

SAMPLE STATISTICS

#24

Cosa facciamo coi dati raccolti? voglio stat. utili.

PRIME STATISTICHE da ottenere per campione ben definito

- media campionaria $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ (tendenza centrale)

- Varianza: $s^2 = \frac{1}{n} \sum_{i=1}^n x_i$ (disconcentramento da media)

- etc...

- deviazione std $d(x) = \sqrt{\frac{1}{m} \sum_{i=1}^m (x_i - \bar{x})^2} = s$ (quanto m dispone)

se $x = \bar{x}$ otengo d_{\min}

DISUGUAGLIANZA CHEBYCHEV (lega media e varianza)

dato campione di n elementi, media \bar{x} e std dev. 's,
ho ret $S_k = \{ x_i \mid \bar{x} - ks < x_i < \bar{x} + ks \}$,

di proporzione $P_k = \frac{|S_k|}{n} : P_k \geq 1 - \frac{1}{k^2}$ con $k \geq 1$

Potrei usare trasformazioni dati, tipo $x'_i = ax_i + b$,
trovando media $a\bar{x} + b$, varianza $a^2 s^2$ e stdvar abs
(simile a convertire sec in minuti $x' = \frac{300 \text{ sec}}{60 \text{ sec}} = 5 \text{ min}$)

LINEAR

STANDARDIZZAZIONE (dati grande $\rightarrow \phi < x < 1$)

uso $a = \frac{1}{s}$ e $b = -\frac{\bar{x}}{s}$ $\rightarrow x'_i = \frac{x_i - \bar{x}}{s}$ $\rightarrow \bar{x}' = \phi$ $\rightarrow s' = 1$

esistono anche NON LINEAR, per vedere comportamento, no dati.

ESERCIZI

Consider a web server with the following system characteristics:

- Single processor with capacity 10^5 op./sec
 - Exponential mean service demand 4×10^4 op./job
 - System utilization 60%.
- By knowing the job size, the service provider adopts a simple Size Based priority scheduling without preemption: jobs with size $\leq \bar{s}$ (or equal) than the average will have the highest priority (class 1); jobs with size greater than the average have the lowest priority (class 2). Determine:
- a. the mean response time for both classes and the global mean response time.
The service provider wants to investigate if a dual core server would improve the service performance.
 - b. Conjecture the behaviour of the performance measures for both classes, by writing the mean waiting and response time definition for the dual core case.

→ Solo formule, non calcoli.

DATI

$$\begin{aligned} \bullet C = 10^5 \text{ op/s} & \quad \bullet \bar{Z} = 4 \cdot 10^4 \text{ op/job} & \bullet P = 0.6 = \lambda E[S] \\ \bullet E[S] = \frac{\bar{Z}}{C} = 0.4 \cdot \frac{E[S]}{E[S]} & \quad \mu = \frac{1}{E[S]} = 2.5 & \quad \lambda = \frac{P}{E[S]} = 1.5 & \quad \bullet \text{SizeB-NP} \end{aligned}$$

ho classe 1 se $\text{size job} \leq E[S]$, altrimenti classe 2.

$$\text{Il testo chiede } E[T_{S_i}] = E[T_{A_i}]^{SB-NP} + E[S_i]^{SB-NP}$$

$$E[S_i] = \int_{x_{k-1}}^{x_k} t \cdot f^n(t) \quad \text{dove } [x_{k-1}, x_k] \text{ sono le size delle classi, e } f^n(t) \text{ è la funzione di densità degli arrivi normalizzata.}$$

$$\text{In questo caso } f^n(t) = \frac{\mu e^{-\mu t}}{F(\frac{1}{\mu}) - F(0)} \quad \text{per classe 1.}$$

$$P_1 = \left[1 - e^{-\mu t} \right]^{\frac{t}{\mu}} = 0,6312$$

$$P_1 + P_2 = 1$$

$$P_2 = \left[1 - e^{-\mu t} \right]^{\frac{t}{\mu}} = 0,36788$$

Calcolo $E[S_1]$ ed $E[S_2]$:

$$E[S_1] = \int_0^{\infty} t \cdot \frac{2,5 \cdot e^{-2,5t}}{P_1} = 0,167453 \text{ s}$$

$$E[S_2] = \int_{\frac{1}{\mu} = 0,4}^{\infty} t \cdot \frac{2,5 \cdot e^{-2,5t}}{P_2} = 0,7933399 \text{ s}$$

$$\text{controllo: } P_1 E[S_1] + P_2 E[S_2] = 0,1056963 + 0,29430 \approx 0,4$$

Adesso calcolo $E[T_{a_1}]$ ed $E[T_{a_2}]$

$$E[T_{a_K}]^{\text{SB-NP}} = \frac{\lambda}{2} \frac{E[S^2]}{\left(1 - \sum_{i=1}^K p_i\right) \left(1 - \sum_{i=1}^{K-1} p_i\right)}$$

$$\text{avvio } E[T_{a_1}] = \frac{\lambda}{2} \frac{E[S^2]}{1 - P_1}, \quad E[T_{a_2}] = \frac{\lambda}{2} \frac{E[S^2]}{(1 - P)(1 - P_1)}$$

Poiché sto in contesto esponenziale + unico servente

$$\frac{\lambda}{2} E[S^2] = P E[S]$$

$$P_K = \lambda \int_{x_{K-1}}^{x_K} t \cdot f(t) dt$$

↳ densità exp NON normalizzata

$$P_{K=1} = P_1 = \lambda \int_0^{\gamma_M} t \cdot \mu e^{-\mu t} dt = 0,158545 = \lambda \cdot \underbrace{P_1}_{\text{PROBABILITÀ}} \cdot E[S_1]$$

$$P_2 = \lambda \int_{\gamma_M}^{\infty} t \cdot \mu e^{-\mu t} dt = 0,441455 = \lambda \cdot \underbrace{P_2}_{\text{PROBABILITÀ}} \cdot E[S_2]$$

