22/03/22

# Performance Modeling of Computer Systems and Networks

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Discrete-Event Simulation
examples

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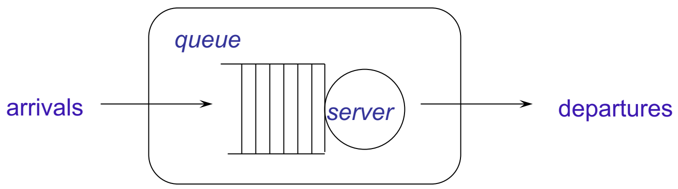
## Single Server Queue




Arrival times: $a_i$

~~15 47 71 111 123 152 166 226 310 320~~

***Pseudo-random generators***

Service times: $s_i$

~~43 36 34 30 38 40 31 29 36 30~~



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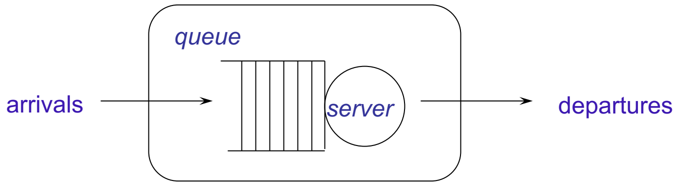
P. 97 discrete



## Single Server Queue




- assume **service times** are between 1.0 and 2.0 minutes
  - The distribution within this range is unknown
  - Without further knowledge, we assume no time is more likely than any other

media varianza

→ Uniform(1.0, 2.0)







## *Exponential distribution*

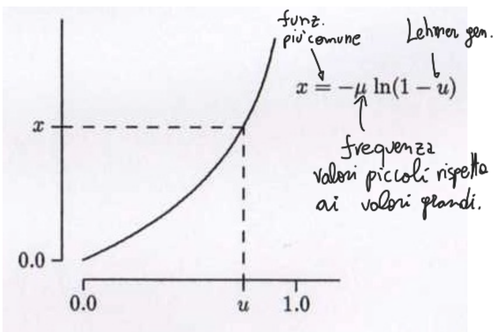
- In general, it is unreasonable to assume that all possible values are equally likely (uniform ha ∀ valore stessa prob, nella realtà non sempre è realistico)
- Frequently, small values are more likely than large values

SOLUZIONE

- We need a non-linear transformation that maps $0.0 \rightarrow 1.0$ to $0.0 \rightarrow \infty$

this is the most frequently used function

$\mu > 0$ is a parameter that "control" the frequency of large values in respect of the small ones



- the transformation is monotone increasing, one-to-one

$$0 < u < 1 \Leftrightarrow 0 < (1 - u) < 1$$
$$\Leftrightarrow -\infty < \ln(1 - u) < 0$$
$$\Leftrightarrow 0 < -\mu \ln(1 - u) < \infty$$
$$\Leftrightarrow 0 < x < \infty$$

```
double Exponential(double μ)          /* use μ > 0.0 */
{
  return (-μ * log(1.0 - Random()));
}
```

Lo ln

- the parameter μ specifies the sample mean

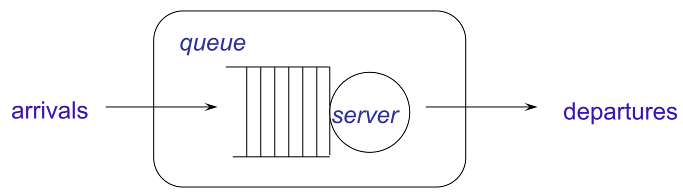




abbiamo visto i tempi di interarrivo, usando TRACCE, ora facciamo senza!



