Performance Modeling of Computer Systems and Networks

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Next Event Simulation
Examples

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1. **Initialize** - set simulation clock and first time of occurrence for each event type
2. **Process current event** - scan event list to determine most imminent event; advance simulation clock; update state
3. **Schedule new events** - new events (if any) are placed in the event list
4. **Terminate** - Continue advancing the clock and handling events until termination condition is satisfied

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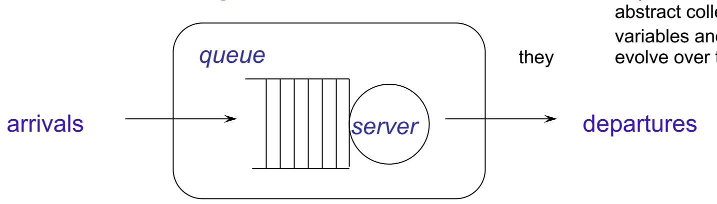
Single Server Queue

- Conceptual model: abstract collection of variables and how they evolve over time



- The *state* is number of jobs in the node at time t: $l(t)$
- Its time-*evolution* is guided by arrival-departure events:
  - An arrival causes $l(t)$ to increase by 1
  - A departure causes $l(t)$ to decrease by 1

3

Single Server Queue

- Specification model: collection of mathematical variables together with logic and equations

The state variable $l(t)$ provides a complete characterization of the state of a ssq

$$l(t) = 0 \Leftrightarrow q(t) = 0 \text{ and } x(t) = 0$$
$$l(t) > 0 \Leftrightarrow q(t) = l(t)\text{-}1 \text{ and } x(t) = 1$$

Da l(t) posso effettivamente definire tutto.
Tutto ciò che è inutile/superfluo non va messo.
Posso scrivere queste formule solo se capisco bene il modello che sto definendo, se non me ne rendo conto allo 'step' prima posso tornare indietro e vedere cosa manca, ma idealmente a questo passo ci arrivo solo dopo aver catturato tutto.

4

l(t) ci dice tutto, non ci serve altro.

- The initial state $l(0)$ can have any non-negative value, typically 0 Ha senso partire da 0, se parto dal sistema vuoto.
  Se partissi da un sistema già stabile, potrei usare altri valori.
- terminal state: any non-negative value
  - Assume at time τ arrival process stopped. Remaining jobs processed before termination
- some mechanism must be used to denote an event impossible
  - Only store possible events in event list
  - Denote impossible events with event time of ∞

se volessi tornare allo stato iniziale, fisso un tempo 'tau' che, superato blocca i nuovi arrivi e fa servire gli ultimi job arrivati in ritardo.

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- The simulation clock (current time) is t
- The terminal ("close the door") time is τ
- The next scheduled arrival time is t_a
- The next scheduled service completion time is t_c
- The number in the node (state variable) is l

Next-Event Simulation

Algorithm

1. **Initialize**: the clock
   the event list (e.g. ssq arrival)
   the system state
2. **Remove** next event from the list
3. **Advance** simulation clock
4. **Process** current event
5. **Schedule** new events (if any) generated from current event
6. Go to 2. until **termination** condition is satisfied

ssq2.c

```
int main(void)
{  long   index     = 0;       /* job index */
   double arrival   = START;   /* arrival time*/
   double delay;               /* delay in queue*/
   double service;              /* service time*/
   double wait;                 /* delay + service*/
   double departure = START;    /* departure time*/
   struct {                     /* sum of ...   */
       double delay;     /*delay times */
       double wait;      /*wait times*/
       double service;   /*service times */
       double interarrival; /*   interarrival times */
   } sum = {0.0, 0.0, 0.0};
PutSeed(123456789);
```

```
while (index < LAST) {
index++;
arrival      = GetArrival();
if (arrival < departure)
    delay  = departure - arrival; /* delay in queue  */
else  delay      = 0.0;            /* no delay  */
service = GetService();
wait  = delay + service;
departure     = arrival + wait;   /* time of departure */
sum.delay    += delay;
sum.wait     += wait;
sum.service += service;  }
```

