10

Positions and Roles

Whereas previous chapters focused on networks as conduits through which important resources and influences flow, this chapter provides a more in-depth account of the positional approach to networks. In doing so, we move away from conceptualizing social structures as more or less cohesive and integrated groups, cliques, and communities and toward a view of social structures as composed of role structures. To use the baseball analogy, in moving toward a more positional view of networks, we shift from seeing teams as interacting individual players with relations with one another to seeing players as enacting the game through an interrelated set of positions on the field that come with role expectations. Thus, as depicted in our view of social structure in Figure 2.3, we begin to move upward and to the right, toward higher levels of structure and greater levels of conceptual abstraction. Doing so requires a different set of methods, which we introduce in this chapter.

10.1 FROM ACTORS TO ROLES

Shakespeare famously said that individuals are "merely players" on the world's stage. His soliloquy focuses on how people perform different roles (schoolboy, lover, soldier, and so on) during the unfolding acts of their lives as they age. Similarly, sociologists often talk about *stages* of the life course (childhood, adolescence, and adulthood), replete with the dual meaning of stages as both periods of time and platforms on which individuals perform. Such dramaturgical metaphors are common, but they take on a special importance in positional approaches to social structure. Erving Goffman is the sociologist who took this metaphor most seriously, showing how the interaction orders of everyday life – going to the doctor, getting a haircut, or celebrating a birthday – all live up to Shakespeare's sociological imagination: individuals coordinate roles and enact them in face-to-face performances, using various entrances and exits, stages, masks, props, scripts, and so on, all afforded to them by the schemas undergirding social structures. For Goffman, social structure is enacted in face-to-face rituals that, when done correctly, produce the sense of connection and belonging that is essential for social cohesion and the solidarity of groups.

Although Goffman was not a network scholar, his work drew on the same social-structural motivation and intellectual lineage behind the development of social network analysis.[1] The role framework is rooted in Durkheimian social anthropology and began to take shape in a more formal sense in a related line of structural anthropological research: the study of kinship systems. Family as an institution is an interrelated and interdependent set of positions. A *position* becomes a *role* through a set of rights and obligations with respect to the other role positions. These roles are social constructions that take on different forms. For example, the Chinese extended family network includes seventy-four unique kinship terms, compared with the twenty-eight commonly used in the West. To accurately describe these roles and how they interrelate as role structures is, in many ways, to abstract from Goffman's face-to-face interactions in a formal way – to move to the level of structures in our version of Hinde's image in Figure 2.3, a move Nadel (1957) proposed as the proper level of social scientific analysis.

This notion that societies are best understood as structures composed of interconnected sets of roles has extended to studies of institutions more broadly. For example, the university is an institution in the same sense as a family in having an interrelated set of roles (e.g., professors and students) serving a social purpose (knowledge creation) in society through various exchanges. As we touched on in Chapter 2, seeing society through the lens of roles is powerful but also has the potential to fall into a type of functionalism because the notion of roles connotes something of a larger *purpose* that a group needs to be fulfilled. Indeed, structural-functionalism was a key aspect of Durkheim's thinking that Talcott Parsons took to great heights in American sociology before it largely collapsed under its own weight. But one need not assume some larger functional purpose for role structures. Today, role-based approaches are largely divorced from grand theories of structural-functionalism and their untestable propositions. Instead, researchers have taken the key insight that actors practice consistent ways of acting vis-à-vis other actors in order to formalize the study of roles and institutions through networks. Whether such roles and institutions have evolved in order to optimize some adaptive capacity, as some contemporary evolutionary approaches suggest (e.g., Christakis 2019; Henrich 2015), is not a position one is required to take in using positional concepts and methods.

[1] In fact, Goffman's dissertation advisor, the social anthropologist W. Lloyd Warner, was part of the original research team studying the wiring room at the Hawthorne Plant.

In what follows, we begin by better conceptualizing positions and roles in social network terms. We then outline the steps involved in undertaking a positional analysis of relational data.

10.2 CONCEPTUALIZING ROLE STRUCTURES

In structural terms, a *role* is a subset of nodes that share a common position. In contrast to the concept of a community, which entails direct ties among individuals, in a role-based conception of social structure, two individuals may have the same role and yet have no direct connection to each other. For example, two men may both be *uncles* in a kinship structure, but being uncles does not make them more or less likely to know each other. Even within the same family, uncles from different sides of a marriage may not be friends and may not even know one another well. Yet, as uncles, they hold the same position with respect to the kinship structure: their siblings have children. Their relation to their siblings' children carries with it a set of culturally determined understandings about how uncles and nieces/nephews should interact. In addition, because uncles have a common set of experiences and pressures related to the role, they likely take on similar behaviors and characteristics, which is why the word "avuncular" connotes a level of kindness and geniality.

The kinship example is familiar, and roles are formally defined relationally, but the core insight of network analysis is that the relational consistency of the kin-like variety carries over to all sorts of social interactions, exchanges, and relations. A position implies a set of persons who are relationally "substitutable," defined by the pattern of relations to other positions. That is, persons in a position share all the same relationships (and lack the same relationships) with others that extend across multiple types of ties. For example, teachers are in a distinct position from students by virtue of the *pattern* of their social and professional relations. The prototypical teacher sends academic interactions to students but does not receive them in return. As isolated managers, teachers also do not typically exchange academic interactions with colleagues. Moreover, teachers are not included in social interactions among students, and students are excluded from the social interactions among teachers in the faculty lounge. Principals are not directly active in instruction but guide teachers in how they should perform in the classroom; however, they are generally excluded from the teachers' and students' social interactions.

When visualized, this pattern of interaction would look something like the depiction in Figure 10.1. As a reduced matrix, our ideal-typical school would look like Table 10.1.

The pattern of relations defines the position. A teacher is the sort of person who instructs students, socializes with teachers, and takes orders from principals. If one knows the relations, then one knows the roles. Students take instruction from teachers and exclude them from their social interactions. Because positions are defined relationally, each role includes a *role*

TABLE 10.1 *Summarized positional relations*

Academic	Principal	Teacher	Student
Principal	0	1	1
Teacher	0	0	1
Student	0	0	0
Social			
Principal	0	0	0
Teacher	0	1	0
Student	0	0	1

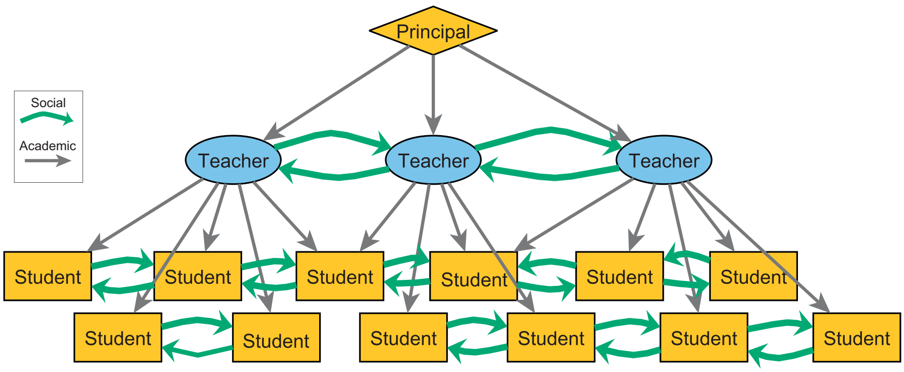

FIGURE 10.1 Stereotypical school role relations

complement: one cannot define a teacher without a student. This is a common feature of structural positions, such as parent–child, husband–wife, cop–robber, client–patron, boss–subordinate, and leader–follower. Social life is characterized by roles in which the position is defined by the relational activity between the positions. When taken together, these roles form a *role system* as a set of interrelated and therefore interdependent positions. A role system is therefore a type of meta-network structure: a higher-order set of social entities (roles) that are dependent on one another, often as part of institutions.

The roles we have been discussing thus far have names; they are institutionalized within a formal framework, and actors that share the setting will have a shared understanding of the expectations and activities of each of these roles. This shared understanding lets us populate the role tables, such as Table 10.1; we understand what teachers, students, and principals do and how they should interact with one another. The core insight from White, Boorman, and Breiger (1976) is that the same tools that allow us to map and abstract formal kinship or organizational relations can be flipped around to inductively *discover* roles, even when we do not know the labels. That is, if one discovers in any given

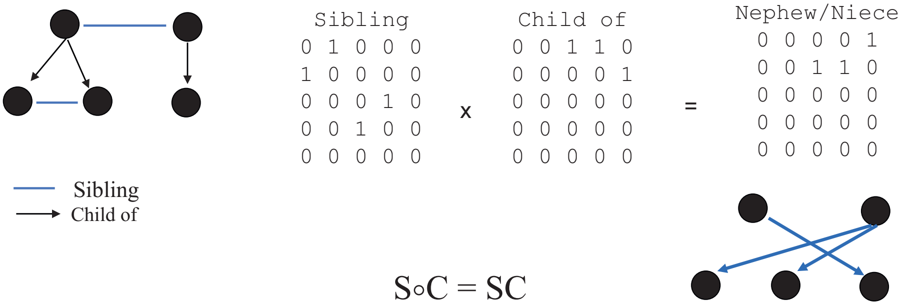

FIGURE 10.2 Roles derived from compound relations

setting that one class of people always acts the same way toward another class of people, then those individuals are enacting a structural role even if they do not have a name for it. Roles may even be emergent within an unfolding interaction and may shift with the actions of different actors. Such roles are evident, for example, as people coordinate getting on and off the subway or move through a crowded grocery store, adopting various roles and attempting to coordinate them more or less successfully without friction and tensions – just as Goffman described in anecdotal terms.

