Performance Modeling of Computer Systems and Networks

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Next Event Simulation

Examples

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- Se devo osservare 3 ore, non devo creare eventi all'inizio e poi spalmarlo nelle 3 ore.
- se processo evento, devo schedulare il prossimo

1. **Initialize** - set simulation clock and first time of occurrence for each event type
2. **Process current event** - scan event list to determine most imminent event; advance simulation clock; update state
3. **Schedule new events -** new events (if any) are placed in the event list
4. **Terminate -** Continue advancing the clock and handling events until termination condition is satisfied

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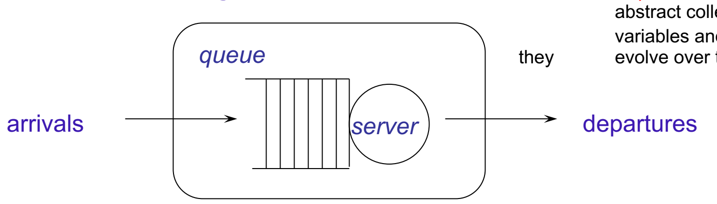
Single Server Queue

- Conceptual model: abstract collection of variables and how they evolve over time



- The *state* is number of jobs in the node at time t: $l(t)$
- Its time-*evolution* is guided by arrival-departure events:
  - An arrival causes $l(t)$ to increase by 1
  - A departure causes $l(t)$ to decrease by 1

Single Server Queue

- Specification model: collection of mathematical variables together with logic and equations

The state variable $l(t)$ provides a complete characterization of the state of a ssq

$$l(t) = 0 \iff q(t) = 0 \text{ and } x(t) = 0$$
$$l(t) > 0 \iff q(t) = l(t)\text{-}1 \text{ and } x(t) = 1$$

l(t) mi dà molte info in coda singola e servente singolo!

senno' errore!

- The initial state $l(0)$ can have any non-negative value, typically 0
- terminal state: any non-negative value
  - Assume at time τ arrival process stopped. Remaining jobs processed before termination
- some mechanism must be used to denote an event impossible
  - Only store possible events in event list
  - Denote impossible events with event time of ∞

- The simulation clock (current time) is t
- The terminal ("close the door") time is τ (non entra più nessuno)
- The next scheduled arrival time is t_a
- The next scheduled service completion time is t_c
- The number in the node (state variable) is l

Next-Event Simulation

Algorithm

1. **Initialize**: the clock
   the event list (e.g. ssq arrival) -> $\ell = 1$
   the system state
2. **Remove** next event from the list
3. **Advance** simulation clock
4. **Process** current event
5. **Schedule** new events (if any) generated from current event
6. Go to 2. until **termination** condition is satisfied

ssq2.c

Vecchio modello NO NEXT-event

```
int main(void)
{ long   index     = 0;      /* job index */
  double arrival   = START;  /* arrival time*/
  double delay;              /* delay in queue*/
  double service;             /* service time*/
  double wait;                /* delay + service*/
  double departure = START;   /* departure time*/
  struct {                    /* sum of ...   */
      double delay;     /*delay times */
      double wait;      /*wait times*/
      double service;   /*service times */
      double interarrival; /*   interarrival times */
  } sum = {0.0, 0.0, 0.0};
PutSeed(123456789);
```

DE simulation
computational model

```
while (index < LAST) {
index++;
arrival      = GetArrival();
if (arrival < departure)
    delay  = departure - arrival; /* delay in queue  */
else  delay      = 0.0;          /* no delay  */
service = GetService();
wait  = delay + service;
departure    = arrival + wait;   /* time of departure */
sum.delay    += delay;
sum.wait     += wait;
sum.service += service;  }
...
```

Il meccanismo di simulazione

Il codice (ed i risultati) sono frutto di un'interazione tra processi il meccanismo di simulazione "dietro" ad esso è l'interazione è tra processi di arrivi e di servizi (come si relazionano)

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Nel NEXT EVENT è più semplice:

Next-Event simulation
ssq

ancora non so quando finisce

tratto un arrivo

Work conserving: server MAI fermo, se c'è qualcuno lo processo

```
I = 0; t = 0.0;
tc = ∞; ta = GetArrival();        /* initialize the event list */
while ((ta < τ) or (I > 0)) {     //condizioni terminazione  arrivo ≥ close the door / se centro NON vuoto dopo τ, lo processo
    t = min(ta, tc);              /* scan the event list */ (evento prossimo: arrivo o completamento?
    if (t == ta) {                /* process an arrival */ (lo assegno al clock)
        I++; (ho un arrivo)
        ta = GetArrival(); consumato un arrivo, ne genero un altro
arrivo successivo
        if (ta > τ )     } close the door se ta_new > τ
            ta = ∞;      }
        if (I == 1) c'è solo lui, sta prendendo servizio, genero nuovo Servizio e completamento
        tc = t + GetService(); //aggiorno clock (errore generare prima gli arrivi e poi prendere i service time)
    }
    else {                        /* process a completion */
        I − −; // uno se ne va
        if (I > 0) // c'è qualcun altro? depature
            tc = t + GetService(); // prossimo completamento
        else // (I-- = 0)                 service time del prossimo job
            tc = ∞; // non ci sono completam.
    }
}
```

2 event types: arrivi, partenze

Algorithm 1

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Program ssq3

- `number` represents $l(t)$ (system state)
  `struct t` represents time
  - `t.arrival, t.completion` event list (t_a, t_c from algorithm 1)
  - `t.current` simulation clock (t from algorithm 1)
  - `t.next` next event time (min(t_a, t_c) from algorithm 1)
  - `t.last` last arrival time
- `struct area` (time-averaged) statistics-gathering structure
  - $\int_0^t l(s)ds$ evaluated as `area.node`
  - $\int_0^t q(s)ds$ evaluated as `area.queue` (funzioni gradino)
  - $\int_0^t x(s)ds$ evaluated as `area.service`

ssq3.c

```
#include <stdio.h>
#include <math.h>
#include "rngs.h"    /* the multi-stream generator */
#define START    0.0
#define STOP    20000.0   /* terminal (close the door) time*/
#define INFINITY (100.0 * STOP) /* must be much larger than STOP */

double Min(double a, double c)
{  if (a < c)     return (a);
    else     return (c);}

double Exponential(double m) …
double Uniform(double a, double b) …
double GetArrival() …
double GetService() …
```

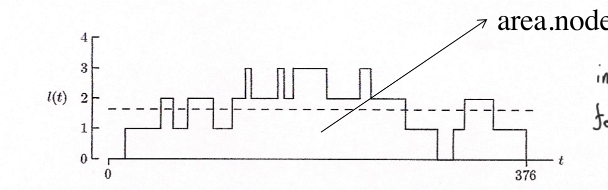
```
 int main(void)
{  struct {
        double arrival;     /* next arrival time */
        double completion; /* next completion time */
        double current; the clock! rrent time */
        double next;        /* next (most imminent) event time */
        double last;        /* last arrival time */
   } t;
   struct {
        double node;   /* time integrated number in the node */
       double queue;   /* time integrated number in the queue */
       double service; /* time integrated number in service */
   } area      = {0.0, 0.0, 0.0};
   long index  = 0;    /* used to count departed job */ (quanti partiti?)
   long number = 0;    /* number in the node */ system state
```

prima c'era solo index

↳ quanti ce ne sono in quell'istante di tempo?


```
PlantSeeds(123456789);
t.current    = START;          /* set the clock */
t.arrival    = GetArrival();   /* schedule the first arrival */
t.completion = INFINITY; /* the first event can't be a completion */
while ((t.arrival < STOP) || (number > 0)) {
   t.next= Min(t.arrival, t.completion); /* next event time   */
   if (number > 0)  {          /* update integrals  */
 l(t)  area.node+= (t.next - t.current) * number;  ← la "riuso"
 q(t)  area.queue+= (t.next - t.current) * (number - 1);
 x(t)  area.service += (t.next - t.current);     }
   t.current = t.next;   advance the clock!
```