…


```
l = 0; t = 0.0;
tc = ∞; ta = GetArrival();     /* initialize the event list */
while ((ta < τ ) or (l > 0)) {
        t = min(ta, tc);       /* scan the event list */
        if (t == ta) {         /* process an arrival */
                l++;
                ta = GetArrival();
                if (ta > τ )
                        ta = ∞;
                if (l == 1)
                tc = t + GetService();
        }
        else {                 /* process a completion */
                l − −;
                if (l > 0)
                        tc = t + GetService();
                else
                        tc = ∞;
        }
}
```

arrival

depature

2 event types: arrival, depature

Algorithm 1

Program ssq3

- `number` represents $l(t)$ (system state)
  `struct t` represents time
  - `t.arrival, t.completion` event list (t_a, t_c from algorithm 1)
  - `t.current` simulation clock (t from algorithm 1)
  - `t.next` next event time (min(t_a, t_c) from algorithm 1)
  - `t.last` last arrival time
- `struct area` (time-averaged) statistics-gathering structure
  - $\int_0^t l(s)ds$ evaluated as `area.node`
  - $\int_0^t q(s)ds$ evaluated as `area.queue`
  - $\int_0^t x(s)ds$ evaluated as `area.service`

ssq3.c

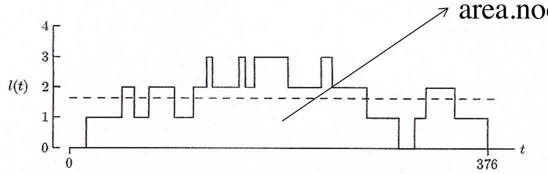
```
#include <stdio.h>
#include <math.h>
#include "rngs.h"    /* the multi-stream generator */
#define START   0.0
#define STOP    20000.0  /* terminal (close the door) time*/
#define INFINITY (100.0 * STOP) /* must be much larger than STOP */

double Min(double a, double c)
{  if (a < c)     return (a);
    else     return (c);}

double Exponential(double m) …
double Uniform(double a, double b) …
double GetArrival() …
double GetService() …
```

```
 int main(void)
{  struct {
        double arrival;     /* next arrival time */
        double completion; /* next completion time */
        double current; the clock! urrent time */
        double next;        /* next (most imminent) event time */
        double last;        /* last arrival time */
   } t;
   struct {
        double node;    /* time integrated number in the node */
        double queue;    /* time integrated number in the queue */
        double service; /* time integrated number in service */
   } area      = {0.0, 0.0, 0.0};
   long index  = 0;     /* used to count departed job */
   long number = 0;     /* number in the node */ system state
```



```
PlantSeeds(123456789);
t.current    = START;          /* set the clock */
t.arrival    = GetArrival();  /* schedule the first arrival */
t.completion = INFINITY; /* the first event can't be a completion */
while ((t.arrival < STOP) || (number > 0)) {
   t.next= Min(t.arrival, t.completion); /* next event time   */
   if (number > 0)  {          /* update integrals  */
        area.node+= (t.next - t.current) * number;
        area.queue+= (t.next - t.current) * (number - 1);
        area.service += (t.next - t.current);    }
   t.current = t.next;  advance the clock!
```

$$\text{area.node} = \int_0^t l(s)\,ds$$



```
if (t.current == t.arrival)  {
   number++;
   t.arrival= GetArrival();
   if (t.arrival > STOP)  {
     t.last= t.current;
     t.arrival    = INFINITY;
   }
   if (number == 1)
        t.completion = t.current + GetService();
}
else {
  index++;
   number--;
   if (number > 0)
        t.completion = t.current + GetService();
   else
        t.completion = INFINITY;
   }
}
```

process an arrival

process a completion


```
printf(" … jobs", index);
printf(" average interarrival time ..", t.last / index);
printf(" average wait …", area.node / index);
printf(" average delay ...", area.queue / index);
printf(" average service time ...", area.service / index);
printf(" average # in the node ... ", area.node / t.current);
printf(" average # in the queue .. ", area.queue / t.current);
printf(" utilization ....", area.service / t.current);
```

