Performance Modeling
of Computer Systems and Networks

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Analytical models

Exercises

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Consider a web server with a mean processing rate of 1.2 job/s.
If the server receives requests with a rate of 0.45 job/s and it has 0.225 enqueued jobs on average, determine:

a) the average utilization
b) the average response time.

During rush hours the arrival rate grows of 20% and the average number of enqueued jobs becomes 0.3681818.
Determine:

c) the performance metrics a) and b)
d) which further increasing in arrival rate makes the server collapsing
e) the performance metrics a) and b) for the limiting case d).

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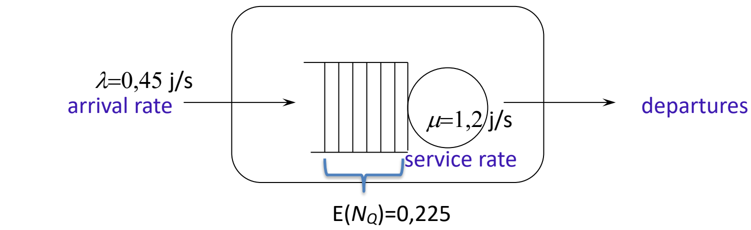
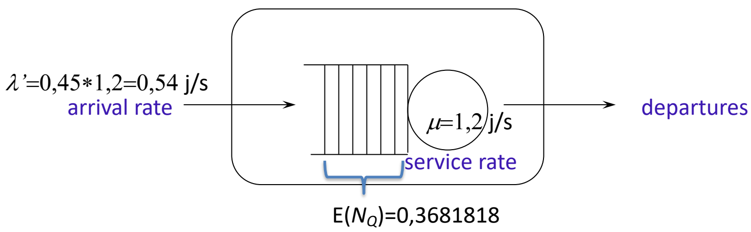


a) the average utilization
b) the average response time.

$$\rho = \lambda / \mu = 0{,}375$$

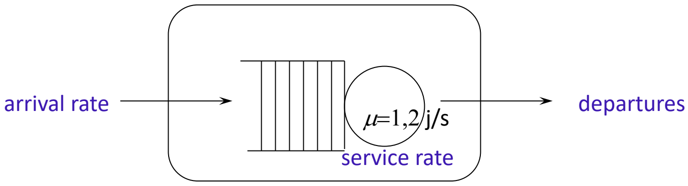
$$E(T_S) = \frac{E(N_S)}{\lambda} = 0{,}6/0{,}45 = 1{,}333333 \text{ s} \qquad E(N_S) = E(N_Q) + \rho = 0{,}225+0{,}375=0{,}6$$



During rush hours the arrival rate grows of 20% and the average number of enqueued jobs becomes 0.3681818.
Determine:
c) the performance metrics a) and b)

$$\rho = \lambda' / \mu = 0{,}45$$

$$E(T_Q) = \frac{E(N_Q)}{\lambda'} = 0{,}681818 \text{ s} \qquad E(T_S) = E(T_Q) + E(S) = 1{,}515151$$



c) which further increasing in arrival rate makes the server collapsing
d) the performance metrics a) and b) for the limiting case d).

$\rho \rightarrow 1 \qquad \lambda' \rightarrow \mu$

$0{,}54 * x = 1{,}2 \qquad x = 2{,}222222$

$E(T_s) = \infty$

Let us consider a server that processes jobs with rate 0.8 jobs/s.
By assuming that the server receives jobs with a rate depending on the time slot as follows:

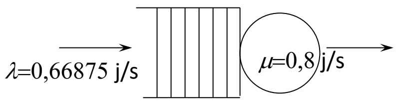
- 8.00 a.m. – 12.00 a.m. average arrival rate 1.5 jobs/s
- 12.00 a.m. – 2.00 p.m. average arrival rate 0.5 jobs/s
- 2.00 p.m. – 7.00 p.m. average arrival rate 1.5 jobs/s
- 7.00 p.m. – 9.00 p.m. average arrival rate 0.5 jobs/s
- 9.00 p.m. – 8.00 a.m. average arrival rate 0.05 jobs/s

Determine:

a) average arrival rate per day (24 hours)
b) average utilization per day
c) average throughput per day
d) average throughput for each time slot

Please, justify and comment the results by indicating the used laws.

a) average arrival rate per day (24 hours)
b) average utilization per day



I time slot:	4 h	1.5 jobs/s
II time slot:	2 h	0.5 jobs/s
III time slot:	5 h	1.5 jobs/s
IV time slot:	2 h	0.5 jobs/s
V time slot:	11 h	0.05 jobs/s

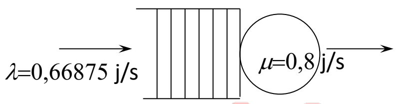
$$\frac{9}{24}\times 1{,}5+\frac{4}{24}\times 0{,}5+\frac{11}{24}\times 0{,}05 = 0{,}66875\ j/s$$

