Performance Modeling of Computer Systems and Networks

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Multi-stream application examples

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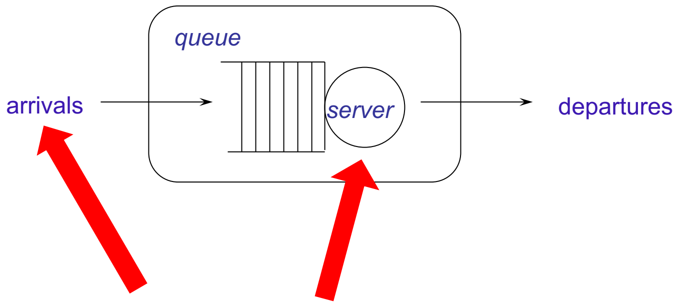
Single Server Queue



1. one stream per stochastic component

2. uncoupled processes → no-overlapped streams

2

Quando uso il multistream (lo userò sempre) non uso la PutSeed, bensì la PlantSeeds, perchè devo 'piantarli' tutti.

Multi-stream RNG
Case study ssq

ssq2 revisited

- Use `rngs` functions for arrivals and services

```
double GetArrival(void) {
static double arrival = START;
SelectStream(0);
arrival += Exponential(2.0);
return (arrival);
}

double GetService(void) {
SelectStream(2);
return (Uniform(1.0, 2.0));
}
```

- include “`rngs.h`” and use PlantSeeds(12345)
  (in place of PutSeed(12345))

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3

arrival and service processes are *uncoupled*

stream 0 for arrivals, stream 1 for services

for 10025 jobs
average interarrival time = 1.99
average wait .... = 3.92
average delay .... = 2.41
average service time .... = 1.50
average # in the node ... = 1.96
average # in the queue .. = 1.21
utilization .... = 0.75

stream 0 for arrivals, stream 2 for services (or e.g. stream 128 to get more separation)

for 10025 jobs
average interarrival time = 1.99
average wait .... = 3.86
average delay .... = 2.36
average service time .... = 1.50
average # in the node ... = 1.93
average # in the queue .. = 1.18
utilization .... = 0.75

Theoretical values

$\bar{r}$	$\bar{w}$	$\bar{d}$	$\bar{s}$	$\bar{l}$	$\bar{q}$	$\bar{x}$
2.00	3.83	2.33	1.50	1.92	1.17	0.75

(in riferimento all'esercizio con Unif(1,2)

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Le cose cambiano, stessi arrivi, distribuzioni etc, ma sto prendendo due pezzi di numeri random diversi!
Ci sono piccole oscillazioni, ma sono normali!

Uncoupling Stochastic Processes

perchè non prenderne una Unif(0,3)?
ha un'altra varianza ancora, sarebbe un "altro caso".

Consider changing the service process to

Uniform(0.0, 1.5) + *Uniform*(0.0, 1.5)

La media è sempre la stessa (1.5), variabilità qui più ampia.
(perchè è come fosse un'unica uniforme da 0 a 3, sommando)

- Without uncoupling, arrival process sequence would change!
- With uncoupling, the service process "sees" exactly the same arrival sequence
- Important variance reduction technique

Se voglio vedere l'effetto della variabilità, devo toccare solo lei,
e devo tenere i flussi diversi. Non posso cambiare più cose insieme!

Theoretical values

$\bar{r}$	$\bar{w}$	$\bar{d}$	$\bar{s}$	$\bar{l}$	$\bar{q}$	$\bar{x}$
2.00	3.83	2.33	1.50	1.92	1.17	0.75

stream 0 for arrivals, stream 1 for services

for 10025 jobs
average interarrival time = 1.99
average wait .... = 4.29
average delay .... = 2.78
average service time .... = 1.51
average # in the node ... = 2.15
average # in the queue .. = 1.40
utilization .... = 0.76

nuovi risultati, alcuni sono cambiati di pochissimo (servizio medio),
altri maggiormente (avg wait).

risultati con uniform(1.0, 2.0)

for 10025 jobs
average interarrival time = 1.99
average wait .... = 3.92
average delay .... = 2.41
average service time .... = 1.50
average # in the node ... = 1.96
average # in the queue .. = 1.21
utilization .... = 0.75

ssq with Multiple Job Types

- Consider multiple job types, each with its own arrival and service process
- Two job types:

arrivi più frequenti service con stessa media, cambia varianza, in classe1 maggiore.