One of the more fascinating and useful features of role structures is that they often involve a compound relation: a relation that is formed by combining two relations. Again, kinship role structures provide the archetype. Consider the compound relation from sibling and "child of" relations. By matrix-multiplying these two relations, the researcher can derive the compound relation of nephew/niece as depicted in Figure 10.2.

Of course, in other cultures, kinship relations might be more or less complex. In the Chinese system, discussed in our introduction to this chapter, relations are built from two primary relations ("parent of" and "married to") compounded by three partitioning attributes (gender, relative age, and relational order). Relations compound differently in the Western systems and are therefore named in one way, whereas Chinese kinship terms are differentiated by parental gender lines (e.g., maternal uncle vs. paternal uncle). Thus, role structures may reveal something of the underlying logic on which social structures are built, although actors may be mostly unaware of this logic.[2]

[2] There is a theoretically rich, albeit somewhat specialized, literature on this approach falling largely under the heading of "semigroup" models. See, in particular, the excellent work of Pattison (1993), White and Reitz (1983), and Wasserman and Faust (1994: ch. 11). This work formalizes the intuitive notions of role sets from Nadel (1957) and structural anthropology via compound relational sets to explicate the logical implications of relational systems.

10.3 INDUCING ROLE STRUCTURES

The primary technique for inducing role structures is called *blockmodeling*. The technique derives its name from the reordering (also called *permuting*) of the rows and columns of a network adjacency matrix into subsets of nodes called *blocks*, the members of which are based on nodes occupying equivalent positions in a network structure.

As with community detection, the goal of blockmodeling is to induce network *subgroups* and *how subgroups interrelate*; however, the results from these two exercises may be quite different. Although some blocks may be communities (with internally cohesive densities of ties), this is not necessarily the case, and it is not the goal of the approach to seek out such cohesive groups. The results of a blockmodel may be surprising, even for those engaged in the network of interactions and relations – as would certainly be the case for those involved in generalized social exchange systems, such as the Kula Ring discussed in Chapter 2 (see Figure 2.7). In such cases, role structures emerge from the combination of local cultural norms rather than an overarching shared understanding. Such self-organizing systems, sometimes called *emergent* phenomena, involve individual agents following similar heuristics forming macrostructures of which they are largely unaware (e.g., schools of fish, flocks of birds, or traffic patterns in Los Angeles). Thus, blockmodeling provides a flexible framework for inducing macrostructures of various kinds from interactional and relational data, the results of which may or may not match what individuals hold to be the important bases of social structure.

To begin to see role systems, all positional approaches must (1) define some measure of equivalence in networks (similarity metric), (2) use the measure to identify subsets of actors that are equivalently positioned (cluster), (3) represent these equivalences by reducing the network to positions and role relations (reduced image), and (4) assess how well cluster solutions represent observed realities (fit).

10.3.1 Defining Equivalence

What does it mean for two nodes to occupy an equivalent position in a network? White, Boorman, and Breiger (1976) revolutionized social network analysis by introducing the main techniques used in formalizing the role-based approach to network structures. Rather than using an *a priori* role set, such as kinship categories, they introduced an inductive method for searching through relations and assigning nodes to equivalent positions (i.e., blocks) with respect to the entire structure of relations in a network. The key insight from this work is that roles can be induced through their *structurally equivalent* sets of ties within a network structure: two actors in the same role will have identical ties to every other actor in the network and therefore can be considered *isomorphic*

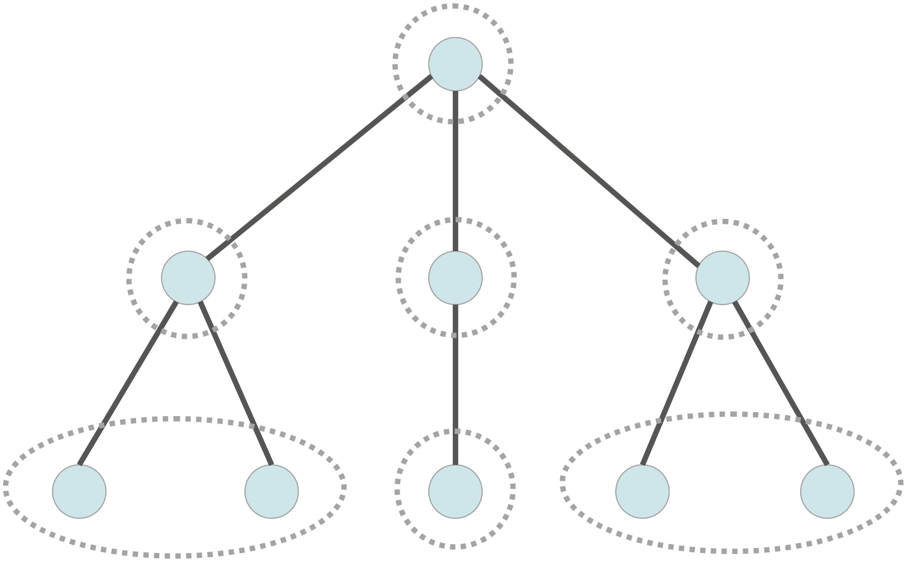

FIGURE 10.3 Structural equivalence in a formal hierarchy

(i.e., having the same form). Consider again a formal hierarchy in an organization, as depicted in a typical chart such as the one depicted in Figure 10.3.

Here, the notion of structural equivalence would lead us to detect seven role positions based on nodes having *identical* relations. Clearly, this reduction is not a very parsimonious or useful because it reduces the number of identified positions only from nine to seven. Moreover, structural equivalence has not really captured what most of us intuitively see as the three positions in the hierarchy. Structural equivalence sets a high bar for determining positions, requiring that actors have *exactly* the same profiles of ties to other actors to be in the same position. Fortunately, more relaxed definitions of structural equivalence also exist in which positions are based on similar profiles of ties, allowing for gradations in determining structural equivalence.

Equivalence can be thought about in at least three alternate ways. One of these is called *automorphic equivalence*. In this formulation, two nodes are considered automorphically equivalent if their positions share all graph-theoretic properties (degree, centrality, reachability, and so on). The positions among one set of nodes map onto those in another set of nodes in the network. This definition relaxes the requirement that nodes be connected with the exact set of other nodes. Using this definition on the network, Figure 10.4 shows that we would induce five positions rather than seven – again, not very useful.

An even more relaxed definition of equivalence is known as *regular equivalence*. In this formulation, two nodes are equivalent and belong in the

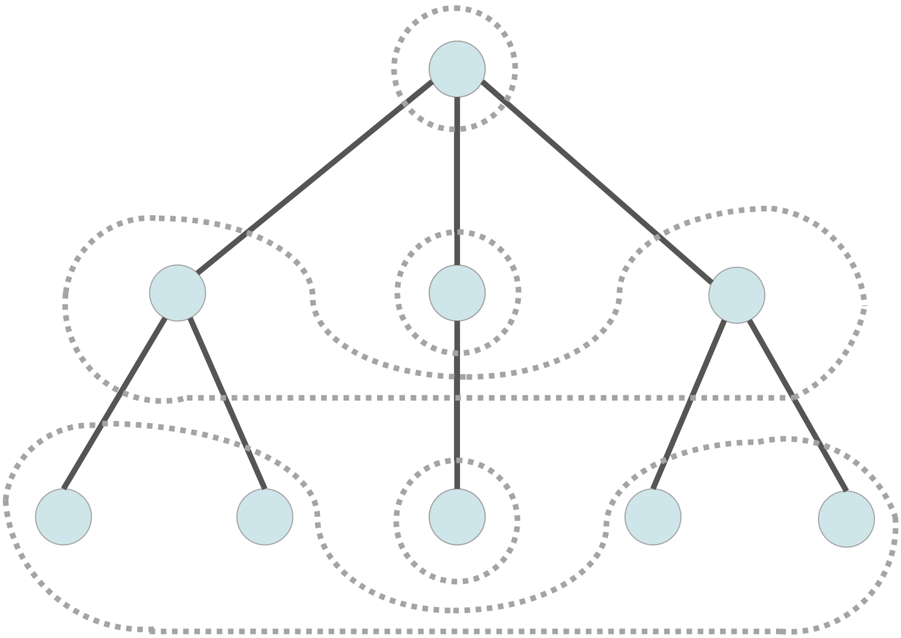

FIGURE 10.4 Automorphic equivalence

same position by having similar profiles of ties to *other equivalent* actors (the definition is deliberately recursive). In Figure 10.1, students are equivalent because they are related to teachers and students, despite having slightly different numbers of ties. Even though some of the actors in the network depicted in Figures 10.3 and 10.4 may not be directly tied to one another and may have different graph-theoretic properties, they share profiles of ties with respect to the types of actors in the formal structure of the network – a clear hierarchy with three levels. There may be multiple ways to partition a network using regular equivalence, and researchers will often use the solution with the fewest positions. Consequently, regular equivalence would provide the most parsimonious role structure, in which actors occupy one of three positions, as shown in Figure 10.5.

