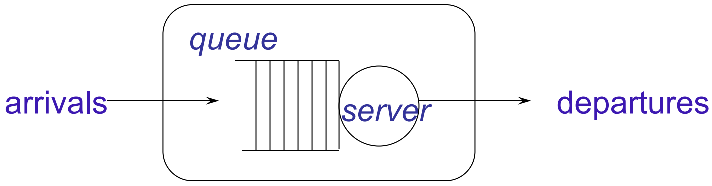
Single server queue



Consider n=10 job and given arrival and service times:

Arrival times:

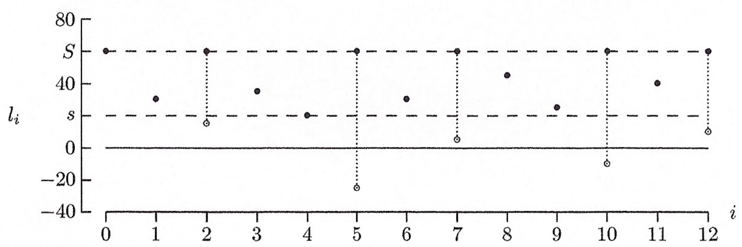
15 47 71 111 123 152 166 226 310 320

Service times:

43 36 34 30 38 40 31 29 36 30

A simple inventory system

i :	1	2	3	4	5	6	7	8	9	10	11	12
d_i :	30	15	25	15	45	30	25	15	20	35	20	30



Cerco usare distribuzioni di probabilità adatte al caso di studio, ma vorrei poter generare anche tante tracce (quindi insiemi di risultati reali, tirati fuori dal sistema). Cioè vorrei poter generare tante altre tracce, ovvero, partendo da un valore random tra 0 a 1, e trasformandolo in una distribuzione di probabilità.

C'è differenza tra variabile random ("reale") e variata random (estratta da generatore)

- ssq1 and sis1 require input data from an outside source
- The usefulness of these programs is limited by amount of available data:
  - What if more data needed?
  - What if the model changed?
  - What if the input data set is small or unavailable?

↓

Random number generator

- It produces real values between 0.0 and 1.0
- The output can be converted to random variate via mathematical transformations

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Performance Modeling of Computer Systems and Networks

Prof. Vittoria de Nitto Personè

Random Number Generators

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Historically there are three types of generators
- table look-up generators (1950)
- hardware generators
- algorithmic (software) generators

Algorithmic generators are widely accepted because they meet all of the following criteria:
- *randomness* - output passes all reasonable statistical tests of randomness
- *controllability* - able to reproduce output, if desired
- *portability* - able to produce the same output on a wide variety of computer systems
- *efficiency* - fast, minimal computer resource requirements
- *documentation* - theoretically analyzed and extensively tested

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Algorithmic Generators

- An *ideal* random number generator produces output such that *each* value in the interval $0.0 < u < 1.0$ is *equally likely* to occur
  equamente distribuiti, anche se tra 0 e 1 ci sono infiniti numeri.
- A *good* random number generator produces output that is (almost) statistically indistinguishable from an ideal generator

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Breve digressione su articolo "MANET Simulation Studies: The incredibles"

Definisco "m", intero primo. Ipotizzo di avere un'urna in cui dentro ci sono numeri da "1" a "m-1". Quando serve "u", compreso tra "1" e "m-1", estraggo un valore "x" dall'urna, e definisco u = x/m.
I possibili valori sono quindi 1/m, 2/m,.... (m-1)/m
Più "m" è grande, più l'insieme è denso nell'intervallo (0,1).
Tuttavia parto da un insieme infinito e devo passare ad uno finito, perchè se ad esempio il numero che voglio si trova tra "2/m" e "3/m", devo approssimare a "3/m", quindi c'è sempre un errore intrinseco.

