SAMPLE STATISTICS

#24

Cosa facciamo coi dati raccolti? voglio stat. utili.

PRIME STATISTICHE da ottenere per campione ben definito

- media campionaria $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ (tendenza centrale)
- Varianza: $s^2 = \frac{1}{n} \sum_{i=1}^{n} x_i$ (discostamento da media)
- etc...
- deviazione std $d(x) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - x)^2} = s$ (quanto si disperde)

se $x = \bar{x}$ ottengo d_{min}

DISUGUAGLIANZA CHEBYCHEV (lega media e varianza)

dato campione di n elementi, media $\bar{x}$ e std dev. s,

ho set $S_K = \{ x_i \mid \bar{x} - Ks < x_i < \bar{x} + Ks \}$,

di proporzione $P_K = \frac{|S_K|}{n}$: $\boxed{P_K \geq 1 - \frac{1}{K^2}}$ con $K > 1$

Posso usare trasformazioni dati, tipo $x_i' = a x_i + b$,

trovando media $a\bar{x} + b$, varianza $a^2 s^2$ e stdvar $|a|s$

(simile a convertire sec in minuti $x' = \frac{300 \text{ sec}}{60 \text{ sec}} = 5 \text{ min}$)

LINEAR STANDARDIZZAZIONE (dati grandi $\rightarrow \emptyset < x < 1$)

uso $a = \frac{1}{s}$ e $b = -\frac{\bar{x}}{s}$ $\longrightarrow$ $x_i' = \frac{x_i - \bar{x}}{s}$ $\rightarrow \bar{x}' = \emptyset$, $s' = 1$

esistono anche NON LINEAR, per vedere comportamento, no dati.

ESERCIZI

Consider a web server with the following system characteristics:
- Single processor with capacity 10^5 op./sec
- Exponential mean service demand $4x10^4$ op./job
- System utilization 60%.

By knowing the job size, the service provider adopts a simple Size Based - priority scheduling without preemption: jobs with size less (or equal) than the average will have the highest priority (class 1); jobs with size greater than the average have the lowest priority (class 2). Determine:

a. the mean response time for both classes and the global mean response time.

The service provider wants to investigate if a dual core server would improve the service performance.

b. Conjecture the behaviour of the performance measures for both classes, by writing the mean waiting and response time definition for the dual core case.

→ solo formule, non calcoli.

DATI

- $C = 10^5$ op/s • $Z = 4 \cdot 10^4$ op/Job • $\rho = 0.6 = \lambda E[S]$
- $E[S] = \frac{Z}{C} = 0.4 \left[\frac{J}{S}\right]$, $\mu = \frac{1}{E[S]} = 2.5$; $\lambda = \frac{\rho}{E[S]} = 1.5$ • SizeB_NP

ho classe 1 se size Job $\leq E[S]$, altrimenti classe 2.

Il testo chiede $E[T_{S_i}]^{SB\_NP} = E[T_{Q_i}]^{SB\_NP} + E[S_i]$

$E[S_i] = \int_{X_{K-1}}^{X_K} t \cdot f^n(t)$ dove $[X_{K-1}, X_K]$ sono le size delle classi, e $f^n(t)$ è la funzione di densità degli arrivi normalizzata.

In questo caso $f^n(t) = \dfrac{\mu e^{-\mu t}}{\underbrace{F(\frac{1}{\mu}) - F(0)}_{p_1}}$ per classe 1.

$$p_1 = \left[1 - e^{-\mu t}\right]_{\emptyset}^{1/\mu} = 0{,}6312$$

$$p_1 + p_2 = 1$$

$$p_2 = \left[1 - e^{-\mu t}\right]_{1/\mu}^{\infty} = 0{,}36788$$

Calcolo $E[S_1]$ ed $E[S_2]$:

$$E[S_1] = \int_0^{0.4} t \cdot \frac{2.5\, e^{-2.5t}}{p_1} = 0{,}167453 \ s$$

$$E[S_2] = \int_{\frac{1}{\mu}=0.4}^{\infty} t \cdot \frac{2{,}5 \cdot e^{-2{,}5t}}{p_2} = 0{,}7999999 \ s$$

controllo: $p_1 E[S_1] + p_2 E[S_2] = 0{,}1056963 + 0{,}29430 \simeq 0{,}4$

Adesso calcolo $E[T_{a_1}]$ ed $E[T_{a_2}]$

$$E[T_{a_K}]^{SB\_NP} = \frac{\frac{\lambda}{2} E[S^2]}{\left(1 - \sum_{i=1}^{K} \rho_i\right)\left(1 - \sum_{i=1}^{K-1} \rho_i\right)}$$

avrò $E[T_{a_1}] = \dfrac{\frac{\lambda}{2} E[S^2]}{1-\rho_1}$, $E[T_{a_2}] = \dfrac{\frac{\lambda}{2} E[S^2]}{(1-\rho)(1-\rho_1)}$

Poiché sto in contesto esponenziale + unico servente

$$\frac{\lambda}{2} E(S^2) = \rho E[S]$$

$$\rho_K = \lambda \int_{x_{K-1}}^{x_K} t \cdot f(t)\,dt$$

↳ densità exp NON normalizzata

$$\rho_{K=1} = \rho_1 = \lambda \int_0^{1/\mu} t \cdot \mu e^{-\mu t}\,dt = 0{,}158545 = \lambda \cdot p_1 \cdot E[S_1]$$

PROBABILITÀ

$$\rho_2 = \lambda \int_{1/\mu}^{\infty} t \cdot \mu e^{-\mu t}\,dt = 0{,}441455 = \lambda \cdot p_2 \cdot E[S_2]$$

VERIFICA: $\sum_{i=1}^{K} \rho_i = 0.6 = \rho = \lambda \int_0^{\infty} t \cdot f(t)\,dt$

