Network Mechanisms and Network Structures

Searching for Mechanisms and ERGM 101

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Outline

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Network Concepts

Let's take stock of the knowelede we acquired in weeks 1 to 5!

Network Theories across the Various Weeks of SMM638

Network theory	**2**	**3**	**4**	**5**	**6**	**8**	**9**	**10**
Value creation		•	•					
Coordination				•				
Network change					•	•	•	•
Contagion					•			•

Groups of Network Theories

Underlying model	Social capital	Social homogeneity
Network flow	Capitalization (value creation)	Contagion
Network architecture	Coordination	Adaptation (network change)

Source is [6, page 47]

Networks as Social Capital: Capitalization and Coordination

How do the capitalization and coordination perspectives differ?

	Capitalization	**Coordination**
Level of analysis	Individual nodes/groups of nodes — aka, the trees in the forest	The network as a whole — aka, the forest
Key tenet	(Information exchange) Networks bring resources to individual nodes	The organization and functioning of organizations/markets depend on the characteristics of its underlying (information exchange) network
Sample problem	What is the best network position for a node or a group of nodes with a given objective function (e.g., innovativeness)?	What is the best reporting structure for an organization with certain characteristics (e.g., a start-up in a high-tech industry)?

Let's focus on the influence of networks on organizational decision-making!

Time for the Ragu Sauce Simulation Game!

Ragu Sauce Simulation Game

Goal

Teams of seven members are asked to write on a piece of paper the recipe for the authentic Italian ragu sauce without using the Internet and without asking to 'externals.'

Setup

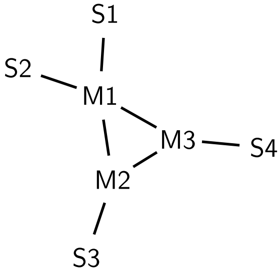
- A team has seven members, a.k.a., nodes
- The information on how to make the sauce is dispersed across four 'source' nodes
  - Two members know sub-sets of ingredients
  - Two members know sub-sets of cooking steps
- The 'manager' nodes — who do not have any specific information on ingredients and cooking steps — are required to collect and synthesize the information from the 'source' nodes

Ragu Sauce Simulation Game

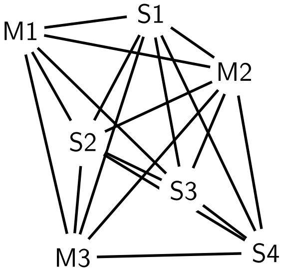
Experimental manipulation

Two teams are supposed to achieve the goal presented in the previous frame. The only difference between the two teams in the network structure. Team **A** has a centralized reporting structure; team **B** has a decentralized reporting structure.

Team A.
Centralized reporting structure



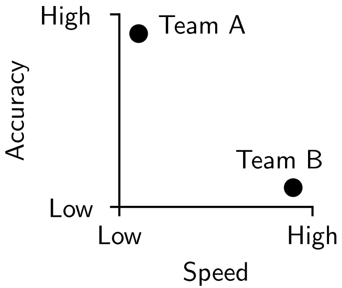
Team B.
Decentralized reporting structure



Ragu Sauce Simulation Game

Main results

Speed Vs. accuracy in decision making



Notes. — Team A is slower in processing information, but less likely to make mistakes. Team B is faster in processing information, but more likely to make mistakes.

Fluid Vs. rigid task partitioning

- In team A, there is roles and responsibilities are distinguishable: manager nodes are responsible for the gathering and integration of dispersed information; source nodes are supposed to share their private information but do not take part in the integration phase
- In team B, the distinction of roles is not visible — potentially, everybody does everything

Ragu Sauce Simulation Game

Generalizable insights

Proposition 3

The characteristic of a reporting network — present in any organization — affects the way in which information is gathered, integrated, and used to make decisions.

Ragu Sauce Simulation Game

Generalizable insights

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Corollario to Proposition 3

The centralization of a reporting network is a key attribute shaping the decision-making process.

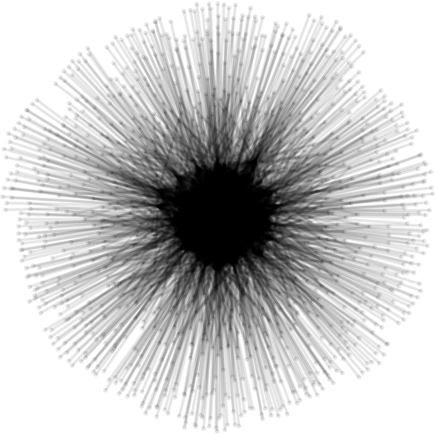
How Do We Measure Network Centralization?

