

Fundamental Network Concepts

Building Blocks of Network Analysis

SMM638 Network Analytics

What is a Graph?

Mathematical Foundation:

A graph G is defined as: $G = \{V, E\}$

Where:

- ▶ $V = [v_1, v_2, \dots, v_i, \dots, v_n]$ (vertices/nodes)
- ▶ $E = [(v_1, v_2), (v_1, v_i), \dots, (v_i, v_j)]$ (edges/links)

In Plain Language:

- ▶ **Vertices (Nodes):** The entities or objects
- ▶ **Edges (Links/Ties):** The connections or relationships
- ▶ **Graph:** The complete structure of nodes and edges

Vertices (Nodes)

Vertices represent the fundamental units in a network

Examples across domains:

- ▶ **Social networks:** People, organizations, groups
- ▶ **Biological networks:** Proteins, genes, organisms
- ▶ **Technological networks:** Computers, routers, devices
- ▶ **Economic networks:** Companies, banks, countries

Node Attributes:

- ▶ Demographic characteristics (age, location)
- ▶ Type or category (customer, supplier, partner)
- ▶ Performance metrics (revenue, citations, activity)
- ▶ Temporal information (founding date, tenure)

Edges (Links/Ties)

Edges encode relationships between nodes

Key Properties:

1. Direction

- ▶ Directed: One-way relationships ($A \rightarrow B$)
- ▶ Undirected: Mutual relationships ($A - B$)

2. Weight

- ▶ Weighted: Strength or frequency of connection
- ▶ Unweighted: Binary presence/absence

3. Sign

- ▶ Positive: Friendship, cooperation, support
- ▶ Negative: Conflict, competition, animosity

4. Type

- ▶ Multiple relationship types (multiplex networks)

Network Relationships

What Constitutes a Connection?

The definition of a relationship determines:

- ▶ What edges mean and how to interpret them
- ▶ Whether relationships are symmetric or asymmetric
- ▶ How to measure or identify connections
- ▶ The substantive interpretation of patterns

Examples:

- ▶ **Social:** Friendship, advice-seeking, collaboration
- ▶ **Economic:** Trade, investment, supply relationships
- ▶ **Information:** Email exchange, citations, hyperlinks
- ▶ **Biological:** Protein interactions, predator-prey

One-Mode Networks

Unipartite Networks: One Type of Node

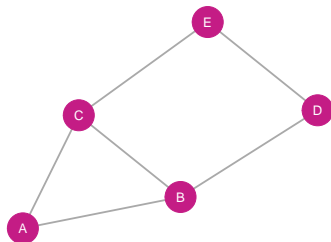
All nodes are of the same type;
connections occur between similar
entities

Common Examples:

- ▶ **Friendship networks:** People
People
- ▶ **Citation networks:** Papers →
Papers
- ▶ **Trade networks:** Countries
Countries
- ▶ **Collaboration networks:**
Scientists Scientists

Characteristics:

- ▶ Adjacency matrix is square ($n \times n$)
- ▶ Can calculate standard network
metrics
- ▶ Direct interpretation of connection



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	0	0	1	1	0

Two-Mode Networks

Bipartite Networks: Two Types of Nodes

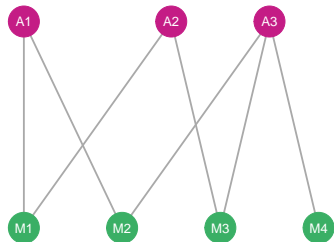
Edges only connect nodes of different types

Common Examples:

- ▶ **Actor-Movie:** Actors Movies
- ▶ **Author-Paper:** Authors
Publications
- ▶ **Customer-Product:** Buyers
Items purchased
- ▶ **Student-Course:** Students
Classes enrolled

Analytical Approaches:

1. Analyze the bipartite structure directly
2. Project onto one-mode networks
(actors actors who shared movies)
3. Examine affiliation patterns



Incidence Matrix:

Actor	M1	M2	M3	M4
A1	1	1	0	0
A2	1	0	1	0
A3	0	1	1	1

Directed Networks

Asymmetric Relationships with Direction

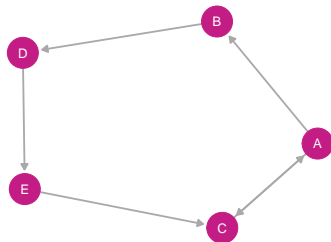
Edges have a source and target: $A \rightarrow B$

Key Examples:

- ▶ **Email networks:** Sender \rightarrow Receiver
- ▶ **Citation networks:** Citing paper \rightarrow Cited paper
- ▶ **Food webs:** Predator \rightarrow Prey
- ▶ **Twitter:** Follower \rightarrow Followed account

Important Distinctions:

- ▶ **In-degree:** Incoming connections (popularity, citations received)
- ▶ **Out-degree:** Outgoing connections (activity, citations made)
- ▶ **Reciprocity:** Do ties go both ways?