VERIFICA: $\sum_{i=1}^5 p_i = 0,6 = P = \lambda \int_0^{\infty} t \cdot f(t) dt$

Trovavo anche λ_1 e λ_2

$$\lambda_1 = \lambda \cdot P_1 = 0,9468 \cdot \frac{\text{OP}}{\text{job}}$$

$$\lambda_2 = \lambda \cdot P_2 = 0,5532 \cdot \frac{\text{OP}}{\text{job}}$$

$$\lambda_1 + \lambda_2 = \lambda \quad \checkmark$$

$$E[T_{Q_1}] = \frac{P E(S)}{(1 - P)} = 0,28522$$

$$E[T_{Q_2}] = \frac{P E(S)}{(1 - P)} = 0,71305$$

OK

allora

$$E[T_{Q_1}]$$

$$E[S_1]$$

$$E[T_{S_1}] = 0,28522 + 0,167453 = 0,452673 \quad \checkmark$$

$$E[T_{S_2}] = 0,71305 + 0,79999 = 1,51304 \quad \checkmark$$

$$E[T_Q] = P_1 E[T_{Q_1}] + P_2 E[T_{Q_2}] = 0,443063 \quad \checkmark$$

$$E[T_S] = E[T_Q] + E[S] = 0,843003 \quad \checkmark$$

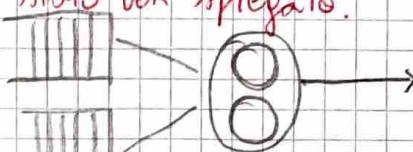
b) Passo ad un dual core, mantenendo le due cloni.
 Poiché sto in un contesto di **scheduling estremo** (le cloni interamente sono FIFO) posso usare ERLAWA.

Alcune osservazioni: NB: Forse invece che P_i del punto a) dovrebbero essere $P_i = (\frac{\lambda_i}{m})E[S_i]$, non è stato ben spiegato.

- NON Ho PRELAZIONE, quindi:

$$E[T_a] = P_1 \cdot \frac{P_a E[S]}{1 - P_1} + P_2 \cdot \frac{P_a E[S]}{(1 - P_1)(1 - P)}$$

non interrotti



attesa di un job
di classe 1 in coda

ovvero ho $E[T_{a,i}]$ che $E[T_{a,2}]$ hanno P_a , poiché i job non vengono interrotti. Forni stato preemptive,
 $E[T_{a,1}]$ avrebbe avuto $P_{a,1}$ (in funzione di P_i)

- Poiché ho 2 core, $E[S_i] = E[S_i]$, ovvero ogni core è dimezzato, con tempi servizi ^m addoppiati; $E[S] = E[S_2] = 2E[S] = 0.8$
- In $E[T_a]$, $E[S] = 0.4$, non uso $E[S_i]$, perché mi interessa tempo di servizio generale (entro job in S_1 e magari esce job da S_2), mentre con $E[S_i]$ guarderei entrate-mate sul singolo core "i".

$$P_a = \frac{(mp)^m}{m! (1-p)} \quad P(0) = \frac{(2 \cdot 0.6)^2}{2! (1-0.6)} \cdot \left[\sum_{i=0}^{m-1} \frac{(mp)^i}{i!} + \frac{(mp)^m}{m! (1-p)} \right]^{-1}$$

$$= \frac{(2 \cdot 0.6)^2}{2 (1-0.6)} \left[1 + 2p + \frac{(2p)^2}{2(1-p)} \right]^{-1} = 0,45 ; \quad P_a \cdot E[S] = 0,18$$

1-0.6312 0.45 0.4

$$\rightarrow E[T_a] = \frac{P_a \cdot E[S]}{1 - P_1} \left[\frac{P_1}{T_1} + \frac{P_2}{(1 - P_1)} \right] = 0,33176$$

0.158545 0.6312 0.6

$$E[T_S] = E[T_a] + E[S] = 0,73176$$

- SB →
- Il responsabile di uno sportello comunale per il rilascio di certificati anagrafici vuole investigare le prestazioni del servizio. Analizzando lo storico dell'attività, si desume che una distribuzione uniforme $Uniform(2, 15)$ può ben caratterizzare il tempo di servizio (espresso in min). Gli utenti, identificati con la propria richiesta, arrivano in modo random con frequenza 0.112 req/min. Si assuma che sia possibile conoscere il tempo di servizio della pratica all'istante di arrivo. Si calcolino i seguenti indici:

- tempi di attesa e risposta per una pratica qualsiasi;
- i tempi di attesa e risposta per classi e globali assumendo di usare un meccanismo prioritario opportunamente scelto (senza prelazione);
- lo slowdown condizionato, per richieste di 5 min e di 10 min, nel caso 1.a;
- lo slowdown condizionato, per richieste di 5 min e di 10 min, nel caso 1.b;

Si commenti al riguardo del vantaggio della soluzione al punto 1.b. Indicare le assunzioni utilizzate per la soluzione.

$$Uniform(2, 15) \quad \lambda = 0.112 \text{ req/MIN}$$

$\begin{matrix} 2 & \\ \downarrow & \\ a & b \end{matrix}$

Abbiamo a che fare con una UNIFORME!

$$f(x) = \frac{1}{b-a} ; F(x) = \frac{x-a}{b-a} ; \text{media} = \frac{a+b}{2} ; \sigma^2 = \frac{(b-a)^2}{12}$$

Svolgimento

$$E[T_a] = \frac{\frac{\lambda}{2} E[S^2]}{1-\rho} \quad \text{NON e' EXP, non uno PES!} \quad \rightsquigarrow \text{formula M/G/1}$$

$$\sigma^2 = E[S^2] - (E[S])^2 = \left(\frac{b-a}{12}\right)^2 = 14,0833 ; E[S] = \frac{15+2}{2} = 8,5 \text{ min}$$

$$\text{allora } E[S^2] = \sigma^2 + (E[S])^2 = 14,0833 + (8,5)^2 = 86,3333$$

$$\text{Da Little } P = \lambda E[S] = 0,952$$

$$E[T_a] = \frac{\frac{0,112}{2} \cdot 86,3333}{1 - 0,952} = 100,722 \text{ min}$$

$$E[T_s] = E[T_a] + E[S] = 109,2222 \text{ min}$$

1.b) Il meccanismo prioritario NP da considerare è SIZE BASED. Posso dividere le classi partendo da $E[S] = 8.5$ ovvero classe 1 (2, 8.5) ; classe 2 (8.5, 15) | è il tempo che potevo usare altrimenti schedule o più classi