- First-thing-first: centralization is a network-level property — that is, it regards the ‘forest’ not the ‘trees’ of a network
- The paper on illicit networks uses the following measures of graph centralization, Freemans's graph centrality index [3] , defined as $\sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]$, where $C_A(n^*)$ is the largest centrality index across the g actors in the network and $C_A(n_i)$ is the centrality index of the i-th actor in the network
- For an overview of graph-centrality measures, see for example [1] and [7, pages 176-177]

!! Be aware that NetworkX does not implement any centralization measure !!

The Trees Mask the Forest Behind a Network

A graph with $N = 2,000$ and $\langle k \rangle = 502.598$

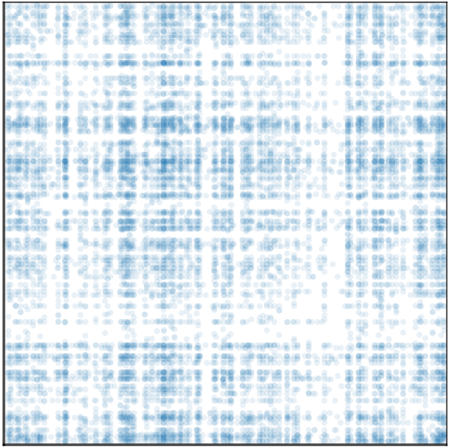


Leading question

- Can you see any type of position in this network visualization?
- That is, can you see specific types of nodes occupying a certain position in the network?

The Trees Mask the Forest Behind a Network

A graph with $N = 2,000$ and $\langle k \rangle = 502.598$

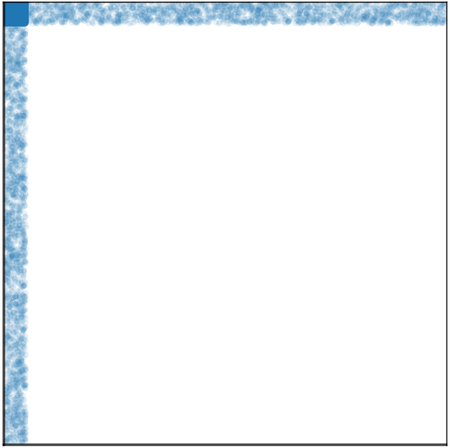


Leading question

- Can you see any type of position in this adjacency matrix?
- That is, can you see specific types of nodes occupying a certain position in the network?

The Trees Mask the Forest Behind a Network

A graph with $N = 2,000$ and $\langle k \rangle = 502.598$



Leading question

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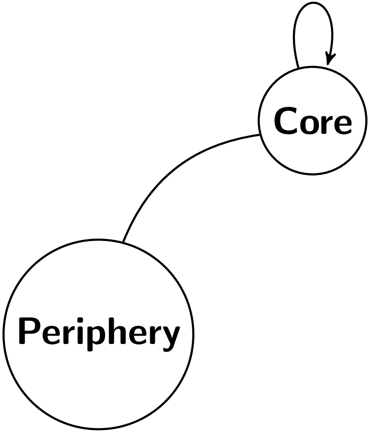
Blockmodeling: The Nuts and Bolts

- The procedure through which an adjacency matrix's rows and columns are permuted to reveal a hidden structure (if any) is called **block-modeling** [5]
- By scrutinizing the outcome of a block-modeling procedure, one can identify the presence of types of nodes in the network — oftentimes called 'positions' — as well as the relationship between them.
- Furthermore, block modeling allows creating a stripped-down representation of a real-world network can — known as reduced form network



Notes. — The reduced-form network representation of a core-periphery network.

Problem Set/Homework

Create the adjacency matrix behind the reporting network of a multi-unit organization with the following attributes

- Ten units
- One manager by unit
- Five middle managers by unit reporting to the unit manager
- Ten employees reporting to each middle manager

Then

- Visualize the adjacency matrix of the network
- Visualize the network
- Get the centralization index of the network

Outline

What Are We Trying to Achieve in the Second Half of the Modeule?

Network theory	**2**	**3**	**4**	**5**	**6**	**8**	**9**	**10**
Value creation		•	•					
Coordination				•				
Network change					•	•	•	•
Contagion						•		•

- In today's session, we will try to discover the mixture of network mechanisms accounting for Soundcloud's network structure

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- In today's session, we will try to discover the mixture of network mechanisms accounting for Soundcloud's network structure
- In the next session, we will learn how to use statistical models to represent a network as a mixture of network mechanisms

Why do Network Mechanisms Matter?