World Views and Synchronization

- ssq2 produces :

```
while (index < LAST) {
index++;
arrival      = GetArrival();
if (arrival < departure)
     delay  = departure - arrival;
else  delay      = 0.0;
service = GetService();
wait  = delay + service;
departure     = arrival + wait;
sum.delay    += delay;
sum.wait     += wait;
sum.service += service;  }
```

$$\sum_{i=1}^{n} d_i \qquad \sum_{i=1}^{n} w_i \qquad \sum_{i=1}^{n} s_i$$

World Views and Synchronization

- ssq2 produces :

$$\sum_{i=1}^{n} w_i$$

```
printf("… jobs", index);
printf("average interarrival time = ", sum.interarrival / index);
printf("average wait ........... = ", sum.wait / index);
printf("average delay .......... = ", sum.delay / index);
printf("average service time .... = ", sum.service / index);
printf("average # in the node ... = ", sum.wait / departure);
printf("average # in the queue .. = ", sum.delay / departure);
printf("utilization ............ = ", sum.service / departure);
```

$$\bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i$$

$$\bar{l} = \frac{n}{c_n}\bar{w}$$

$$\bar{q} = \frac{n}{c_n}\bar{d}$$

$$\bar{x} = \frac{n}{c_n}\bar{s}$$

$$\frac{\sum_{i=1}^{n} w_i}{c_n} = \frac{n\bar{w}}{c_n}$$

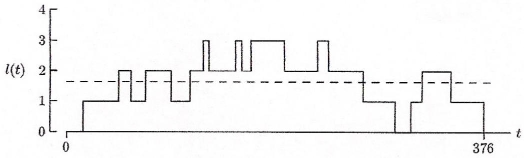
- ssq3 produces :

```
PlantSeeds(123456789);
t.current    = START;
t.arrival    = GetArrival();
t.completion = INFINITY;
while ((t.arrival < STOP) || (number > 0)) {
    t.next= Min(t.arrival, t.completion);
    if (number > 0)  {
        area.node+= (t.next - t.current) * number;
        area.queue+= (t.next - t.current) * (number - 1);
        area.service += (t.next - t.current);    }
    t.current        = t.next;
```

$\int_0^\tau l(t)dt$ ← area.node

$\int_0^\tau q(t)dt$ ← area.queue

$\int_0^\tau x(t)dt$ ← area.service



$$\tau \bar{l} = \int_0^\tau l(t)dt$$

```
printf(" … jobs", index);
printf(" average interarrival time ..", t.last / index);
printf(" average wait …", area.node / index);
printf(" average delay ...", area.queue / index);
printf(" average service time ...", area.service / index);
printf(" average # in the node ... ", area.node / t.current);
```

$$\bar{w} = \frac{\tau}{n}\bar{l}$$

$$\bar{l} = \frac{1}{\tau}\int_0^\tau l(t)dt$$

```
printf(" average # in the queue .. ", area.queue / t.current);
printf(" utilization ....", area.service / t.current);
```

World Views and Synchronization

- programs ssq2 and ssq3 simulate exactly the same system
- The two have different *world views*
  - ssq2 naturally produces job-averaged statistics (based upon *process-interaction*)
  - ssq3 naturally produces time-averaged statistics (based upon *event-scheduling*)

World Views and Synchronization

The programs should produce exactly the same statistics

- in ssq2 random variates are always generated in the alternating order:

$$a_1, s_1, a_2, s_2, \ldots$$

```
while (index < LAST) {
index++;
arrival      = GetArrival();
if (arrival < departure)
     delay  = departure - arrival;
else  delay      = 0.0;
service = GetService();
wait  = delay + service;
departure     = arrival + wait;
sum.delay    += delay;
sum.wait     += wait;
sum.service += service;  }
```

World Views and Synchronization

• in ssq3 the order cannot be known a priori

```
while ((t_a < τ ) or (l > 0)) {
        t = min(t_a, t_c);     /* scan the event list */
        if (t == t_a) {        /* process an arrival */
                l++;
                t_a = GetArrival();
                if (t_a > τ )
                        t_a = ∞;
                if (l == 1)
                t_c = t + GetService();
        }
        else {                 /* process a completion */
                l – –;
                if (l > 0)
                        t_c = t + GetService();
    .....
```


World Views and Synchronization

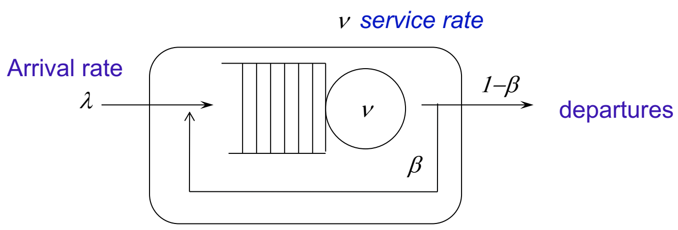
The programs should produce exactly the same statistics

