Performance Modeling of Computer Systems and Networks

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Performance Sensitivity to the Service time distribution

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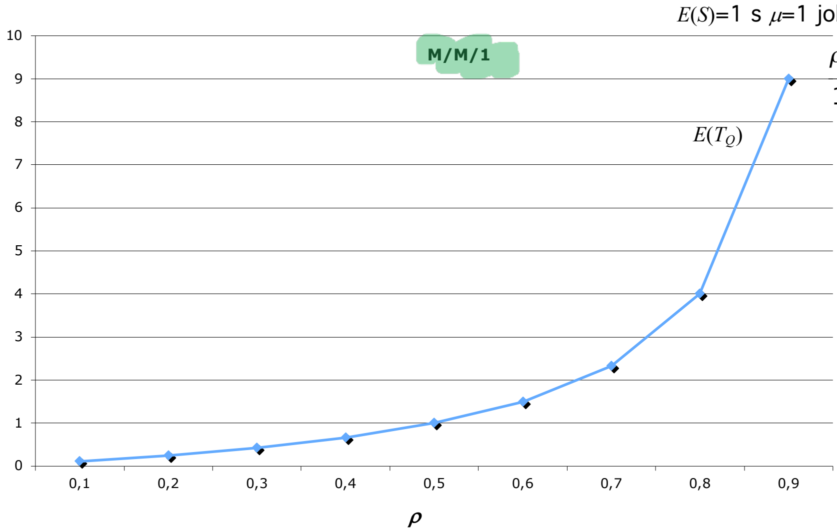
Analytical models
M/G/1

Expected waiting time as load grows

$E(S)$=1 s μ=1 job/s

$$\frac{\rho E(S)}{1-\rho}$$



What's happen if the service time distribution changes?

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The Khinchin Pollaczek equation (KP)

$$E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1 + C^2\right], \qquad E(T_Q) = \frac{\rho}{1-\rho}\,\frac{C^2+1}{2}\,E(S)$$

$$C^2(S) = \frac{\sigma^2(S)}{E^2(S)}$$

Expected waiting time in an M/G/1 queue can be huge, even under very low utilization ρ, if C^2 is huge. (se C^2 grande, anche con ρ piccolo, posso avere tempi di attesa rilevanti, poiché ho alta variabilità)

D $\longrightarrow$ $C^2=0$

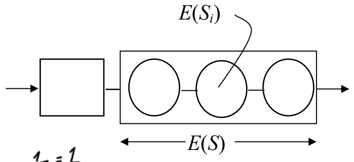
M $\longrightarrow$ $C^2=1$

$E_k \longrightarrow C^2 = \frac{1}{k}$

$H_2 \longrightarrow C^2 = g(p) = \frac{1}{2p(1-p)} - 1$

Expected waiting time as load grows: Erlang case

$E(S)$=0.5 s μ=2 job/s



$\frac{1}{\mu} = \frac{1}{2}$

$$E(S_i) = \frac{0.5}{3} = 0.166666666 \quad \text{s}$$

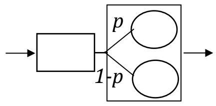
$K=3$ ≐ tempo medio nello stato "i".

$$\sigma^2(S) = \frac{1}{k}\left(\frac{1}{\mu}\right)^2 = 0.0833333$$

Expected waiting time as load grows: Hyperexponential case

$E(S)$=0.5 s μ=2 job/s

$2p\mu$=0.8 job/s



$2(1-p)\mu$=3.2 job/s

p=0.2

in media, il 20% del traffico riceve servizio con tempo $\frac{1}{3.2}$, mentre il restante $\frac{1}{0.8}$ (tempo maggiore)

$$\sigma^2(S) = g(p)\left(\frac{1}{\mu}\right)^2 = 0.53125$$

varianza

$$g(p) = \frac{1}{2p(1-p)} - 1 = 2.125$$

The Khinchin Pollaczek equation (KP)

$$g(p) = \frac{1}{2p(1-p)} - 1 \qquad E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1 + C^2\right], \quad E(T_Q) = \frac{\rho}{1-\rho}\,\frac{C^2+1}{2}\,E(S)$$