1. Network mechanisms can explain why networks look how they look!
2. If we know the network mechanisms, we can predict the evolution of a network. E.g.,
   - Who will pay attention to which market offers
   - Who will date whom
   - Who will adopt or reject whom's opinion

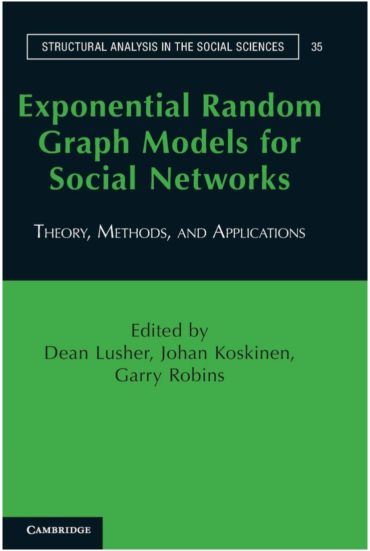
What are ERGMs?

Exponential Random Graph Models (ERGMs) are probabilistic models to appreciate the structure of a network [4]

The regression function underlying ERGM is as follows:

$$P(Y = y) = \frac{\exp(\theta^T g(y))}{k(\theta)} \quad (1)$$

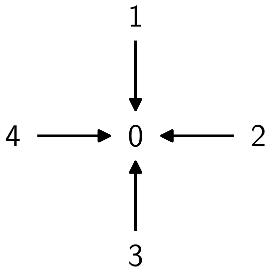
where Y is a latent class of networks (i.e., a set of homogeneous networks that is unknown to the researcher), y is the observed network, θ is a regression parameter vector, $g(y)$ is the vector of regression covariates (i.e., network statistics), and $c(\theta)$ is a normalizing constant.



Network Mechanisms in the Followership Network

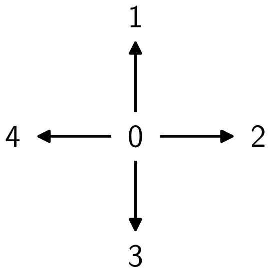
Popularity effects

In-degree popularity (preferential attachment)



Synopsis: certain nodes may develop a cumulative advantage — a so-called rich-get-richer effect

Out-degree popularity



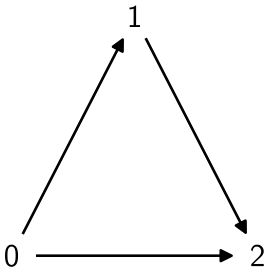
Synopsis: certain nodes may develop a tendency to connect with many others

Network Mechanisms in the Followership Network

Reciprocity effect

$$0 \longleftrightarrow 1$$

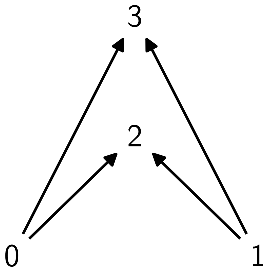
Synopsis: pairs of nodes may show a tendency to follow each other — that is, to reciprocate each other's following choice



Synopsis: "My following's following is my following"

Network Mechanisms in the Followership Network

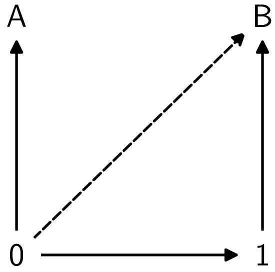
Balance



Synopsis: Pairs of (connected or disconnected) nodes may show the tendency to share 'followings'

Cross-Level Effects: From Following to Taste and Back

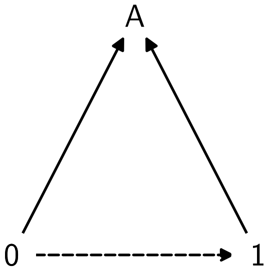
Homophily – Membership closure



Synopsis: 1 introduces 0 to music style B; $\{A, B\}$ is the set of music styles; $\{0, 1\}$ is the set of users; 'vertical' ties can be intended as a bipartite graph linking users and music styles via 'likes,' 'comments,' or 'reposts' (mainly, user preferences); dashed arrows denote ties at risk of emerging based on membership closure

Cross-Level Effects: From Following to Preferences and Back

Homophily – Focal closure



Synopsis: 0 follows 1 because of shared taste/preferences; $\{A, B\}$ is the set of music styles; $\{0, 1\}$ is the set of users; 'vertical' ties can be intended as a bipartite graph linking users and music styles via 'likes,' 'comments,' or 'reposts'; dashed arrows denote ties at risk of emerging based on focal closure

Grouping Explanation of Network Ties

In ERGMs, it is customary to group effects into two families:

