

## Performance Modeling of Computer Systems and Networks

Prof. Vittoria de Nitto Personè

Operational analysis:  
Queueing Networks

Università degli studi di Roma Tor Vergata  
Department of Civil Engineering and Computer Science Engineering

Copyright © Vittoria de Nitto Personè, 2021  
<https://creativecommons.org/licenses/by-nc-nd/4.0/>



1

Da un punto di vista degli analisti di sistema si è creato dello scetticismo, perché il modello era regolamentato da un approccio stazionario, le ipotesi richiedevano indipendenza stocastica tra i job, etc...

Analytical models  
*conceptual model*

## Analytical models

*Queueing network (QN) modelling* is a particular approach to computer system (CS) modeling in which the CS is represented as a *network of queues* which is evaluated *analytically*

- Many analysts experienced doubts on its accuracy
- A series of assumptions:
  - ✓ the system is modeled by a *stationary stochastic process*;
  - ✓ jobs are *stochastically independent*;
  - ✓ job steps from device to device follow a *Markov chain*;
  - ✓ the system is in stochastic equilibrium;
  - ✓ the service time requirements at each device conform to an *exponential distribution*;
  - ✓ the system is *ergodic*, i.e. long-term time averages converge to the values computed for stochastic equilibrium

Prof. Vittoria de Nitto Personè

2

2

Molte assunzioni erano "forzate" rispetto al caso in analisi.

## Analytical models

Some of these concepts are difficult and **cannot be proved to hold by observing the system in a finite time period**  
Most can be disproved empirically

- ✓ parameters change over time
- ✓ jobs are dependent
- ✓ systems are observable only for short periods
- ✓ ...

## Analytical models

In applying or validating the results of Markovian QN theory, **analysts substituted operational values for stochastic parameters**

**directly measured**

The Markovian QN equations are also valid among operational variables

They hold under different assumptions and **apply to a large class of real systems**<sup>1</sup>

<sup>1</sup>

BUZEN, J.P. "Operational analysis: the key to the new generation of performance prediction tools," in Proc. IEEE COMPCON, 1976, IEEE, New York.  
DENNING, P. J.; AND BUZEN, J. P. "Operational analysis of queueing networks," in Proc. Third Int. Symp. Computer Performance Modeling, Measurement, and Evaluation, 1977, North-Holland Publ. Co., Amsterdam, The Netherlands.

Operational Analysis

## Operational Analysis

Three **operational principles**:

1. All quantities should be *precisely measurable* and all assumptions should be *directly testable*
2. The system must be *flow balanced*
3. The devices must be *homogeneous*,  
i.e., the routing must be independent of queue lengths ( $q_i$ )  
the mean service time at a given device must not depend on  $q_i$  of other devices

**the same mathematical equations**

but the operational assumptions can be tested

↓

much more confidence and understanding of the QN technology

Prof. Vittoria de Nitto Personè 5

5

Una ipotesi è testabile operazionalmente se può essere misurata da misure sul sistema

Analytical models  
Operational analysis

## Operational Analysis

Def.  
Hypotheses whose veracity can be established beyond doubt by measurement will be called *operationally testable*

Operational analysis provides a rigorous mathematical discipline for studying CS performance based solely on operationally testable hypotheses

two basic components:

- a system (real or hypothetical)
- a (finite) time period

↘ the observation period

Prof. Vittoria de Nitto Personè 6

6

Le componenti sono il sistema su cui faccio le misure, e un intervallo di osservazione per il sistema.

Analytical models  
Operational analysis

## Operational Analysis

According to the *operational* approach, let us consider

**Basic quantities**

$T$	the length of the observation period (op)
$A$	the number of arrivals during op
$B$	the total amount of time during which the system is busy $B \leq T$ during op
$C$	the number of completions during op

$A/T$  = frequenza di arrivo, cioè  $\lambda$   
 $C/T$  = frequenza di uscita, cioè  $X$  (ma per ora non lo chiamiamo throughput)  
 $B/T$  = utilizzazione  $U$   
 $B/C$  = servizio medio  $S$