Sappiamo che $E[S_i] = \int_{x_{k-1}}^{x_k} t \cdot f_i^n(t) \frac{P_i}{P_i}$

$$P_1 = F(8.5) - F(2) = \frac{8.5 - 2}{15 - 2} - \frac{2 - 2}{15 - 2} = 0.5 = P_2$$

NB: F ed f sono relative all' UNIFORME !

$$f(t) = \frac{1}{13} \text{ (è costante)}$$

$$E[S_1] = \int_2^{8.5} t \cdot \frac{1}{13} \cdot \frac{1}{0.5} = \frac{1}{6.5} t^2 \Big|_2^{8.5} = 5.25 \text{ min}$$

$$E[S_2] = \int_{8.5}^{15} \frac{1}{6.5} \cdot t = 11.75 \text{ min}$$

$$\lambda_1 = \lambda \cdot P_1 = 0.056 = \lambda_2 \quad (P_1 = P_2)$$

$$\rho_1 = \lambda_1 E[S_1] = 0.294$$

$$\rho_2 = \lambda_2 E[S_2] = 0.658 \quad \Rightarrow \rho_1 + \rho_2 = 0.952 = \rho$$

$$E[T_{Q_1}] = \frac{\frac{0.112}{2} \cdot 36.333}{1 - 0.294} = 6.84 \text{ min}$$

$$E[T_{Q_2}] = \frac{\frac{0.112}{2} \cdot 86.333}{(1 - 0.294)(1 - 0.952)} = 142.6604 \text{ min}$$

$$E[T_{\alpha}] = 0.5(6.84) + 0.5(192.6604) = 74.75 \text{ min} < \text{NO SB}$$

$$E[T_{S_1}] = E[T_{\alpha_1}] + E[S_1] = 12,09797$$

$$E[T_{S_2}] = E[T_{\alpha_2}] + E[S_2] = 154.41604$$

$$E[T_S]^{glob} = 0.5 E[T_{S_1}] + 0.5 \cdot E[T_{S_2}] = 83.257005$$

1.c)

$$sd = \frac{E[T_\alpha] + x}{x}$$

indip da x

$\rightarrow x=5 \rightarrow sd(5) = 21.1444$

$\rightarrow x=10 \rightarrow sd(10) = 10,0722$

2.d) qui lavoro sulle due cloni ($E[T_{\alpha_1}]$ ed $E[T_{\alpha_2}]$)

$$sd(x) = \frac{E[T_{\alpha_1}] + x}{x} \rightarrow x=5 \rightarrow sd(5) = 2.369594$$

$$sd(x) = \frac{E[T_{\alpha_2}] + x}{x} \rightarrow x=10 \rightarrow sd(10) = 15.266$$

NB: se $x=5$ va con $E[T_{\alpha_1}]$, poiché comprende nize da 2 a 8.5, $x=10$ va nella seconda classe (8.5, 15)

Non posso fare $x=10$ con $E[T_{\alpha_1}]$

Consider a web server with processing capacity $C = 10^5$ op/sec. The server receives requests with a mean rate 2 req/sec. The requests have different demand Z . Consider the following intervals:

- ◆ $Z < 20.000$ op
- ◆ $20.000 \text{ op} \leq Z < 40.000$ op
- ◆ $Z \geq 40.000$ op

$$Z = 379 \text{ demand}$$

By assuming that:

- i. the mean size is 40.000 op, characterized by an exponential distribution;
- j. the arrival rate is characterized by a Poisson process;

Define a management mechanism of the server to satisfy the following QoS requirements:

1. Mean response time ≤ 1.5 s for all requests
2. Mean waiting time ≤ 0.5 s, for $Z < 40.000$ op.ni.

Evaluate

- a. The mean throughput for the server with the chosen management mechanism;
- b. The mean conditional slowdown for jobs with size $x=0.1$ s, 0.3 s
- c. Compare the mean slowdown obtained in b. with the corresponding mean slowdown for FIFO and PS scheduling.

Please comment all the obtained results.

Svolgimento

$$E[S] = \frac{\bar{Z}}{C} = \frac{4 \cdot 10^4}{10^5} = 0.4 \text{ s} \rightarrow \rho = \lambda \cdot E[S] = 0.8$$

diviso in 3 domi (basandosi sugli intervalli): Size Based

$$C_1 (0, 0.2) \\ \frac{1}{2} \\ \frac{2 \cdot 10^4}{10^5}$$

$$C_2 (0.2, 0.4)$$

$$C_3 (0.4, \infty)$$

$$\text{calcolo } \mu = \frac{1}{E[S]} = 2.5$$

$$P_1 = [1 - e^{-\mu t}]^{0.2} = 0.393469$$

$$P_2 = [1 - e^{-\mu t}]^{0.4}_{0.2} = 0.23865$$

$$\sum_{i=1}^3 P_i = 1 \quad \checkmark$$

$$P_3 = [1 - e^{-\mu t}]^{\infty}_{0.4} = 0.367873$$

$$E[S_1] = \int_0^{0.2} t \cdot \frac{\mu e^{-\mu t}}{P_1} = 0.0917013 \text{ s}$$

$$E[S_2] = \int_{0.2}^{0.4} t \cdot \frac{\mu e^{-\mu t}}{P_2} = 0.291701 \text{ s}$$

$$E[S_3] = \int_{0.4}^{\infty} t \cdot \frac{\mu e^{-\mu t}}{P_3} = 0.8 \text{ s}$$