## Single Server Queue




Arrival times: $a_i$

- use the exponential function for the interarrival times

$a_i = a_{i-1} + \mathit{Exponential}(\mu)$; $i = 1, 2, 3, \ldots, n$

Service times: $s_i$

*Uniform*(1.0, 2.0)

usava le tracce

- program ssq2 is an extension of ssq1
  - arrival times are drawn from *Exponential*(2.0)
  - service times are drawn from *Uniform*(1.0, 2.0)





trace-driven simulation

```
#include <stdio.h>

#define FILENAME   "ssq1.dat"    /* input data file */
#define START      0.0

  double GetArrival(FILE *fp)   /* read an arrival time */
{   double a;
    fscanf(fp, "%lf", &a);
    return (a);}

double GetService(FILE *fp)   /* read a service time */
{   double s;
    fscanf(fp, "%lf\n", &s);
    return (s);}
```

## ssq2.c

distribution-driven simulation

```
#include <stdio.h>
#include <math.h>
#include "rng.h"
#define LAST         10000L    /* number of jobs processed */
#define START        0.0

double Exponential(double m)                    /* ------*
{return (-m * log(1.0 - Random())); }              m > 0.0
                                                 ------ */
double Uniform(double a, double b)              /* ------*
{return (a + (b - a) * Random());    }               a < b
                                                * ------*/
 double GetArrival(void)
{static double arrival = START;
 arrival += Exponential(2.0); <- genero tempo interarrivo
 return (arrival);}

double GetService(void)
 {return (Uniform(1.0, 2.0));}
```



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- the program generates all first-order statistics

$$\bar{r}, \bar{w}, \bar{d}, \bar{s}, \bar{l}, \bar{q}, \bar{x}$$

;



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trace-driven simulation = distribution-driven simulation

```
int main(void)
{  FILE   *fp;                     /* input data file */
   long   index     = 0;          /* job index */
   double arrival   = START;      /* arrival time*/
   double delay;                  /* delay in queue*/
   double service;                 /* service time*/
   double wait;                    /* delay + service*/
       double departure = START;     /* departure time*/
   struct {                        /* sum of ...   */
       double delay;      /*delay times */
       double wait;       /*wait times*/
       double service;    /*service times */
       double interarrival; /*   interarrival times */
   } sum = {0.0, 0.0, 0.0};
```





trace-driven simulation

```
fp = fopen(FILENAME, "r");
if (fp == NULL) {
   fprintf(stderr,"Cannot open input file %s\n",FILENAME);
   return (1);  }
while (!feof(fp)) {
```

distribution-driven simulation

STATIC

```
PutSeed(123456789);
while (index < LAST) {
```

ultimo Job
che vogliamo simulare

```
                        trace-driven: fp
while (index < LAST) {
   index++;
   arrival       = GetArrival();
   if (arrival < departure)
        delay  = departure - arrival; /* delay in queue  */
   else  delay       = 0.0;            /* no delay  */
   service = GetService();
   wait  = delay + service;
   departure     = arrival + wait;   /* time of departure */
   sum.delay    += delay;
   sum.wait     += wait;
   sum.service += service;
}
sum.interarrival = arrival - START;}
```



```
printf("\nfor %ld jobs\n", index);
printf("   average interarrival time = %6.2f\n", sum.interarrival / index);
printf("   average wait ............ = %6.2f\n", sum.wait / index);
printf("   average delay ........... = %6.2f\n", sum.delay / index);
printf("   average service time .... = %6.2f\n", sum.service / index);
printf("   average # in the node ... = %6.2f\n", sum.wait / departure);
printf("   average # in the queue .. = %6.2f\n", sum.delay / departure);
printf("   utilization ............. = %6.2f\n", sum.service / departure);
return (0);}
```

ESEMPIO SINGLE SERVER QUEUE

$\lambda \rightarrow$ [queue] $\mu \rightarrow$

• coda $\infty$ • servente singolo • $\lambda = 0{,}5$ J/s esponenziale • $E(S) = 1.5$ s, Unif$(a=1, b=2)$

SVOLGIMENTO

01/04/21

Generale M/G/1 (Unif)

• Con Little $E(T_q) = \frac{\rho}{1-\rho}\left(\frac{c^2+1}{2}\right) \cdot E(S)$ ; media : $\bar{x} = E(\bar{S}) = \frac{a+b}{2} = 1{,}5$ s ; VARIANZA $\sigma^2(x) = E(S^2) = \frac{(b-a)^2}{12} = \frac{1}{12}$

• calcolo $\rho = \lambda E(S) = 0{,}75$ e $E(T_q) = 2{,}3$ s (se uso $c^2 = \frac{1}{12} \cdot \frac{1}{(1{,}5)^2} = 0{,}037037$) tempo attesa medio in coda. $E(T_s) = E(T_q) + E(S) = 2{,}\bar{3} + 1{,}5 = 3{,}83$ s.