Trovo anche λ_1 e λ_2

$\lambda_1 = \lambda \cdot p_1 = 0{,}9468 \ \frac{op}{job}$ $\qquad$ $\lambda_2 = \lambda \cdot p_2 = 0{,}5532 \ \frac{op}{job}$

$\lambda_1 + \lambda_2 = \lambda$ ✓

$$E[T_{q_1}] = \frac{\rho E(S)}{(1 - 0.158545)} = 0{,}28522$$

$$E[T_{q_2}] = \frac{\rho E[S]}{(1 - 0.158545)(1 - 0.6)} = 0{,}71305$$

OK

allora

$$E[T_{S_1}] = \underbrace{0{,}28522}_{E[T_{q_1}]} + \underbrace{0{,}167453}_{E[S_1]} = 0{,}452673 \quad 1$$

$$E[T_{S_2}] = 0{,}71305 + 0.79999 = 1{,}51304 \quad 1$$

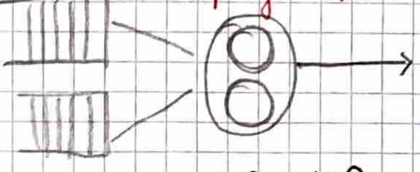
$$E[T_q] = p_1 E[T_{q_1}] + p_2 E[T_{q_2}] = 0{,}443003 \quad 1$$

$$E[T_S] = E[T_q] + E[S] = 0{,}843003 \quad 1$$

b) Passo ad un dual core, mantenendo le due classi.
Poichè sto in un contesto di scheduling astratto (le classi internamente sono FIFO) posso usare ERLANG.

Alcune osservazioni:

NB: FORSE invece che ρ_1 del punto a) dovevo usare $\rho_1 = \left(\frac{\lambda_1}{m}\right)E[S_1]$, non è stato ben spiegato.



SB_NP

- NON HO PRELAZIONE, quindi:

$$E[T_Q] = \rho_1 \cdot \underbrace{\boxed{\frac{P_Q\, E[S]}{1-\rho_1}}}_{\text{attesa di un job di classe 1 in coda}} + \rho_2 \cdot \frac{\overbrace{P_Q\, E[S]}^{\text{NON interrotti}}}{(1-\rho_1)(1-\rho)}$$

ovvero sia $E[T_{Q,1}]$ che $E[T_{Q,2}]$ hanno P_Q, poichè i job non vengono interrotti. Fosse stato preemptive, $E[T_{Q,1}]$ avrebbe avuto P_{Q1} (in funzione di ρ_1)

- Poichè ho 2 core, $E[S_1] = \frac{E[S_i]}{m}$, ovvero ogni core è dimezzato, con tempi servizi raddoppiati; $E[S_1]=E[S_2]=2E[S]=0.8$

- in $E[T_Q]$, $E[S]=0.4$, NON uso $E[S_i]$, perchè mi interessa tempo di servizio generale (entra job in S_1 e magari esce job da S_2), mentre con $E[S_i]$ guarderei entrate-uscite nel singolo core "i".

- $$P_Q = \frac{(m\rho)^m}{m!(1-\rho)}\, p(0) = \frac{(2\cdot 0.6)^2}{2!(1-0.6)} \cdot \left[\sum_{i=0}^{m-1} \frac{(m\rho)^i}{i!} + \frac{(m\rho)^m}{m!(1-\rho)}\right]^{-1}$$

$$= \frac{(2\cdot 0{,}6)^2}{2(1-0.6)}\left[1 + 2\rho + \frac{(2\rho)^2}{2(1-\rho)}\right]^{-1} = 0{,}45\ ; \quad \underset{0.45}{P_Q} \cdot \underset{0.4}{E[S]} = 0{,}18$$

$$\rightarrow E[T_Q] = \underbrace{\frac{P_Q \cdot E[S]}{(1-\rho_1)}}_{0.158545}\left[\underset{0.6312}{\rho_1} + \frac{\overset{1-0.6312}{\rho_2}}{\underset{\hookrightarrow 0.6}{(1-\rho_1)}}\right] = 0{,}33176$$

$$E[T_S] = E[T_Q] + \overset{0.4}{E[S]} = 0{,}73176$$

1. Il responsabile di uno sportello comunale per il rilascio di certificati anagrafici vuole investigare le prestazioni del servizio. Analizzando lo storico dell'attività, si desume che una distribuzione uniforme *Uniform*(2, 15)[1] può ben caratterizzare il tempo di servizio (espresso in min). Gli utenti, identificati con la propria richiesta, arrivano in modo random con frequenza 0.112 req/min. Si assuma che sia possibile conoscere il tempo di servizio della pratica all'istante di arrivo. Si calcolino i seguenti indici:

SB →

- 1.a. tempi di attesa e risposta per una pratica qualsiasi;
- 1.b. i tempi di attesa e risposta per classi e globali assumendo di usare un meccanismo prioritario opportunamente scelto (senza prelazione);
- 1.c. lo *slowdown* condizionato, per richieste di 5 min e di 10 min, nel caso 1.a;
- 1.d. lo *slowdown* condizionato, per richieste di 5 min e di 10 min, nel caso 1.b;

Si commenti al riguardo del vantaggio della soluzione al punto 1.b. Indicare le assunzioni utilizzate per la soluzione.

$Unif(\underset{a}{2}, \underset{b}{15})$ $\qquad \lambda = 0.112$ req/MIN

Abbiamo a che fare con una UNIFORME!

$f(x) = \frac{1}{b-a}$; $F(x) = \frac{x-a}{b-a}$; $\underset{E[S]}{media} = \frac{a+b}{2}$; $\sigma^2 = \frac{(b-a)^2}{12}$

SVOLGIMENTO

$$E[T_a] = \frac{\frac{\lambda}{2} E[S^2]}{1-\rho}$$

NON è Exp, non uso $\rho E[S]$!