  –Class 0: *Exponential*(4.0) interarrivals, *Uniform*(1.0, 3.0) service

  –Classe 1: *Exponential*(6.0) interarrivals, *Uniform*(0.0, 4.0) service

- Use `rngs` to allocate a different stream to each stochastic process

errore frequente: devo sequenziare istanti di arrivo di due classi. A seconda dell'istante di arrivo che stiamo 'trattando' dobbiamo simulare il successivo. Un errore sarebbe generare TUTTI INSIEME GLI ARRIVI in uno stesso momento.

E' un errore quindi generare subito e tutti insieme istanti di arrivo e tempi di servizio.

Arrival process

```
double GetArrival(int *j)          /* j corrisponds to job type */
{
const double mean[2] = {4.0, 6.0};
static double arrival[2] = {START, START};
static int init = 1;
double temp;
if (init) {
        SelectStream(0);
        arrival[0] += Exponential(mean[0]);
        SelectStream(1);
        arrival[1] += Exponential(mean[1]);
        init = 0;
        }
if (arrival[0] <= arrival[1])
        *j = 0;
else
        *j = 1;
temp = arrival[*j];
SelectStream(*j);
arrival[*j] += Exponential(mean[*j]);
return (temp);
}
```

media 4

media 6

li genero via via, infatti ora parto sono dai due START.

trovo quale è "venuto" prima

The first arrival is of class 0!

se sto consumando arrival 0, devo generare il prossimo arrivo.

- streams 0 and 1 are used for interarrival times of class 0 and class 1 jobs respectively

Ad ogni istante di tempo della simulazione, ho array con i prossimi istanti di arrivo. Come abbiamo detto prima, non li genero tutti, ma solo quando ne sto consumando uno!

Service process

```
double GetService(int j)        j = 0 or 1, è la classe.
{
  const double min[2] = {1.0, 0.0};
  const double max[2] = {3.0, 4.0};
  SelectStream(j + 2);  mi sposto sullo stream 2 o 3, a seconda di j
  return (Uniform(min[j], max[j]));
}                    Unif[1,3] or Unif[0,4]
```

- *j* corrisponds to the job type (0 or 1)
- streams 2 and 3 are used for service times of class 0 and 1 respectively
- All four simulated stochastic processes are uncoupled!
- Any process could be changed without altering the random sequence of others!

Consistency checks

- The "teorethical" steady-state statistics are

$\bar{r}$	$\bar{w}$	$\bar{d}$	$\bar{s}$	$\bar{l}$	$\bar{q}$	$\bar{x}$
2.40	7.92	5.92	2.00	3.30	2.47	0.83

exact analytical results,
No simulation!

- obvious consistency checks: $\bar{w} = \bar{d} + \bar{s}$ $\quad \bar{l} = \bar{q} + \bar{x}$
- other consistency checks:
  - both job types have avg service time of 2.0 → $\bar{s} = 2.00$
  - arrival rate should be
    1/4 + 1/6 = 5/12 → $\bar{r} = 12/5 = 2.40$
  - $\bar{x}$ should be ratio of arrival to service rates

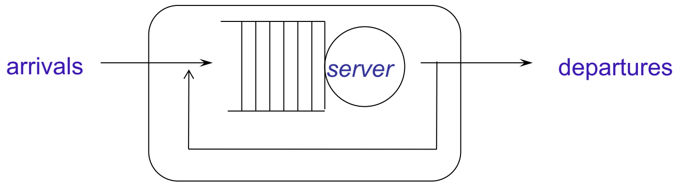
$$\frac{5/12}{1/2} = 5/6 \cong 0.83$$

Exercises

- Exercises: 3.2.3, 3.2.4, 3.2.7

un job che arriva al sistema potrebbe richiedere 'servizi' ulteriori.
Il completamento del servizio non è detto che corrisponda alla partenza.
Qui una partenza corrisponde al completamento di un numero di servizi richiesti.
C'è quindi completamento singolo e totale.