• Uso LITTLE per i risultati : $E(N_q) = 1{,}1667$ ; $E(N_s) = 1{,}9167 = 1{,}1667 + \rho$

NB: $\frac{\rho}{1-\rho}\left(\frac{c^2+1}{2}\right)E(S) \underset{\rho = \lambda E(S)}{=} \frac{\lambda}{\rho(1-\rho)}\left(\frac{c^2+1}{2}\right)E^2(S) \underset{c^2 = \frac{\sigma^2(S)}{E^2(S)}}{=} \frac{\lambda}{2\rho(1-\rho)}\left[\frac{\sigma^2}{E^2(S)}+1\right]E^2(S) = \frac{\lambda \cdot E(S^2)}{2\rho(1-\rho)}$

DIVERSI!

$\wedge . \wedge$

• $E(N_s)$ ci dice che ho in media quasi 2 job nel centro

• servente occupato per il 75%

• un job che chiede $E(S) = 1{,}5$ attende $E(T_s) = 3.8 \approx$ doppio!

p.60 appunti



## Example 1

- The "theoretical" averages using *Exponential*(2.0) (rate 0.5 j/s) arrivals and *Uniform*(1.0, 2.0) (rate 0.67) service times are

| $\bar{r}$ | $\bar{w}$ | $\bar{d}$ | $\bar{s}$ | $\bar{l}$ | $\bar{q}$ | $\bar{x}$ |
|---|---|---|---|---|---|---|
| 2.00 | 3.83 | 2.33 | 1.50 | 1.92 | 1.17 | 0.75 |

exact analytical results, No simulation!

server risponde ogni · Job chiede ogni 1.5 · in media, 2 Job

- Although the server is busy 75% of the time, on average there are approximately 2 jobs in the service node
- A job can expect to spend more time in the queue than in service
- To achieve these averages, many jobs must pass through node

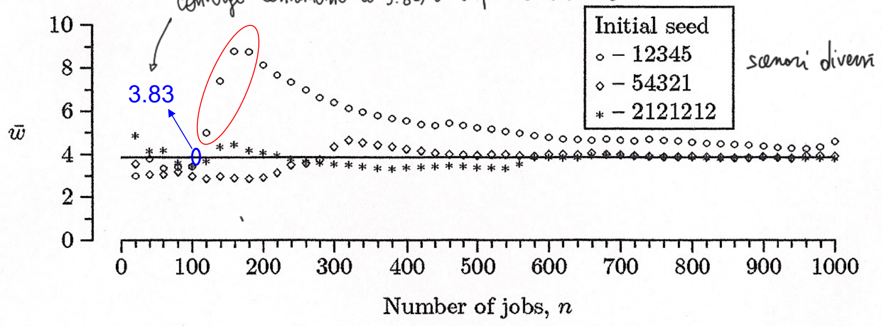
(simulando posso vedere il comportamento per molti job, osservando quando le statistiche tendono a questi valori asintotici)





- The accumulated average wait was printed every 20 job

converge lentamente a 3.83, e dipende dal seme




semi diversi

- The convergence of $w$ is slow, erratic, and dependent on the initial seed

(quando diventa stazionario)

- the program can be used to study the *steady-state* behavior
  - Will the statistics converge independent of the initial seed?
  - How many jobs does it take to achieve steady-state behavior?