→ formula M/G/1

$$\sigma^2 = E[S^2] - (E[S])^2 = \frac{(b-a)^2}{12} = 14,0833 \quad ; \quad E[S] = \frac{15+2}{2} = 8,5 \text{ min}$$

allora $E[S^2] = \sigma^2 + (E[S])^2 = 14.0833 + (8.5)^2 = 86.3333$

Da Little $\rho = \lambda E[S] = 0.952$

$$E[T_a] = \frac{\frac{0.112}{2} \cdot 86.3333}{1 - 0.952} = 100,722 \text{ min}$$

$$E[T_s] = E[T_a] + E[S] = 109,2222 \text{ min}$$

1.6) Il meccanismo prioritario NP da considerare è SIZE BASED. Posso dividere le classi partendo da $E[S]=8.5$ ovvero classe 1 $[2, 8.5)$; classe 2 $[8.5, 15)$ (è il + semplice, potevo usare altri schedule o più classi)

Sappiamo che $E[S_i] = \int_{x_{k-1}}^{x_k} t \cdot \frac{f^N(t)}{p_i}$

$$p_1 = F(8.5) - F(2) = \frac{8.5-2}{15-2} - \frac{2-2}{15-2} = 0.5 = p_2$$

NB: F ed f sono relative all' UNIFORME!

$$f(t) = \frac{1}{13} \quad \text{(è costante)}$$

$$E[S_1] = \int_2^{8.5} t \cdot \frac{1}{13} \cdot \frac{1}{0.5} = \frac{1}{6.5} t^2 \Big|_2^{8.5} = 5.25 \text{ min}$$

$$E[S_2] = \int_{8.5}^{15} \frac{1}{6.5} \cdot t = 11.75 \text{ min}$$

$$\lambda_1 = \lambda \cdot p_1 = 0.056 = \lambda_2 \quad (p_1 = p_2)$$

$$\rho_1 = \lambda_1 E[S_1] = 0.294$$

$$\rho_2 = \lambda_2 E[S_2] = 0.658$$

$$\rho_1 + \rho_2 = 0.952 = \rho$$

$$E[T_{q_1}] = \frac{\frac{0.112}{2} \cdot 86.3333}{1 - 0.294} = 6.84 \text{ min}$$

$$E[T_{q_2}] = \frac{\frac{0.112}{2} \cdot 86.3333}{(1-0.294)(1-0.952)} = 142,6604 \text{ min}$$

$E[T_a] = 0.5(6.84) + 0.5(142.6604) = 74.75$ min < NO SB

$E[T_{s_1}] = E[T_{a_1}] + E[S_1] = 12,09797$

$E[T_{s_2}] = E[T_{a_2}] + E[S_2] = 154.41604$

$E[T_s]^{glob} = 0.5\, E[T_{s_1}] + 0.5 \cdot E[T_{s_2}] = 83.257005$

1.c)

$$sd = \frac{\overbrace{E[T_a]}^{\text{indip da } x} + x}{x} \quad \begin{cases} x = 5 \rightarrow sd(5) = 21.1444 \\ x = 10 \rightarrow sd(10) = 10,0722 \end{cases}$$

1.d) qui lavoro sulle due classi ($E(T_{a_1})$ ed $E(T_{a_2})$)

$$sd(x) = \frac{E[T_{a_1}] + x}{x} \rightarrow x = 5 \rightarrow sd(5) = 2.369594$$

$$sd(x) = \frac{E[T_{a_2}] + x}{x} \rightarrow x = 10 \rightarrow sd(10) = 15.266$$

NB: se $x = 5$ vo con $E[T_{a_1}]$, poiché comprende size da 2 a 8.5, $x = 10$ vo nella seconda classe (8.5, 15)

Non posso fare $x = 10$ con $E[T_{a_1}]$

Consider a web server with processing capacity $C = 10^5$ op/sec. The server receives requests with a mean rate 2 req/sec. The requests have different demand Z. Consider the following intervals:

- $Z < 20.000$ op
- $20.000 \text{ op} \leq Z < 40.000$ op
- $Z \geq 40.000$ op

By assuming that: Z = avg demand

i. the mean size is 40.000 op, characterized by an exponential distribution;
j. the arrival rate is characterized by a Poisson process;

Define a management mechanism of the server to satisfy the following QoS requirements:

1. Mean response time ≤ 1.5 s for all requests
2. Mean waiting time ≤ 0.5 s, for $Z < 40.000$ op.ni.

Evaluate

a. The mean *throughput* for the server with the chosen management mechanism;
b. The mean *conditional slowdown* for jobs with size x=0.1 s, 0.3 s
c. Compare the mean slowdown obtained in b. with the corresponding mean slowdown for FIFO and PS scheduling.

Please comment all the obtained results.

SVOLGIMENTO

$$E[S] = \frac{Z}{C} = \frac{4 \cdot 10^4}{10^5} = 0.4\,s \longrightarrow \rho = \lambda \cdot E[S] = 0.8$$

divido in 3 classi (basandomi sugli intervalli): Size Based

$C_1\ (0, 0.2)$ $\quad C_2\ (0.2, 0.4)$ $\quad C_3\ (0.4, \infty)$

$$0.2 = \frac{2 \cdot 10^4}{10^5}$$

calcolo $\mu = \frac{1}{E[S]} = 2.5$

$$P_1 = \left[1 - e^{-\mu t}\right]_0^{0.2} = 0.393469$$

$$P_2 = \left[1 - e^{-\mu t}\right]_{0.2}^{0.4} = 0.238651 \qquad \sum_{i=1}^{3} P_i = 1 \checkmark$$