→ to do so requires rngs

```
 double GetArrival()
{ static double arrival = START;
  SelectStream(0);
  arrival += Exponential(2.0);
  return (arrival);}

double GetService()
{ SelectStream(1);
  return (Uniform(0.0, 1.5)+Uniform(0.0, 1.5));}
```

Model Extensions

SSQ with immediate feedback



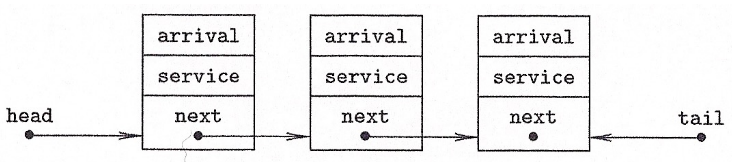
job index	1	2	3	4	5	.	6	.	7	8	.	9	…
Arrival/feedback	1	3	4	7	10	**13**	14	**15**	19	24	**26**	30	…
service	9	3	2	4	7	5	6	3	4	6	3	7	…
departure	10	**13**	**15**	19	**26**	31	37	**40**	44	50	53	**60**	…

Model Extensions

SSQ with immediate feedback

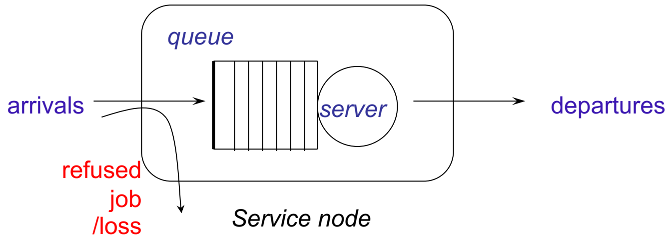
```
else {                    /* process a completion */
   if (GetFeedback() == 0) {   /* this statement is new */
         index++;
         number--;}
   if (number > 0)
         t.completion = t.current + GetService();
   else    t.completion = INFINITY;
```

- alternate queue disciplines
  - it is necessary to add a dynamic-queue data structure



- +2 supporting queue functions: Enqueue, Dequeue

Model Extensions



Assumption:

for the finite *capacity case*,
only the accepted jobs are considered

BUT
the *loss probability* could be of interest!

Model Extensions

SSQ with finite capacity

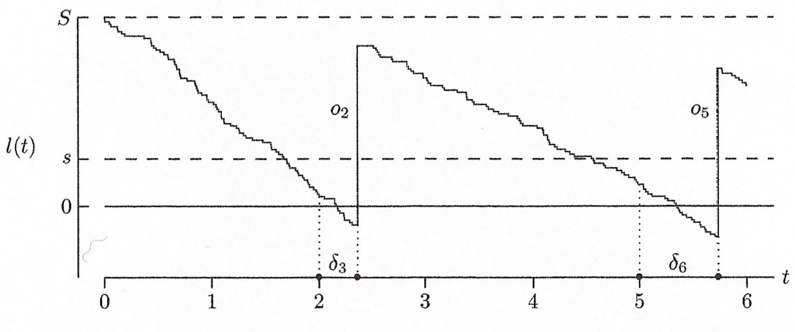
```
if (t.current == t.arrival) {          /* process an arrival */

   if (number < CAPACITY) {
      number++;
      if (number == 1)
         t.completion = t.current + GetService();
   }
   else
      reject++;
   t.arrival = GetArrival();
   if (t.arrival > STOP) {
      t.last = t.current;
      t.arrival = INFINITY;
   }
}
```

Random Sampling

- The structure of ssq3 facilitates adding sampling
- Add a sampling event to the event list
  - Sample deterministically, every δ time units
  - Sample Randomly, every *Exponential*(δ) time units

A Simple Inventory System with Delivery Lag

Two changes relative to sis2
- *Uniform*(0,1) lag between inventory review and order delivery
- More realistic demand model
  - Demand instances for a single item occur *at random*
  - Average rate is λ demand instances per time interval
  - Time between demand instances is *Exponential*($1/\lambda$)



Comparison of Demand Models

sis2: used an *aggregate* demand for each time interval, generated as an *Equilikely*(10,50) random variate