Finally, whereas the preceding definitions of equivalence focus on deriving positions using information garnered from the full network structure, another way to think of equivalence is more local and centered on each node and its pattern of ties within its local "neighborhood" of nodes. If two nodes share a similar structural position within their own distinct regions in a structure, this is sometimes called *role equivalence* or *local equivalence*. Here, the researcher considers only the direct ties for each node and compares these with respect to how isomorphic the structural properties are with one another. As we will discuss and demonstrate later in this chapter, triads once again play a vital role,

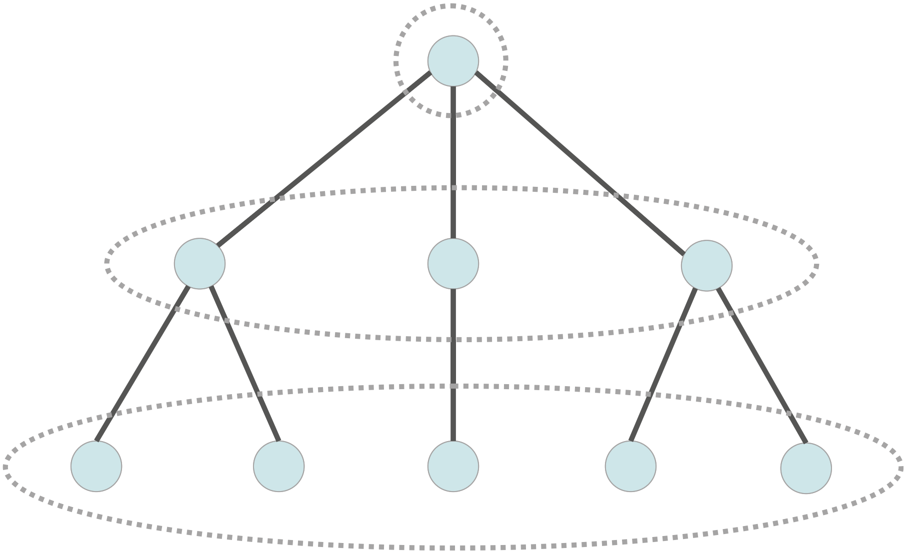

FIGURE 10.5 Regular equivalence

in this case using the sixteen isomorphism classes of triads to determine role equivalence.

10.3.2 Identify Subsets of Structurally Equivalent Actors

Conceptualizing what one means by equivalence is a first step in deriving roles. Subsequently, though, one must operationalize this conceptualization using network data to produce a representation of a role structure. To better illustrate this operationalization, we offer an example of structural equivalence using a relatively simple hierarchy depicted in Figure 10.6.

Using the strictest definition of equivalence – structural equivalence – we compare each pair of nodes with respect to their ties to every other node in the network. In this definition, only pairs of nodes (dyads) that have *exactly* the same profiles of ties are structurally equivalent. These are placed in the same block. In the given network, each pair of nodes has 12 (= n – 2) possible ties to other nodes in the network, each of which are either the same or different (i.e., matched or unmatched). These blocks are derived by comparing the row profiles of nodes i and j in the adjacency matrix. In this example, we use only row profiles because the network is symmetric. Afterward, the rows and columns are rearranged (permuted) into blocks in which two nodes are equivalent only if all twelve ties are matched. This procedure reduces the structure to six blocks. The rows and columns in Table 10.2 reflect this reordering.

TABLE 10.2 *Permuted adjacency matrix based on the blockmodel of hierarchy in Figure 10.6*

	1	2	3		4		5				6			
1	.	1	1	1	0	0	0	0	0	0	0	0	0	0
2	1	.	0	0	1	1	0	0	0	0	0	0	0	0
3	1	0	.	1	0	0	1	1	1	1	0	0	0	0
	1	0	1	.	0	0	1	1	1	1	0	0	0	0
4	0	1	0	0	.	1	0	0	0	0	1	1	1	1
	0	1	0	0	1	.	0	0	0	0	1	1	1	1
5	0	0	1	1	0	0	.	0	0	0	0	0	0	0
	0	0	1	1	0	0	0	.	0	0	0	0	0	0
	0	0	1	1	0	0	0	0	.	0	0	0	0	0
	0	0	1	1	0	0	0	0	0	.	0	0	0	0
6	0	0	0	0	1	1	0	0	0	0	.	0	0	0
	0	0	0	0	1	1	0	0	0	0	0	.	0	0
	0	0	0	0	1	1	0	0	0	0	0	0	.	0
	0	0	0	0	1	1	0	0	0	0	0	0	0	.

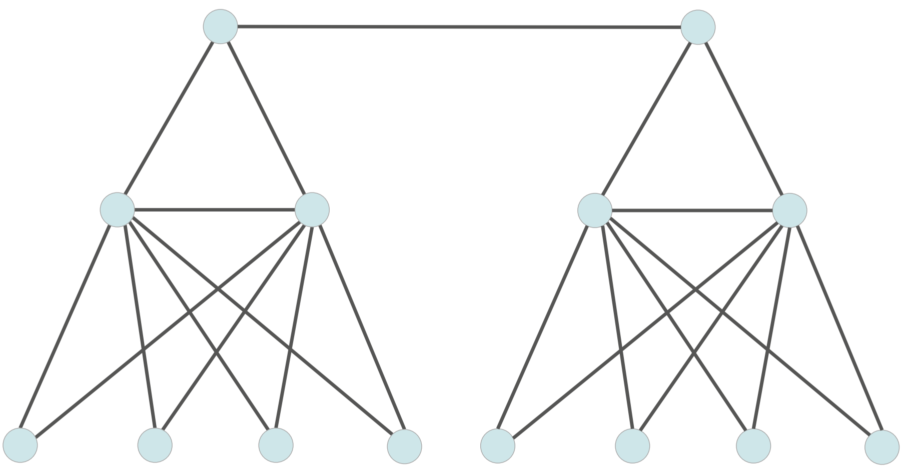

FIGURE 10.6 A typical hierarchical structure

10.3.3 Representing Equivalences as Positions

Once nodes have been assigned to blocks, the next step is to represent the positions as a set of more or less interrelated roles. To see roles and role structures more clearly, one further reduces the permuted network in Table 10.2 to a simplified network – in this case, a network with six nodes (blocks) and relations. Here, we do so based on the existence of ties within and between blocks to reveal the underlying role structure. A very simple way to

TABLE 10.3 *Image matrix reduction of the blockmodel solution in Table 10.2*

	1	2	3	4	5	6
1	0	1	1	0	0	0
2	1	0	0	1	0	0
3	1	0	1	0	1	0
4	0	1	0	1	0	1
5	0	0	1	0	0	0
6	0	0	0	1	0	0

generate a role structure is to assign a 1 to any cells in the reduced network representing a block that has ties within that block or linking it with another block, and a 0 otherwise. This produces what is called an *image matrix* of within- and between-block relations, as shown in Table 10.3.

In practice, one rarely finds a perfect fit in which the data emerge as clearly as in Table 10.3, having all 1s and all 0s in the blocks. In many cases, rather than a binary image matrix, one would create a density table in which each cell is the proportion of possible ties within and between blocks, typically requiring the researcher to determine a cutoff for what is considered a "0 block" and a "1 block" in producing the image matrix. Therefore, reducing the permuted adjacency matrix into a macrostructure entails making a judgment call based on one of two more relaxed rules: (1) the *lean fit* rule in which a block is given a 0 only if all of its cells are 0 while all other blocks receive a 1, or (2) the *density fit* rule in which a block is given a 1 if the density of ties within that block is above a threshold (generally, the average density of the entire network) or a count that exceeds random expectation. In our estimation of the literature, either the density fit rule (+1 standard deviation above the network density when ties are valued) or counts beyond statistical expectation appear to be the most defensible.

Blockmodeling culminates in the projection of a new network based on the image matrix, which reveals the *reduced macrostructure* of the original adjacency matrix. In this new network, the blocks are nodes, and the ties exist where relations within and among equivalent blocks meet a certain threshold. Thus, the macrostructure of the original network in Figure 10.6 can be seen in Figure 10.7. The original hierarchy is reduced to six role positions, two of which (positions 3 and 4) are internally cohesive, having nodes tied to one another within that block. In this case, the ties within and among blocks are dichotomized. In a density table, however, the ties in the macrostructure may be valued and given weight according to the degree of equivalence within and between blocks.