← funzioni gradino, $\int_a^b$ semplice!



$$\text{area.node} = \int_0^t l(s)\,ds$$

integrali gradino, facili da calcolare

```
if (t.current == t.arrival)  {
   number++;
   t.arrival= GetArrival();
   if (t.arrival > STOP)  {
     t.last= t.current;
     t.arrival    = INFINITY;
   }
   if (number == 1)
        t.completion = t.current + GetService();
}
else {
  index++;
   number--;
   if (number > 0)
        t.completion = t.current + GetService();
   else
        t.completion = INFINITY;
   }
}
```

process an arrival

≈ a prima, ma in C

process a completion


```
printf(" … jobs", index);
printf(" average interarrival time ..", t.last / index);
printf(" average wait …", area.node / index);
printf(" average delay ...", area.queue / index);
printf(" average service time ...", area.service / index);
printf(" average # in the node ... ", area.node / t.current);
printf(" average # in the queue .. ", area.queue / t.current);
printf(" utilization ....", area.service / t.current);
```

World Views and Synchronization

- ssq2 produces :

```
while (index < LAST) {
index++;
arrival      = GetArrival();
if (arrival < departure)
      delay  = departure - arrival;
else  delay       = 0.0;
service = GetService();
wait  = delay + service;
departure    = arrival + wait;
sum.delay    += delay;
sum.wait     += wait;
sum.service += service;  }
```

$\sum_{i=1}^{n} d_i$

$\sum_{i=1}^{n} w_i$

$\sum_{i=1}^{n} s_i$

statistiche job average

World Views and Synchronization

$\sum_{i=1}^{n} w_i$

- ssq2 produces :

```
printf("… jobs", index);
printf("average interarrival time = ", sum.interarrival / index);
printf("average wait ........... = ", sum.wait / index);
printf("average delay .......... = ", sum.delay / index);
printf("average service time .... = ", sum.service / index);
printf("average # in the node ... = ", sum.wait / departure);
printf("average # in the queue .. = ", sum.delay / departure);
printf("utilization ............ = ", sum.service / departure);
```

$\bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i$

$\bar{l} = \frac{n}{c_n}\bar{w}$

$\bar{q} = \frac{n}{c_n}\bar{d}$

$\bar{x} = \frac{n}{c_n}\bar{s}$

$$\frac{\sum_{i=1}^{n} w_i}{c_n} = \frac{n\bar{w}}{c_n}$$

Qui si usa approccio next-event, ha rilevanza il tempo.

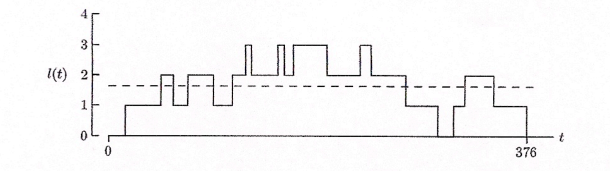
- ssq3 produces :

```
PlantSeeds(123456789);
t.current    = START;
t.arrival    = GetArrival();
t.completion = INFINITY;
while ((t.arrival < STOP) || (number > 0)) {
   t.next= Min(t.arrival, t.completion);
   if (number > 0)  {
        area.node+= (t.next - t.current) * number;
        area.queue+= (t.next - t.current) * (number - 1);
        area.service += (t.next - t.current);    }
   t.current       = t.next;
```

$\int_0^\tau l(t)dt$ ← area.node

$\int_0^\tau q(t)dt$ ← area.queue

$\int_0^\tau x(t)dt$ ← area.service



Gli integrali sono semplici (rettangoli), calcolo "quanto tempo sono stati" in un certo stato.