• calcolo P

$$P_1 = \lambda \int_0^{0.2} t \mu e^{-\mu t} dt = 0.0721632$$

$$P_2 = 2 \int_{0.2}^{0.4} t \cdot \mu e^{-\mu t} = 0.13923 \quad \sum_{i=1}^3 P_i = P = 0.8 \checkmark$$

$$P_3 = 2 \int_{0.4}^{\infty} t \mu e^{-\mu t} = 0.588607$$

• calcolo $E[T_{\alpha_i}]$

$$E[T_{\alpha_1}] = \frac{P E[S]}{1 - P_1} = 0.34288 \text{ s}$$

vincolo

rispettato

$$E[T_{\alpha_2}] = \frac{\lambda E[S]}{(1 - P_1 - P_2)(1 - P)} = 0.437338 \text{ s}$$

$$E[T_{\alpha_3}] = \frac{P E[S]}{(1 - P)(1 - P_1 - P_2)} = 2.0288945 \text{ s}$$

non era
richiesto

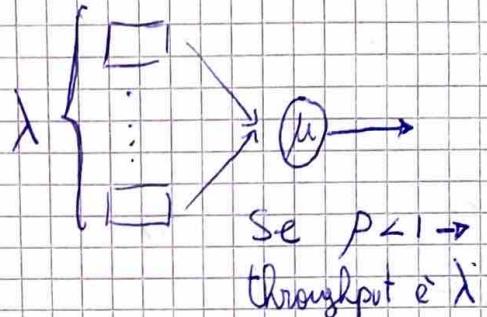
$$E[T_a] = \sum p_i E[T_{a,i}] = 0.988032295$$

$$E[S] = \sum p_i E[S_i] = 0.4$$

$$E[T_s] = E[T_a] + E[S] = 1.38803 < 1.5 \checkmark$$

a) throughput medio

$\mu > \lambda$ e $p < 1$, stabile.



b) sd (0.1) in $E[T_{a,1}] = \frac{0.344888 + 1}{0.1} = 4,4488$

sd (0.3) in $E[T_{a,2}] = \frac{0.437338}{0.3} + 1 = 2,4577$

c) sd FIFO = $\frac{\frac{1}{2} E[S^2]}{x(1-p)} + 1 = \frac{p E[S]}{x(1-p)} + 1 = \frac{0.8 \cdot 0.4}{0.1(0.2)} + 1 = 17$

da $x = 0.3$ ho $5.\overline{3} + 1 = 6.\overline{3}$

d) PS (fair, riguale 1 job) = $\frac{1}{1-p} = \frac{1}{1-0.8} = 5$

- 1.2. Consider a single-core server hosting a web service. Requests arrive to the server according to a Poisson, with an average inter-arrival time of 200 ms.
- Knowing that the maximum buffer size is $N = 4$ (including the jobs in service) and that each request requires on average 200 ms of processing time, compute the throughput of the system.
 - Consider a CPU upgrade to a faster single-core processor which can process a request in 150 ms. Compute the throughput of the upgraded system.
 - Consider a CPU upgrade to a slower quad-core processor, which can process a request in 300 ms using one of its processor cores. Compute the throughput of the upgraded system.

Svolgimento

$$\text{interarrivo} = 200 \text{ ms} \rightarrow \lambda = \frac{1}{200} = 0,005 \left[\frac{\text{J}}{\text{ms}} \right]$$

$$N = 4 \quad E[S] = 200 \text{ ms} \rightarrow \mu = \frac{1}{E[S]} = 0,005 \left[\frac{\text{J}}{\text{ms}} \right]$$

1) $\lambda' = \lambda(1 - P_{\text{loss}})$ è throughput (capacità finita, non è solo λ)

avendo SINGLE CORE) con $C = 4$ ($\lambda = \mu$)

$$\pi_4 = \left(\frac{\lambda}{\mu}\right)^4 \pi_0 \quad \text{con } \pi_0 = \frac{1}{\sum_{i=0}^4 \left(\frac{\lambda}{\mu}\right)^i} = \frac{1}{5}$$

$$\text{allora } \pi_4 = 1^4 \cdot \frac{1}{5} = \frac{1}{5} = P_{\text{loss}}$$

$$\text{e } \lambda' = 0,005 \left(1 - \frac{1}{5}\right) = 0,004 \text{ J/ms} = 4 \text{ J/s}$$

2) request in 150 ms $\rightarrow \mu = 0.00666666$

tranne che per μ , lo calcolo come prima:

$$\Pi_0 = \frac{1}{\sum_{i=0}^4 \left(\frac{0.005}{0.006}\right)^i} = 0.327785$$

$$\Pi_4 = P_{Loss} = \Pi_0 \cdot \left(\frac{0.005}{0.006}\right)^4 = 0.103713$$

$$\lambda = 0.005(1 - 0.103713) = 0.00448143 \text{ J/ms} = 4.48 \text{ J/s}$$

3) qui ho 4 core (= 4 serventi), ciascuno ha req. 300 ms

$$-\mu = 0.003333 \left[\frac{\text{J}}{\text{ms}} \right] \quad P = \frac{\lambda}{m\mu} = 0.375$$

non ho 'coda' (1 sob A core). Ma solo erlong - B !

$$P_{Loss} = \Pi_4 = \frac{\left(\frac{\lambda}{\mu}\right)^4 / 4!}{\sum_{j=0}^4 \left(\frac{\lambda}{\mu}\right)^j / j!} = 0.048$$

$$\text{e quindi } \lambda' = \lambda(1 - P_{Loss}) = 4.76 \text{ J/s}$$

$$P = \frac{\lambda}{\mu} = 0.4$$

$$\frac{1}{\mu} = E[S] = 0,00001$$

Esercizio 1

- single processor capacity $10^5 \text{ op/s} = \mu$
- domanda media $\exp 9 \cdot 10^5 \text{ op/job} = \lambda$
- syst. utilization $60\% = P$? ?
- Size based without preemption 2 classi
 - $c_1 \rightarrow \text{job} \leq E[S]$
 - $c_2 \rightarrow \text{job} > E[S]$
- a) mean response time per c_1 e c_2 , global response time.

Svolgimento

$$E(T_{S_1})^{SB-NP} = E(T_{a_1})^{SB-NP} + E[S_1]$$

$$\cdot E(S_1) = \int_{X_{K-1}}^{X_K} t \cdot f^n(t) dt = \int_{X_{K-1}}^{X_K} t \cdot \frac{\mu e^{-\mu t}}{F(X_K) - F(X_{K-1})}$$

$$= \int_0^{\frac{1}{\mu}} t \cdot \frac{\mu e^{-\mu t}}{F(\frac{1}{\mu}) - F(0)} = \int_0^{\frac{1}{\mu}} t \cdot \frac{\mu e^{-\mu t}}{P_1} dt$$