But is the structure in Figure 10.7 the only possible macrostructure? As we outlined earlier, the different definitions of equivalence often provide very different block solutions, and structural equivalence frequently sets the bar too high for providing helpful reductions in the complexity of a network. In

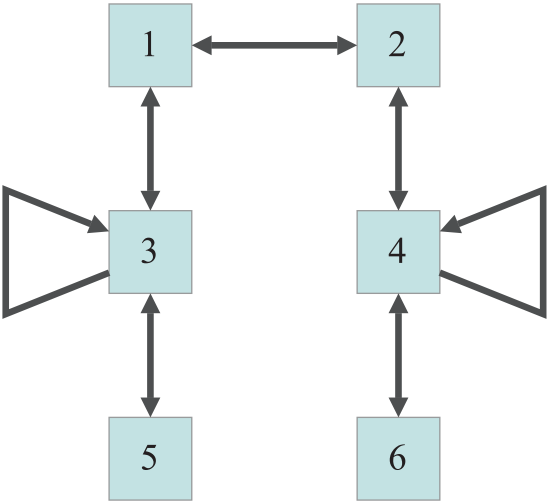

FIGURE 10.7 Reduced macrostructure of hierarchy in Figure 10.6

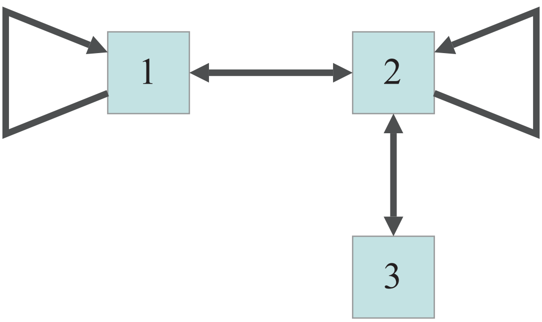

FIGURE 10.8 Reduced macrostructure of hierarchy in Figure 10.6 based on regular equivalence

this case, more relaxed definitions would produce results that are similar but even more parsimonious. For example, using the more relaxed definition of regular equivalence would further reduce the role structure's complexity to only three roles, as depicted in Figure 10.8. There, positions 1–2 of Figure 10.8 are collapsed into 1; 3–4, into 2; and 5–6, into 3. Notably, positions 3 and 4 are treated the same because they have the same pattern of ties even though the ties are sent to different specific actors. The researcher must gauge which reduction is more useful in addressing the research question or report on various solutions to derive a more complete rendering of a role structure.

While our illustration used only a single form of relation, blockmodeling is often at its most useful in examining multi-relational systems (e.g., friendship,

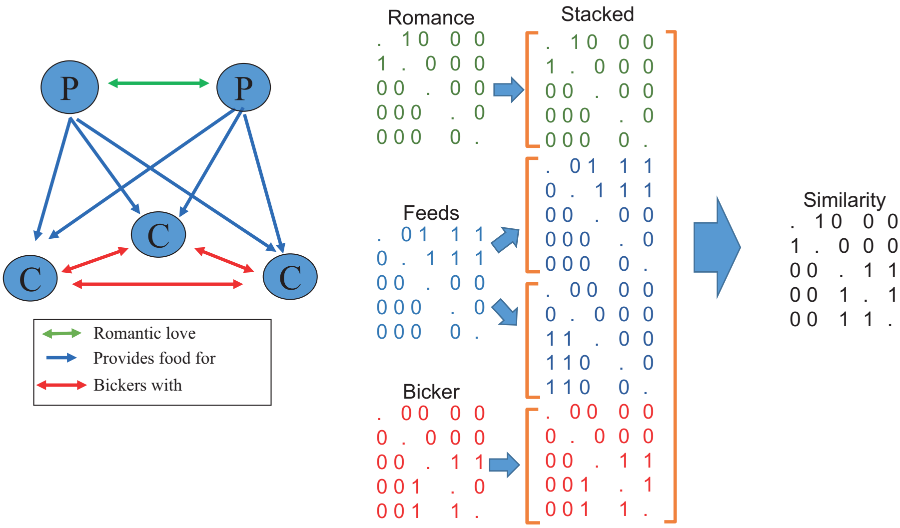

FIGURE 10.9 Illustrating stacking multiple relations within a family exchange network

advice-seeking, frequency of contact). In such cases, the researcher stacks the various matrices prior to running the measure of equivalence. In the case of directed ties, both directions of the ties (*ij* and *ji*) are included as separate matrices in the stack, as shown in Figure 10.9.

Similarities among nodes are thereby calculated across all the relations simultaneously. The blockmodeling technique is also appropriate for affiliation networks – that is, two-mode networks, such as the southern women's groups described in Chapter 3. As we cover in greater detail in the next chapter, when the researcher is interested in two modes of structure, the goal might be somewhat different than inducing a role structure per se; the researcher may instead want to see how two modes (e.g., individuals and groups) interrelate.

So far, we have covered three of the four steps in performing a positional analysis. The final step is assessing how well such derived positions represent/fit an observed reality. Before detailing this final step, we need to discuss several issues regarding choices in how researchers derive structurally equivalent positions.

10.4 ALGORITHMS FOR DERIVING BLOCKS

The distinct conceptualizations of equivalence discussed so far are, in reality, nested: structurally equivalent actors are automorphically equivalent; automorphically equivalent actors are regularly equivalent; and structurally equivalent and automorphically equivalent actors are regularly equivalent. Thus, issues arise when trying to operationalize a single equivalence definition. Automorphic and regular equivalence are more difficult and computationally intensive to measure than structural equivalence, and they require iteratively searching over possible block assignments for sets that have the same graph-theoretic patterns (White & Reitz 1983; see also White 2005). Further, because true structural equivalence is rarely found in real-world networks, most studies do not even try to adhere to the strict definition of structural equivalence.

Thus, the example depicted in Figure 10.6 is meant as a simple illustration of blockmodeling using a clear hierarchy. In practice, blockmodeling is most frequently used with network data on informal relations in which roles are less institutionalized and not determined by authority relations. Such cases make the task of deriving positions more difficult, requiring computational approaches; that is, the positions cannot be derived intuitively through visual inspection. We discuss some of the computational approaches next.

10.4.1 CONCOR

In their original algorithm for identifying blocks, White, Boorman, and Breiger (1976) used the correlation across the columns of a stacked multi-relational network as the operationalization of structural equivalence. The clustering algorithm they used is called CONCOR because it created subgroups based on the convergence (CON) of iterated correlations (COR) among the ties within a network. CONCOR correlates each actor's tie profile with every other actor's tie profile, then correlates actor's correlation profiles, and so on over numerous iterations (i.e., correlating the correlations) until the matrix consists of all symmetric *ij* relations of –1s and 1s. When permuted, the resulting matrix will neatly divide into two groups: all +1s within the block and all –1s between blocks, splitting the network into two subgroups. The process is then repeated within each of these two subgroups anew, separately further dividing each cluster in two. Thus, the procedure would produce a kind of nested structure of subgroups, as illustrated in Figure 10.10.

Despite compelling applications across a number of classic studies, the CONCOR algorithm is less frequently used today. For one, the procedure tends to operate on the number of splits, and it therefore produces an even number of subgroups unless manually directed to split a particular cluster further, and investigators are often reluctant to play such a subjective role in guiding the clustering. Moreover, the overall mathematical properties of the procedure – that is, what iterative correlations of correlations reflect – are not clearly understood. From our vantage, CONCOR seems underutilized and has certain advantages. First, the top-down perspective on divisions can be quite useful for coarser understandings of social structures. Second, the procedure of convergence of correlations appears to be akin to singular value decomposition in which the network is split along poles of the first eigenvector; then, within those halves, it is split on the most suited eigenvector again; and so on. In a way, CONCOR offers

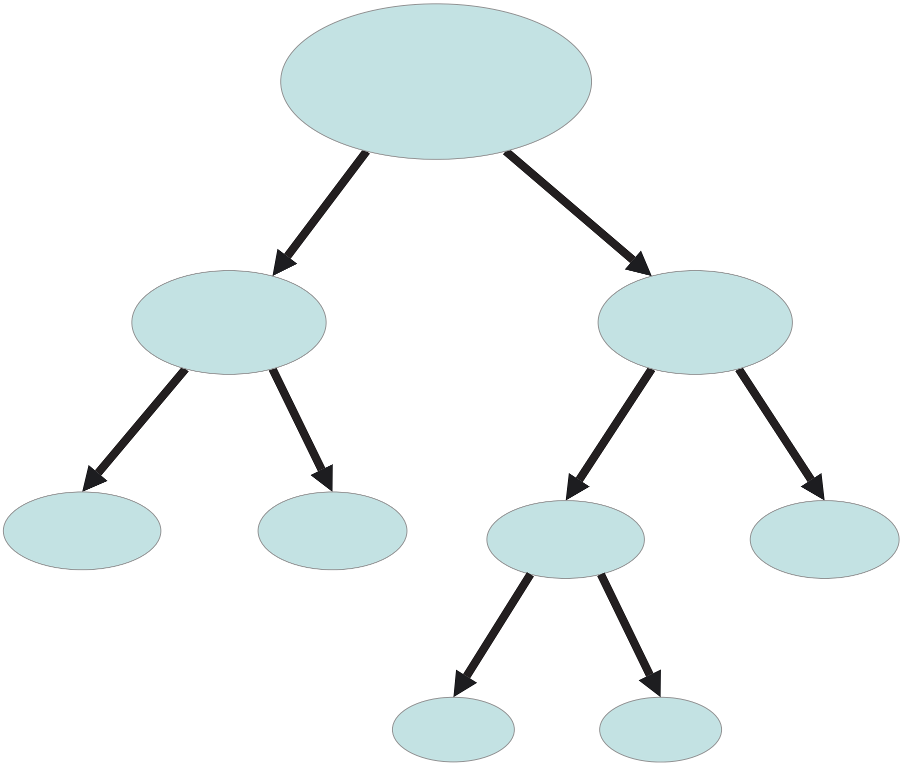

FIGURE 10.10 Illustration of the CONCOR algorithm

an embedded series of eigenvector poles and has a fractal sort of quality to it. We encourage further work on CONCOR and explorations of its intuitive appeal.

