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Performance Modeling of Computer Systems and Networks

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Priority scheduling

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1

Analytical models
priority scheduling

Service classes

- (Multimedia traffic)
- Quality of Service (QoS)
- Penalties

The proper scheduling policy can improve
performance of a server tremendously.
It costs nothing to alter your scheduling policy
(no money, no new hardware), so the
performance gain comes for free.

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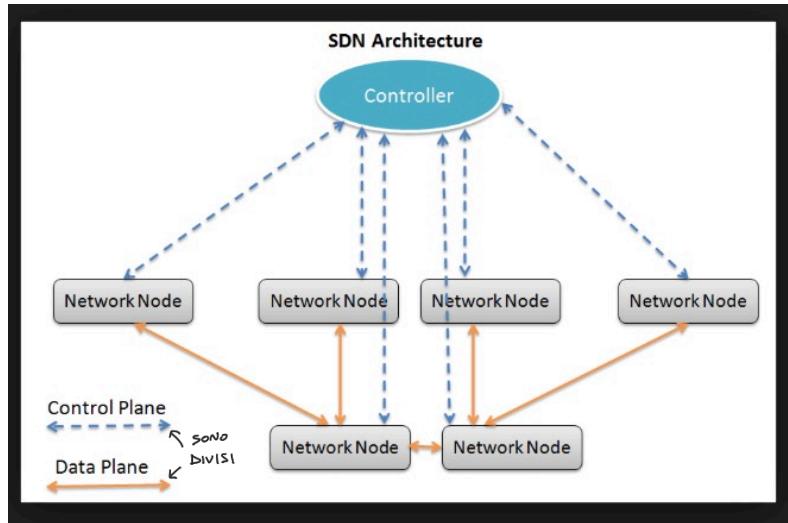
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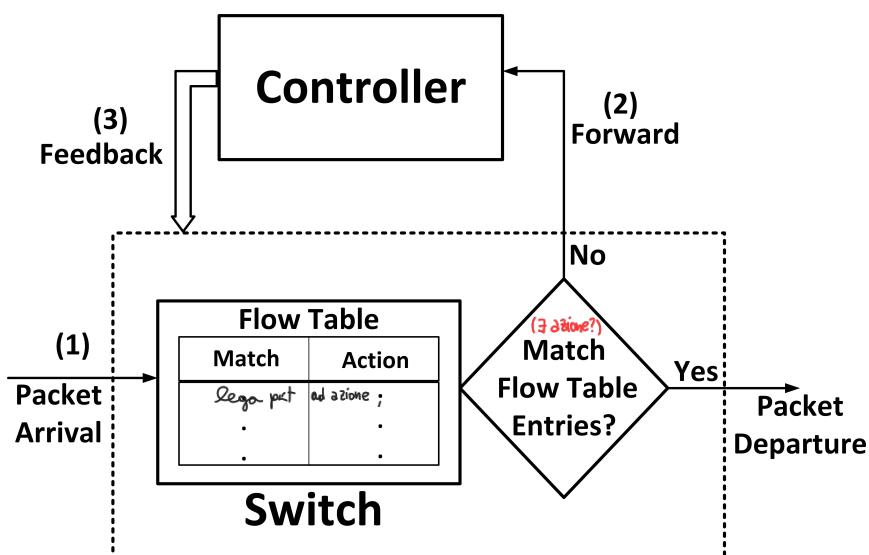
1

SDN

architettura per la realizzazione di reti di telecomunicazioni
il piano di controllo della rete e quello di trasporto dei dati sono separati logicamente



3



4

Ho un Priority Scheduling perchè, se esiste un'associazione $\langle \text{packetID}, \text{azione} \rangle$ nella tabella, allora il pkt può partire (coda partenze). Altrimenti, se tale associazione non esiste, il controllore, tramite feedback, ne verifica il perchè (coda feedback).

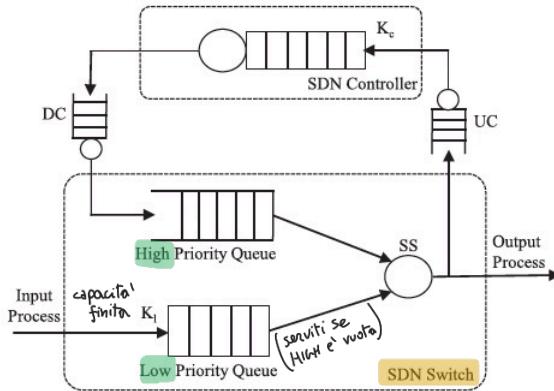


Fig. 1. The PQ-based SDN system architecture.

