Fundamental Network Concepts

Building Blocks of Network Analysis

SMM638 Network Analytics

What is a Graph?

Mathematical Foundation:

A graph G is defined as: $G = \{V, E\}$

Where:

- $V = [v_1, v_2, ..., v_i, ..., v_n]$ (vertices/nodes)
- $E = [(v_1, v_2), (v_1, v_i), ..., (v_i, v_j)]$ (edges/links)

In Plain Language:

- **Vertices (Nodes)**: The entities or objects
- **Edges (Links/Ties)**: The connections or relationships
- **Graph**: The complete structure of nodes and edges

Vertices (Nodes)

Vertices represent the fundamental units in a network

Examples across domains:

- **Social networks**: People, organizations, groups
- **Biological networks**: Proteins, genes, organisms
- **Technological networks**: Computers, routers, devices
- **Economic networks**: Companies, banks, countries

Node Attributes:

- Demographic characteristics (age, location)
- Type or category (customer, supplier, partner)
- Performance metrics (revenue, citations, activity)
- Temporal information (founding date, tenure)

Edges (Links/Ties)

Edges encode relationships between nodes

Key Properties:

1. **Direction**
   - Directed: One-way relationships (A $\rightarrow$ B)
   - Undirected: Mutual relationships (A — B)
2. **Weight**
   - Weighted: Strength or frequency of connection
   - Unweighted: Binary presence/absence
3. **Sign**
   - Positive: Friendship, cooperation, support
   - Negative: Conflict, competition, animosity
4. **Type**
   - Multiple relationship types (multiplex networks)

Network Relationships

What Constitutes a Connection?

The definition of a relationship determines:

- What edges mean and how to interpret them
- Whether relationships are symmetric or asymmetric
- How to measure or identify connections
- The substantive interpretation of patterns

Examples:

- **Social**: Friendship, advice-seeking, collaboration
- **Economic**: Trade, investment, supply relationships
- **Information**: Email exchange, citations, hyperlinks
- **Biological**: Protein interactions, predator-prey

One-Mode Networks

Unipartite Networks: One Type of Node

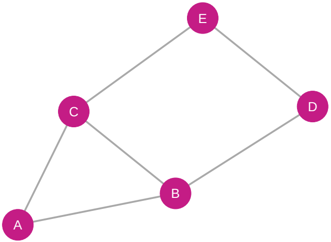
All nodes are of the same type; connections occur between similar entities

Common Examples:

- **Friendship networks**: People People
- **Citation networks**: Papers → Papers
- **Trade networks**: Countries Countries
- **Collaboration networks**: Scientists Scientists

Characteristics:

- Adjacency matrix is square ($n \times n$)
- Can calculate standard network metrics
- Direct interpretation of connection



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	0	0	1	1	0

Two-Mode Networks

Bipartite Networks: Two Types of Nodes

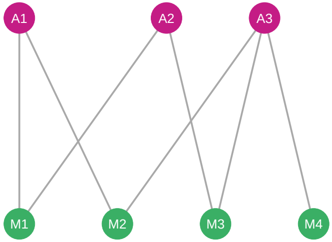
Edges only connect nodes of different types

Common Examples:

- **Actor-Movie**: Actors Movies
- **Author-Paper**: Authors Publications
- **Customer-Product**: Buyers Items purchased
- **Student-Course**: Students Classes enrolled

Analytical Approaches:

1. Analyze the bipartite structure directly
2. Project onto one-mode networks (actors actors who shared movies)
3. Examine affiliation patterns



Incidence Matrix:

Actor	M1	M2	M3	M4
A1	1	1	0	0
A2	1	0	1	0
A3	0	1	1	1

Directed Networks

Asymmetric Relationships with Direction

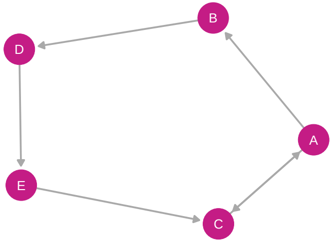
Edges have a source and target: $A \rightarrow B$

Key Examples:

- **Email networks**: Sender $\rightarrow$ Receiver
- **Citation networks**: Citing paper $\rightarrow$ Cited paper
- **Food webs**: Predator $\rightarrow$ Prey
- **Twitter**: Follower $\rightarrow$ Followed account

Important Distinctions:

- **In-degree**: Incoming connections (popularity, citations received)
- **Out-degree**: Outgoing connections (activity, citations made)
- Reciprocity: Do ties go both ways?