10.4.2 Hierarchical Cluster Analysis

One of the most useful tools in helping to place nodes in the same role or position in a network is *hierarchical cluster analysis* (HCA).[3] HCA is a flexible tool that first clusters the most similar cases in a data set and then proceeds in stepwise fashion, assigning the next most similar cases, and so on. Depending on how one determines similarity (or dissimilarity), the results of HCA can be used to derive any number of conceptually important groupings of nodes. When the determination of similarity is based on direct ties among nodes,

[3] We explain HCA here as it has been traditionally used in many network applications. But any reasonably robust contemporary clustering tool will also suffice, and new models based on machine learning routines are likely promising. The key here it to understand the logic of similarity across patterns, and HCA does a particularly nice job of illustrating that.

HCA is used to derive cohesive subgroups and to detect communities. When the determination of similarity is based on tie profiles (correlation or inverse Euclidean distance), then HCA is used to derive positions of structural similarity and their interrelation as roles. Whereas CONCOR is a divisive method that splits wholes into finer subgroups, many of the most popular clustering routines are agglomerative, starting with individual nodes and lumping cases together if they are similar.

Let's walk through a simple example to illustrate how HCA works. The first step in an HCA is to array nodes as more or less proximate in a social space. How one determines this social space is key to determining what clusters emerge. The dimensions of that space may be predefined in some way, or they may be entirely relational (i.e., determined by the similarity of the network ties). Using predefined dimensions can be a quick and "cheap" (i.e., requiring no true network data collection) way to cluster individuals when relational data are difficult to collect but node-level data are available. For example, a researcher interested in adolescent subgroups might draw on node-level characteristics known to be meaningful in some way to students – say, taste in music. Each dimension of the music space could be a music genre or an artist, and individuals would be points based on their degree of liking or disliking these musical forms; adolescents with similar musical preferences would be relatively closer to one another. Of course, this approach assumes that the researcher has a good idea of which dimensions of social space are most important.

The next step in an HCA concerns the actual clustering routine. After arraying individuals in social space and deriving pairwise distances among all the points of actors in such a space (e.g., correlation or Euclidean distance), HCA uses one of several algorithms to iteratively join the points at increasing levels of distance – hence the "hierarchical" aspect of the cluster analysis procedure. These dyadic distances are then used to identify clusters of actors who are relatively close to one another in social space. In the process, this technique also yields a view of how much the clusters overlap with one another at various degrees of structural similarity.

Consider the following toy example as a general model of how hierarchical clustering works.[4] Imagine again the example of the high school, with the research goal of detecting subgroups in a high school. A simple cluster analysis could array individual students within a two-dimensional social space, based on their proximity/distance to one another according to two social qualities – say, perceptions of intelligence and status – as a two-dimensional space illustrated in the first panel of Figure 10.11. Here, our intuition tells us that there are at least two and perhaps four clusters. But can one formalize this intuition in a way that generalizes to more complicated cases?

[4] For more detailed accounts, see Aldenderfer and Blashfield (1984) and Wasserman and Faust (1994).

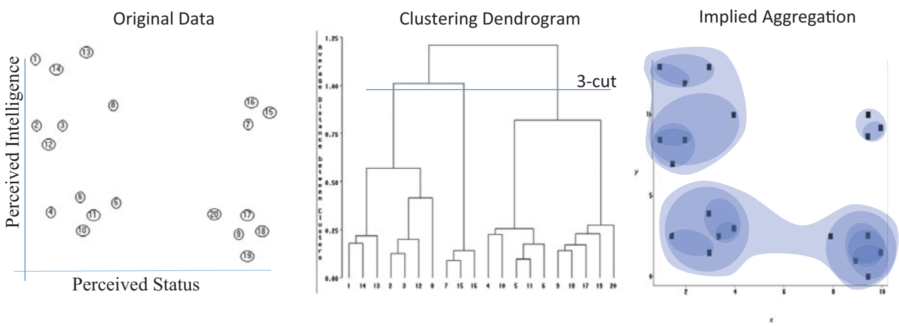

FIGURE 10.11 Illustrating hierarchical cluster analysis

HCA works by joining two or more points that are the closest to one another. These two points are often treated as a single point, with the value equal to the centroid of those initial points, although how points are agglomerated depends on clustering algorithms we will soon discuss. Using one of these several algorithms, HCA repeats this process of hierarchically joining (agglomerating) points until all of them are joined within a single cluster at a maximal distance in the space. The information in this *clustering schedule* is contained within a tree-like or roots-like structure called a *dendrogram* (shown in the second panel of Figure 10.11): the points that are proximate are first joined with one another at increasing distances (shown as the *y*-axis) until the maximum distance, at which time all points are joined in a single cluster. The dendrogram reflects the order in which cases are joined together. The third panel of Figure 10.11 layers this simple ordering as a topographical shading under the original nodes up to the three-cut. Actors with similar combinations of intelligence and status are grouped together, and the implied aggregation offers a visual illustration that helps reveal the structure of subgroups and the degree to which they overlap. At this school, at least, we see that the cluster of students with the highest status is also, well, not the smartest, and vice versa. Of course, it would be very interesting to see whether friendships tend to be concentrated within these clusters or whether they bridge such groupings.

The most important issue in performing an HCA concerns *why* the researcher is using this procedure: whether seeking to detect cohesive subgroups (as discussed in Chapter 8) or trying to uncover structurally equivalent blocks (as discussed in this chapter). The research goal determines the construction of the similarity/distance matrix that is then clustered by the HCA. If the researcher is using HCA in community detection, the similarities among nodes should be based on their direct and indirect ties to one another through one or more sets of relations. One might use the length of geodesics as the basis for arraying nodes as proximate or distant in social space. If the researcher is more interested in an alternative to CONCOR in finding structural equivalence, then the *ij* similarities should be based on shared profiles of ties to all other actors in the network, whether or not *i* and *j* have a direct tie or indirect ties. Of course, in practice, structurally equivalent actors in a network often do have direct ties, and consequently, blocks may sometimes be communities.

Three more technical choices are necessary to perform an HCA: (1) measurement, (2) the clustering algorithm, and (3) cluster cutoffs.

10.4.2.1 Measurement

To perform an HCA, one must first measure the distances between nodes with a distance or similarity metric (generally one is just the inverse of the other). Measuring *ij* distances based on ties or shared characteristics requires the correct choice of a metric based on the levels of measurement (nominal, ordinal, interval, or ratio). The toy example in Figure 10.11 translate node positions in a two-dimensional space into *ij* Euclidean distances because we assume that status and intelligence were at least ordinal measures. This metric may be extended to multiple dimensions when using node-level characteristics, but it does not generalize to binary data. Consequently, when determining *ij* distances based on profiles of binary ties, one may need to use techniques specifically tailored for dichotomous data (tetrachoric correlations or Jaccard matching algorithm) to induce dyadic distances. Moreover, one may want to reflect on whether metrics are sensitive to inflated 0s (reflecting many null ties). When ties are valued at a higher level of measurement (e.g., the number of times *ij* have lunch together in a month), the researcher may use a Pearson's correlation to derive similarities among nodes. Results are often robust to the choice of distance metric, but it is important to select an appropriate measure for one's data.

10.4.2.2 Clustering Algorithm

The second methodological decision concerns the algorithm used to join nodes into clusters. Typically, these clustering algorithms identify sets of nodes that meet a distance threshold as defined in various ways. The *single-link method* ascertains whether members' lowest similarity score meets the threshold (so that members have at least one such best value of similarity). The *average-link method* uses the centroid, or the average distance among cluster members, as a cutoff (used in the preceding example). The *complete-link method* uses the farthest distance to all members of a cluster as the cutoff (so that all members have at least that level of similarity). And *Ward's method* identifies clusters so as to minimize distance variance within clusters, which tends to produce highly distinct clusters. As with all unsupervised clustering algorithms, users should test alternatives for robustness.

Different methods tend to produce clusters with somewhat different structural characteristics. For example, the single-link (also called *nearest-neighbor*) method tends to produce clusters that are long and stringy in the space as it chains together nodes that are closest. Perhaps the most consistently used method in social network analysis, complete-link hierarchical clustering is the most conservative because it seems to afford the most sensible and conservative estimation of structural positions. The important point is that the choice of a clustering algorithm can greatly influence the derived clusters, and researchers will often probe the sensitivity of results to alternate clustering routines.

10.4.2.3 Cluster Cutoffs

The third decision in HCA concerns the optimal number of clusters to represent a social structure, which will depend on precisely where one decides to draw the line across the clustering schedule dendrogram. Again, there is no one correct choice for such an exploratory procedure, and presenting the dendrogram with the research results will be useful in most cases. Statistical techniques can help inform the choice of a cutoff point, but determining the appropriate number of clusters will be heavily influenced by the researcher's judgment, and this decision-making process should be clear in the research report. This decision can be facilitated with a statistic for variance explained, which can be derived from how well the cluster assignments at a given cutoff help explain the original dyadic distance matrix used as input for the cluster analysis. The more the clusters account for the distances, the better the fit. This can also be done using the modularity score for different numbers of clusters. The researcher can produce what is called a *scree plot* or *elbow plot* of the number of clusters by the variance explained or modularity score, looking for any sharp change or elbow-like bend in the plot as a good indicator of the optimal cutoff point. Alternatively, one can apply the same modularity scoring discussed in Chapter 8 to the similarity matrix and choose the cutoff that maximizes modularity. Moreover, while not commonly implemented in default settings in statistical programs, with HCA one can manually cut deeper along one branch than another if doing so makes substantive sense and is justified.