$$P_3 = \left[1 - e^{-\mu t}\right]_{0.4}^{\infty} = 0.367879$$

$$E[S_1] = \int_0^{0.2} t \cdot \frac{\mu e^{-\mu t}}{p_1} = 0.0917013\ s$$

$$E[S_2] = \int_{0.2}^{0.4} t \frac{\mu e^{-\mu t}}{p_2} = 0.291701\ s$$

$$E[S_3] = \int_{0.4}^{\infty} t \frac{\mu e^{-\mu t}}{p_3} = 0.8\ s$$

- calcolo ρ

$$\rho_1 = \underset{\underset{2}{\shortparallel}}{\lambda} \int_0^{0.2} t \mu e^{-\mu t}\, dt = 0.0721632$$

$$\rho_2 = 2 \int_{0.2}^{0.4} t \cdot \mu e^{-\mu t} = 0.13923 \qquad \sum_{i=1}^{3} \rho_i = \rho = 0.8 \checkmark$$

$$\rho_3 = 2 \int_{0.4}^{\infty} t \mu e^{-\mu t} = 0.588607$$

- calcolo $E[T_{a_i}]$

$$E[T_{a_1}] = \frac{\rho E[S]}{1-\rho_1} = 0.34888\ s$$

$$E[T_{a_2}] = \frac{\rho E[S]}{(1-\rho_1-\rho_2)(1-\rho_1)} = 0.437338\ s$$

vincolo rispettato

$$E[T_{a_3}] = \frac{\rho E[S]}{(1-\rho)(1-\rho_1-\rho_2)} = 2.0288945\ s$$

non era richiesto

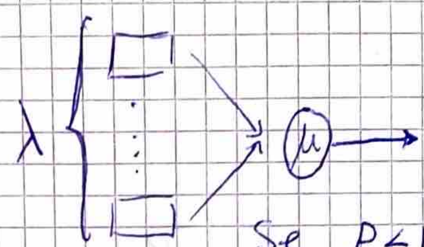
$E[T_Q] = \sum p_i E[T_{Q_i}] = 0{,}988032295$

$E[S] = \sum p_i E[S_i] = 0.4$

$E[T_S] = E[T_Q] + E[S] = 1.38803 < 1.5$ ✓

a) throughput medio

$\mu > \lambda$ e $\rho < 1$, stabile.



Se $\rho < 1 \rightarrow$ throughput è λ

b) sd (0.1) in $E[T_{Q_1}] = \dfrac{0.344888}{0.1} + 1 = 4{,}4488$

sd (0.3) in $E[T_{Q_2}] = \dfrac{0.437338}{0.3} + 1 = 2{,}4577$

c) sd FIFO $= \dfrac{\frac{\lambda}{2} E[S^2]}{x\,(1-\rho)} + 1 = \dfrac{\rho E[S]}{x(1-\rho)} + 1 = \dfrac{0.8 \cdot 0.4}{\underset{\overset{\parallel}{x}}{0.1}(0.2)} + 1 = 17$

se $x = 0.3$ ho $5.\overline{3} + 1 = 6.\overline{3}$

d) PS (fair, uguale $\forall$ job) $= \dfrac{1}{1-\rho} = \dfrac{1}{1-0.8} = 5$

1.2. Consider a single-core server hosting a web service. Requests arrive to the server according to a Poisson, with an average inter-arrival time of 200 ms.

1. a. Knowing that the maximum buffer size is N = 4 (including the jobs in service) and that each request requires on average 200 ms of processing time, compute the throughput of the system.
2. b. Consider a CPU upgrade to a faster single-core processor which can process a request in 150 ms. Compute the throughput of the upgraded system.
3. c. Consider a CPU upgrade to a slower quad-core processor, which can process a request in 300 ms using one of its processor cores. Compute the throughput of the upgraded system.

SVOLGIMENTO

- interarrivo $= 200 ms \longrightarrow \lambda = \frac{1}{200} = 0{,}005 \left[\frac{J}{ms}\right]$
- $N = 4$ $\quad E[S] = 200 ms \longrightarrow \mu = \frac{1}{E[S]} = 0{,}005 \left[\frac{J}{ms}\right]$

1) $\lambda' = \lambda(1 - P_{LOSS})$ è throughput (capacità finita, non è solo λ)

essendo SINGLE CORE, con $C = 4$ $\quad (\lambda = \mu)$

$$\Pi_4 = \left(\frac{\lambda}{\mu}\right)^4 \Pi_0 \quad \text{con } \Pi_0 = \frac{1}{\sum_{i=0}^{4} \left(\frac{\lambda}{\mu}\right)^i} = \frac{1}{5}$$

allora $\Pi_4 = 1^4 \cdot \frac{1}{5} = \frac{1}{5} = P_{LOSS}$

e $\lambda' = 0.005\left(1 - \frac{1}{5}\right) = 0.004\ J/ms = 4\ J/s$

2) request in 150 ms $\rightarrow \mu = 0.00666666$

tranne che per μ, lo calcolo come prima:

$$\pi_0 = \frac{1}{\sum^{4} \left(\frac{0.005}{0.00\overline{6}}\right)^i} = 0.327785$$

$$\pi_4 = P_{LOSS} = \pi_0 \cdot \left(\frac{0.005}{0.00\overline{6}}\right)^4 = 0.103713$$

$$\lambda' = 0.005(1 - 0.1343798) = 0.00448193\ J/ms = 4{,}48\ J/s$$

3) qui ho 4 core (= 4 serventi), ciascuno ha req. 300 ms

$$-\ \mu = 0.003333 \left[\frac{J}{ms}\right] \qquad \rho = \frac{\lambda}{m\mu} = 0.375$$

non ho coda (1 job $\forall$ core). Uso erlang-B!

$$P_{LOSS} = \pi_4 = \frac{\left(\frac{\lambda}{\mu}\right)^4 / 4!}{\sum_{j=0}^{4} \left(\frac{\lambda}{\mu}\right)^j / j!} = 0.048$$

e quindi $\lambda' = \lambda(1 - P_{LOSS}) = 4{,}76\ J/S$

Sotto c'è monnezza!!

$$\rho = \frac{\lambda}{\mu} = 0.4$$

$$\frac{1}{\mu} = E[S] = 0{,}00001$$

esercizio **1**

- single processor capacity 10^5 op/s $= \mu$
- domanda media exp $4 \cdot 10^5$ op/job $= \lambda$
- syst. utilization $60\% = \rho$??
- Size based without preemption, 2 classi: $c_1 \to$ job $\leq E[S]$ ($= \frac{1}{\mu}$), $c_2 \to$ job $> E[S]$

a) mean response time per c_1 e c_2, global response time.