è una media pesata in base agli slot:

$$\rho = \lambda / \mu = 0{,}835937$$

$$\frac{(1.5\cdot 4)+(0{,}5\cdot 2)+(1{,}5\cdot 5)+(0{,}5\cdot 2)+(0{,}05\cdot 11)}{24} = 0{,}66875\ J/S$$

c) average throughput per day



I time slot:	4 h	1.5 jobs/s	$\lambda > \mu$
II time slot:	2 h	0.5 jobs/s	$\lambda < \mu$
III time slot:	5 h	1.5 jobs/s	$\lambda > \mu$
IV time slot:	2 h	0.5 jobs/s	$\lambda < \mu$
V time slot:	11 h	0.05 jobs/s	$\lambda < \mu$

$$\cancel{\lambda < \mu \longrightarrow X = \lambda}$$
$$\cancel{\lambda > \mu \longrightarrow X = \mu}$$

tale considerazione vale se si parte da una coda VUOTA, qui dobbiamo vedere se lo slot T+1 deve smaltire anche i Job dello slot T

$$\underbrace{\frac{9}{24}\times 0{,}8}_{\text{ore in cui } \lambda<\mu} + \underbrace{\frac{4}{24}\times 0{,}5}_{\text{ore con } \lambda>\mu \text{ e } \lambda=0{,}5} + \underbrace{\frac{11}{24}\times 0{,}05}_{\text{ore con } \lambda>\mu \text{ e } \lambda=0{,}05} = 0{,}4062496\ j/s < 0{,}66875\ j/s$$

???

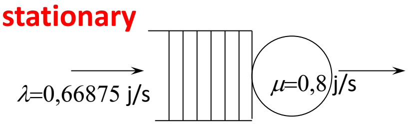
vediamo che, in questo caso (SBAGLIATO) throughput < arrivi, cioè alcuni Job rimangono in coda!

The system is not stationary!!!

(NON STAZIONARIO : entra X, esce Y<X, NO FLOW BALANCE

The system is not stationary

d) average throughput for each time slot



I time slot:	4 h = $\underbrace{14.400}_{\text{secondi in 4H}}$ x1,5=	21.600	jobs arrived	NON STAZIONARIO
	14.400x0,8=	11.520	jobs served	
		10.080	jobs queue!!	rimasti in coda
II time slot:		10.080	job in queue	NON STAZIONARIO
	2 h =7.200x0.5=	3.600	job arrived	
	7.200x0.8=	5.760	job served	
		7.920	jobs in queue	
III time slot:		7.920	jobs in queue	NON STAZIONARIO
	5 h =18.400x1.5=	27.000	jobs arrived	
	18.400x0.8=	14.400	jobs served	
		20.520	jobs in queue	

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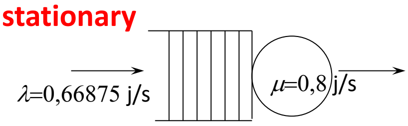
The system is not stationary

d) average throughput for each time slot



IV time slot:		20.520	jobs in queue	NON STAZIONARIO
	2 h =7.200x0.5=	3.600	job arrived	
	7.200x0.8=	5.760	job served	
		18.360	jobs in queue	
V time slot:		18.360	jobs in queue	STAZIONARIO, LI HO SERVITI TUTTI!
	11 h =39.600x0.05=	1.980	jobs arrived	
		20.340	all served!!!	

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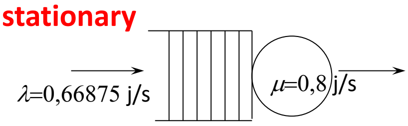
The system is not stationary

d) average throughput for each time slot



server never empty!!

I time slot:	4 h = 14.400 x1,5=	21.600	jobs arrived	**X=0,8 j/s**
	14.400x0,8=	11.520	jobs served	
		10.080	jobs in queue!!	
II time slot:		10.080	job in queue	
	2 h =7.200x0.5=	3.600	job arrived	**X=0,8 j/s**
	7.200x0.8=	5.760	job served	
		7.920	jobs in queue	
III time slot:		7.920	jobs in queue	
	5 h =18.400x1.5=	27.000	jobs arrived	**X=0,8 j/s**
	18.400x0.8=	14.400	jobs served	
		20.520	jobs in queue	

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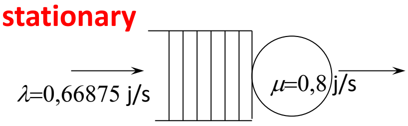
The system is not stationary

d) average throughput for each time slot



IV time slot:		20.520	jobs in queue	
	2 h =7.200x0.5=	3.600	job arrived	**X=0,8 j/s**
	7.200x0.8=	5.760	job served	
		18.360	jobs in queue	
V time slot:		18.360	jobs in queue	
	11 h =39.600x0.05=	1.980	jobs arrived	**X=** 20.340/39600= 0,513636 j/s
		20.340	all served!!!	

STAZIONARIO

Throughput $X = \lambda$ = tasso arrivo = $\frac{\text{Job arrivati TOT}}{\text{secondi TOT nell'intervallo di osservazione}}$

$$\frac{13}{24}\times 0,8+\frac{11}{24}\times 0,5136363 = 0,6687499\ j/s$$

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