Miao W., Min G., Wu Y., Wang H. and Hu J., 2016. Performance Modelling and Analysis of Software-Defined Networking under Bursty Multimedia Traffic. *ACM Trans. Multimedia Comput. Commun. Appl.*, Vol. 12, No. 5s, Article 77.

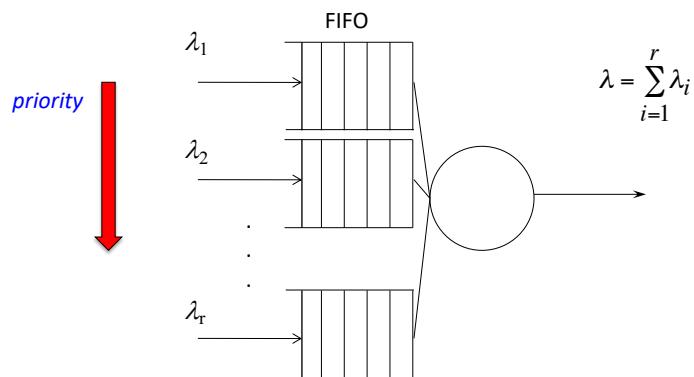
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5

5

Analytical models
priority scheduling

Priority classes

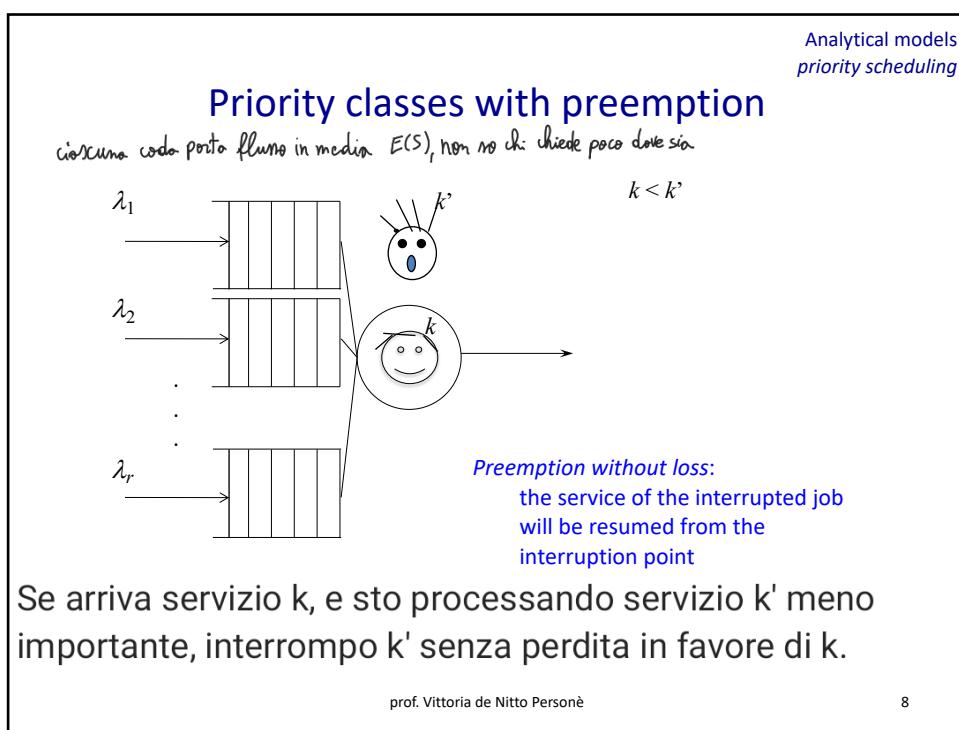
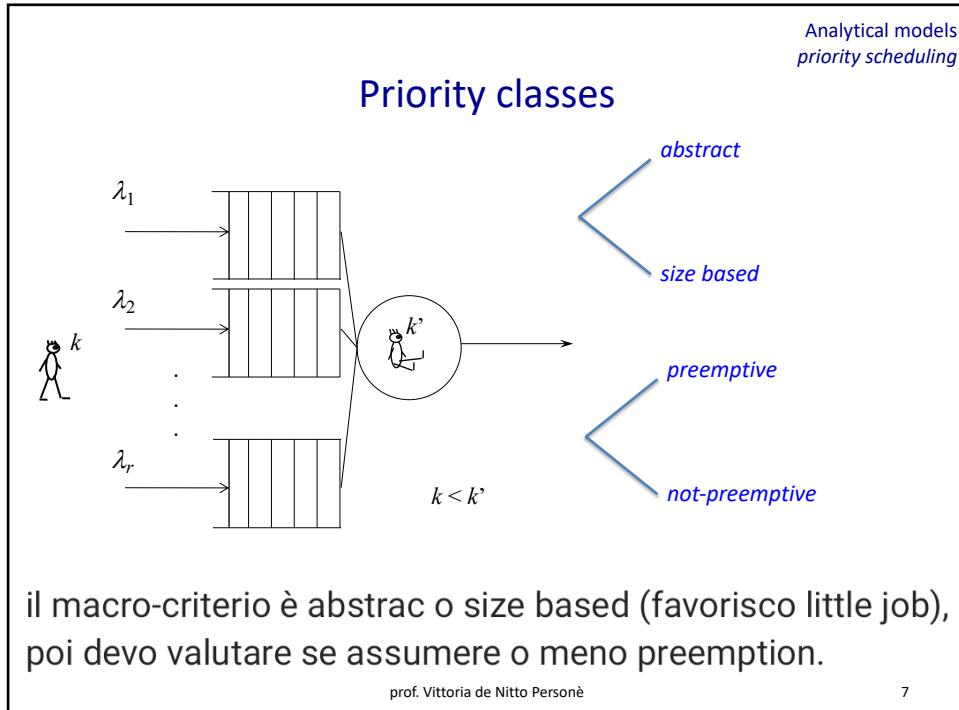