Single Server Queue with feedback



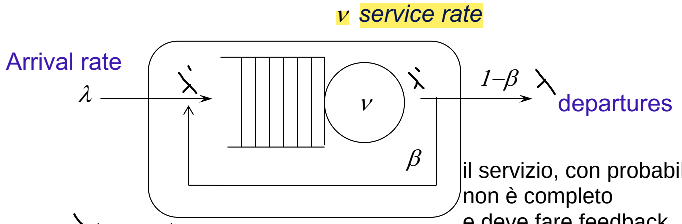
- If the service a job receives is incomplete or unsatisfactory, the job feeds back
- Completion of service and departure now have <u>different meanings</u>

Non so a priori quante volte richiede feedback un job. E' probabilistico.

Model Considerations

"ni" quindi 1/ni è tempo di servizio per UN SOLO passaggio.

In coda non entra solo lambda,
ma anche i job che tornando indietro causa feedback.
E' interessante trovare il valore di beta
che porta al collasso il sistema.



il servizio, con probabilità Beta, non è completo
e deve fare feedback

$$\lambda = \grave{\lambda}(1-\beta)$$

- When feedback occurs the job joins the queue consistent with the queue discipline
- The decision to depart or feed back is random with *feedback probability* β

Come calcolare l'utilizzazione? perchè non è più semplicemente lambda/ni $\rho = \frac{\grave{\lambda}}{\nu}$

Model Considerations

- Feedback is independent of past history
- In theory, a job may feed back arbitrarily many times
- Jobs from outside the system are merged with jobs from the feedback process
- The steady-state request-for-service rate is larger than λ by the positive additive factor $\beta\bar{x}\nu$
- Note that $\bar{s}$ increases with feedback but $1/\nu$ is the average service time per request

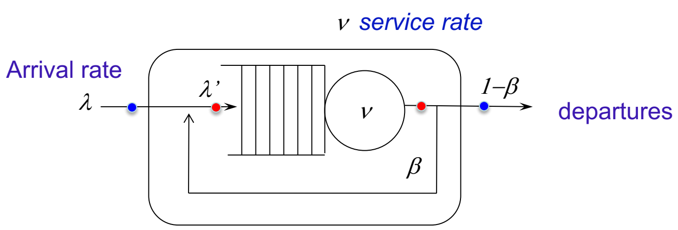
Flow Balance and Saturation

- Jobs flow into the service node at the average rate of λ
- To remain flow balanced jobs must flow out of the service node at the same average rate
- The average rate at which jobs flow out of the service node is

$$\bar{x}(1-\beta)\nu$$

- Flow balance is achieved when $\lambda = \bar{x}(1-\beta)\nu$
- Saturation occurs when $\bar{x}=1$ or as $\beta \to 1 - \lambda/\nu$

Coda con feedback



$\rho = \lambda' / \nu$ $\rho = \lambda/(1-\beta)\nu$

Saturation:

$\rho \to 1$

$\lambda/(1-\beta)\nu \to 1$

$\beta \to 1 - \lambda/\nu$

Flow Balance: $\lambda = \lambda'(1-\beta)$

$\lambda' = \lambda/(1-\beta)$

- Feedback is independent of past history
- In theory, a job may feed back arbitrarily many times

```
int GetFeedback(double beta)        /* 0.0 <= β < 1.0 */
{
    SelectStream(2);
    if (Random() < beta)
        return (1);                 /* feedback*/
    else
        return (0);                 /* no feedback */
}
```