- Aggregate demand per time interval is random
- Within an interval, time between demand instances is constant
- Example: if aggregate demand is 25, inter-demand time is 1/25=0.04

- Now using *Exponential*($1/\lambda$) inter-demand times
  - Demand is modeled as an arrival process
  - Average demand per time interval is λ

Specification Level: States and Notation

- The simulation clock is t (real-valued)
- The terminal time is τ (integer-valued)
- Current inventory level is $l(t)$ (integer-valued)
- Amount of inventory on order, if any, is $o(t)$ (integer-valued)
  - Necessary due to delivery lag
- $l(t)$ and $o(t)$ provide complete state description
- Initial state is assumed to be $l(0)=\mathrm{S}$ and $o(0)=0$
- Terminal state is assumed to be $l(\tau)=\mathrm{S}$ and $o(\tau)=0$
- Cost to bring $l(\mathrm{t})$ to S at simulation end (with no lag) must be included in accumulated statistics

Specification Level: Events

Three types of events can change the system state

- A *demand* for an item at time t
  - $l(t)$ decreases by 1
- An inventory *review* at integer-valued time t
  - If $l(t) \geq s \rightarrow o(t)=0$
  - If $l(t) < s \rightarrow o(t)=S-l(t)$
- An *arrival* of an inventory replenishment order at time t
  - $l(t)$ increases by $o(t)$
  - $o(t)$ becomes 0

Algorithm 2: initialization

Time variables used for event list:

- t_d: next scheduled inventory *demand*
- t_r: next scheduled inventory *review*
- t_a: next scheduled inventory *arrival*

∞ denotes impossible events

```
l = S;                  /* initialize inventory level */
o = 0;                  /* initialize amount on order */
t = 0.0;                /* initialize simulation clock */
td = GetDemand();       /* initialize event list */
tr = t + 1.0;           /* initialize event list */
ta = ∞;                 /* initialize event list */
```

Algorithm 2: main loop

3 types of events:
demand, review, arrival

```
while (t < τ) {
        t = min(t_d , t_r , t_a);      /* scan the event list */
        if (t == t_d) {                /* process an inventory demand */
            l--;                       demand
            t_d = GetDemand();
        }
        else if (t == t_r ) {          /* process an inventory review */
            if (l < s) {               review
                o = S - l;
                δ= GetLag();
                t_a = t + δ ;
            }
            t_r += 1.0;
        }
        else {                         /* process an inventory arrival */
            l += o;                    arrival
            o = 0;
            t_a = ∞;
        }
}
```


Program sis3

- implements algorithm 2

correspond to t_d, t_r, t_a

```
while (t.current < STOP) {
  t.next = Min(t.demand, t.review, t.arrive);
  if (inventory > 0)
   sum.holding  += (t.next - t.current) * inventory;
  else
    sum.shortage -= (t.next - t.current) * inventory;
  t.current = t.next;
  if (t.current == t.demand) {
    sum.demand++; /* process an inventory demand */
    inventory--;
    t.demand = GetDemand();    }
  else    ………
```

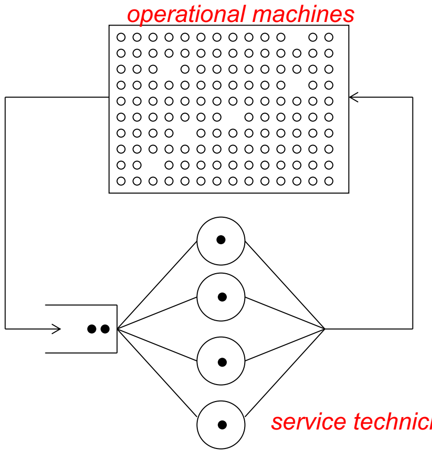
Program sis3

- State variables `inventory` and `order` correspond to $l(t)$ and $o(t)$
    - `t.next` next event instant (min(t_d , t_r , t_a) in algorithm 2)
    - `t.last` last arrival instant

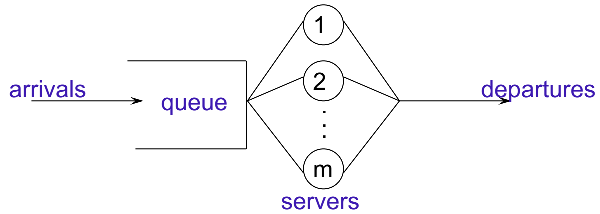
`sum.hold` and `sum.short` accumulate the time-integrated holding and shortage integrals