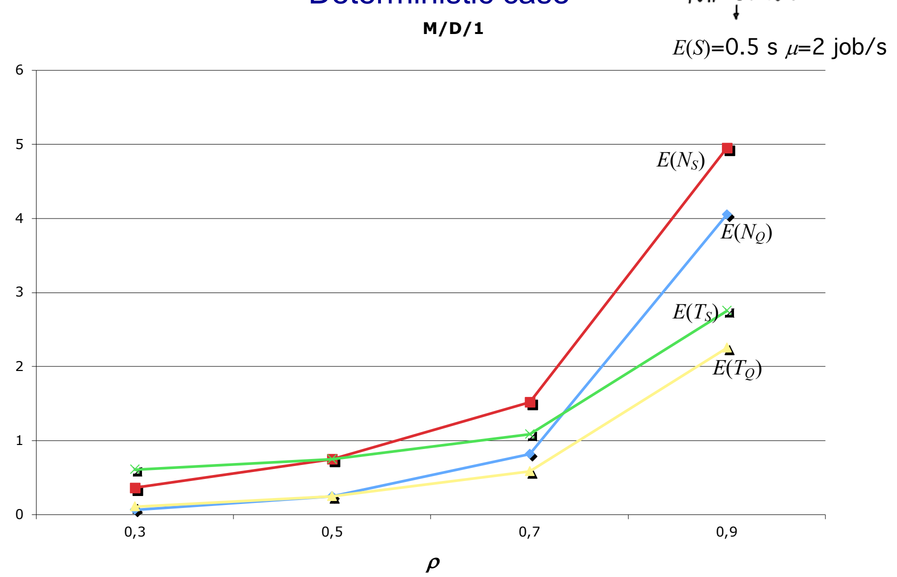
Service time	$E(N_Q)$	$E(T_Q)$
Deterministic, M/D/1	$\frac{\rho^2}{2(1-\rho)}$	$\frac{\rho E(S)}{2(1-\rho)}$
Markovian, M/M/1	$\frac{\rho^2}{1-\rho}$	$\frac{\rho E(S)}{1-\rho}$
K-Erlang, $M/E_k/1$ $\sigma^2(S) = \frac{E(S)^2}{k}$	$\frac{\rho^2}{2(1-\rho)}\left(1+\frac{1}{k}\right)$	$\frac{\rho E(S)}{2(1-\rho)}\left(1+\frac{1}{k}\right)$
Hyperexpo, $M/H_2/1$ $\sigma^2(S) = E(S)^2 g(p)$	$\frac{\rho^2}{2(1-\rho)}(1+g(p))$	$\frac{\rho E(S)}{2(1-\rho)}(1+g(p))$