In determining the number of clusters/blocks, keep in mind that the general goal of blockmodeling is to identify a *simplified* rendering of structural reality that explains as much observed variance as possible. The challenge most researchers face is determining how much complexity to introduce, balanced against ensuring comprehensibility. For large social structures, more clusters and positions will capture more variance. However, just as readers cannot comprehend an endless regression table of variables and plots, they cannot comprehend a social structure with a complex set of many interrelated positions. By contrast, explaining 10 percent of the variance of a large social system based on, say, five positions is a valuable contribution. Thus, we have found that it is best to examine cluster profiles – that is, patterns of the base relations/variables – at each level and determine substantively what distinguishes the parent cluster from the two that emerge from it. If the difference reflects a similar pattern but different level, then it is probably not worth introducing yet another category. But, if the split reveals a different type of pattern, then it is probably worth keeping. In short, although it is true that everything matters, some things matter more, and, at any rate, the human mind can comprehend and remember no more than a few of those things.

10.4.2.4 Caveats and Scaling Up HCA

As with any tool, HCA comes with caveats. When using cluster analysis, one should remain aware that the procedure will find clusters in any data, even completely randomly generated data. Consequently, the derived clusters should be met with initial suspicion. Because cluster analysis is presented here as an inductive and exploratory method, the researcher will likely decide on the method based on substantive issues of the domain in question and will likely use more than one approach to probe the sensitivity of results to alternatives. For example, in their now famous paper on the Medici family's ascent to political power in Renaissance Florence, Padgett and Ansell (1993) derived their informative block structure and account by first measuring structural equivalence via correlation, then using complete-link hierarchical clustering, and setting a correlation level cutoff at 0.50. One could argue that these choices made sense because historical records are fuzzy and the authors were analyzing a world that is potentially a sea of noise with islands of structural coherence. As such, a high correlation threshold drops many of the noisy signals of association out. Moreover, what remained were sets of families with the same name and relational efforts (their clusters). Once again, their approach shows that method problems are often theory problems in disguise.

As we discuss in greater detail in the final chapter, these clustering methods have some challenges when being scaled up to very large networks. Some clustering techniques can easily handle millions of cases, but they will not use criteria based on all-pairs distances as outlined in the examples discussed in this chapter. For such giant cases, we generally recommend modeling in two stages: (1) use a fast and efficient, but likely imprecise, method to generate many small clusters and then (2) apply a more nuanced method to the resulting clusters.

10.4.3 Other Measurement Approaches

One technique that we believe is quite interpretable and applicable is blockmodeling based on role equivalence (Burt 1990) or *local* patterns of ties in an individual's personal network (Mandel 1983). As we promised in Chapter 7, triads will make a few cameos throughout this book, and this is one of those times. In this approach, profiles of similarities of actors are based on their position in the sixteen isomorphic classes of triads (see Figure 7.4). The goal is to derive blocks using information in each node's local tie neighborhood. From the standpoint of each node, one can see $\frac{(n-1)(n-2)}{2}$ triads that characterize its position, where n is the number of nodes in the network. Each of the triads must be one of the sixteen types already outlined. However, we can go beyond these sixteen types by taking the position of one node – that is, by looking at each triad separately from the standpoint of each *i*, *j*, and *k*. When one centers on each node and looks at the triad from that node's standpoint, for many of the sixteen types of triads, a node can occupy positions that are structurally distinct. Consider the 012 triad, in which only a single directed tie exists. In this situation, each node occupies a very distinct position: the receiver of the tie, the sender of the tie, and the isolate. Overall, there are thirty-six unique positions a node may occupy within its local triadic environment. These are shown in Figure 10.12 as the variations on each of the original sixteen isomorphic

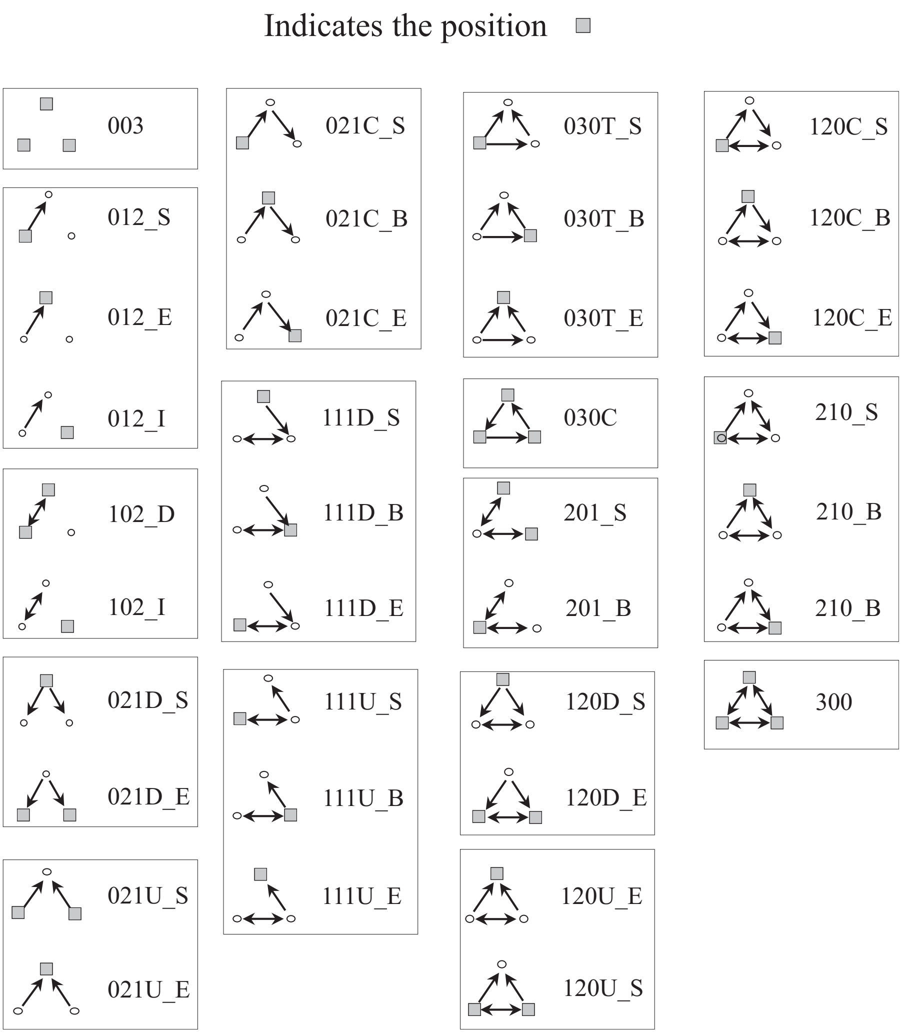

FIGURE 10.12 Triad-position census for deriving role equivalence

classes. Nodes depicted as shaded squares indicate the possible unique positions in each isomorphic class.

Each node within a network therefore has its involvement profile across the thirty-six types. A dyadic measure of role equivalence is derived by comparing each pair of nodes with respect to how similar or different they are in their triad profiles. This effectively compares each person with all other pairs of people in the network, and thus we can compare two people by how similarly they relate to all others. Nodes who have the same pattern of ties will have similar scores. If, in addition, we add global location indicators (such as closeness and betweenness centrality scores), then we get not only pattern but also location and, as such, doing so becomes a very fast and effective measure of automorphic equivalence. This similarity matrix can then be used in CONCOR or HCA to derive blocks, which may then be shown to interrelate in some fashion reflective of a more general role structure. One can accommodate multiple relational networks by either enumerating the multi-relational triadic positions (i.e., all patterns of three actors and *k* relations observed in the network, which is a nontrivial task) or simply repeating the thirty-six elements described here for each of the *k* relations (which is much faster and usually sufficient, particularly if one also adds some cross-relation indictors, such as actor-level correlation across relations). If the set of relations is small, the former provides greater detail, but the latter is much simpler to calculate.

As an example, consider friendship nomination data within a single small elementary class as given in panel (a) of Figure 10.13. Here, thirty-four students naturally divide into four communities (represented by the shaded area in the sociogram). We first enumerated each person's involvement in the 36-element triad-position vector and then submitted it to a hierarchical (Ward's variance) cluster analysis. The optimal solution, as judged by the modularity score applied to the underlying similarity matrix, was a seven-cluster solution. We then examined these clusters in turn to see what sorts of triadic involvement were most common in each position. Panel (b) of Figure 10.13 displays the triadic involvement profiles for four of the seven positions. In Position 1 (orange) are the "outsiders," who are characterized by having very low overall involvement. Cluster 3 (red) generally represents "wannabes," who send ties to others with higher status than themselves but do not receive reciprocal ties. Cluster 5 (pink) is characterized by involvement in the reciprocal relations of otherwise hierarchical triads (such as the 120u_S position). Cluster 7, effectively the structural opposite of Cluster 1, is the group that is generally much more involved in the network and significantly more likely to be in the closed triad (300) and on the receiving end of the hierarchical triads (030T_E, 120D_E).