Machine Shop Model

1. Goals
2. Conceptual model
3. Specification model
4. Computational model
5. Verify
6. Validate



Conceptual model: MSQ



Servers in a multi-server service node are called *service channels*

- m is the number of servers
- The *server index* is $s = 1, 2, \ldots, \mathrm{m}$

The *state* includes:

the number of jobs in the node at time t: $l(t)$
the state of each server at at time t: $x_s(\mathrm{t})$

Specification Model: States and Notation

$l(t)$ denotes the number of jobs in the service node at time t

- If $l(t) \geq m$, all servers are busy and $q(t) = l(t) - m$
- If $l(t) < m$, some servers are idle
- If servers are distinct, need to know which servers are idle

For *s* = 1, 2, . . . , *m* define

$x_s(\mathrm{t})$: the number of jobs in service (0 or 1) at server *s* at time t

The complete state description is $l(t), x_1(t), x_2(t), \ldots, x_m(t)$

$$q(t) = l(t) - \sum_{s=1}^{m} x_s(t)$$

Specification Model: Events

What types of events can change state variables
$l(t), x_1(t), x_2(t), \ldots, x_m(t)$?

- *arrival* at time t
  - $l(t)$ increases by 1
  - If $l(t) < m$, an idle server s is selected, and $x_s(t)$ becomes 1
    else all servers are busy

- A *completion of service* by server s at time t
  - $l(t)$ decreases by 1
  - if $l(t) \geq m$, a job is selected from the queue to enter service
    else $x_s(t)$ becomes 0

→ m+1 event types

Specification Model: Additional Assumptions

- The <u>initial</u> <u>state</u> is an empty node
  - $l(0) = 0$
  - $x_1(0)=x_2(0)= \ldots = x_m(0)=0$
  - The <u>first</u> <u>event</u> must be an arrival

- The arrival process is turned off at time τ
  - The node continues operation after time τ until empty
  - The <u>terminal state</u> is an empty node
  - The <u>last event</u> is a completion of service

For simplicity, all servers are independent and *statistically identical*

- *Equity* selection is the server selection rule (lowest-utilized)

All of these assumptions can be relaxed

Event list

1 on, 0 off
1 busy, 0 idle

	0	t	x	arrival
	1	t	x	completion by 1
	2	t	x	completion by 2
m=4	3	t	x	completion by 3
	4	t	x	completion by 4

- can be organized as an array of m + 1 event types
- field t: scheduled time of next occurrence for the event
- field x: current *activity status* of the event

- for large m $\longrightarrow$ alternate event-list structures

Program msq

Implements this next-event multi-server service node simulation model

- `number` state variable $l(t)$
- state variables $x_1(t), x_2(t), \ldots, x_m(t)$ are part of the event list
- `area` time-integrated statistic $\int_0^t l(\theta)d\theta$
- `sum` array, records for each server
  - the sum of service times
  - the number served
- function `NextEvent` searches the event list to find the next event
- function `FindOne` searches the event list to find the longest-idle server (because equity selection is used)

program msq.c

```
typedef struct {
  double t;
  int    x;
} event_list[SERVERS + 1];
…
    int NextEvent(event_list event)
    { int e;
      int i = 0;
      while (event[i].x == 0)
        i++;
      e = i;
      while (i < SERVERS) {
        i++;
        if ((event[i].x == 1) && (event[i].t < event[e].t))
           e = i;  }
      return (e);}
```

is the first active event, assume it is the next

look for the next active event

if it is previous, update e

programma msq.c

```
 int FindOne(event_list event)
{ int s;
  int i = 1;
  while (event[i].x == 1)
    i++;
  s = i;
  while (i < SERVERS) {
     i++;
     if ((event[i].x == 0) && (event[i].t < event[s].t))
     s = i;  }
  return (s);}
```

first server idle

look for the next idle

if its completion is previous,
it is idle since more time

Exercises

- 5.1.1, 5.1.2, 5.1.3
- 5.2.1, 5.2.2,
- 5.2.8: modify program msq to allow for a finite capacity (max r jobs); a. draw a histogram of the time between lost jobs at the node; b. comment on the shape of this histogram.