$$\tau \bar{l} = \int_0^\tau l(t)dt$$

```
printf(" … jobs", index);
printf(" average interarrival time ..", t.last / index);
printf(" average wait …", area.node / index);
printf(" average delay ...", area.queue / index);
printf(" average service time ...", area.service / index);
printf(" average # in the node ... ", area.node / t.current);
```

$$\bar{w} = \frac{\tau}{n}\bar{l}$$

$$\bar{l} = \frac{1}{\tau}\int_0^\tau l(t)dt$$

```
printf(" average # in the queue .. ", area.queue / t.current);
printf(" utilization ....", area.service / t.current);
```

World Views and Synchronization

- programs ssq2 and ssq3 simulate exactly the same system
- The two have different *world views*
  - ssq2 naturally produces job-averaged statistics (based upon *process-interaction*)
  - ssq3 naturally produces time-averaged statistics (based upon *event-scheduling*)

Se prendo due sistemi uguali, e uso due approcci diversi, devo ottenere stessi indici, altrimenti c'è un problema.

ssq2 produce arrivo e successivo servizio (arrivo-servizio a1-s1, a2-s2,..)

World Views and Synchronization

The programs should produce exactly the same statistics

- in ssq2 random variates are always generated in the alternating order:

$$a_1, s_1, a_2, s_2, \ldots$$

```
while (index < LAST) {
index++;
arrival      = GetArrival();
if (arrival < departure)
      delay  = departure - arrival;
else  delay      = 0.0;
service = GetService();
wait  = delay + service;
departure     = arrival + wait;
sum.delay    += delay;
sum.wait     += wait;
sum.service += service;  }
```

ssq3 dipende dal tempo, non è detto che avrò sempre a1-s1, a2-s2,... non posso dire quale sarà l'ordine.

Next-Event simulation
ssq

World Views and Synchronization

- in ssq3 the order cannot be known a priori

```
while ((ta < τ ) or (l > 0)) {
        t = min(ta, tc);     /* scan the event list */
        if (t == ta) {       /* process an arrival */
                l++;
                ta = GetArrival();
                if (ta > τ )
                        ta = ∞;
                if (l == 1)
                tc = t + GetService();
        }
        else {               /* process a completion */
                l − −;
                if (l > 0)
                        tc = t + GetService();
   .....
```

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Solo se 23 disaccoppio i due processi posso avere le stesse statistiche.
I flussi devono essere divisi, cioè multistream (quantità e ordine arrivi/servizi non mi interessa! perchè i "numeri" sono quelli.)

Next-Event simulation
ssq

World Views and Synchronization

The programs should produce exactly the same statistics

→ to do so requires rngs

```
 double GetArrival()
{ static double arrival = START;
  SelectStream(0);
  arrival += Exponential(2.0);
  return (arrival);}

double GetService()
{ SelectStream(1);
  return (Uniform(0.0, 1.5)+Uniform(0.0, 1.5));}
```

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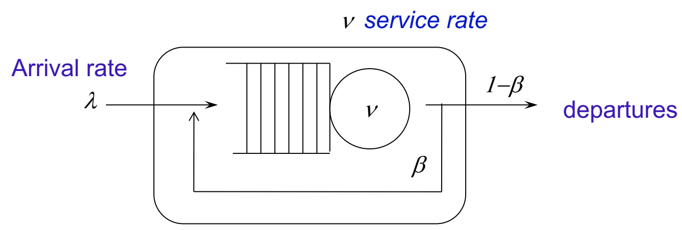
Per aumentare la variabilità uso due uniformi con due stream diversi, per i tempi di servizio. Confronto poi sugli stessi istanti di arrivo. E' fattibile per il multistream, separando e poi potendo mettere insieme.