- **Endogenous explanation of network ties** derived from the internal structure of relations as the bases for how networks evolve (e.g., in-degree centrality)
- **Exogenous explanation of network ties** are independent of the network under study, like institutional factors (e.g., an anti-trust action tackling on alliances among airline companies) or the attributes of the nodes in the network (e.g., an individual's personality traits)

Outline

ERGM Key Features (1 out of 2)

- In the ERGM framework, observed networks are mixtures of effects
- To estimate the mixture of effects that might have generated the observed network at hand, ERGMs rely on the following general form

$$P(Y = y|X) = \frac{\exp(\theta^T g(y))}{k(\theta)} \tag{2}$$

where Y is the set of all networks of the class, y is a single observation from that class, X is a matrix of attributes for the nodes, $g(y, x)$ is a vector of network statistics, θ is the vector of coefficients for those statistics, and $k(\theta)$ is a normalizing constant that ensures that the probability distribution properly sums to 1.

ERGM Key Features (2 out of 2)

- The numerator of Eq. 32 can be expressed as follows

$$[\theta_1, \theta_2, \ldots, \theta_n] \begin{bmatrix} g_y \\ g_y \\ \vdots \\ g_y \end{bmatrix} = \sum_{i=1}^{n} \theta_i \cdot g_i(y) \tag{3}$$

  which reflects the probability of observing a particular network in a set of networks as a function of many $g(y)$.

- The main difficulty in estimating Eq 32 is $k(\theta)$ since enumerating all the graphs of a given class to ensure completeness may be impossible
  - The solution to this problem exceeds the scope of SMM638
  - FYI, Frank and Strauss [2], dealt with the problem by building on the Hammersly–Clifford theorem for dependence graphs, which they used to create what they called a 'Markov random graph.' The dependency graph can be used to specify which elements of Y are nonindependent and to model them by considering each edge in the network as a random variable linked by shared nodes

A More Manageable Representation of Eq. 3

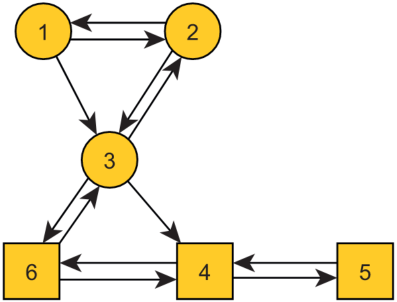
- Eq. 3 can be dispensed in terms of the log odds of an edge:

$$logit(Y_{ij} = 1) = \theta^T \cdot \delta[g(y, X)]_{ij} \quad (4)$$

where Y_{ij} is the dyad, and $\delta[g(y, X)]_{ij}$ is the change in $g(y, X)$ when the value of only the ij dyad is changed from 0 to 1. The δ here, known as 'change statistics,' allow the estimation of the probability model based on any configuration of the networks

  - To do so, one simply calculates the value of $g(y, X)$ with the edge set to 1, recalculates with the edge set to 0, and computes the difference between these two quantities
  - In practice, the two separate calculations are not necessarily required; one can often know the effect of the change on the difference

Estmating $logit(Y_{ij} = 1)$: Iteration Examples



i	j	Tied	Edges	Recip	Trans	Shape
1	2	1	1	1	2	1
1	3	1	1	0	3	1
1	4	0	1	0	1	0
1	5	0	1	0	0	0
1	6	0	1	0	2	0
2	1	1	1	1	1	1
2	3	1	1	1	2	1
2	4	0	1	0	2	0
2	5	0	1	0	0	0
2	6	0	1	0	3	0
3	1	0	1	1	3	1
3	2	1	1	1	1	1
3	4	1	1	0	3	0
3	5	0	1	0	2	0
3	6	1	1	1	2	0

References

[1] Stephen P Borgatti and Martin G Everett. "A Graph-Theoretic Perspective on Centrality". In: *Social networks* 28.4 (2006), pp. 466–484.

[2] Ove Frank and David Strauss. "Markov graphs". In: *Journal of the american Statistical association* 81.395 (1986), pp. 832–842.

[3] Linton C Freeman. "Centrality in Social Networks: Conceptual Clarification". In: *Social networks* 1.3 (1979), pp. 215–239.

[4] Dean Lusher, Johan Koskinen, and Garry Robins. *Exponential random graph models for social networks: Theory, methods, and applications*. Cambridge University Press, 2013.

[5] Tiago P Peixoto. "Efficient Monte Carlo and Greedy Heuristic for the Inference of Stochastic Block Models". In: *Physical Review E* 89.1 (2014), p. 012804.

[6] John Scott and Peter J Carrington. *The SAGE Handbook of Social Network Analysis*. SAGE publications, 2011.

[7] Stanley Wasserman, Katherine Faust, et al. "Social Network Analysis: Methods and Applications". In: (1994).