Statistical considerations

- Index i=1, 2, 3, … counts jobs that enter the service node
  - fed-back jobs are not recounted
- Using this indexing, all job-averaged statistics remain valid
- We must update delay times, wait times and service times for each feedback
- Jobs from outside the system are merged with jobs from the feedback process
- The steady-state request-for-service rate is larger than λ by the positive additive factor $\beta \bar{x} \nu$
- Note that $\bar{s}$ increases with feedback but $1/\nu$ is the average service time per request

Algorithm and Data Structure Considerations

job index	1	2
Arrival/feedback	1	3
service	9	3
departure	10	**13**

Algorithm and Data Structure Considerations

job index	1	2	3	4	5	.	6
Arrival/feedback	1	3	4	7	10		14
service	9	3	2	4	7		6
departure	10	**13**	**15**	19	**26**		37

Algorithm and Data Structure Considerations

job index	1	2	3	4	5	.	6	.	7	8	.	9 ...
Arrival/feedback	1	3	4	7	10	**13**	14	**15**	19	24	**26**	30 ...
service	9	3	2	4	7	5	6	3	4	6	3	7 ...
departure	10	**13**	**15**	19	**26**	31	37	**40**	44	50	53	**60** ...

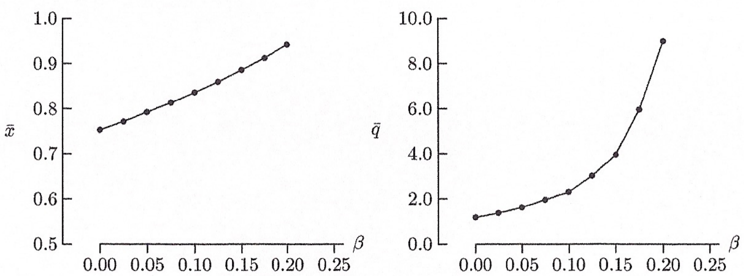
At the computational level, some algorithm and data structure is necessary

Theoretical values

$\bar{r}$	$\bar{w}$	$\bar{d}$	$\bar{s}$	$\bar{l}$	$\bar{q}$	$\bar{x}$
2.00	3.83	2.33	1.50	1.92	1.17	0.75

Program ssq2 was modified to incorporate immediate feedback

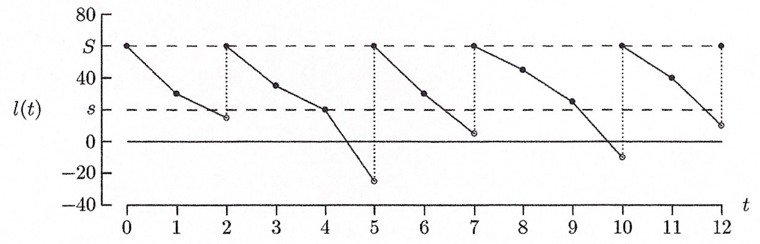
- interarrivals = *Exponential*(2.0)
- service times = *Uniform*(1.0, 2.0)



- It appears saturation is achieved as $\beta \to 0.25$

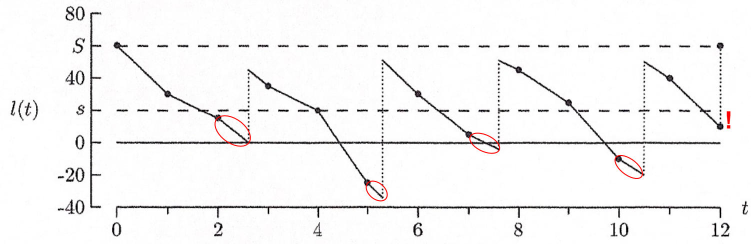
Inventory system with delivery lag

- *delivery lag* (dl) occurs when orders are not delivered immediately
- Lag is assumed to be random and independent of order size
- Without lag, inventory jumps occur only at inventory review times