(è il grafico di prima, con 5/5 simulazioni diverse)

- the program can be used to study the *transient* behavior (fase iniziale curve)
  - Fix the number of jobs processed and replicate the program with the initial state fixed (2000 Job nel grafico, tutte le simulazioni partono da stessa condiz.)
  - Each replication uses a different initial rng seed (cambio semi)





## Steady-state analysis



| | $\bar{r}$ | $\bar{w}$ | $\bar{d}$ | $\bar{s}$ | $\bar{l}$ | $\bar{q}$ | $\bar{x}$ | |
|---|---|---|---|---|---|---|---|---|
| theoretical | 2.00 | 3.83 | 2.33 | 1.50 | 1.92 | 1.17 | 0.75 | seed=123456789 |
| n=10 | 2.85 | 1.74 | 0.39 | 1.35 | 0.59 | 0.13 | 0.45 | |
| n=100 | 2.06 | 3.16 | 1.67 | 1.48 | 1.50 | 0.80 | 0.71 | |
| n=1000 | 2.03 | 3.44 | 1.94 | 1.50 | 1.69 | 0.96 | 0.74 | |
| n=10000 | 2.02 | 3.86 | 2.36 | 1.50 | 1.91 | 1.17 | 0.74 | |
| n=100000 | 2.00 | 3.85 | 2.35 | 1.50 | 1.92 | 1.17 | 0.75 | |
| n=1000000 | 2.00 | 3.81 | 2.31 | 1.50 | 1.90 | 1.15 | 0.75 | |
| n=1000000 | 2.00 | 3.84 | 2.34 | 1.50 | 1.92 | 1.17 | 0.75 | seed=1 |

inizio transiente

POCHI, comportamento molto variabile

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| n=10 | 2.13 | 2.36 | 0.75 | 1.62 | 1.02 | 0.32 | 0.69 | seed=1 |
| n=10 | 1.48 | 2.37 | 0.87 | 1.50 | 1.24 | 0.46 | 0.79 | seed=987654321 |
| n=10 | 1.49 | 1.89 | 0.49 | 1.40 | 1.12 | 0.29 | 0.83 | seed=2121212121 |

Transient analysis

## Simulating an initial steady state

Inizio stazionarietà

$\bar{d} = 2.33$ (delay, attesa in coda)

departure=3, il sistema parte in stato stabile (departure = 3), e NON da VUOTO

$a_1 = a_0 + \text{expo}(\underset{\text{media}}{2}) = 0 + 0.8 = 0.8$

$0.8 < 3$ (c'è attesa)

$d_1 = 3 - 0.8 = 2.2$ ≈ attesa media in coda

servizio

$s_1 = \text{Uniform}(1,2) = 1.3$

$w_1 = 2.2 + 1.3 = 3.5$ (tempo risposta: attesa + servizio)

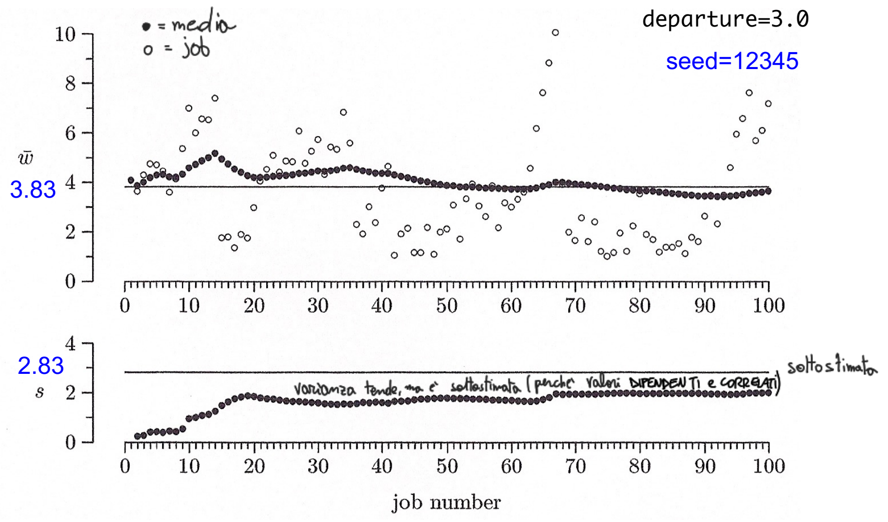
$c_1 = 0.8 + 3.5 = 4.3$ (quando Job1 esce dal centro)