Conceptual Model

- Choose a *large* positive integer *m*>0. This defines the set

  $\chi_m = \{1, 2, \ldots m-1\}$
- Fill a (conceptual) urn with the elements of χ_m
- Each time a random number *u* is needed, draw an integer x "at random" from the urn and let *u* =x/*m*

- Each draw *simulates* a sample of an independent identically distributed sequence of *Uniform*(0, 1)

- The possible values are 1/*m*, 2/*m*, … (*m*-1)/*m*

- It is important that *m* be large so that the possible values are densely distributed between 0.0 and 1.0

Conceptual Model

- 0.0 and 1.0 are impossible

  This is important for some random variates

- the same probability for each draw→ replacement of the drawn element

- for practical reasons, we will draw without replacement

  If *m* is large and the number of draws is small relative to *m*, then the distinction is largely irrelevant

Noi usiamo generatore di Lehmer, che, dato un x, ne genera un altro mediante: g(x) = ax mod m   0 < g(x) < m (0 escluso perchè m primo)

m = modulo; a = moltiplicatore; x0 = seme iniziale

Lehmer Generator

- is defined in terms of two fixed parameters:
  - *modulus m*, a fixed large prime integer
  - *multiplier a*, a fixed integer in χ_m
- the possible values are 1/*m*, 2/*m*, … (*m*-1)/*m*

The integer sequence x_0, x_1, … is defined by the iterative equation

$$x_{i+1} = g(x_i)$$

with

$$g(x) = ax \bmod m$$

$x_0 \in \chi_m$ is called *initial seed*

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Esempio: m = 7; a = 3; x0 = 1 χ_7 = {1,2,3,4,5,6}

avrò quindi x0 =1, x1= 3*1 mod 7 = 3;   x2=2 ; x3 = 6; x4 = 4 ; x5= 12 mod 7 = 5;   x6 = 15 mod 7 = 1

E' moltiplicatore full-period, perchè ho generato tutti i numeri tra 1 e 6

- Because of the mod operator, 0 ≤ g(x) < *m*
- 0 must not occur
  - since *m* is prime, g(x) ≠ 0 if $x \in \chi_m$
  - if $x_0 \in \chi_m$, then $x_i \in \chi_m$ for all i≥0
- IF the multiplier and prime modulus are chosen properly, a Lehmer generator is statistically indistinguishable from drawing from χ_m with replacement
- NOTE, there is nothing random about a Lehmer generator

→ pseudo-random generator

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Se a=2, x0 = 1 abbiamo: x0=1, x1=2, x2=4, x3=1 quindi rinizia la sequenza, non è full period.
Se a=4, x0 = 1 abbiamo: x0=1, x1=4, x2=2, x3=1 non è full period

Parameter Considerations

- the choice of *m* is dictated, in part, by system considerations
  - on a system with 32-bit 2's complement integer arithmetic, $2^{31}-1$ is a natural choice (it is prime!)
  - with 16-bit or 64-bit integer representation, the choice is not obvious (the maxes are not prime)
  - in general, we want to choose *m* to be the largest representable prime integer

- Given *m*, the choice of *a* must be made with great care

Come abbiamo visto prima, la scelta di 'a' è fondamentale.

- For a chosen (a, *m*) pair, does the function g(·) generate a **full-period** sequence?

- If a full period sequence is generated, how random does the sequence appear to be?

- Can *ax* mod *m* be evaluated efficiently and correctly?
  - Integer overflow can occur when computing *ax*

ax generalmente molto grande, quindi quando lo calcolo devo evitare overflow.

Full Period Multipliers

- If we pick any initial seed $x_0 \in \chi_m$ and generate the sequence x_0, x_1, x_2, … then x_0 will occur again
- Further x_0 will reappear at index p that is either $m - 1$ or a divisor of $m - 1$

We are interested in choosing full-period (FP) multipliers where p = m-1

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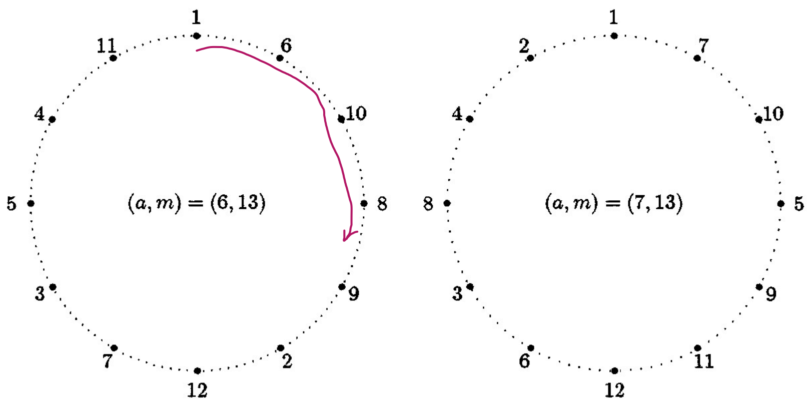
Full-period multipliers generate a virtual circular list with m-1 distinct elements. genero i primi m-1=12 interi, in ordine diverso.