SVOLGIMENTO

$$E(T_{S_1})^{SB-NP} = E(T_{Q_1})^{SB-NP} + E[S_1]$$

$$\cdot E(S_1) = \int_{x_{k-1}}^{x_k} t \cdot \underbrace{f^n(t)}_{\text{NORMALIZZATA}}\, dt = \int_{x_{k-1}}^{x_k} t \cdot \frac{\mu e^{-\mu t}}{F(x_k) - F(x_{k-1})}$$

$$= \int_0^{1/\mu} t \cdot \frac{\mu e^{-\mu t}}{F(\frac{1}{\mu}) - F(0)} = \int_0^{1/\mu} t \cdot \frac{\mu e^{-\mu t}}{\rho_1}\, dt$$

[chi è "F"? è f. distr. arrivi $\to$ exp "$1 - e^{-\mu t}$"

$$\rho_1 = [1 - e^{-\mu t}]\Big|_{\emptyset}^{1/\mu} = 1 - e^{-\mu \frac{1}{\mu}} - [1 - 1] = 0{,}63212$$

$$\rho_2 = [1 - e^{-\mu t}]_{\emptyset}^{\infty} = 0{,}36788 \longrightarrow \text{check: } \boxed{\rho_1 + \rho_2 = 1}$$

$$\to E[S_1] = \int_0^{\frac{1}{10^5}} t \cdot \frac{(10^5) \cdot e^{-(10^5 \cdot t)}}{0{,}63212} = 4{,}18024 \cdot 10^{-6}$$

$$E[S_2] = \int_{\frac{1}{10^5}}^{\infty} t \cdot \frac{10^5 e^{-(10^5) t}}{0,36788} dt = \frac{10^5}{0,36788} \int_{\frac{1}{10^5}}^{\infty} \underbrace{t}_{f} \cdot \underbrace{e^{-10^5 \cdot t}}_{g'} dt$$

per parti

$$= 271828 \cdot \int_{\frac{1}{10^5}}^{\infty} e^{-10^5 t} \cdot t \, dt$$

per parti $\begin{cases} f = t \\ g' = e^{-10^5 \cdot t} \, dg \end{cases}$

$f' = 1\,dt \qquad g = \frac{-1}{10^5} \cdot e^{-10^5 t}$

$$= 271828 \left[t \cdot \frac{-1}{10^5} e^{-10^5 \cdot t} - \int_{\frac{1}{10^5}}^{\infty} 1 \cdot \frac{-1}{10^5} e^{-10^5 t} dt \right] = 0,00002$$

Check

$$E[S] = p_1 E(S_1) + p_2 E(S_2) \stackrel{?}{=} 10^{-5} \quad \text{Ok}$$

$$= [0,63212 \cdot 4 \cdot 18024 \cdot 10^{-6}] + [0.36788 \cdot 0.00002]$$

$$= 0,00001 \quad \text{Ok}$$

1.c) slowdown condiz. nel caso 1a per richieste di 5 min e 10 min

INDIP da size

$$Sd = \frac{(E[T_Q] + 5)}{5} = 21.1445$$

$5 = x$ (quanto chiede)

$$Sd = \frac{E[T_Q] + x}{x} \quad \Bigg| \; E[T_Q] = 100.722, x = 10 \quad = 10.0722$$

1.d) slowdown ~~con~~ nel caso 2b.

$$Sd = \frac{\overbrace{6.84}^{E[T_Q]} + 5}{5} = 2.369594 \quad \Bigg] \; Q_1$$

$$\cancel{Sd = \frac{6.84 + 10}{10} = 1,684}$$

$$\cancel{Sd = \frac{142.6604 + 5}{5} = 29.5338}$$

$$Sd = \frac{142.6604 + 10}{10} = 15.2666 \quad \Bigg] \; Q_2$$

es.3)

$\mu = 10^5$ op/s $= C$ $\quad \lambda = 2$ req/s, tali richieste

possono essere $\begin{cases} z < 20.000 \text{ op} \\ 20000 \le z < 40000 \text{ op} \\ z \ge 40000 \text{ op} \end{cases}$

assumo che:

- arrival rate è Poisson
- mean size è 40000 op (exp) = z (avg demand)

devo soddisfare: $\begin{cases} E(T_S) \le 1.5\,s\ \forall \text{req} \\ E(T_Q) \le 0.5\,s, \text{ se } z < 40000 \end{cases}$

- valuta throughput
- slowdown $x = 0.1s$, $0.3s$ (mean)
- comparalo con mean slowdown FIFO e PS.

$\frac{2 \cdot 10^4}{10^5} = 0.2\,s$ $\quad \rho = 2 \cdot 0,2 = 0,4$

$E[S] = 0.2$ $\quad \mu = 5$

$\rho = \lambda E[S]$

$E[S] = 0.4$ $\quad \mu = 2.5$

$E[S] > 0.4$

$\lambda = \frac{\rho}{E[S]}$

$E[S] = 0.4\,s$ $\quad \mu = 2.5$ $\frac{J}{s}$

$\rho = 0.8 = \frac{\lambda}{2.5}$ $\quad$??