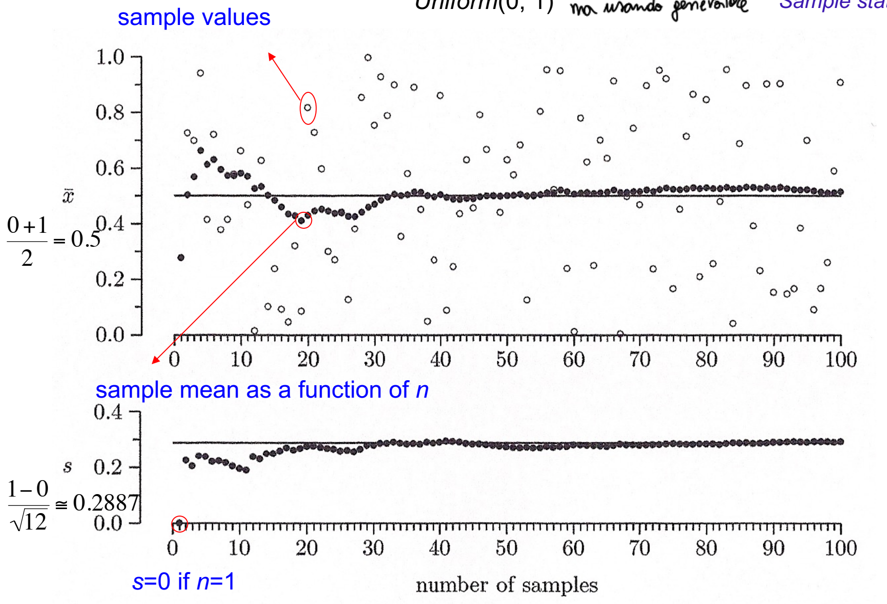


DE simulation

*Sample statistics*



*Uniform*(0, 1) stesso esempio, ma usando generatore










## Serial correlation

- *Independence*: each $x_i$ value does not depend on any other point
- Time-sequenced DES output is typically not independent
- E.g.: wait times of consecutive jobs have positive *serial correlation*
- Example: Consider output from ssq2
  - *Exponential*(2) interarrivals, *Uniform*(1,2) service
- wait times $w_1, w_2, \ldots, w_{100}$, have high positive serial correlation
  - The correlation produces a bias in the standard deviation

p.101 Discrete



## Example 2

- assume that jobs arrive at random with a steady-state arrival rate of 0.5 jobs per minute
- assume that Job service times are "composite" with two components:
  - the *number* of service tasks is 1 + *Geometric*(0.9) → media = $\frac{1}{0.9}$
  - the *time* (in minutes) per task is *Uniform*(0.1, 0.2) → media = $\frac{0.1+0.2}{2}$ = 0,15

```
double GetService(void)
{
long k;
double sum = 0.0;
long tasks = 1 + Geometric(0.9);
for (k = 0; k < tasks; k++)
    sum += Uniform(0.1, 0.2);
return (sum);
}
```







- The theoretical steady-state statistics for this model are

popolazione media

| $\bar{r}$ | $\bar{w}$ | $\bar{d}$ | $\bar{s}$ | $\bar{l}$ | $\bar{q}$ | $\bar{x}$ | |
|---|---|---|---|---|---|---|---|
| 2.00 | 5.77 | 4.27 | 1.50 | 2.89 | 2.14 | 0.75 | exact analytical results, No simulation! |
| " | (3.83 | 2.33 | " | 1.92 | 1.17) | " | ← prodotti da ssq2 |

- The arrival rate, service rate, and utilization are identical to the previous case (example 1)
- The other four statistics are significantly larger
- performance measures are sensitive to the choice of service time distribution

Discrete-event simulation
*Case study sis*

## *A simple inventory system* (revisione)


- Distributes items from current inventory to customers
- Customer demand is discrete
- Simple: one type of item

  – Inventory review is periodic
  – Items are ordered, if necessary, only at review times
  – (s, S) are the min,max inventory levels, 0≤s<S