Analytical models
M/G/1

Expected waiting time as load grows: Deterministic case

tutti chiedono

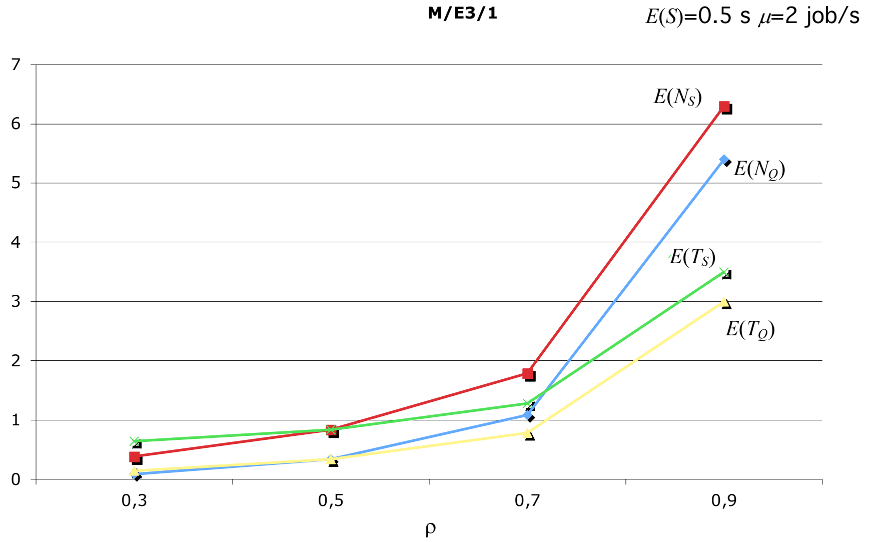
$E(S)$=0.5 s μ=2 job/s



Analytical models
M/G/1

Expected waiting time as load grows: Erlang case

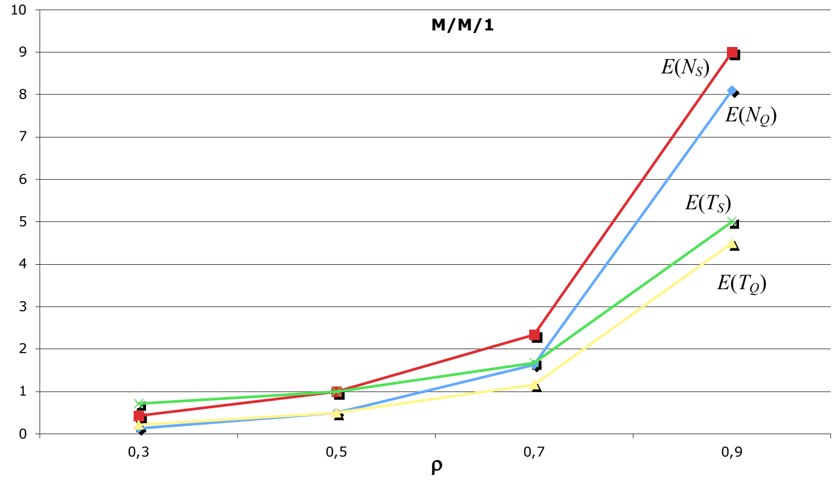
$E(S)$=0.5 s μ=2 job/s



Analytical models
M/G/1

Expected waiting time as load grows: Exponential case

$E(S)$=0.5 s μ=2 job/s



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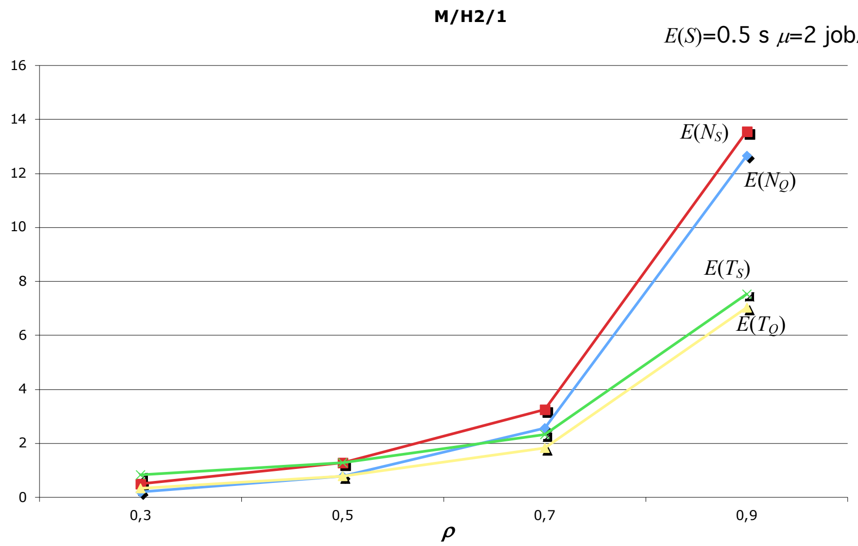
Analytical models
M/G/1

Expected waiting time as load grows: Hyperexponential case

$E(S)$=0.5 s μ=2 job/s



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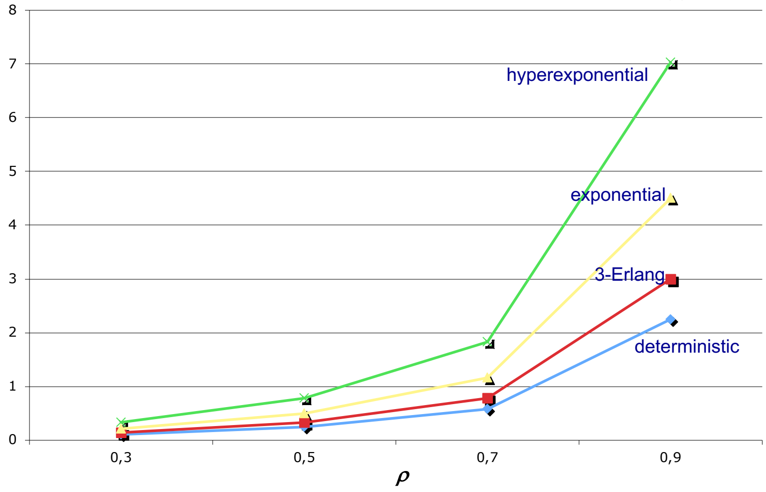
Expected waiting time sensitivity to the service time distribution