Prof. Vittoria de Nitto Personè 7

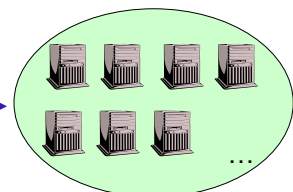
7 Osserviamo che:  $U = C/C \cdot B/T = C/T \cdot B/C = X \cdot S$ , legge dell'utilizzazione.  
 Non ho fatto nessuna assunzione di equilibrio tra flussi, possiamo infatti derivare l'utilizzazione per una certa finestra, senza bisogno dell'equilibrio.  
 E' valida per ogni finestra temporale. Ma cosa vuol dire bilanciamento dei flussi?  
 Sarebbe  $A=C$ , tutto ciò che arriva è uguale a ciò che esce nella stessa finestra temporale  $T$ .  
 Essa è una condizione facilmente testabile.  
 Tale assunzione è detta "job flow balance", allora  $A/T = C/T$ , allora  $\lambda = X$ , ovvero solo sotto condizione di job balance. Allora l'utilizzazione è scrivibile come  
 $U = \lambda \cdot S$ , probabilità che ad un certo istante di tempo il sistema sia occupato.

Non è detto che il JFB valga per ogni finestra temporale, tuttavia si può fare sempre questa assunzione, senza particolari errori. Esso è valutabile con  $(A-C)/C$ , se  $A=C$  ovviamente ho errore 0, e cresce se  $A$  si discosta da  $C$ .

## Multi Resources Systems 1

- a collection of servers, working together for incoming requests
- homogeneous servers

*requests* →



Data centers  
Server farm

*What is the minimum number of servers needed to guarantee that only a small fraction of jobs are delayed?*

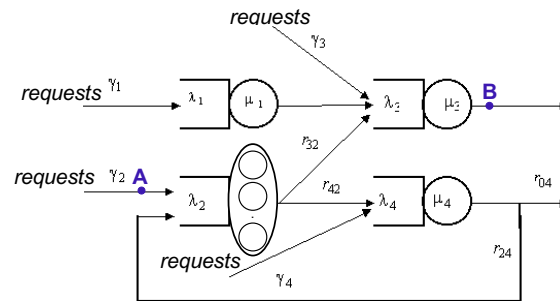
*Is a single central queue superior to having a queue at each server?*

Prof. Vittoria de Nitto Personè 8

8

## Multi Resources Systems 2

- a collection of different resources, connected, where incoming requests can require services more than once to each resources



Which is the "load" of each resource?

Which is the time spent on each resource?

Which is the time to go from point A to point B?  
(end-to-end response time)