Ogni coda ha flusso lambda(i), e la somma è lambda.
Quali sono i criteri per formare queste 'classi'?

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6

6



Analytical models
priority scheduling

Abstract priority without preemption

$$\begin{aligned} S_k & \\ E(S_k) = \frac{1}{\mu_k} & \quad \sigma^2(S_k) \\ \rho_k = \lambda_k E(S_k) & \quad \rho = \sum_{i=1}^r \rho_i \\ \text{abstract + No preemption} & \\ E(S_k) = E(S) = \frac{1}{\mu}, \quad \sigma^2(S_k) = \sigma^2(S), \forall k & \\ \rho_k = \lambda_k E(S) & \end{aligned}$$

Ogni classe k ha propri parametri. Iniziamo ad esaminare dal caso abstract-No prelazione, dove i tempi di servizio sono uguali per tutti.

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9

Analytical models
priority scheduling

Abstract priority without preemption

local performance measures

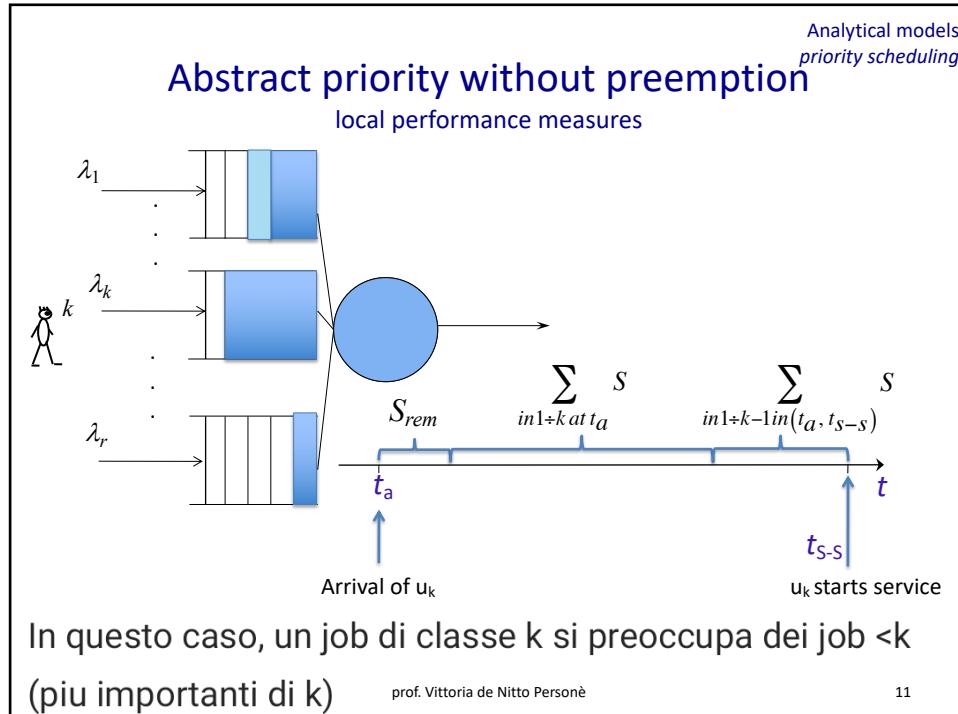
$E(T_{Q_k})?$

t_a t
Arrival of u_k u_k Starts service

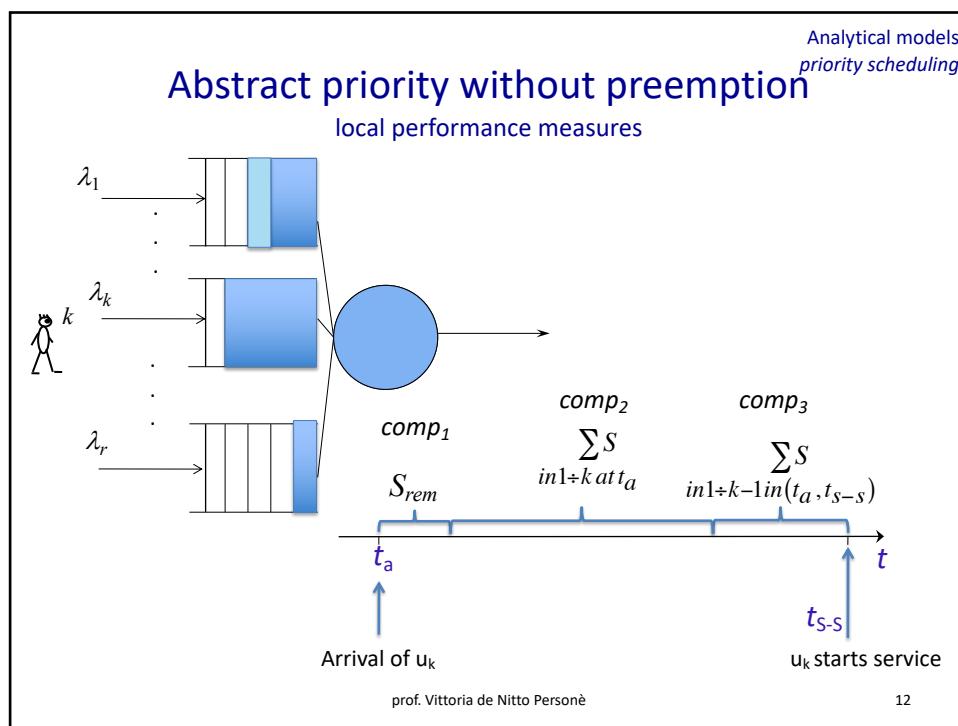
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10

10



11



12

cosa trova il job di classe 'k'? : arriva a T_a , aspetta il completamento del job in esecuzione (no prelazione) + tempo per completare servizi più importanti (da 1 a $k-1$) + quelli che arrivano dopo k ma sono più importanti di lui! (quelli che arrivano finché non prende servizio, quando lo prende non viene più superato).

Analytical models
priority scheduling