A mixing matrix provides a better sense of the macrostructure implied by these activities. Panel (c) of Figure 10.13 provides the relative interaction (observed over expected) of each position with every other position, and

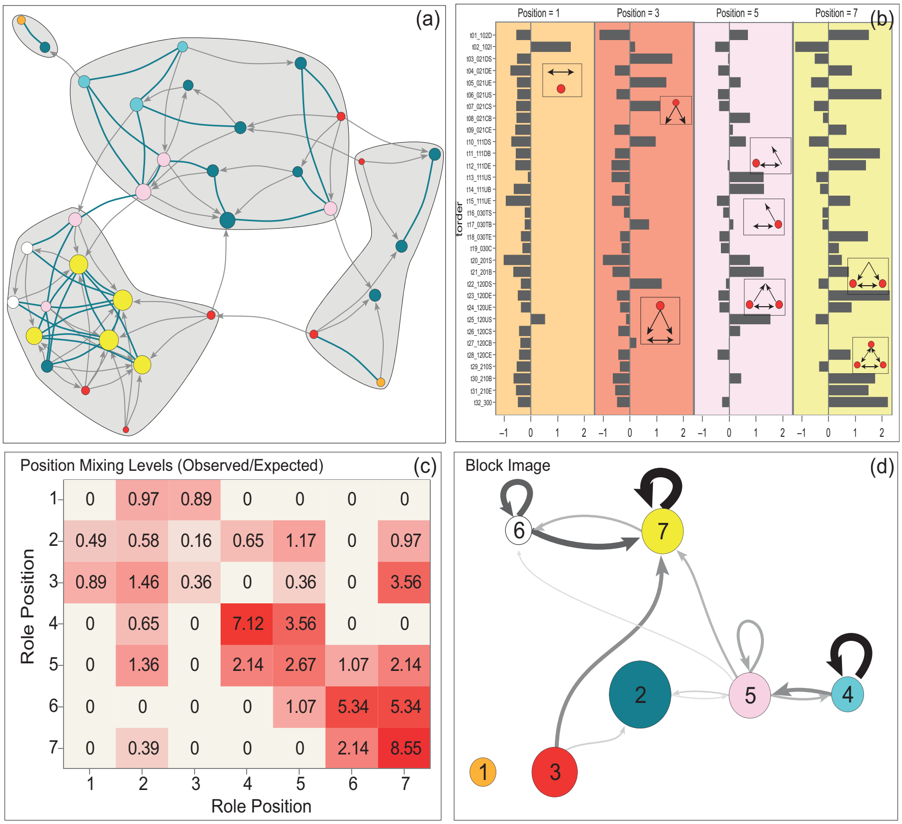

FIGURE 10.13 (a–d) Role positions in a classroom friendship network

panel (d) translates that into an image network. Here, it is clear that Position 7 is a "leading crowd" sort of position (Coleman 1961), and Position 6 is a set of close lieutenants. Positions 4 and 5 are both self-recognized (4 more so than 5), and they are closely tied to each other. Position 4 is effectively eschewing the status structure and focused internally, whereas Position 5 provides an (asymmetric, aspirational) bridge to the leading crowd.

A seven-cluster solution to a 34-person network is perhaps not an overly parsimonious reduction. Having reviewed the positions, we might reasonably collapse 7 and 6 into one "leader" position (because 6 has only two people) and perhaps combine 4 and 5, depending on how much the aspirational aspect matters for the research question at hand. Again, the exploratory and descriptive nature of these models is aimed at providing insight, so a five-position solution might be sufficient. Of course, any derived role structure requires a final step of analysis – namely, assessing its fit.

10.5 ASSESSING ROLE STRUCTURES

Finally, we can ask how well a role structure fits the data. In general, any role analysis that helps make sense of the data and informs our substantive or theoretical understanding is probably a good one. Often, assessing the usefulness of a blockmodel entails a comparative framework: by looking at networks over time or across regions or institutional domains, the researcher begins to see structural forces at work in shaping macrostructures. For example, consider the issue of partisan politics and the extent to which parties have become more divided, which was visualized in Chapter 5 (see Figure 5.18). The image was derived from using CONCOR to blockmodel US senators based on their voting behavior; senators voting in the same way on the same legislation have a positive tie. By repeating the blockmodeling procedure on each Congress, a growing fragmentation appears as two more internally cohesive voting positions align with the two political parties.

Or consider the two high school friendship networks depicted in Figure 10.14. Jefferson High School provides students with a good boundary for social relations. Students there are somewhat clustered along grade levels, but the school appears mainly cohesive. Sunshine High School, on the other hand, does not provide a good boundary for social relations. There, students cluster heavily along racial lines. When macrostructures of these two schools are induced, role equivalence and clustering create a reduced block image of school social structures. We can see a clear difference between the schools. In the reduced block images, the width of ties is proportional to the ratio of cell density to mean cell density (similarity of role equivalence profiles). Jefferson High School has a core region with three groups of popular and internally cohesive students that constitute roughly 34 percent of the school; an additional two groups of noncohesive outsiders make up 32 percent of the school; and two equivalently peripheral groups represent roughly 33 percent of the school. In contrast, Sunshine High School has an exceedingly small core of internally cohesive popular students, while the remaining groups are noncohesive and peripheral in some way.

Another way to assess a role structure is through a confirmatory approach. In this case, the researcher has reason to believe that a role structure may conform to a specific structural form – for example, a core–periphery structure. By comparing the observed role structure to an idealized version of a core–periphery block structure with the same number of blocks, a researcher can assess the fit of the proposed structure to the actual role structure. Prior research has identified a variety of these ideal forms, as shown in panel (a) of Figure 10.15. Here, early work by White, Boorman, and Breiger (1976; also see Bearman 1997) defined a series of pairwise patterns as ideal forms of exchange and governance. Such patterns include a caucus model, polarization, passive–active positions, patronage–clientage, center–periphery, pure deference, and independent deference structures. As shown in panel (b), structural equivalence

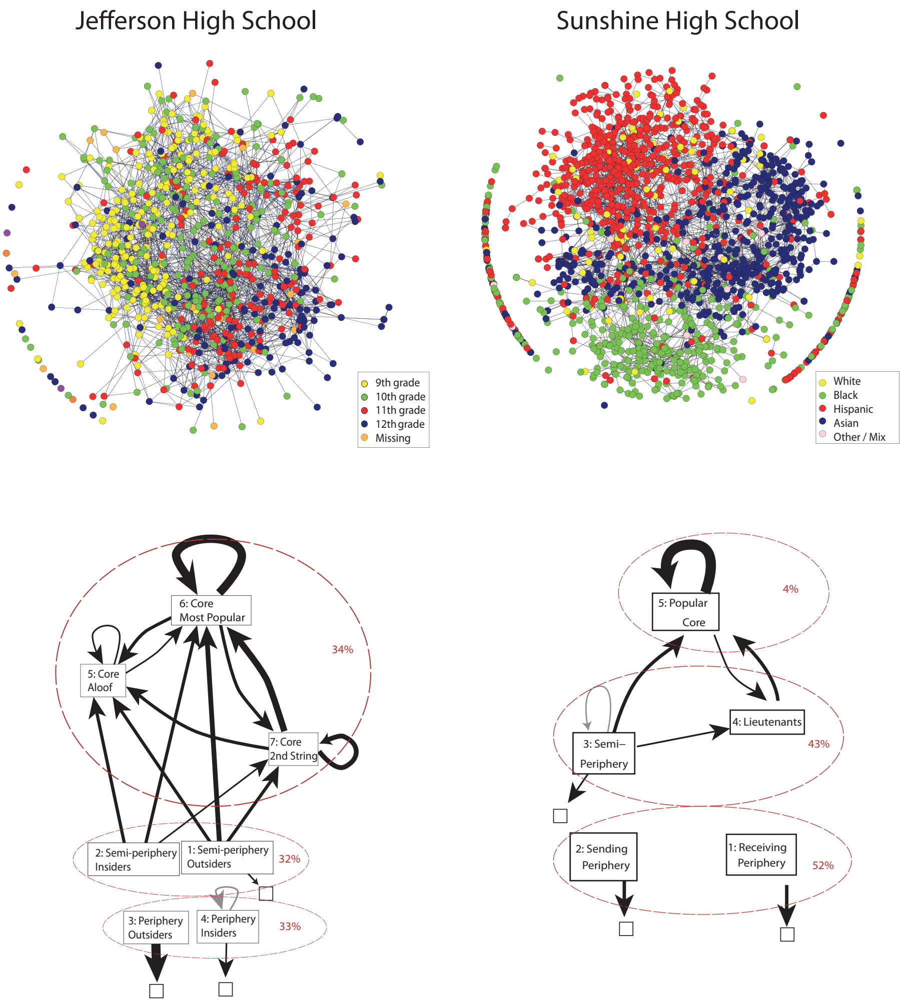

FIGURE 10.14 Comparing role structures in two high schools

has also been extended to larger configurations of positions (Wasserman and Faust 1994) that reflect structures composed of multiple cohesive subgroups, pole positions (see matrix A1 in panel (b) of Figure 10.15), a center and periphery, two cores, centralization, simple hierarchy, looping, and transitivity.

To ascertain whether these idealized structures pertain to each case, one can examine how well the idealized block image corresponds to the observed reduced macrostructure. One can do this cursorily by comparing idealized and reduced block images with the same number of positions. Or one can expand the idealized structure to token ties, assigning 1s and 0s to ties landing in the appropriate positions, and then compare those hypothetical relations to

(a)

$z = \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$	$u = \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$	No role structure for this type of tie.
$c = \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix}$	$d = \begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}$	Caucus model. Single clique with autonomous identity.
$x = \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}$	$u = \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix}$	Pure deference. Niether block has a coherent identity without the other.
$p = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$	$n = \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$	Balanced or polarized. Positive within and hostility between.
$g = \begin{matrix} 1 & 1 \\ 0 & 0 \end{matrix}$	$s = \begin{matrix} 0 & 0 \\ 1 & 1 \end{matrix}$	Passive and active blocks.
$h = \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix}$	$t = \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix}$	Patronage – client systems with two roles. Deference flows up.
$e = \begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix}$	$f = \begin{matrix} 0 & 1 \\ 1 & 1 \end{matrix}$	Center – periphery with autonomous core surrounded by isolates.
$v = \begin{matrix} 1 & 0 \\ 1 & 1 \end{matrix}$	$w = \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}$	Deference structure where deferring blocks have social identities.