[chi è "F"? è f.distr. arrivo $\rightarrow \exp$ "1 - $e^{-\mu t}$ "]

$$P_1 = [1 - e^{-\mu t}] \Big|_0^{\frac{1}{\mu}} = 1 - e^{-\mu \frac{1}{\mu}} - [1 - 1] = 0,63212$$

$$P_2 = [1 - e^{-\mu t}] \Big|_0^\infty = 0,36788 \rightarrow \text{check: } P_1 + P_2 = 1$$

$$\rightarrow E[S_1] = \int_0^{10^5} t \cdot \frac{(10^5) \cdot e^{-(10^5 \cdot t)}}{0,63212} = 4,18024 \cdot 10^{-6}$$

$$E[S_2] = \int_{\frac{1}{10^5}}^{\infty} t \cdot \frac{10^5 e^{-10^5 t}}{0,36788} dt = \frac{10^5}{0,36788} \int_{\frac{1}{10^5}}^{\infty} t \cdot e^{-10^5 t} dt$$

for parts

~~$$= -\frac{1}{10^5} e^{-10^5 t} + C \quad \text{from } 0,36788$$~~

~~121~~

$$\frac{1}{10^5} \left[-t \cdot e^{-10^5 t} - \int 1 \cdot e^{-10^5 t} dt \right] = \frac{1}{10^5} \left[-t \cdot e^{-10^5 t} - \frac{1}{10^5} e^{-10^5 t} \right]$$

$$= 271828 \cdot \int_{\frac{1}{10^5}}^{\infty} e^{-10^5 t} \cdot t dt \quad \text{per parti} \quad \begin{cases} f = t \\ g' = e^{-10^5 t} \end{cases}$$

\downarrow

$$f = t dt \quad g = -\frac{1}{10^5} e^{-10^5 t}$$

$$= 271828 \left[t \cdot -\frac{1}{10^5} e^{-10^5 t} - \int_{\frac{1}{10^5}}^{\infty} 1 \cdot -\frac{1}{10^5} e^{-10^5 t} dt \right] = 0,00002$$

Ergebnis

check

$$E[S] = p_1 E(S_1) + p_2 E(S_2) \stackrel{?}{=} 10^{-5} \quad \text{OK}$$

$$= [0,63212 \cdot 4 \cdot 18024 \cdot 10^{-6}] + [0,36788 \cdot 0,00002]$$

$$= 0,00001 \quad \text{OK}$$

1.c) slowdown condiz. nel caso 1 per richieste di 5 min e 10 min indip da size

$$Sd = \frac{(\overbrace{E[T_a]}^{\text{INDIP da size}} + 5)}{5} \times (\text{quanto chiede})$$

$$Sd = \frac{E[T_a] + x}{x} \quad | E[T_a] = 100.722, x = 10 \\ = 10.0722$$

1.d) slowdown con nel caso 2b.

$$Sd = \frac{\overbrace{E[T_{a_1}]}^{6.84} + 5}{5} = 2.369594 \quad] Q_1$$

$$\cancel{Sd = \frac{6.84 + 10}{10} = 1.684}$$

~~$$Sd = \frac{142.6604 + 10}{10} = 15.26604 \quad] Q_2$$~~

$$Sd = \frac{142.6604 + 10}{10} = 15.26604 \quad] Q_2$$

es. 3)

$$\mu = 10^5 \text{ op/s} = C \quad \lambda = 2 \text{ req/s}, \text{ tali richieste}$$

sono vere

$$\begin{cases} z < 20.000 \text{ op} \\ 20.000 \leq z \leq 40.000 \text{ op} \\ z \geq 40.000 \text{ op} \end{cases}$$

ottimizzare:

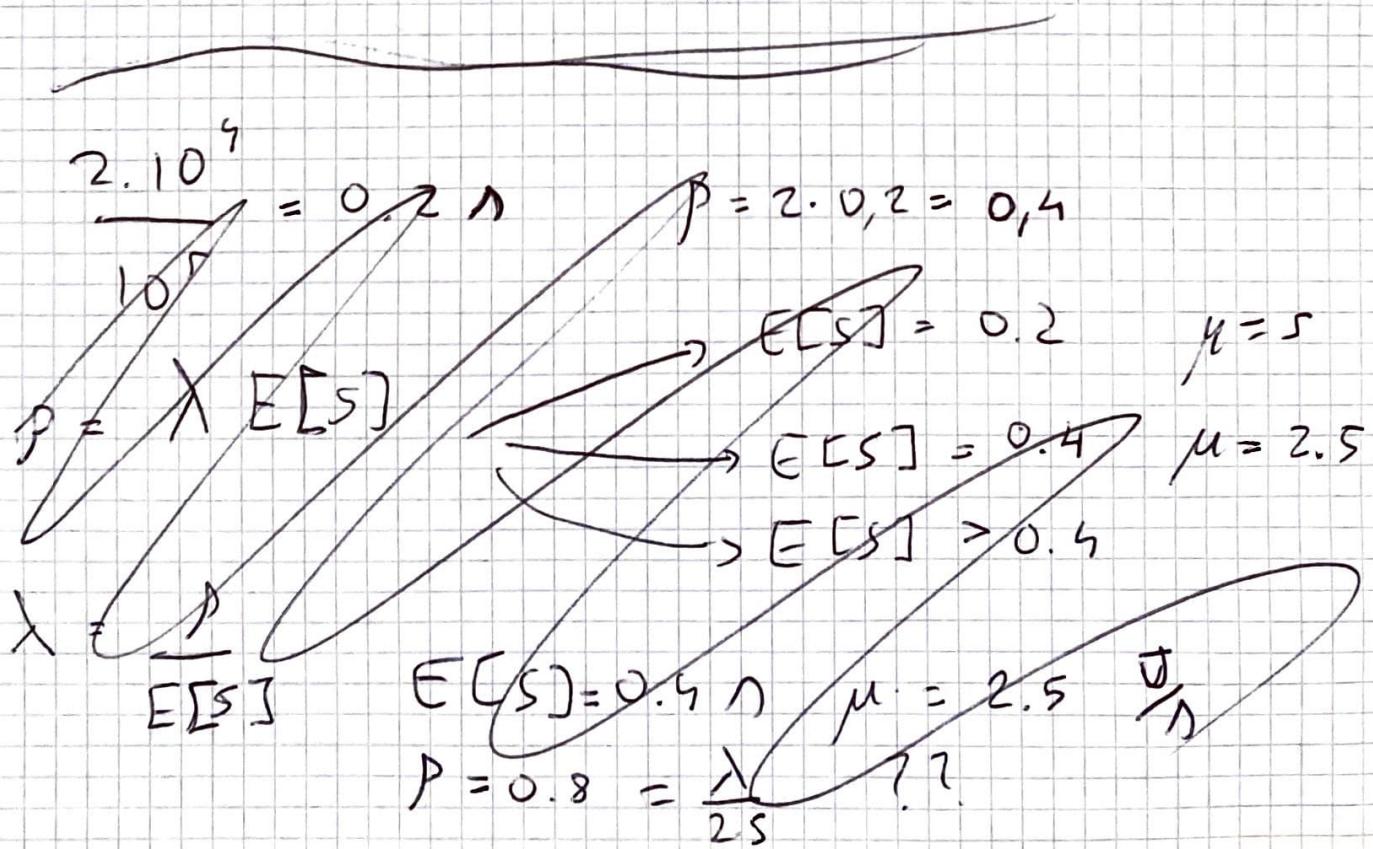
- arrival rate è Poisson
- Mean size è 40000 op (exp) = z (avg demand)