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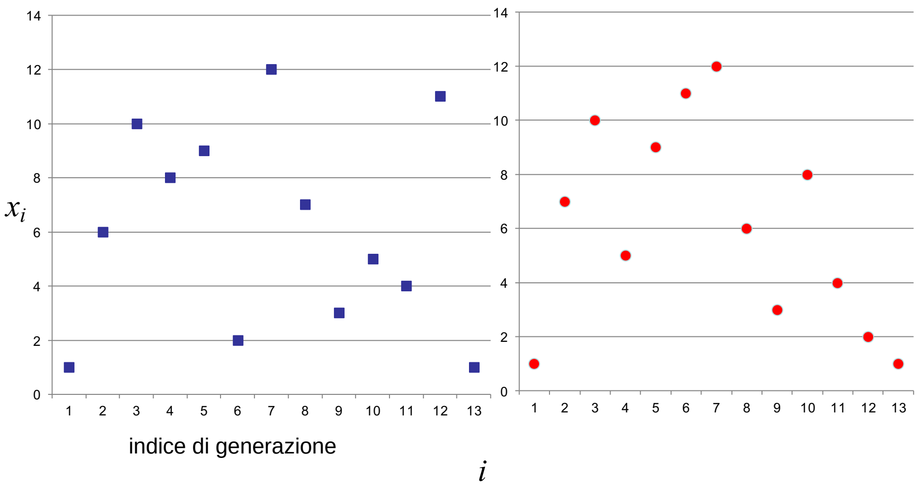
Se uso una 'sottosequenza' per fare qualcosa, non devo utilizzarlo anche per altro.

graficamente sparsi, sembra random!

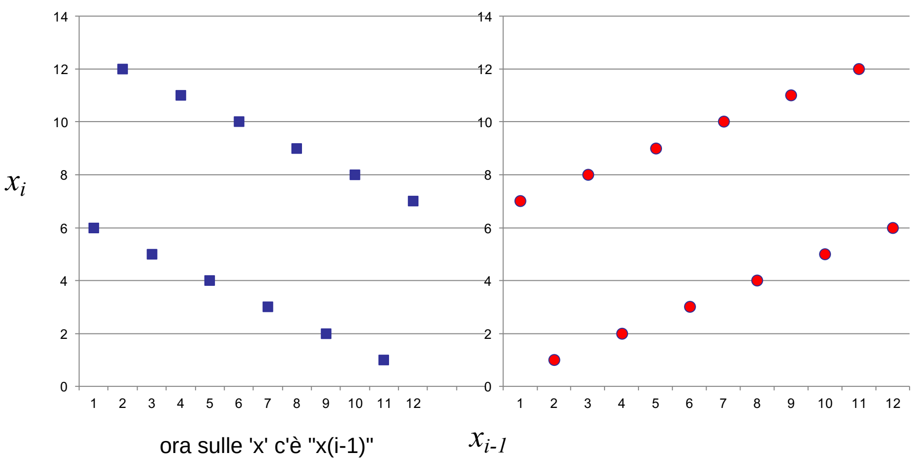
m=13

a=6, x_0=1 $\qquad$ a=7, x_0=1

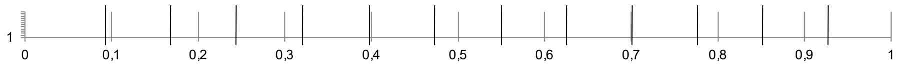


m=13

a=6, x_0=1 $\qquad$ a=7, x_0=1



Le "barre più lunghe" sono i 12 numeri di prima divisi 'm'=13



0,076923077	0,461538462	0,769230769	0,615384615	0,692307692	0,153846154
0,923076923	0,538461538	0,230769231	0,384615385	0,307692308	0,846153846

se mi servisse un numero vicino a 0,076923077 ma diverso da 0,076923077;
sarei obbligato ad utilizzare 0,076923077.

0,076923077	0,538461538	0,769230769	0,384615385	0,692307692	0,846153846
0,923076923	0,461538462	0,230769231	0,615384615	0,307692308	0,153846154

12 **6** **3** **8** **4** **2**

questi sono i numeri "interi" abbiamo 12 6 3 e 8 4 2, ovvero sono multipli tra loro.