$E(T_Q)$

$E(S)$=0.5 s μ=2 job/s



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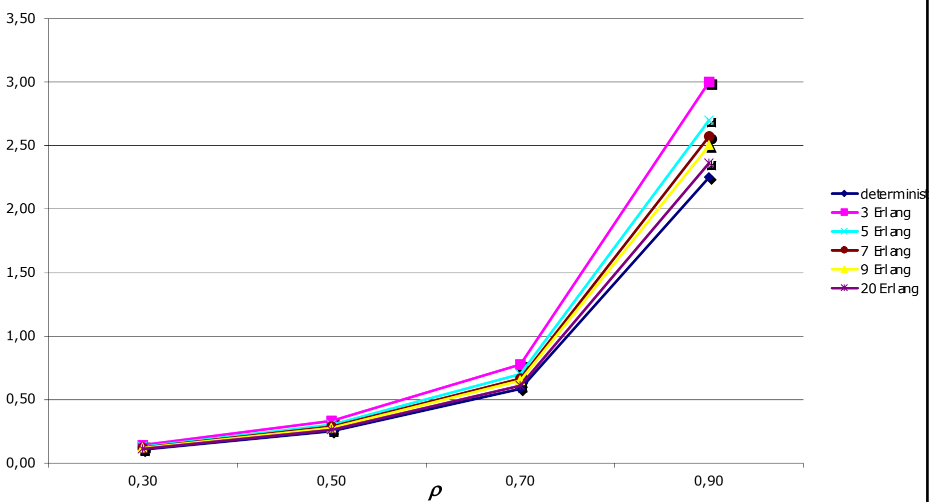
k-Erlang study for different k

$E(T_Q)$

$E(S)$=0.5 s μ=2 job/s



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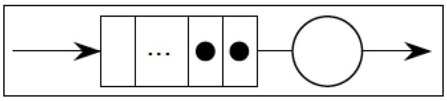
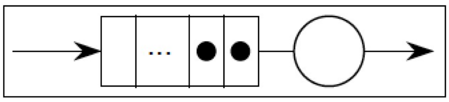
A TP system accepts and processes a stream of transactions, mediated through a (large) buffer:



- Transactions arrive "randomly" at some specified rate (= interarrivo esponenziale)
- The TP server is capable of servicing transactions at a given service *rate*

Q: If both the arrival rate and service rate are doubled, what happens to the mea[n] response time?



- The arrival rate is 15tps $= \lambda$, $\lambda' = 15 \cdot 1{,}10 = 16{,}5$
- The mean service time per transaction is 58.37ms $= \frac{1}{\mu} = 0{,}058375 \rightarrow \mu = 17{,}13$

Q: What happens to the mean response time if the arrival rate increases by 10%?

$\rightarrow \rho = \frac{\lambda}{\mu} = \frac{15}{17{,}13} = 87{,}56\ \%$, $\rho' = 96{,}31\%$

SVOLGIMENTO

Porto da KP generale, poiché non so la distribuzione!

$$E(T_Q) = \frac{\rho}{1-\rho}\frac{C^2+1}{2}E(S)$$

INVARIATO:
cambia solo tasso arrivi

$$E(T_{Q'}) = \frac{\rho'}{1-\rho'}\frac{C^2+1}{2}E(S)$$

$$\frac{E(T_Q)}{E(T_{Q'})} \cong 0,27 \cong \frac{1}{3,7}$$

$E(T_{Q'}) = 3,7 \cdot E(T_Q)$

$$\begin{cases} E(T_S) = E(T_Q) + E(S) \\ E(T_S') = 3,7 \cdot E(T_Q) + E(S) \end{cases}$$

Heavy tail distributions properties

esponenziale $\longrightarrow$ memoryless
failure rate costante

Heavy tail $\longrightarrow$ failure rate decrescente

(Pareto: $r(x) = \alpha / x$, $x > 1$)

Where they are

Jobs Unix

Sizes files websites $\alpha \approx 1.1$

Internet topology

Packet n° IP flows 1% → 50%

Natural phenomena

Pareto

$$f(x) = \alpha k^{\alpha} x^{-\alpha-1} \qquad k \le x < \infty$$

α , parametro di forma

$$E[X] = \frac{\alpha k}{\alpha - 1} \qquad \alpha > 1$$

$$\sigma^2[X] = \frac{\alpha k^2}{(\alpha-1)^2(\alpha-2)} \qquad \alpha > 2$$

combino e trovo E[Ta]

Bounded Pareto

$$f(x) = \alpha x^{-\alpha-1} \frac{k^{\alpha}}{1-\left(\frac{k}{p}\right)^{\alpha}} \qquad k \le x \le p, 0 < \alpha < 2$$

se mi allontano da $\alpha = 2$ mi allontano dal caso peggiore ($\alpha << 2$ heavy tail)

(Vilfredo Pareto, 15 July 1848 – 19 August 1923, economista e sociologo)

Pareto

$$E(T_Q) = \frac{\rho}{1-\rho}\frac{C^2+1}{2}E(S)$$

$$C^2(S) = \frac{\sigma^2(S)}{E^2(S)}$$

$$E[T_Q] = \frac{\rho E[S]}{1-\rho}\frac{1+\alpha(\alpha-2)}{2\alpha(\alpha-2)}$$