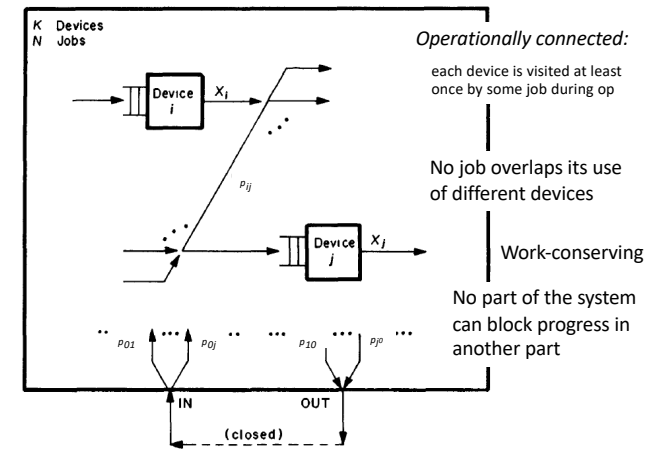
Prof. Vittoria de Nitto Personè

9

9

Operational Analysis

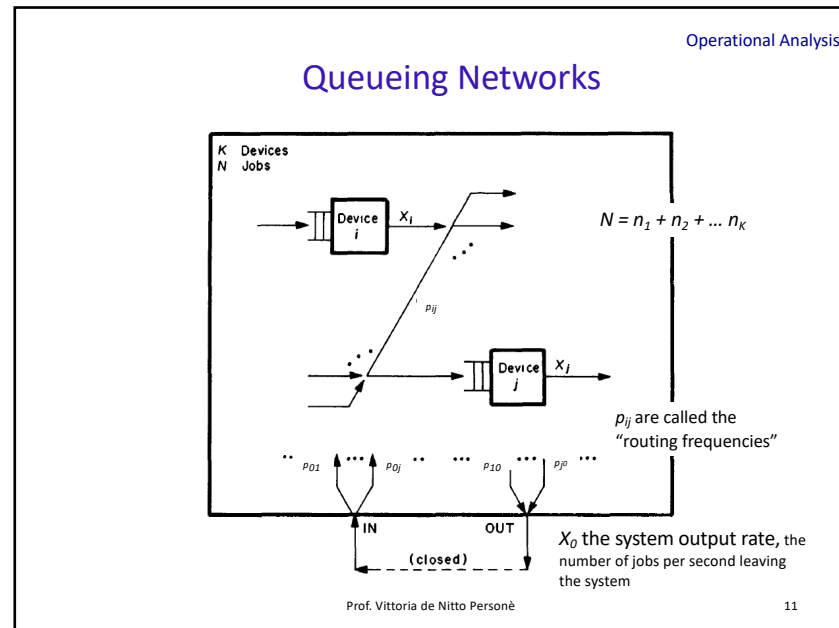
## Queueing Networks



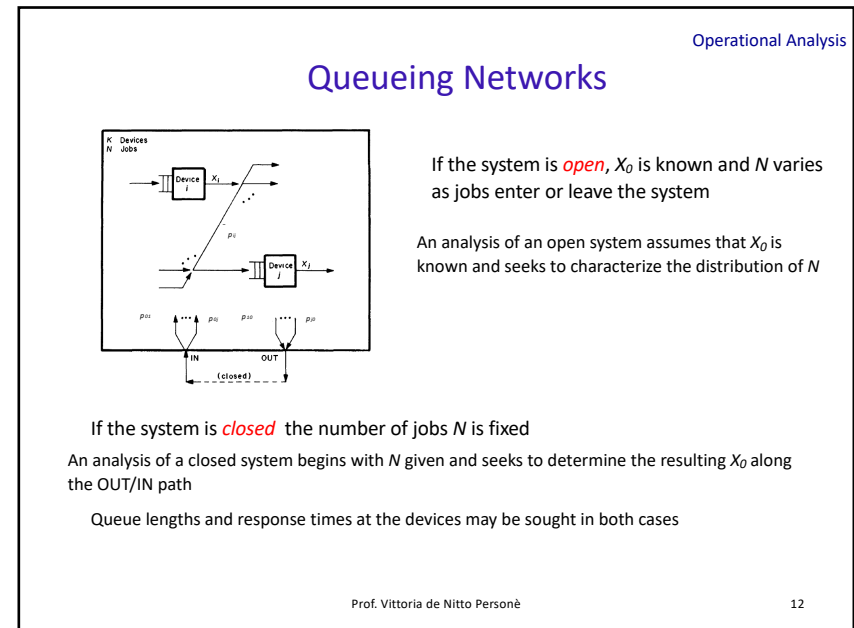
Prof. Vittoria de Nitto Personè

10

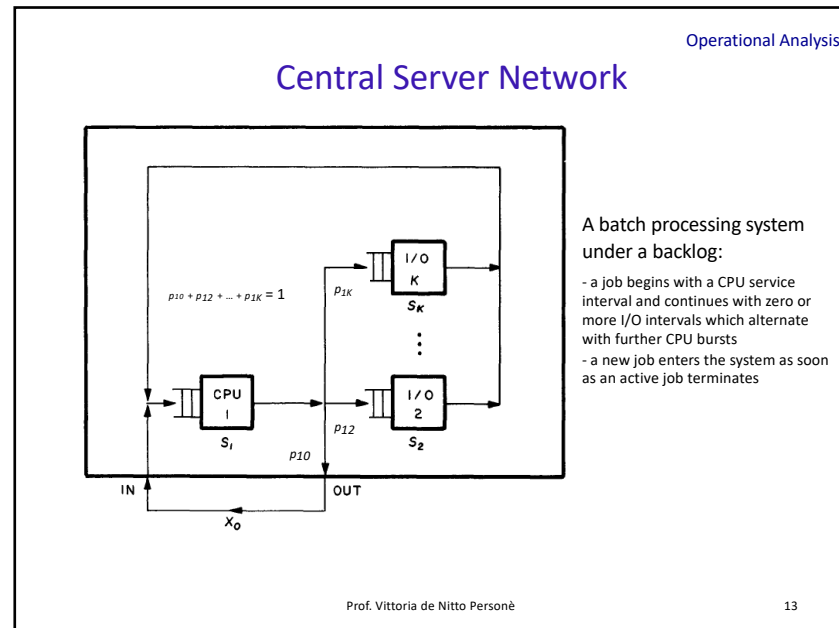
10



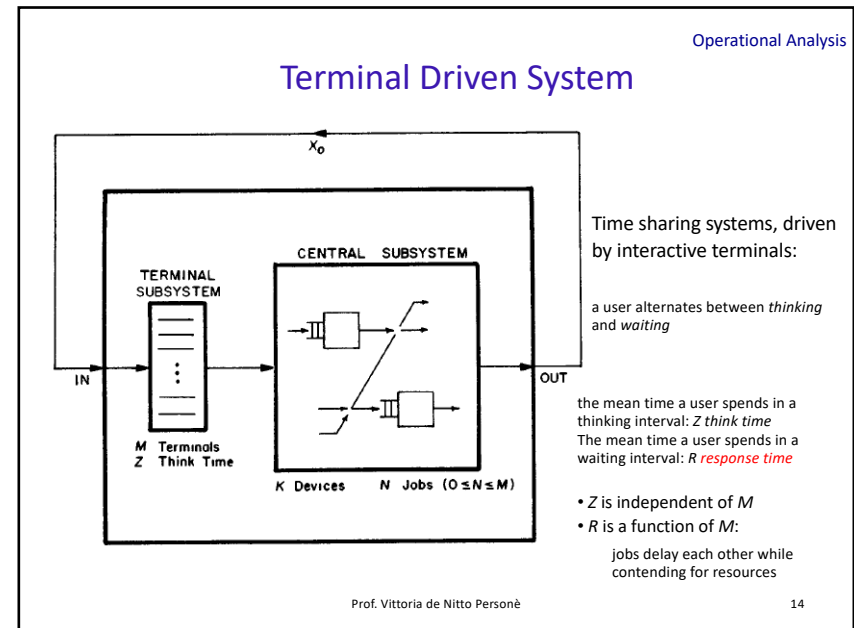
11



12



13



14

## Queueing Networks

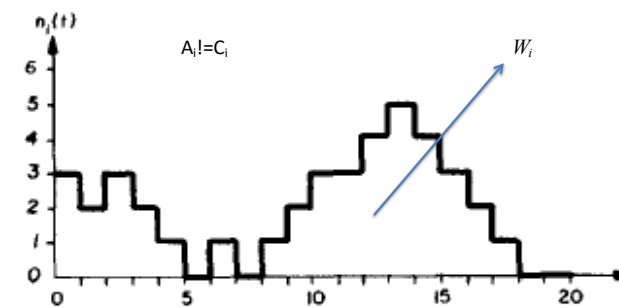
The system is measured for an op of  $T$  seconds, the following data are collected for each device  $i=1, 2, \dots, K$

- $A_i$  the number of arrivals;
- $B_i$  total busy time, during which  $n_i > 0$
- $C_{ij}$  number of times a job requests service at device  $j$  immediately after completing a service request at device  $i$ ;  
note that it is possible  $C_{ii} > 0$ .

If we treat the “outside world” as device “0”, we can define also

- $A_{0j}$  number of jobs whose first service request is for device  $j$ ;
- $C_{i0}$  number of jobs whose last service request is for device  $i$ .

## Queueing Networks





Operational analysis

## Operational Equations

$$X_j = \sum_{i=0}^K X_i p_{ij} \quad \text{Job Flow Balance equations}$$

$$\begin{cases} V_0 = 1 \\ V_j = p_{0j} + \sum_{i=1}^K V_i p_{ij} \end{cases} \quad \text{Visit Ratio equations}$$

Prof. Vittoria de Nitto Personè

17

17

Operational analysis

## Operational Equations

$$U_i = X_i S_i \quad \text{Utilization Law}$$

$$\bar{n}_i = X_i R_i \quad \text{Little's Law}$$

$$X_0 = \sum_{i=0}^K X_i p_{i0} \quad \text{Output Flow Law}$$

$$R = \sum_{i=1}^K V_i R_i \quad \text{General Response Time Law}$$

$$R = M / X_0 - Z \quad \text{Interactive Response Time Formula} \\ \text{(Assumes flow balance)}$$

Prof. Vittoria de Nitto Personè

18

18

