$ signifies the column with the dependent variable information (aka, whether a tie is present or absent).



i	j	Tied	Edges	Recip	Trans	Shape
1	2	1	1	1	2	1
1	3	1	1	0	3	1
1	4	0	1	0	1	0
1	5	0	1	0	0	0
1	6	0	1	0	2	0
2	1	1	1	1	1	1
2	3	1	1	1	2	1
2	4	0	1	0	2	0
2	5	0	1	0	0	0
2	6	0	1	0	3	0
3	1	0	1	1	3	1
3	2	1	1	1	1	1
3	4	1	1	0	3	0
3	5	0	1	0	2	0
3	6	1	1	1	2	0

Figure 2: Thought experiments regarding the impact of $i \rightarrow j$ on model effects. Source is Rawlings et al. (2023, p. 320)

Arrange network data

ERGM estimation