$\alpha > 2$

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Pareto study as load grows

$E(S)$=0.5 s μ=2 job/s

$E(T_Q)$

ro list ρ	α = 2,01	α = 2,05	α = 2,1	α = 2,15	determ	3-Erlang	expo	hyper
0,3	5,437633262	1,152439024	0,617346939	0,439368771	0,107	0,142	0,213	0,333
0,5	12,68781095	2,68902439	1,44047619	1,025193798	0,25	0,333	0,5	0,781
0,7	30	6,274390244	3,361111111	2,392118863	0,583	0,778	1,167	1,823
0,9	114,1902985	24,20121951	12,96428571	9,226744186	2,25	3	4,5	7,031

(minimo per α=2,01)

k=0.2512 k=0.2619

I tempi, anche al variare di α, esplodono con $\alpha \approx 2$!!

con carico ρ=0,3, passando a ρ=0,7, il tempo attesa nella coda passa da 0,333 a 1,823 (5,5 volte in più), fino a 7,031 con ρ=0,9. Tutto ciò con E(S)=0,5 (piccolo)

$$\underset{0,5}{E[S]} = \frac{\alpha k}{\alpha - 1} \xrightarrow[\text{fisso } \alpha]{} k_{MIN} = \frac{\alpha-1}{\alpha}E[S]$$

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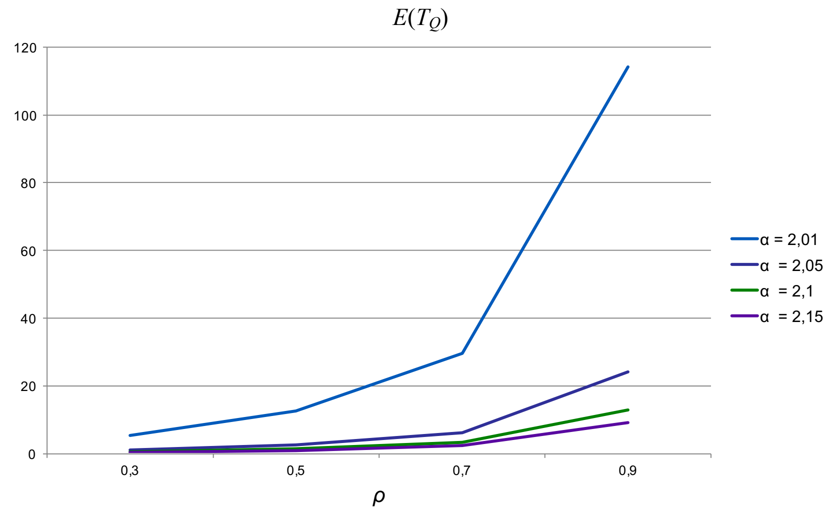
Tra le distribuzioni, la peggiore e' l'hyperesponenziale, ma le Pareto, in questi test, sono ancora peggio!! La variabilità impatta fortemente!

Pareto study as load grows



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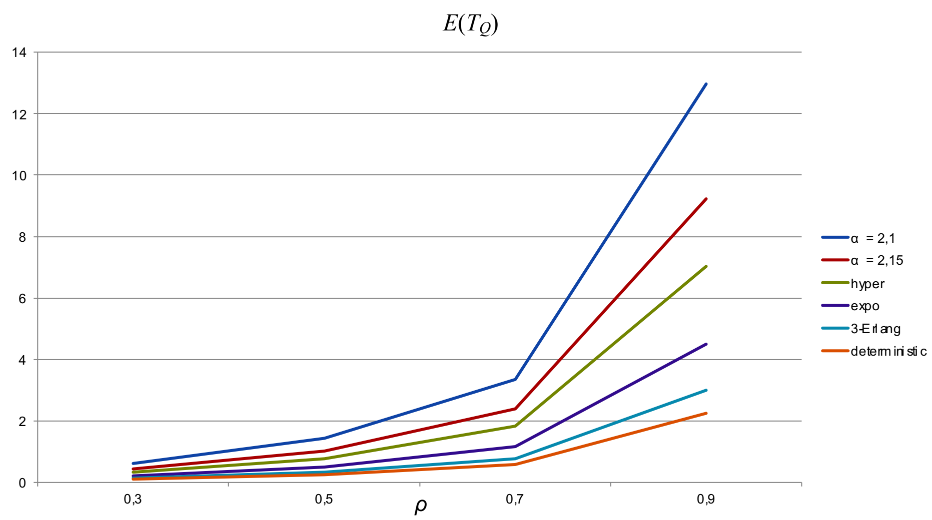
Pareto study as load grows



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