Model Extensions

SSQ with immediate feedback



job index	1	2	3	4	5	.	6	.	7	8	.	9	…
Arrival/feedback	1	3	4	7	10	**13**	14	**15**	19	24	**26**	30	…
service	9	3	2	4	7	5	6	3	4	6	3	7	…
departure	10	**13**	**15**	19	**26**	31	37	**40**	44	50	53	**60**	…

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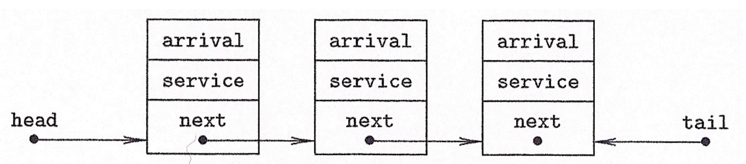
Questo era il caso feedback, alcuni job, come il n.2, "tornano indietro", con NextEvent facile:

Model Extensions

SSQ with immediate feedback

```
else {                    /* process a completion */
   if (GetFeedback() == 0) {   /* this statement is new */
        index++;
        number--;}
   if (number > 0)
        t.completion = t.current + GetService();
   else    t.completion = INFINITY;
```

- alternate queue disciplines
  - it is necessary to add a dynamic-queue data structure



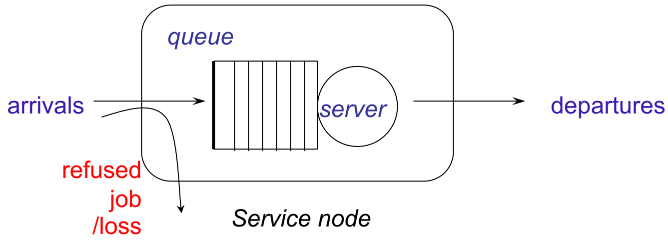
- +2 supporting queue functions: Enqueue, Dequeue

Implemento la 26 riga in rosso, ovvero quando un job completa, mi chiedo se fa feedback, se non lo fa, esso esce, aumento job index (cioè quelli completati), e decremento number (quelli nel sistema). Se fa feedback, ho sempre stessi job, va nella coda. Per il resto è tutto uguale: prenderò il primo nella coda, lo mando in esecuzione etc, quello che ha fatto feedback aspetterà il suo turno. Questo caso FIFO, se voglio altri schedule uso altre struct.

Qui, voglio contare job persi (arrivati ma rifiutati), che ci dice la qualità. Aggiungerli ad una next-event è semplice!

Discrete-event simulation
conceptual model

Model Extensions



Assumption:
for the finite *capacity case*,
only the accepted jobs are considered

BUT
the *loss probability* could be of interest!

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Le righe in 27 rosso vedono se number < capacità (number job max), se è inferiore incremento il numero di job, e se questo job arrivato è l'unico allora lo completo. altrimenti aumento il numero di reject.

Next-Event simulation
ssq with finite capacity

Model Extensions

SSQ with finite capacity

```
if (t.current == t.arrival) {          /* process an arrival */

    if (number < CAPACITY) {
        number++;
        if (number == 1)
            t.completion = t.current + GetService();
    }
    else
        reject++;
    t.arrival = GetArrival();
    if (t.arrival > STOP) {
        t.last = t.current;
        t.arrival = INFINITY;
    }
}
```

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L'evento di campionamento: posso decidere ogni quanto, può essere deterministico o no.

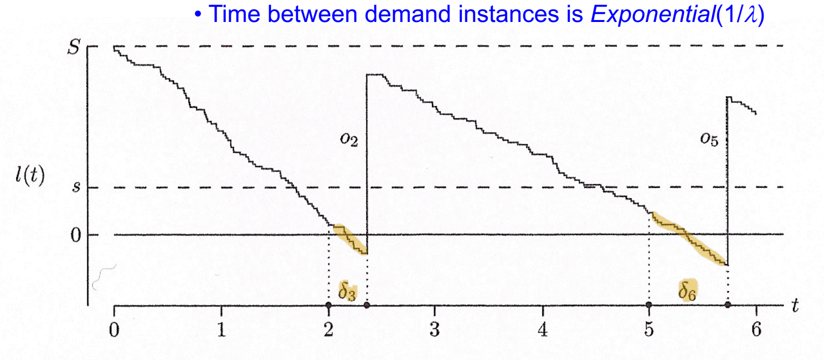
Random Sampling

- The structure of ssq3 facilitates adding sampling
- Add a sampling event to the event list
  - Sample deterministically, every δ time units
  - Sample Randomly, every *Exponential*(δ) time units