Ora necessito di $E[T_{Q_1}]$ ed $E[T_{Q_2}]$

$$E[T_{Q_1}]^{SB-NP}_{K=1} = \frac{\frac{\lambda}{2}E[S^2]}{\left(1-\sum_{i=1}^{K}\rho_i\right)\left(1-\sum_{i=1}^{K-1}\rho_i\right)}$$

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2}E[S^2]}{1-\rho_1} \qquad E[T_{Q_2}] = \frac{\frac{\lambda}{2}E[S^2]}{(1-\rho)(1-\rho_1)}$$

$\frac{\lambda}{2}E(S^2) = \rho E(S)$ se EXP, ma con priority classes ho in realtà $\rho_{a_1}E[S]$ e $\rho_a E[S]$ perché c'è priorità, devo calcolare

- ρ_1 • ρ_{a_1} • ρ_a

$$\rho_{K=1} = \lambda\int_{x_{K-1}}^{x_K} t\cdot \underbrace{f(t)}_{\text{NON NORMALIZ.}}dt \quad \text{è RHO}$$

$$\rho_{K=} = \lambda\int_0^{1/\mu} t\cdot \mu e^{-\mu t}dt = (4\cdot 10^4)\int_0^{10^{-5}} t\cdot 10^5\cdot e^{-10^5 t}dt$$

0.1057

$$= 1{,}0570\cdot 10^{-6} = \underline{\lambda\cdot 0.63212\cdot E[S_1]} ✓$$

$$\rho_2 = 2{,}9430\cdot 10^{-6}$$

$$\sum_{i=1}^{K}\rho_i = \lambda\int_0^{x_k} tf(t) = 0.4 \quad \text{Consistente}$$

$$\left.\begin{array}{l}\lambda_1 = \lambda \cdot 0{,}63212 = 25284.8 \\ \lambda_2 = 14715.2\end{array}\right] = 4\cdot 10^4 \text{ OK}$$

$$E[T_{a_1}] = \frac{\rho E(S)}{(1-1.057\cdot 10^{-6})} = \cancel{4\cdot 10^{-6}} = 4{,}4727\cdot 10^{-6}$$

$$E[T_{a_2}] = \frac{\rho E(S)}{(1-1.057\cdot 10^{-6})(1-0.4)} = 7{,}4576\cdot 10^{-6}$$

$$E[T_{s_1}]^{SB\_NP} = 4{,}4727\cdot 10^{-6} + 4{,}18024\cdot 10^{-6} = 8{,}6529\cdot 10^{-6}$$

$$E[T_{s_2}] = = 7{,}4576\cdot 10^{-6} + 0{,}00002 = 27{,}4576\cdot 10^{-6}$$

$$E(T_a) = p_1 E(T_{a_1}) + p_2 E(T_{a_2}) = 5{,}5707\cdot 10^{-6}$$

$$E(T_s) = E(T_a) + \underbrace{E(S)}_{0.4} = 0.400005570785012$$

Con dual core? ho 2 core → uso erlang

$$E[T_a] = \underbrace{\rho_1 \frac{P_{a1} E[S]}{1-\rho_1}}_{\text{classe 1}} + \underbrace{\rho_2 \frac{P_a\, E[S]}{(1-\rho)(1-\rho_1)}}_{\text{classe 2}}$$

M/M/m

$m = 2$

$$P_a = \frac{(m\rho)^m}{m!\,(1-\rho)}\, p(0) \Big|_{\substack{\rho=0.4\\ m=2}} = 0.22857$$

$$p(0) = \sum = \left[1 + 2\rho + \frac{(2\rho)^2}{2(1-\rho)}\right]^{-1} \Big|_{\rho=0.4} = 0{,}428571$$

P_a con viene OK 0.22857

$$P_{a_1} = \frac{(m\rho_1)^m}{m!\,(1-\rho_1)}\, p(0) = 0.03430$$

$\rho = 0.141$

$\rho_1 = 0.14$

$p_1(0) = 0.80880$

$\rho = 0.10570$

$P_{a_1} = 0{,}02020$

$E[T_a] = 0{,}05134$ $\qquad E[T_s] = E[T_a] + 0.4 = 0{,}45134$

FINE

es. 2

- tempo servizio $\sim$ Unif $(\overset{a}{2}, \overset{b}{15})$ (in m)
- arrivi random $\lambda = 0.112$ req/min.
- Posso sapere tempo servizio all'istante di arrivo.

Calcolare

1) tempi attesa e risposta per pratica qualsiasi

NB: $f(x) = \frac{1}{b-a}$, $F(x) = \frac{x-a}{b-a}$, medio $= \frac{a+b}{2}$

Varianza $= \frac{(b-a)^2}{12}$

$$M/U/1$$

arrivi

$$E[T_a] = \frac{\frac{\lambda}{2} E[S^2]}{1-\rho} \quad \text{per } M/G/1$$

$\lambda = 0.112$; $E[S] = \frac{a+b}{2} = 8,5$ min

$$\sigma^2 = E[X^2] - E[X]^2 = \frac{(b-a)^2}{12} = 14.0833$$

$$\rightarrow E[X^2] = 14,0833 + (8,5)^2 = 86.3333 = E[S^2]$$

Da Little: $\rho = \lambda \cdot E[S] = 0,952$

$$\rightarrow E[T_a] = \frac{\frac{0.112}{2} \cdot 86.3333}{1 - 0,952} = 100,722 \text{ min}$$

$$\rightarrow E[T_s] = E[T_a] + E[S] = 109,2222$$

1.6) tempi attesa / risposta classi - globali con meccanismo prioritario a scelta NO prelazione.