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	1	0
C	1	0	0	0	0
D	0	0	0	0	1
E	0	0	1	0	0

Undirected Networks

Symmetric Relationships Without Direction

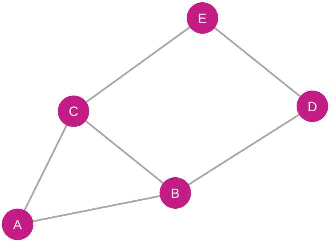
Edges represent mutual connections: $A — B$

Key Examples:

- **Friendship networks**: Mutual friendships
- **Co-authorship**: Joint publications
- **Infrastructure**: Roads, power grids, railways
- **Protein interactions**: Molecular binding

Characteristics:

- Connection implies reciprocal relationship
- Single degree measure (not in/out)
- Simpler mathematical properties
- Adjacency matrix is symmetric



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	0	0	1	1	0

Signed Networks

Edges Carry Positive or Negative Valence

Relationships can be friendly or hostile

Positive Edges (+):

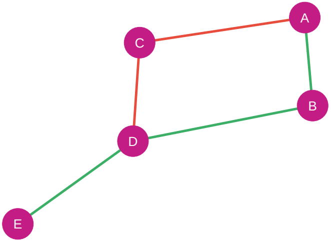
- Friendship, alliance, cooperation
- Support, endorsement, trust

Negative Edges (−):

- Animosity, conflict, competition
- Opposition, distrust, rivalry

Applications:

- Social balance theory (enemy of my enemy is my friend)
- Coalition formation in politics
- Opinion polarization dynamics
- Organizational conflict analysis



Signed Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	-1	0	0
B	1	0	0	1	0
C	-1	0	0	-1	0
D	0	1	-1	0	1
E	0	0	0	1	0

Weighted Networks

Edge Strength Varies Continuously

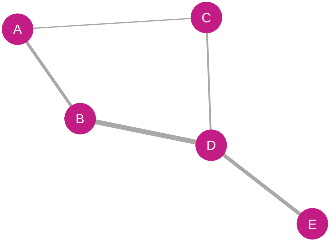
Weights represent connection intensity, frequency, or capacity

Examples:

- **Communication**: Number of messages exchanged
- **Transportation**: Traffic volume, distance, capacity
- **Financial**: Transaction amounts, investment size
- **Neural**: Synaptic strength between neurons

Analytical Implications:

- Can identify strong vs. weak ties
- Weighted centrality measures
- Flow and capacity analysis
- More nuanced than binary networks



Weighted Adjacency Matrix:

Node	A	B	C	D	E
A	0	5	2	0	0
B	5	0	0	8	0
C	2	0	0	3	0
D	0	8	3	0	6
E	0	0	0	6	0

Unweighted Networks

Binary: Connection Present or Absent

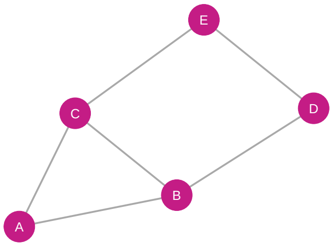
All edges treated equally (0 or 1)

Characteristics:

- Simpler to collect and analyze
- Focus on topology, not intensity
- May lose important information
- Standard network metrics apply directly

When Appropriate:

- Relationship strength unclear or unmeasurable
- Presence/absence is the key question
- Simplification aids interpretation
- Preliminary exploratory analysis



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	0	0	1	1	0

Dyads

The Simplest Network Substructure

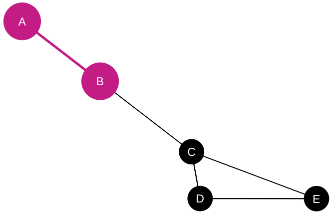
A dyad consists of two nodes and potential edge(s) between them

Types in Directed Networks:

1. **Null dyad**: No connection (0 edges)
2. **Asymmetric dyad**: One-way connection (1 edge)
3. **Mutual/Reciprocal dyad**: Two-way connection (2 edges)

Analytical Value:

- Foundation for reciprocity analysis
- Building block of larger structures
- Pairwise relationship dynamics
- Simplest unit of social interaction



Adjacency Matrix (Binary, Dyad Highlighted):

Node

A

B

C

D

E

A

0

1

Triads

Three Nodes and Their Connections

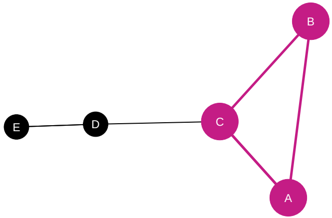
Triads are fundamental for understanding:

Key Concepts:

- **Transitivity**: "Friend of friend is friend" (A→B, B→C, A→C)
- **Structural balance**: Stability of positive/negative relationships
- **Clustering**: Local cohesion patterns
- **Network motifs**: Recurring small-scale patterns

Example Patterns:

- Open triad: A→B, B→C (no A→C)
- Closed triad: A→B, B→C, C→A (triangle)
- Balanced triad: Signs follow balance theory rules



Adjacency Matrix (Binary, Triad Highlighted):

Node
A
B
C
D
E
A
0
1

Key Takeaways

Caution

Core Building Blocks:

1. Networks = Nodes + Edges + Relationships
2. Direction matters: Symmetric vs. Asymmetric
3. Weights capture relationship intensity
4. Signs represent positive/negative ties
5. Mode determines what connects to what

Tip

Analytical Foundation:

- Choice of representation affects analysis
- Different network types require different methods
- Substructures (dyads, triads) reveal patterns
- Complex networks require sophisticated approaches

Next: We'll use these concepts to measure and analyze real networks