Nell'approccio delivery lag ho un contesto piu complicato, perchè l'ordine non arriva subito

A Simple Inventory System with Delivery Lag

Two changes relative to sis2
- *Uniform*(0,1) lag between inventory review and order delivery
- More realistic demand model
  - Demand instances for a single item occur *at random*
  - Average rate is λ demand instances per time interval
  - Time between demand instances is *Exponential*($1/\lambda$)



Modelli di "domanda": prima si usava equilikely per generare un valore da "spalmare" uniformemente (tempo inter-demand). Le 25 richieste di articoli arrivano ogni 0.04 (unif).

Comparison of Demand Models

sis2: used an *aggregate* demand for each time interval, generated as an *Equilikely*(10,50) random variate

- Aggregate demand per time interval is random
- Within an interval, time between demand instances is constant
- Example: if aggregate demand is 25, inter-demand time is 1/25=0.04

- Now using *Exponential*($1/\lambda$) inter-demand times
  - Demand is modeled as an arrival process
  - Average demand per time interval is λ

Voglio usare un 31 modello diverso, più reale, usando l'esponenziale. le domande sono modellate come processi di arrivi (per singolo articolo). Stiamo sempre parlando di INVENTORY SYSTEM.

Specification Level: States and Notation

- The simulation clock is t (real-valued)
- The terminal time is τ (integer-valued)
- Current inventory level is $l(t)$ (integer-valued)
- Amount of inventory on order, if any, is $o(t)$ (integer-valued)
  - Necessary due to delivery lag
- $l(t)$ and $o(t)$ provide complete state description
- (NON sempre necessario) Initial state is assumed to be $l(0)$=S and $o(0)$=0 uguali
- Terminal state is assumed to be $l(\tau)$=S and $o(\tau)$=0
- Cost to bring $l(t)$ to S at simulation end (with no lag) must be included in accumulated statistics

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Poichè c'è anche delivery lag, o(t) è fondamentale per descrivere lo stato completamente. (con loro due posso fare tutto).
L'ultimo ordine che riporta il sistema ad essere completamente pieno, avrà un suo costo incluso nei costi di sistema finale, cioè inclusi nelle statistiche.

Con next-event, definiti gli eventi e gli algoritmi che li gestiscono, ho 3 eventi:

Specification Level: Events

Three types of events can change the system state

- A *demand* for an item at time t
  - $l(t)$ decreases by 1
- An inventory *review* at integer-valued time t
  - If $l(t) \geq s \rightarrow o(t)=0$
  - If $l(t) < s \rightarrow o(t)=S-l(t)$

(aumento scorte per stare sopra il minimo)

- An *arrival* of an inventory replenishment order at time t
  - $l(t)$ increases by $o(t)$
  - $o(t)$ becomes 0

Algorithm 2: initialization

Time variables used for event list:

- t_d: next scheduled inventory *demand*
- t_r: next scheduled inventory *review*
- t_a: next scheduled inventory *arrival*

∞ denotes impossible events

```
l = S;                /* initialize inventory level */
o = 0;                /* initialize amount on order */
t = 0.0;              /* initialize simulation clock */
t_d = GetDemand();    /* initialize event list */
t_r = t + 1.0;        /* initialize event list */
t_a = ∞;              /* initialize event list */
```

demand:
se arriva domanda, decremento le scorte e genero prossivo evento di domanda (consumo evento, genero evento dopo).

review: si genera lag, perchè I<s (Tr = revisione, Ta = arrivo merci) se sotto soglia, devo fare ordine, e definisco prossimo arrivo merci. devo generare anche next Tr. (scandito periodicamente, in week).