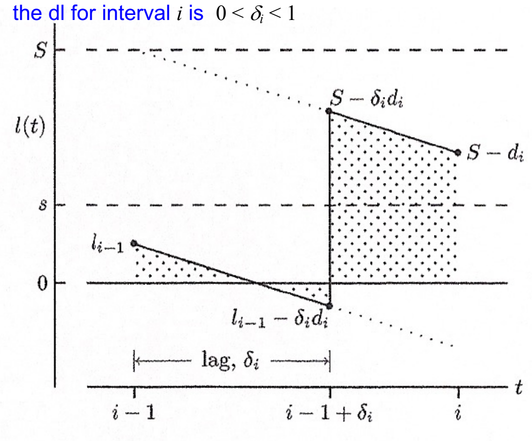
Inventory system with delivery lag

- With delivery lag, inventory jumps occur at arbitrary times



- The last order is assumed to have no lag
- We assume that orders are delivered before the next inventory review
- With this assumption, there is no change to the specification model

- If $l_{i-1} \geq s$ the equations for $\bar{l}_i^+$ and $\bar{l}_i^-$ remain correct
- If $l_{i-1} < s$, the time-averaged holding and shortage intervals must be modified
  - the dl for interval i is $0 < \delta_i < 1$

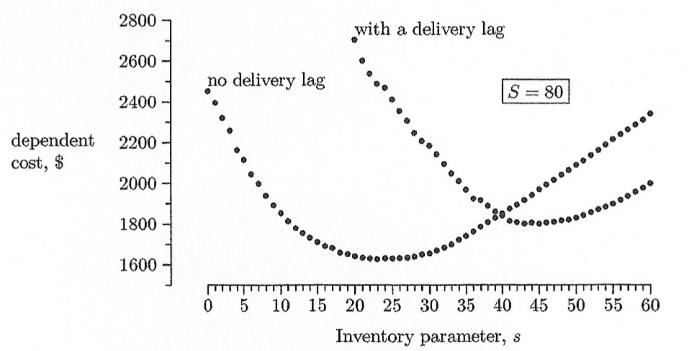


Consistency Checks

- It is fundamentally important to verify extended models with the parent model (before the extension)
  - Set system parameters to special values
- Set $\beta = 0$ for the ssq with feedback
  - Verify that all statistics agree with parent
- Using the library `rngs` facilitates this kind of comparison
- It is a good practice to check for intuitive “small-perturbation” consistency
  - Use a small, but non-zero β and check that appropriate statistics are slightly larger

- For the InvSys with delivery lag, $\delta_i = 0.0$ iff no order during i^{th} interval, $0 < \delta_i < 1.0$ otherwise
- The InvSys is *lag-free* iff $\delta_i = 0.0$ for all i
- If (*S*, *s*) are fixed then, even with small dl:
  - $\bar{o}, \bar{d}, \bar{u}$ are the same regardless of delivery lag
  - Compared to the lag-free system, $\bar{l}_i^+$ will decrease
- Compared to the lag-free system, $\bar{l}_i^-$ will increase or remain unchanged

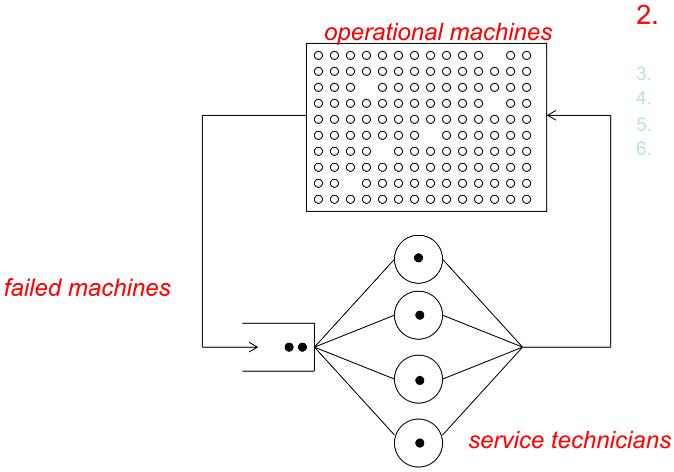
- Delivery lags are indipendent *Uniform*(0.0, 1.0) random variates