dove richiedere:

$$E(T_s) \leq 1.5 \text{ s} \quad \forall \text{req}$$
$$E(T_q) \leq 0.5 \text{ s}, \text{ se } z \leq 40.000$$

• Voluto throughput

- slowdown $\times = 0.15, 0.35$ (mean)
- confronto con mean slowdown FIFO e PS.



Ora ricento di $E[T_{Q_1}]$ ed $E[T_{Q_2}]$

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2} E[S^2]}{(1 - \sum_{i=1}^K p_i) \left(1 - \sum_{i=1}^{K-1} p_i\right)}$$

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2} E[S^2]}{1 - p_1}$$

$$E[T_{Q_2}] = \frac{\frac{\lambda}{2} E[S^2]}{(1-p)(1-\lambda)}$$

$\frac{\lambda}{2} E(S^2) = \rho E(S)$ ne EXP, ma con priority classes

ho in realtà $p_{Q_1} E[S]$ e $p_{Q_2} E[S]$ perché c'è priorità, devo calcolare

$$\bullet P_1 \quad \bullet P_{Q_1} \quad \bullet P_Q$$

$$D_{K=1} = \lambda \int_{x_{K-1}}^{x_K} t \cdot f(t) dt \quad \text{e' RHO}$$

~~NON NORMALIZZ.~~

$$D_K = \lambda \int_0^{\infty} t \cdot \mu e^{-\mu t} dt = (4 \cdot 10^4) \int_0^{\infty} t \cdot 10^5 \cdot e^{-10^5 t} dt$$

$$0.1657 = 1,0570 \cdot 10^{-6} = \lambda \cdot 0.63212 \cdot E[S] \checkmark$$

$$P_2 = 2,9430 \cdot 10^{-6}$$

$$\sum_{i=1}^K p_i = \lambda \int_0^{\infty} t f(t) = 0.4 \quad \text{Consistente}$$

$$\lambda_1 = \lambda \cdot 0,63212 = 25284,8 \quad] = 4 \cdot 10^4 \text{ OK}$$

$$\lambda_2 = 14715,2$$

$$E[T_{Q_1}] = \frac{\rho E(S)}{(1 - 1.057 \cdot 10^{-6})} = 47076 = 4,727 \cdot 10^{-6}$$

$$E[T_{Q_2}] = \frac{\rho E(S)}{(1 - 1.057 \cdot 10^{-6})(1 - 0.4)} = 7,4576 \cdot 10^{-6}$$

$$E[T_S]^{SB-NP} = 4,727 \cdot 10^{-6} + 7,18024 \cdot 10^{-6} = 8,6529 \cdot 10^{-6}$$

$$E[T_S] = 7,4576 \cdot 10^{-6} + 0,00002 = 77,4576 \cdot 10^{-6}$$

$$E(T_Q) = p_1 E(T_{Q_1}) + p_2 E(T_{Q_2}) = 5,5707 \cdot 10^{-6}$$

$$E(T_S) = E(T_Q) + E(S) = 0.400005570785012$$

9
0,4

Can dual core? has 2 core \rightarrow no erlang

$$E[T_a] = p_1 \underbrace{\frac{P_{a1} E(S)}{1-p_1}}_{\text{done 1}} + p_2 \underbrace{\frac{P_a E[S]}{(1-p)(1-p_1)}}_{\text{done 2}} \quad M/M/m \quad m=2$$

$$P_a = \frac{(m \cdot p)^m}{m! (1-p)} \quad p(0) = 0.22857$$

$$P(0) = \sum_{k=0}^1 = \left[1 + 2p + \frac{(2p)^2}{2(1-p)} \right]^{-1} \quad \begin{matrix} p=0.4 \\ m=2 \\ p=0.4 \end{matrix} = 0.42857$$

↓
P_{a1} can viene OK 0.22857

$$P_{a1} = \frac{(m p_1)^m}{m! (1-p_1)} \quad p(0) = 0.03430$$

↓
 $p_1 = 0.14$

$$p(0) = 0.80880$$

↓
 $p = 0.10570$

$$P_{a1} = 0.02020$$

$$E[T_a] = 0.05134$$

$$E[T_s] = E[T_a] + 0.4 = 0.45134$$

FINE

Q1.2

• tempo servizio $\sim \text{Unif}(\frac{a}{2}, \frac{b}{2})$ (in m)

• arrivi random $\lambda = 0.112 \text{ req/min.}$

• Posso sapere tempo servizio all'istante di arrivo.