S → fisso
s → variabile per vedere i benefit

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P.102

Discrete-Event Simulation
*case study sis*

### Simply Inventory System

- Program sis2 has randomly generated demands using an *Equilikely*(a, b) random variate (non realistico) → passo alla geometrica (anche lei non perfetta)
- Using random data, we can study transient and steady-state behaviors

sis1.c

```
#include <stdio.h>

~~#define FILENAME  "sis1.dat"~~
#define MINIMUM   20
#define MAXIMUM   80
#define STOP     100
#define sqr(x)    ((x) * (x))

long GetDemand(FILE *fp)
{
    long d;
    fscanf(fp, "%ld¥n", &d);
    return (d);}
```

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sis2.c

distribution driven

```
#include <stdio.h>
#include "rng.h"

#define MINIMUM   20
#define MAXIMUM   80
#define STOP     100      /* 100 weeks = about 2 years*/
#define sqr(x)    ((x) * (x))


long Equilikely(long a, long b)
{  return (a + (long) ((b - a + 1) * Random()));}

long GetDemand(void)
{
        return (Equilikely(10, 50)); }
```

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```
int main(void)
{
   long index     = 0;
   long inventory = MAXIMUM;
   long demand;
   long order;
   struct {
       double setup;
       double holding;  /*inventory hold (+) */
       double shortage; /*inventory short (-) */
       double order;
       double demand;
   } sum = { 0.0, 0.0, 0.0, 0.0, 0.0 };

    PutSeed(123456789);
```

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```
while (index < STOP) {
  index++;
  if (inventory < MINIMUM) {
       order = MAXIMUM - inventory;
       sum.setup++;
       sum.order    += order;
  }
  else order         = 0;
  inventory += order;     /* there is no delivery lag */    demand  =
  GetDemand();
  sum.demand     += demand;
  if (inventory > demand)
       sum.holding  += (inventory - 0.5 * demand);
  else {
       sum.holding  += sqr(inventory) / (2.0 * demand);
       sum.shortage += sqr(demand - inventory) / (2.0 * demand);
  }
  inventory      -= demand;
}
```





```
if (inventory < MAXIMUM) {
       order = MAXIMUM - inventory;
       sum.setup++;
       sum.order+= order;
       inventory      += order;
  }

...
```

for 100 time intervals with an average demand of 27.68
and policy parameters (s, S) = (20, 80)

average order ........... = 27.68
setup frequency .......... = 0.36
average holding level .... = 44.81
average shortage level ... = 0.14

sis2.dat, con $n=100$ e $(s, S) = (20, 80)$, restituisce:

$\bar{o} = \bar{d} = 29,29$ $\quad \bar{u} = 0,39$ $\quad \bar{l}^{+} = 42,40$ $\quad \bar{l}^{-} = 0,25$

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```c
int main(void)
{  long seed;
   long index     = 0;
   long inventory = MAXIMUM;
   long demand;
   long order;
   struct {
       double setup;
       double holding;  /*inventory held (+) */
       double shortage; /*   inventory short (-)   */
       double order;
       double demand;
   } sum = { 0.0, 0.0, 0.0, 0.0, 0.0 };