- delivery lag causes $\bar{l}_i^+$ to decrease and $\bar{l}_i^-$ to increase or remain the same
- with C_{hold}= $25 and C_{short}= $700, cause shift up and to the left

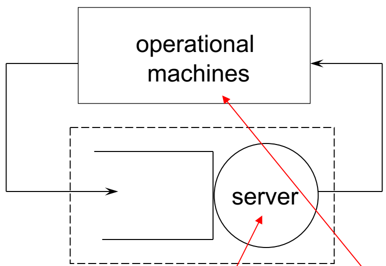
Machine Shop Model

1. Goals
2. **Conceptual model**
3. Specification model
4. Computational model
5. Verify
6. Validate



Machine-shop, ssq model

The machine shop model is closed because there are a finite number of machines in the system



- Assume repair times are *Uniform*(1.0, 2.0) random variates
- There are *M* machines that fail after an *Exponential*(100.0) random variate

Program `ssms`

- program `ssms` simulates a Single Server Machine-Shop
- the library `rngs` is used to uncouple the random process
- the failure process is defined by the array failures
  - a O(M) search is used to find the next failure
  - alternate data structures can be used to increase computational efficiency


```
 double GetFailure(void)
{   SelectStream(0);
    return (Exponential(100.0));}

double NextFailure(double failure[], int *m)
{   int     i = 0;
    double t = failure[0];

    *m = i;
    for (i = 1; i < M; i++)
        if (failure[i] < t) {
           t = failure[i];
          *m = i;
        }
    return (t);
}

double GetService(void)
{   SelectStream(1);
    return (Uniform(1.0, 2.0));
}
```

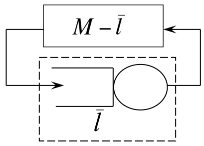
```c
int main(void)
{
  long   index     = 0;      /* job (machine failure) index */
  double arrival   = START; /* time of arrival (failure)   */
  double delay;             /* delay in repair  queue      */
  double service;           /* service (repair) time       */
  double wait;              /* delay + service             */
  double departure = START; /* time of service completion  */
  int    m;                 /* machine index 0,1,...(M-1)  */
  double failure[M];        /* list of next failure times  */
  struct {                  /* sum of ...            */
    double wait;            /*   wait times          */
    double delay;           /*   delay times         */
    double service;         /*   service times       */
    double interarrival;    /*   interarrival times  */
  } sum = {0.0, 0.0, 0.0};
```



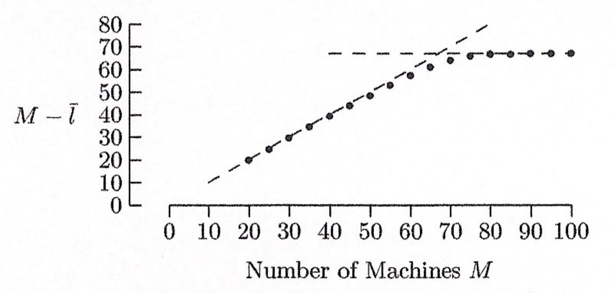
```c
PlantSeeds(123456789);

for (m = 0; m < M; m++)          /* initial failures */
failure[m] = START + GetFailure();

while (index < LAST) {
   index++;
   arrival      = NextFailure(failure, &m);
   if (arrival < departure)
       delay      = departure - arrival;
   else
       delay      = 0.0;
   service      = GetService();
   wait         = delay + service;
   departure    = arrival + wait;            /* completion of service  */
   failure[m]   = departure + GetFailure(); /* next failure, machine m */
   sum.wait    += wait;
   sum.delay   += delay;
   sum.service += service;
}
sum.interarrival = arrival - START;
```



- The time-averaged number of working machines is $M - \bar{l}$



- For small values of M the time-averaged number of operational machines is essentially M
- For large values of M this value is essentially constant at approximately 67

Exercises

- Exercises: 3.3.2, 3.3.3, 3.3.4, 3.3.7