Size based, 2 classi $\to$ (2, 8.5) ; (8.5, 15)

$$E(S_1) = \int_2^{8.5} t \cdot \frac{\mu e^{-\mu t}}{F(8.5)-F(2)} \quad \boxed{NO}$$

~~$F(8.5) - F(2) = 1 - e^{-\mu t}\Big|_2^{8.5}$~~

$$F(8.5) - F(2) = \frac{8.5-2}{15-2} - \frac{2-2}{15-2} = \frac{6.5}{13} = 0.5 = p_1$$

$$f(t) = \frac{1}{b-a} = \frac{1}{13}$$

$$E(S_1) = \int_2^{8.5} t \cdot \frac{f(t)}{\underbrace{F(8.5)-F(2)}_{p_1}} = \frac{1}{13} \cdot 0.5 \int_2^{8.5} t = \frac{0.5}{13} \frac{t^2}{2}\Big|_2^{8.5} = 5.25$$

$$E(S_2) = \int_{8.5}^{15} \frac{1}{13} \cdot t \cdot \frac{1}{\underbrace{0.5}_{p_2}} = \frac{1}{6.5} \cdot \frac{t^2}{2}\Big|_{8.5}^{15} = 11.75$$

$$\lambda_1 = \lambda \cdot p_1 = 0.056 = \lambda \cdot p_2 = \lambda_2$$

$$\rho_1 = \lambda_1 E[S_1] = 0.294$$

$$\rho_2 = \lambda_2 E[S_2] = 0.658$$

$\longrightarrow \rho = 0.952$

$$E[T_{q_1}] = \frac{\rho E[S]}{1-0.294} \rightarrow E(S) = 0.5(5.25 + 11.75)$$

No

solo se EXP

$$E[T_{q_1}] = \frac{\frac{\lambda}{2} E(S^2)}{1-0.294} =$$

$$\frac{\frac{0.112}{2}(86.333)}{1-0.294} = 6.84 \text{ min}$$

$$E[T_{q_2}] = \frac{\frac{\lambda}{2} E[S^2]}{(1-0.294)(1-\rho)} = \frac{E[T_{q_1}]}{1-0.952} = 142{,}6604 \text{ min}$$

$E[T_q] = 0.5(6.84 + 142{,}6604) = 74{,}75$ min < No SB

$\hookrightarrow P_1 = P_2$ (equiprob)

~~E[Ts] = E[Tq] + E~~

$$E[T_{s_1}] = E[T_{q_1}] + E[S_1] = 12{,}09797$$

$$E[T_{s_2}] = E[T_{q_2}] + E[S_2] = 154.41604$$

$\hookrightarrow E[T_s]^{glob} = 0.5[12.09797 + 154.41604]$

$= 83.257005$ ridotta

SVOLGIMENTO

$$E(S) = \frac{z}{C} = \frac{4 \cdot 10^4}{10^5} = 0.4\ s \longrightarrow \rho = \lambda \cdot E(S) = 2 \cdot 0.4 = 0.8$$

$C \rightarrow$ costante

necessito di 3 classi (ho 3 divisioni)

$C_1\ (0, 0.2)$ $\quad C_2\ (0.2, 0.4)$ $\quad C_3\ (0.4, \infty)$

C_1

$$\mu = \frac{1}{E[S]} = 2.5$$

$$P_1 = \left[1 - e^{-\mu t}\right]_0^{0.2} = 0.393469$$

$$P_2 = \left[1 - e^{-\mu t}\right]_{0.2}^{0.4} = 0.238651$$

$$\Sigma = 1$$

$$\mu = \frac{1}{E[S]}$$

$$P_3 = \left[1 - e^{-\mu t}\right]_{0.4}^{\infty} = 1 - 0 - \left[1 - e^{-2.5 \cdot 0.4}\right] = 0.367879$$

$$E(S_1) = \int_0^{0.2} t \cdot \frac{\mu e^{-\mu t}}{P_1} = 0{,}0917013\ s$$

$$E(S_2) = \int_{0.2}^{0.4} \text{"} \frac{\text{"}}{P_2} = 0.291701\ s$$

$$E(S_3) = \int_{0.4}^{\infty} \text{"} \frac{\text{"}}{P_3} = 0.8\ s$$

$$\rho_1 = \lambda \int_0^{0.2} t \cdot \mu e^{-\mu t}\, dt = 0.0721632$$

$$\rho_2 = \overset{2}{\overset{\shortparallel}{\lambda}} \int_{0.2}^{0.4} t\, \mu e^{-\mu t}\, dt = 0.13923$$

$$\rho_3 = \lambda \int_{0.4}^{\infty} t\, \mu e^{-\mu t}\, dt = 0.588607$$

$\Sigma = 0.8 = \rho$

ok

$$E(T_{a_1}) = \frac{\rho\, E[S]}{(1-\rho_1)} = 0.344888 \wedge$$

$$E(T_{a_2}) = \frac{\rho\, E[S]}{(1-\rho_1-\rho_2)(1-\rho_1)} = 0.437338 \wedge$$

$$E(T_{a_3}) = \frac{\rho\, E[S]}{(1-\rho)(1-\rho_1-\rho_2)} = 2.0288995 \wedge$$

rispettato

$$E[T_a] = \sum p_i\, E(T_{a_i}) = 0.988032295$$

$$E[S] = \sum p_i\, E[S_i] = 0.4$$

$$E[T_s] = E[T_a] + E[S] = 1.38803$$ ✓ Rispettato

a) throughput

$\mu > \lambda$ stabile, $\rho < 1$, throughput $= \lambda$ 2 req/s ?

b) $sd(0.1) = \frac{0.344888}{0.1} + 1 = 4{,}4488$
(ETa_1)

$sd(0.3) = \frac{0.437338}{0.3} + 1 = 2.4577$
(ETa_2)

FIFO

$0.1)\ \frac{\frac{\lambda}{2}[E[S^2]]}{0.1(1-\rho)} + 1 = \frac{\rho E[S]}{0.1(1-0.8)} + 1 = \frac{0.8 \cdot 0.4}{0.1(0.2)} + 1 = 17$

$0.3)\ 5.\overline{3} + 1 = 6.\overline{3}$

PS

$\frac{1}{1-\rho} = 5 \ \forall$ job
$\rho = 0.8$

$\boxed{Ok}$

---

4)