Calcolare

• tempi attesa e risposta per pratica qualcosa

NB: $f(x) = \frac{1}{b-a}$, $F(x) = \frac{x-a}{b-a}$, medie $= \frac{a+b}{2}$

$$\text{Varianza} = \frac{(b-a)^2}{12}$$

M/U/1

$$E[T_a] = \frac{\frac{\lambda}{2} E[S^2]}{1-\rho} \quad \text{per M/G/1}$$

$$\lambda = 0.112 ; E[S] = \frac{a+b}{2} = 8.5 \text{ min}$$

$$\sigma^2 = E[X^2] - E[X]^2 = \frac{(b-a)^2}{12} = 14.0833$$

$$\rightarrow E[X^2] = 14.0833 + (8.5)^2 \cdot 86.3333 = E[S^2]$$

Ds Little: ~~$\rho = E[S]$~~ = 0.952

$$\rightarrow E[T_a] = \frac{0.112 \cdot 86.3333}{1-0.952} = 100.722 \text{ min}$$

$$\rightarrow E[T_s] = E[T_a] + E[S] = 109.2222$$

1.6) tempi attesi / risposte clam - globali con meccanismi preventivi a netto NO prelievi.

size based, 2 clam $\rightarrow (2, 8.5)$
 $\rightarrow (8.5, 15)$

$$E(S_1) = \int_2^{8.5} t \cdot \frac{\mu e^{-\mu t}}{F(8.5) - F(2)} dt$$

$$\frac{F(8.5) - F(2)}{F(8.5) - F(2)} = 1 - e^{-\mu t} \Big|_2 = P_1$$

$$F(8.5) - F(2) = \frac{8.5 - 2}{15 - 2} - \frac{2 - 2}{18 - 2} = \frac{6.5}{13} = 0.5$$

$$f(t) = \frac{1}{6-\alpha} = \frac{1}{13}$$

$$E(S_1) = \int_2^{8.5} t \cdot \frac{f(t)}{F(8.5) - F(2)} dt = \frac{1}{13} \cdot 0.5 \int_2^{8.5} t dt = \frac{0.5}{13} \frac{t^2}{2} \Big|_2^{8.5} = 5.25$$

$$E(S_2) = \int_{8.5}^{15} \frac{1}{13} \cdot t \cdot \frac{1}{0.5} dt = \frac{1}{6.5} \cdot \frac{t^2}{2} \Big|_{8.5}^{15} = 11.75$$

$$\lambda_1 = \lambda \cdot p_1 = 0.056 = \lambda \cdot p_2 = \lambda_2$$

$$P_1 = \lambda_1 E[S_1] = 0.294 \quad \boxed{P = 0.952}$$

$$P_2 = \lambda_2 E[S_2] = 0.658$$

$$E[T_{Q_1}] = \frac{E[S]}{1 - 0.294} \rightarrow E(S) = 0.5(5.25 + 11.75)$$

No

No late exp

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2} E(S^2)}{1 - 0.294} =$$

$$\frac{\frac{0.112}{2} (86.333)}{1 - 0.294} = 6.84 \text{ min}$$

$$E[T_{Q_2}] = \frac{\frac{\lambda}{2} E(S^2)}{(1 - 0.294)(1 - \rho)} = \frac{E[T_{Q_1}]}{1 - 0.952} = 142,6604 \text{ min}$$

$$E[T_Q] = 0.5(6.84 + 142,6604) = 74,75 \text{ min} < \text{No SB}$$

$\hookrightarrow P_1 = P_2 \text{ (equiprof)}$

~~E(TS) < E(TQ) and~~

$$E[T_{S_1}] = E[T_{Q_1}] + E[S_1] = 12,09797$$

$$E[T_{S_2}] = E[T_{Q_2}] + E[S_2] = 154.41604$$

$$\hookrightarrow E[T_S]^{\text{glob}} = \cancel{E[T_S]} + 0.5[12.09797 + 154.41604]$$

$$= 83.257005 \text{ ridotta}$$

SVOLGIMENTO

$$E(S) = \frac{c}{\lambda} = \frac{4 \cdot 10^4}{10^5} = 0.4 \text{ A} \rightarrow \rho = \lambda \cdot E(S) = 2 \cdot 0.4 = 0.8$$

L
→ costante

nei punti di 3 classi (ha 3 divisioni)

$$C_1(0, 0.2) \quad C_2(0.2, 0.4) \quad C_3(0.4, \infty)$$

$$\mu = \frac{1}{E(S)} = 2.5$$

~~E(S)~~

$$P_1 = [1 - e^{-\mu t}]_0^{0.2} = 0.393469$$

$$P_2 = [1 - e^{-\mu t}]_{0.2}^{0.4} = 0.238651 \quad \sum = 1$$

$$\mu = \frac{1}{E(S)}$$

$$P_3 = [1 - e^{-\mu t}]_{0.4}^{\infty} = 1 - 0 - [1 - e^{-2.5 \cdot 0.4}] = 0.367879$$

$$E(S_1) = \int_0^{0.2} t \cdot \frac{\mu e^{-\mu t}}{P_1} = 0.0917013 \text{ A}$$

$$E(S_2) = \int_{0.2}^{0.4} \frac{t \cdot \mu e^{-\mu t}}{P_2} = 0.2917013 \text{ A}$$

$$E(S_3) = \int_{0.4}^{\infty} \frac{t \cdot \mu e^{-\mu t}}{P_3} = 0.8 \text{ A}$$

$$P_1 = \lambda \int_0^{0.2} t \cdot \mu e^{-\mu t} dt = 0.0721632$$

$$P_2 = \lambda \int_{0.2}^{0.4} t \cdot \mu e^{-\mu t} dt = 0.13923$$

$$P_3 = \lambda \int_{0.4}^{\infty} t \cdot \mu e^{-\mu t} dt = 0.588607$$

OK

$$E(T_{a_1}) = \frac{P E[S]}{(1 - P_1)} = 0.344838 \wedge$$

$$E(T_{a_2}) = \frac{P E[S]}{(1 - P_1 - P_2)(1 - P_1)} = 0.437338 \wedge$$

$$E(T_{a_3}) = \frac{P E[S]}{(1 - P)(1 - P_1 - P_2)} = 2.0288895 \wedge$$

rispettato

$$E[T_a] = \sum p_i E(T_{a_i}) = 0.988032295$$

$$E[S] = \sum p_i E[S_i] = 0.4$$

$$E[T_S] = E[T_a] + E[S] = 1.38803 \quad \checkmark \text{ Rispettato}$$

a) throughput

$\mu > \lambda$ stable, $P < 1$, throughput = λ 2 req/s ?