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	1	0
C	1	0	0	0	0
D	0	0	0	0	1
E	0	0	1	0	0

Undirected Networks

Symmetric Relationships Without Direction

Edges represent mutual connections:

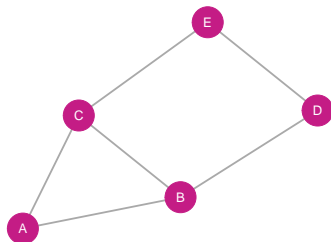
$A-B$

Key Examples:

- ▶ **Friendship networks:** Mutual friendships
- ▶ **Co-authorship:** Joint publications
- ▶ **Infrastructure:** Roads, power grids, railways
- ▶ **Protein interactions:** Molecular binding

Characteristics:

- ▶ Connection implies reciprocal relationship
- ▶ Single degree measure (not in/out)
- ▶ Simpler mathematical properties
- ▶ Adjacency matrix is symmetric



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	0	0	1	1	0

Signed Networks

Edges Carry Positive or Negative Valence

Relationships can be friendly or hostile

Positive Edges (+):

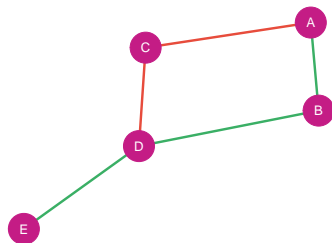
- ▶ Friendship, alliance, cooperation
- ▶ Support, endorsement, trust

Negative Edges (—):

- ▶ Animosity, conflict, competition
- ▶ Opposition, distrust, rivalry

Applications:

- ▶ Social balance theory (enemy of my enemy is my friend)
- ▶ Coalition formation in politics
- ▶ Opinion polarization dynamics
- ▶ Organizational conflict analysis



Signed Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	-1	0	0
B	1	0	0	1	0
C	-1	0	0	-1	0
D	0	1	-1	0	1
E	0	0	0	1	0

Weighted Networks

Edge Strength Varies Continuously

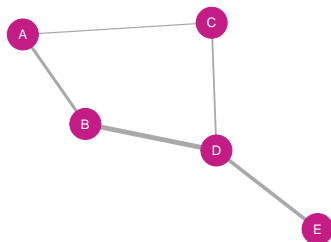
Weights represent connection intensity, frequency, or capacity

Examples:

- ▶ **Communication:** Number of messages exchanged
- ▶ **Transportation:** Traffic volume, distance, capacity
- ▶ **Financial:** Transaction amounts, investment size
- ▶ **Neural:** Synaptic strength between neurons

Analytical Implications:

- ▶ Can identify strong vs. weak ties
- ▶ Weighted centrality measures
- ▶ Flow and capacity analysis
- ▶ More nuanced than binary networks



Weighted Adjacency Matrix:

Node	A	B	C	D	E
A	0	5	2	0	0
B	5	0	0	8	0
C	2	0	0	3	0
D	0	8	3	0	6
E	0	0	0	6	0

Unweighted Networks

Binary: Connection Present or Absent

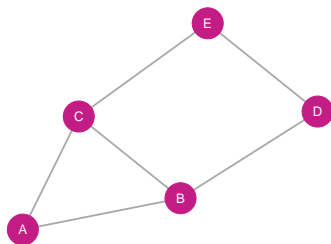
All edges treated equally (0 or 1)

Characteristics:

- ▶ Simpler to collect and analyze
- ▶ Focus on topology, not intensity
- ▶ May lose important information
- ▶ Standard network metrics apply directly

When Appropriate:

- ▶ Relationship strength unclear or unmeasurable
- ▶ Presence/absence is the key question
- ▶ Simplification aids interpretation
- ▶ Preliminary exploratory analysis



Adjacency Matrix:

Node	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	1	1	0	0	1
D	0	1	0	0	1
E	0	0	1	1	0

Dyads

The Simplest Network Substructure

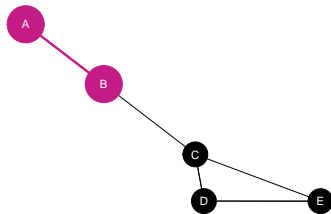
A dyad consists of two nodes and potential edge(s) between them

Types in Directed Networks:

1. **Null dyad:** No connection (0 edges)
2. **Asymmetric dyad:** One-way connection (1 edge)
3. **Mutual/Reciprocal dyad:** Two-way connection (2 edges)

Analytical Value:

- ▶ Foundation for reciprocity analysis
- ▶ Building block of larger structures
- ▶ Pairwise relationship dynamics
- ▶ Simplest unit of social interaction



Adjacency Matrix (Binary, Dyad Highlighted):

Node

A

B

C

D

E

A

0

1

Triads

Three Nodes and Their Connections

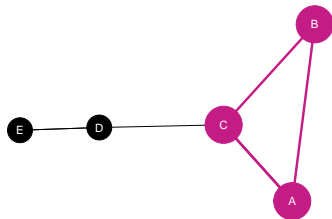
Triads are fundamental for understanding:

Key Concepts:

- ▶ **Transitivity:** “Friend of friend is friend” ($A \rightarrow B$, $B \rightarrow C$, $A \rightarrow C$)
- ▶ **Structural balance:** Stability of positive/negative relationships
- ▶ **Clustering:** Local cohesion patterns
- ▶ **Network motifs:** Recurring small-scale patterns

Example Patterns:

- ▶ Open triad: $A \rightarrow B$, $B \rightarrow C$ (no $A \rightarrow C$)
- ▶ Closed triad: $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$ (triangle)
- ▶ Balanced triad: Signs follow balance theory rules



Adjacency Matrix (Binary, Triad Highlighted):

Node

A

B

C

D

E

A

0

1

Key Takeaways



Caution

Core Building Blocks:

1. Networks = Nodes + Edges + Relationships
2. Direction matters:
Symmetric
vs. Asymmetric
3. Weights capture
relationship intensity
4. Signs represent
positive/negative ties
5. Mode determines what
connects to what



Tip

Analytical Foundation:

- ▶ Choice of representation affects analysis
- ▶ Different network types require different methods
- ▶ Substructures (dyads, triads) reveal patterns
- ▶ Complex networks require sophisticated approaches

Next: We'll use these concepts to measure and analyze real networks