FIGURE 10.15 Generalized blockmodels with four or more positions. (a) Sixteen possible two-by-two binary matrices (White, Boorman, & Breiger 1976) and (b) ideal images for more than four positions (Wasserman & Faust 1994)

the observed ones. For example, the reduced image matrix can be used to predict the presence or absence of a tie between all *ij* pairs in the network. Thus, one would ask what a predicted network would look like if only the image matrix were used as a basis for determining the probability of a tie between *i* and *j*. For example, if *i* is in Block 1 and *j* is in Block 2, should one expect a tie between *i* and *j*? One could then compare the predicted network to the observed network, using that as a model fit statistic. This comparison could be done using parametric procedures, such as quadratic assignment procedure correlation or regression (outlined in Chapter 13). The *generalized blockmodeling* approach identified by Doreian, Batagelj, and Ferligoj (2005) takes this idea as its starting point and expands on the multiple ways that nodes can be related generally and methods for matching an observed network to the target.

Most of our examples in this chapter (and the R tutorial) concern static block models with uniplex and multiplex ties. However, blockmodels can be used to depict structural change and potentially test idealized forms of structural change. White, Boorman, and Breiger (1976) offered one such example: they looked at the role structure of a network in its last observed period and imposed prior networks on this reduced image to ascertain when the current structure came into being. Arguably, one could reverse this and ask when the initial role structure disappeared by using the reduced block matrix for the starting network and seeing when the network reproduces or departs from this initial form.

(b)

A. Cohesive Subgroups					A1. Separate Groups			
1	0	0	0		0	0	0	1
0	1	0	0		0	0	1	0
0	0	1	0		0	1	0	0
0	0	0	1		1	0	0	0

B. Center-Periphery					B1. Two Cores			
1	1	1	1		1	1	1	1
1	0	0	0		1	0	0	1
1	0	0	0		1	0	0	1
1	0	0	0		1	1	1	1

C. Centralized

1	1	1	1		1	0	0	0
0	0	0	0	or	1	0	0	0
0	0	0	0		1	0	0	0
0	0	0	0		1	0	0	0

D. Hierarchy					D1. Looping			
0	1	0	0		0	1	0	0
0	0	1	0		0	0	1	0
0	0	0	1		0	0	0	1
0	0	0	0		1	0	0	0

E. Transitivity

0	1	1	1		0	0	0	0
0	0	1	1	or	1	0	0	0
0	0	0	1		1	1	0	0
0	0	0	0		1	1	1	0

FIGURE 10.15 (*cont.*)

Another means of blockmodeling social network change is more complex. The researcher can take a series of network panels and stack them much like when multiple types of ties are present. Then the researcher performs the blockmodeling procedure on that series of matrices, identifying sets of nodes with similar network careers – that is, individuals who have similar ties and lack similar ties at each time period. This approach often results in more blocks and can become quite complex because it is no longer merely identifying static positions but rather subsets of the positions that have certain careers across other positions. The payoff here is the revelation of longitudinal identities (e.g., careers, stories), akin to what social network theorists like Harrison White and Charles Tilly posited. In sociology and studies of social structure, such longitudinal analyses can become quite interesting, as we will discuss in Chapter 16 regarding multilayered networks.

Regardless, the more types of ties and the more network panels, the more complex the clustering solutions become. Because of this added complexity, we recommend using CONCOR or coarser block solutions that capture the most variance in ties over time with the least number of positions. (As Chapter 8 clarifies, that is where cluster methods have the greatest consensus – not in the minutia of fine-grained distinctions.) Bear in mind two things here. First, explaining 5–10 percent of the variance in millions of associations is an amazing accomplishment, so selecting more parsimonious clustering solutions is acceptable. Second, showing four positions over time or with multiple types of ties is already difficult to follow and comprehend, and extending that exploration to twenty positions may be really hard to follow without substantial simplification. Although social structure may prove complex in reality, network scholars need to make sure their explanations are accessible and that readers can use these explanations to develop their own understanding and further their expertise.

Last, as with any data analysis, researchers should build confidence in their results by engaging in robustness and sensitivity checks. For blockmodeling, such checks would focus on what measure of equivalence was used, how the cutoff for the number of derived blocks was determined, and whether the final depiction is comprehensible and furthers the reader's understanding of the phenomenon. Using more than one measure of equivalence and clustering method may be informative and should generally produce similar results. But be mindful to select a depiction that others can comprehend!

10.6 UNITING ROLES AND COMMUNITIES

Role-based approaches may seem quite far afield of the community-based approaches we covered in previous chapters. However, theoretically, the two approaches to social integration born from these approaches (i.e., structural equivalence and cohesion) are clearly connected, and Goffman's dramaturgical metaphors can help us think about their underlying unity. If you've ever been in a play, you were given a role and a script and, through countless rehearsals, learned to coordinate performances under a director's vision. In doing so, you probably came to like and feel more connected to your fellow performers; perhaps you became close friends or even romantically involved. The theater is awash in stories of actors who form lasting ties to those with whom they have performed. At times, tensions arise and may even become acute, but professional actors must continue to perform roles alongside others with whom they have harsh feelings in real life. By enacting the underlying scripts (i.e., schemas) associated with their roles, people produce a sense of connection in themselves and to others, which leads to a flow of emotional energy when these performances are successful. Success in our everyday performances tends to breed future successes, such that individuals who have greater facility at performing certain roles will tend to receive the type of emotional charge that builds confidence, confirms their identity, and eases integration into the social collectivity (Collins 2014). Those who are less successful (because of any number of endogenous and exogenous factors) experience greater isolation and shame, which may lead them to seek out other outsiders with whom they identify or even to seek revenge on the group they feel has rejected them (Scheff 1988). Thus, roles and performances can be deadly enactments in which even violence itself takes on a ritual and performative character.

Methodologically, cohesion and structural equivalence are not as distinct as one might think. The two core tasks for detecting positions are defining relational similarity and clustering the population to reveal role positions. Methods for both abound, and our goal here has been to illustrate the most common approaches with an eye toward empowering researchers to employ the techniques best suited to their problem. The technical similarities to community detection are clear: just as there are myriad ways to cluster direct relations to find communities, there are many ways to cluster profile similarity matrices to reveal roles (and, in fact, one can apply some of the community detection tools directly to similarity matrices). Once positions are identified, there are two common ways to use the results. Most often, one uses positions to describe and explain other activities, noting that people in one position behave differently than people in another position. This provides rich insights into how positions shape incentives and constraints for individual action. Somewhat less common, but potentially much deeper, are models that play out the system-level results of compound role relations. Just as one's "brother's daughter" is a "niece," sequences of relations form a system that can be explicated formally using role algebras like the one in Figure 10.2 (Pattison 1993). We only hinted at this strategy earlier, given that R tools are much less well developed. We encourage readers to take up this challenge because it represents the full promise of White, Boorman, and Breiger's original operationalization of role systems.

10.7 SUMMARY

In this chapter, we shifted our attention away from connectionist approaches to seeing social structures as interrelated sets of roles enacted through many scripts and traced through relational exchanges. This approach once again echoes Durkheim's initial sociological dilemma about what holds a society together. However, whereas the communities discussed in Chapter 8 operationalize mechanical solidarity, role systems and blockmodels operationalize organic solidarity – that is, how differentiated positions may or may not form a connected whole. This brings up theoretically and methodologically fascinating issues. Several of these issues are the bases for ongoing research and will undoubtedly form important future directions, some of which we touch on in Chapter 16.

SUGGESTED FURTHER READING

Bearman, Peter. 1997. "Generalized Exchange." *American Journal of Sociology* 102: 1383–415. (An exemplar use of blockmodeling to identify positions in a kinship exchange structure, demonstrating that systemic social action occurs even when actors are unaware of the reasons for their actions.)

Doreian, Patrick, Vladimir Batagelj, and Anuska Ferligoj. 2004. *Generalized Blockmodeling*. New York: Cambridge University Press. (Provides a thorough overview of blockmodeling and role analysis starting with the conventional notions of equivalence outlined in this chapter, but then extends to new equivalency frameworks based on different types of flows across positions.)

Leifer, Eric M. 1988. "Interaction Preludes to Role Setting: Exploratory Local Action." *American Sociological Review* 53: 865–78. (Presents an elegant theory of ambiguity that provides insights into how indeterminant action creates the conditions necessary for role exchange in settings where roles are not prescribed.)

Pattison, Philippa. 1993. *Algebraic Models for Social Networks*. New York: Cambridge University Press. (An exceptional text on using formal models for role relations for both complete and incomplete networks; an approach that holds true promise for identifying relational implications and systems-level regularities across settings.)

White, Harrison C., Scott A. Boorman, and Ronald L. Breiger. 1976. "Social Structure from Multiple Networks. I. Blockmodels of Roles and Positions." *American Journal of Sociology* 81(4): 730–80. (Foundational work on using network methods to identify informal role systems.)