b) $S_d(0.1) = \frac{0.34488}{0.1} + 1 = 4,4488$

$$S_d(0.3) = \frac{0.437338}{0.3} + 1 = 2.4577$$

FIFO

$$0.1 \left(\frac{\frac{\lambda}{2} [E[S^2]]}{0.1(1-P)} + 1 \right) = \frac{P E[S]}{0.1(1-0.8)} + 1 = \frac{0.8 \cdot 0.9}{0.1(0.2)} + 1 = 17$$

$$0.3 \left(\frac{5}{3} + 1 \right) = 6.3$$

PS

$$\frac{1}{1-P} = 5 \text{ A job}$$

OK

4)

- arriv Poisson, λ = interarrivo = 200 ms

$$\rightarrow \frac{1 \text{ job}}{200 \text{ ms}} = 0,005 \text{ j/ms} \quad \lambda$$

- SIZE Buff = 4 (incluso Job servizio)
- $E[S] = 200 \text{ ms} \rightarrow \mu = \frac{1}{E[S]} = 0,005 \text{ j/ms}$

con coda (3 job in coda) \rightarrow erlang c

$$\lambda' = \lambda(1 - p_{\text{loss}})$$

$$\cancel{P_{\text{loss}}} = \frac{(mp)^m}{m! (1-p)} \cancel{P(0)} \quad | \quad \begin{array}{l} m=3 \\ P=1 \end{array}$$

N_c

$(C=3)(\gamma-1)$ $\lambda = \mu = 0.005$ $C=4$

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \pi_0 = \pi_0 = \frac{1}{\sum_{i=0}^3 \left(\frac{0.005}{0.005}\right)^i} = \frac{1}{4}$$

$$\lambda' = 3 \left(1 - \frac{1}{4}\right) = 3 \left(\frac{3}{4}\right) = 4 \text{ j/s}$$

$$\lambda' = 0.005 \left(1 - \frac{1}{4}\right) = 0.004 \text{ j/ms} = 4 \text{ j/s}$$

b) Request in 150 ms ~~edge~~ $\Rightarrow \mu = 0.006$

$$\sum_i \pi_i = \left(\frac{\lambda}{\mu}\right)^i \pi_0 =$$

$$\rightarrow \cancel{\pi_3} \left(\frac{0.005}{0.006} \right)^3 \pi_0 = 0.5787 \cdot \pi_0 = 0.186283$$

$$\pi_0 = \frac{1}{\sum_{i=0}^3 \left(\frac{0.005}{0.006}\right)^i} = \frac{1}{1 + 0.833 + 0.6944 + 0.5787 + 0.4822} = 0.3219$$

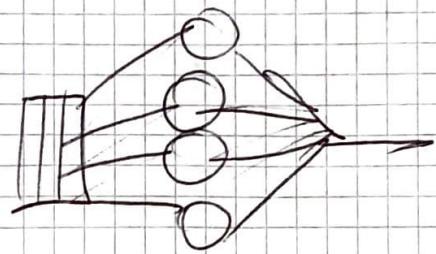
$$\rightarrow \lambda = 0.005 \cdot 3 \left(1 - 0.186283 \right) =$$

$$\pi_0 = \frac{1}{\sum_{i=0}^4 \left(\frac{0.005}{0.006}\right)^i} = 0.27864$$

$$\pi_4 = p_{loss} = \left(\frac{0.005}{0.006}\right)^4 \cdot \pi_0 = 0.134375$$

$$\lambda = 0.005 \left(1 - 0.134375 \right) = 0.00432 \text{ s/ms} = 4,328 \text{ 1/s}$$

c) 4 core, request 300 ms mundo um core.
throughput?



$$\mu' = 0.0033 \text{ ms}^{-1}$$

$$\lambda = 0.005 \text{ s/ms}$$

$$P_{\text{ar}} = \frac{4 \left(\frac{\lambda}{\mu'} \right)^4 p(0)}{4! (1 - \frac{\lambda}{\mu'})}$$

$$P = \frac{\lambda}{\mu'} = 0.3750 \text{ OK}$$

$$P(0) = \sum_{i=0}^3 \left[\frac{(4P)^i}{i!} + \frac{(4P)^4}{m! (1-P)} \right]^{-1} =$$

$$= \left\{ \left[1 + \frac{(4P)^4}{4! (1-P)} \right] + \left[4P + \frac{(4P)^2}{3! (1-P)} \right] + \left[\frac{(4P)^2}{2} + \frac{(4P)^3}{4! (1-P)} \right] \right.$$

$$+ \left. \left[\frac{(4P)^3}{3!} + \frac{4P}{4! (1-P)} \right] \right\}^{-1}$$

$$= \left\{ 1 + 4P + \frac{16P^2}{2} + \frac{64P^3}{6} + 4 \left(\frac{4P}{4! (1-P)} \right)^4 \right\}^{-1} = 0.180587$$

$$P_{\text{ar}} = 0.00095 \rightarrow \lambda = \lambda (1 - 0.0095) = 0.0049525$$

$$\rightarrow 4,95 \text{ s/n}$$

erlang B

$$\pi_4 = (0.3750)^4 \frac{1}{4!} \pi_0 = 0$$

$$\pi_0 = \frac{1}{\sum_{i=0}^4 \left(\frac{0.3750}{m} \right)^i} = \frac{1}{0.629669} = 0.07375$$

$$\pi_4 = \frac{(0.3750)^4}{4!}$$

$$\frac{\left(\frac{\lambda}{m}\right)^m / m!}{\sum_{i=0}^m \left(\frac{\lambda}{m}\right)^i / i!} \text{ OK}$$

$$\frac{0.005}{0.00333}$$

Ü.1 NOTG

$$C = 20^5 \text{ op/h}$$

$$Z = 4 \cdot 10^4$$

$$\rho = 0.6 = \lambda E[S]$$

$$E[S] = \frac{Z}{C} = 0.4$$

$$\mu = \frac{1}{E[S]} = 2.5$$

$$\rho = 0.6 = \lambda \cdot 0.4 \rightarrow \lambda = \frac{0.6}{0.4} = 1.5$$

clane 2

$$\leq 0.4 = E[S]$$

$$P_1 = 0.6312$$

$$E[S_1] = 0.1674531$$

$$\frac{1}{\mu} = E[S]$$

$$P_1 E[S_1] = 0,1056963$$

clane 2

$$> 0.4 = E[S]$$

$$P_2 = 0.36788$$

$$E[S_2] = 0.799989$$

$$P_2 E[S_2] = 0,29430$$