Next-Event simulation
InvSys

Algorithm 2: main loop

3 types of events:
demand, review, arrival

```
while (t < τ) {
    t = min(td , tr , ta);      /* scan the event list */
    if (t == td) {             /* process an inventory demand */
        l--;                   demand
        td = GetDemand();
    }
    else if (t == tr ) {       /* process an inventory review */
        if (l < s) {           review
            o = S - l;
            δ= GetLag();
            ta = t + δ ;
        }
        tr += 1.0;
    }
    else {                     /* process an inventory arrival */
        l += o;                arrival
        o = 0;
        ta = ∞;
    }
}
```

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arrival merce: incremento/aggiorno (consumo un Ta, solo quando viene fatto nuovo ordine posso definirlo, per ora è infinito=impossibile)

Next-Event simulation
InvSys

Program sis3

- implements algorithm 2

correspond to t_d , t_r , t_a

```
while (t.current < STOP) {
  t.next = Min(t.demand, t.review, t.arrive);
  if (inventory > 0)
   sum.holding  += (t.next - t.current) * inventory;
  else
    sum.shortage -= (t.next - t.current) * inventory;
  t.current = t.next;
  if (t.current == t.demand) {
    sum.demand++; /* process an inventory demand */
    inventory--;
    t.demand = GetDemand();    }
  else    ………
```

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Anche qui sopra ho funzioni gradino. Facili da calcolare, ovvero quantità*tempo.

Program sis3

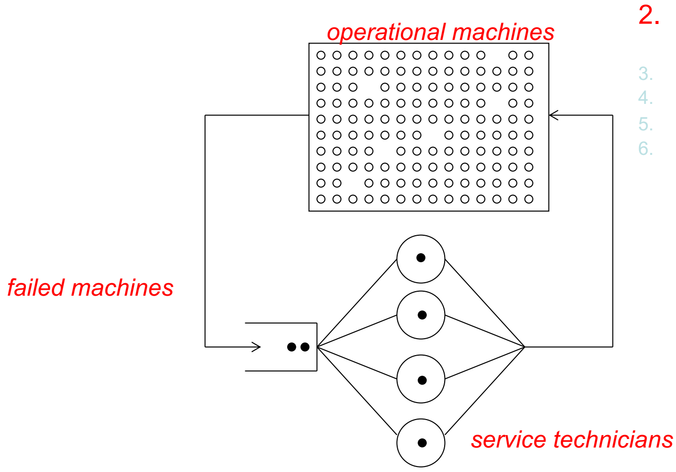
- State variables `inventory` and `order` correspond to $l(t)$ and $o(t)$
  - `t.next` next event instant (min(t_d , t_r , t_a) in algorithm 2)
  - `t.last` last arrival instant

`sum.hold` and `sum.short` accumulate the time-integrated holding and shortage integrals

Coda a servente singolo, con arrivi = guasti machine, e servizi = riparazioni.

Machine Shop Model

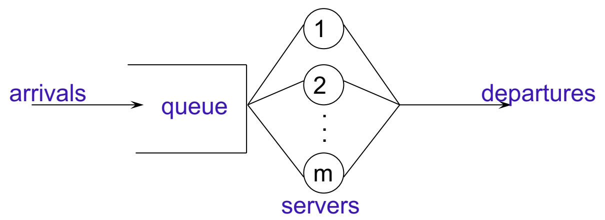
1. Goals
2. **Conceptual model**
3. Specification model
4. Computational model
5. Verify
6. Validate



Lo stato è numero di job (come coda singola), e lo stato di ogni server per ogni tempo (anche se dipende da "l(t) < m" o ">m"). 09/04/21

DE simulation
Next-Event Simulation

Conceptual model: MSQ



Servers in a multi-server service node are called *service channels*

- m is the number of servers
- The *server index* is $s = 1, 2, \ldots, m$

The *state* includes:
the number of jobs in the node at time t: $l(t)$
the state of each server at at time t: $x_s(t)$

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DE simulation
Next-Event Simulation

Specification Model: States and Notation

$l(t)$ denotes the number of jobs in the service node at time t

- If $l(t) \geq m$, all servers are busy and $q(t) = l(t) - m$
- If $l(t) < m$, some servers are idle
- If servers are distinct, need to know which servers are idle

For $s = 1, 2, \ldots, m$ define
$x_s(t)$: the number of jobs in service (0 or 1) at server s at time t