PutSeed(-1); //clock sistema
GetSeed(&seed);
printf("\nwith an initial seed of %ld", seed);
```

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for 100 time intervals with an average demand of 27.68
and policy parameters (s, S) = (20, 80)

average order ........... = 27.68
setup frequency .......... = 0.36
average holding level .... = 44.81
average shortage level ... = 0.14

with an initial seed of 1333437895 (seed = -1)
for 100 time intervals with an average demand of 31.00
and policy parameters (s, S) = (20, 80)

average order ........... = 31.00
setup frequency .......... = 0.40
average holding level .... = 43.39
average shortage level ... = 0.37







## Simply Inventory System

equilikely(a = 10, b = 50) ⟶ average demand = $\frac{a+b}{2} = 30$

if (a, b) = (10, 50) and (s, S) = (20, 80), then the approximate steady-state satistics are

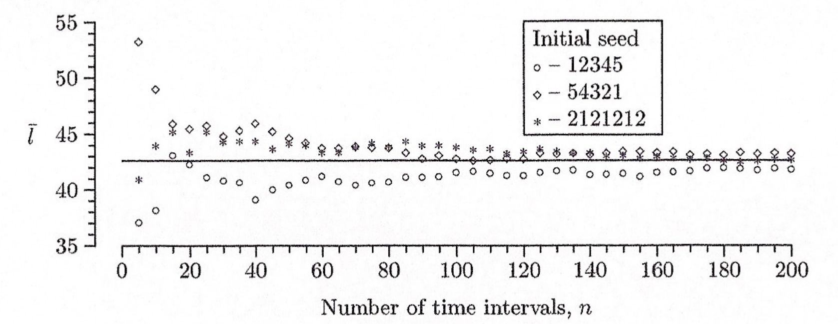
| $\bar{d}$ | $\bar{o}$ | $\bar{u}$ | $\bar{l}^+$ | $\bar{l}^-$ |
|---|---|---|---|---|
| 30.00 | 30.00 | 0.39 | 42.86 | 0.26 |

(valore teorico, prodotto da sis2)

(trace-driven (con la traccia)

$\bar{o} = \bar{d} = 29.29$ $\quad \bar{u} = 0.39$ $\quad \bar{l}^+ = 42.40$ $\quad \bar{l}^- = 0.25$ )

The average inventory level $\bar{l} = \bar{l}^+ - \bar{l}^-$
approaches steady state after several hundred time intervals




Convergence is slow, erratic, and dependent on the initial seed

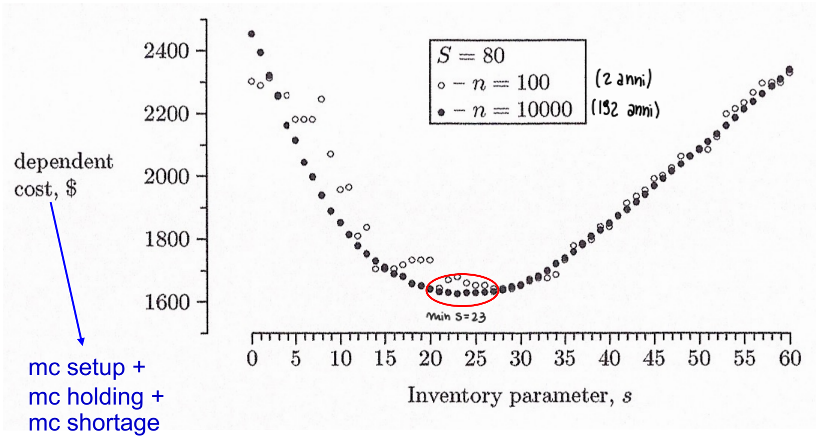






- using a fixed initial seed guarantees the exact same demand sequence (in the example 12345), devo cambiare solo la s, non tocco altri valori!
  - any changes to the system are caused solely by the change of s
  - a steady state study of this system is unreasonable:
    - all parameters would have to remain fixed for many years
    - when n=100, we simulate approximately 2 years
    - when n=10000, we simulate approximately 192 years

- if we fix S, we can find the optimal cost by varying s
- recall that the dependent cost ignores the fixed cost of each item




Abbiamo visto il problema dell'ottimizzazione di funzione non nota con certezza. Si parla di stochastic optimization





P. 104 discrete



## Statistical Considerations
(sect. 3.1.3)

- *variance reduction:*

  (intuitive approach) use the same random numbers

  (We kept the same initial seed and changed only s)

NOTE:
transient behavior will always have some inherent uncertainty (poiché prodotto da una discrete event simulation DIPENDE dalla sequenza di variabili random generate durante l'esecuzione del programma)

- *Robust Estimation*:

  when a data point is not sensitive to small changes in assumptions

  - values of s close to 23 have essentially the same cost
  - Would the cost be more sensitive to changes in S or other assumed values?

- se S varia in un intorno di 80, o average demand cambia in un intorno di 30, o introduco delivery lags, la stima s = 23 è robusta <=> s rimane nell'intorno di 23 anche con queste assunzioni. In generale ROBUST ESTIMATORS sono insensibili al modello assunto.

# Exercises

- derive analytical results on p.12 (example 1 slide)
- study program ssq2.c; run it and compare output with the results on p.12
- Exercises: 3.1.1, 3.1.2, 3.1.4, 3.1.5, 3.1.6