- arrivi Poisson, ~~M~~ interarrivo = 200 ms

$\rightarrow \frac{1 \text{ job}}{200 \text{ ms}} = 0{,}005 \text{ j/ms } \lambda$

- SIZE Buff = 4 (incluso Job servizio)
- $E[S] = 200$ ms $\longrightarrow \mu = \frac{1}{E[S]} = 0{,}005$ j/ms

con coda (3 job in coda) $\rightarrow$ erlang c

$\lambda' = \lambda(1 - P_{LOSS})$

$$\cancel{P_{LOSS} = \frac{(m\rho)^m \, p(0)}{m!\,(1-\rho)} = \quad \rho = 1 \text{ (NO)} \quad m = 1}$$

NO

$C = 3$ $\boxed{(4-1)}$ $\quad \lambda = \mu = 0.005 \qquad C = 4$

$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \pi_0 \qquad \pi_0 = \frac{1}{\sum_{i=0}^{3} \left(\frac{0{,}005}{0{,}005}\right)^i} = \cancel{\frac{1}{3}} = \frac{1}{4}$

$\lambda' = 3\left(1 - \frac{1}{3}\right) = 3\left(\frac{4}{3}\right) = 4$ j/s

$\lambda' = 0.005\left(1 - \frac{1}{5}\right) = 0.004$ j/ms $= 4$ j/s

b) request in 150 ms → $\mu = 0.006$

↳ $\pi_i = \left(\frac{\lambda}{\mu}\right)^i \pi_0 =$

$$\cancel{\pi_3 = \left(\frac{0.005}{0.006}\right)^3 \pi_0 = 0.5787 \cdot \pi_0 = 0.186283}$$

$$\cancel{\pi_0 = \frac{1}{\sum_{i=0}^{3}\left(\frac{0.005}{0.006}\right)^i} = \frac{1}{1 + 0.833 + 0.6944 + 0.5787 + 0.4822} = 0.3219}$$

$$\cancel{\lambda' = 3(1 - 0.186283) =}$$

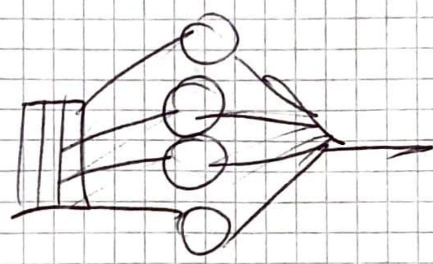
$$\pi_0 = \frac{1}{\sum^{4}\left(\frac{0.005}{0.006}\right)^i} = 0.27864$$

$$\pi_4 = P_{LOSS} = \left(\frac{0.005}{0.006}\right)^4 \cdot \pi_0 = 0.134375$$

$$\lambda' = 0.005(1 - 0.134375) = 0.00432 \text{ j/ms} = 4{,}328 \text{ j/s}$$

c) 4 core, request 300 ms usando un core.

Throughput?



$\mu' = 0.0033$ J/ms

$\lambda = 0.005$ J/ms

$$P_Q = \frac{4\left(\frac{\lambda}{\mu'}\right)^4 p(0)}{4!\left(1-\frac{\lambda}{\mu'}\right)}$$

$$\rho = \frac{\lambda}{m\mu} = 0.3750 \quad \text{OK}$$

$$p(0) = \sum_{i=0}^{3}\left[\frac{(4\rho)^i}{i!} + \frac{(4\rho)^4}{m!(1-\rho)}\right]^{-1} =$$

$$= \left\{\underbrace{\left[1 + \frac{(4\rho)^4}{4!(1-\rho)}\right]}_{i=0} + \underbrace{\left[4\rho + \frac{(4\rho)^4}{4!(1-\rho)}\right]}_{i=1} + \underbrace{\left[\frac{(4\rho)^2}{2} + \frac{(4\rho)^4}{4!(1-\rho)}\right]}_{i=2}\right.$$

$$\left. + \underbrace{\left[\frac{(4\rho)^3}{3!} + \frac{4\rho}{4!(1-\rho)}\right]}_{i=3}\right\}^{-1}$$

$$= \left\{1 + 4\rho + \frac{16\rho^2}{2} + \frac{64\rho^3}{6} + 4\left(\frac{(4\rho)^4}{4!(1-\rho)}\right)\right\}^{-1} = 0,180587$$

$P_Q = 0,00095 \longrightarrow \lambda' = \lambda(1-0.0095) = 0,0049525$

$\longrightarrow 4,95$ J/s

erlang B

$$\pi_4 = (0.3750)^4 \frac{1}{4!} \pi_0 = 0$$

$$\pi_0 = \frac{1}{\sum_{i=0}^{4} \left(\frac{\lambda}{\mu}\right)^i} = \cancel{0.629669}$$

0.07375

$$\cancel{\pi_4 = (0.3750)^4 / 4!}$$

$$\frac{\left(\frac{\lambda}{\mu}\right)^{m=4} / m!}{\sum_{i=0}^{m} \left(\frac{\lambda}{\mu}\right)^i / i!}$$

OK

$$\frac{0.005 \text{ ua}}{0.00333}$$

es.1 NOTE

$C = 10^5$ op/h $\quad Z = 4 \cdot 10^4 \quad \rho = 0.6 = \lambda E[S]$

$$E[S] = \frac{Z}{C} = 0.4$$

$$\mu = \frac{1}{E[S]} = 2.5 \qquad \rho = 0.6 = \lambda \cdot 0.4 \rightarrow \lambda = \frac{0.6}{0.4} = 1.5$$

classe 1

$\leq 0.4 = E[S]$

$P_1 = 0.6312$

$E[S_1] = 0.1674531$

$\frac{1}{\mu} = E[S]$

$P_1 E[S_1] = 0{,}1056963$

classe 2

$> 0.4 = E[S]$

$P_2 = 0.36788$

$E[S_2] = 0.799999$

$P_2 E[S_2] = 0{,}29430$