The complete state description is $l(t), x_1(t), x_2(t), \ldots, x_m(t)$

$$q(t) = l(t) - \sum_{s=1}^{m} x_s(t)$$

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Le variabili di stato devono essere il minimo indispensabile, per rappresentare in maniera completa e non ambigua lo stato in ogni 't', devo evitare variabili superflue.
nb: gli indici sono variabili di output, non possono essere variabili di stato.

eventi: Quali tipi di eventi possono cambiare lo stato?
usando sia l(t) che q(t), l'arrivo li cambia tutti e due, l'arrivo deve cambiare una cosa.

Specification Model: Events

What types of events can change state variables
$l(t), x_1(t), x_2(t), \ldots, x_m(t)$?

- *arrival* at time t
  - $l(t)$ increases by 1
  - If $l(t) < m$, an idle server s is selected, and $x_s(t)$ becomes 1
    else all servers are busy

server numero 's' è usato → setto a '1'.

- A *completion of service* by server s at time t
  - $l(t)$ decreases by 1
  - if $l(t) \geq m$, a job is selected from the queue to enter service
    else $x_s(t)$ becomes 0

poiché un server si è liberato, se $l(t) \geq m$ (cioè ho più job che server), un job può entrare in servizio nel job appena liberato

→ m+1 event types, in realtà ho due tipologie: { • ARRIVI • COMPLETAMENTO

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Specification Model: Additional Assumptions

- The initial state is an empty node
  - $l(0) = 0$
  - $x_1(0)=x_2(0)= \ldots = x_m(0)=0$
  - The first event must be an arrival

- The arrival process is turned off at time τ
  - The node continues operation after time τ until empty
  - The terminal state is an empty node
  - The last event is a completion of service

For simplicity, all servers are independent and *statistically identical*

- *Equity* selection is the server selection rule (lowest-utilized)

All of these assumptions can be relaxed

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con 4 server va bene array, con m molto più grandi converrebbe l'uso di liste.

Event list

1 on, 0 off

1 busy, 0 idle

	0	*t*	x	arrival
	1	*t*	x	completion by 1
	2	*t*	x	completion by 2
m=4	3	*t*	x	completion by 3
	4	*t*	x	completion by 4

- can be organized as an array of m + 1 event types
- field t: scheduled time of next occurrence for the event
- field x: current *activity status* of the event

- for large m → alternate event-list structures

Program msq

Implements this next-event multi-server service node simulation model

- `number` state variable $l(t)$
- state variables $x_1(t), x_2(t), \ldots, x_m(t)$ are part of the event list
- `area` time-integrated statistic $\int_0^t l(\theta)d\theta$
- `sum` array, records for each server
  - the sum of service times
  - the number served
- function `NextEvent` searches the event list to find the next event
- function `FindOne` searches the event list to find the longest-idle server (because equity selection is used)

program msq.c

```
typedef struct {
  double t;
  int    x;
} event_list[SERVERS + 1];
…
    int NextEvent(event_list event)
    { int e;
      int i = 0;
      while (event[i].x == 0)
        i++;
      e = i;
      while (i < SERVERS) {
        i++;
        if ((event[i].x == 1) && (event[i].t < event[e].t))
           e = i;  }
      return (e);}
```

is the first active event, assume it is the next

look for the next active event

if it is previous, update e

programma msq.c

```
 int FindOne(event_list event)
{ int s;
  int i = 1;
  while (event[i].x == 1)
    i++;
  s = i;
  while (i < SERVERS) {
     i++;
     if ((event[i].x == 0) && (event[i].t < event[s].t))
     s = i;  }
  return (s);}
```

first server idle

look for the next idle

if its completion is previous,
it is idle since more time

Exercises

- 5.1.1, 5.1.2, 5.1.3
- 5.2.1, 5.2.2,
- 5.2.8: modify program msq to allow for a finite capacity (max r jobs); a. draw a histogram of the time between lost jobs at the node; b. comment on the shape of this histogram.