Abstract priority without preemption

local performance measures

$\lambda_1 \rightarrow$ [Queue] → Central Circle

\vdots

$\lambda_k \rightarrow$ [Queue] → Central Circle

\vdots

$\lambda_r \rightarrow$ [Queue] → Central Circle

comp₁: $E(S_{rem}) = \frac{\lambda}{2} E(S^2)$

comp₂: proportional to the load of queues 1:k
 $= \frac{1}{1 - \sum_{i=1}^k \rho_i}$

comp₃: proportional to the load of queues 1:k-1
 $= \frac{1}{1 - \sum_{i=1}^{k-1} \rho_i}$

da KP $E(T_{Q_k}) = \frac{\lambda}{2} E(S^2) / (1 - \rho)$, quindi mi aspetto qualcosa di simile! (infatti i componenti sono simili)

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13

Analytical models
priority scheduling

Abstract priority without preemption

local performance measures

$\lambda_1 \rightarrow$ [Queue] → Central Circle

\vdots

$\lambda_k \rightarrow$ [Queue] → Central Circle

\vdots

$\lambda_r \rightarrow$ [Queue] → Central Circle

$E(T_{Q_k})^{NP_priority} = \frac{\lambda}{2} E(S^2) / \left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k-1} \rho_i\right)$

$E(T_{Q_k}) \leq E(T_{Q_{k+1}})$

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14

Analytical models
priority scheduling

Abstract priority without preemption

local performance measures

$$E(T_{Q_k}) \leq E(T_{Q_{k+1}})$$

$$\frac{\frac{\lambda}{2} E(S^2)}{\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k-1} \rho_i\right)} \leq \frac{\frac{\lambda}{2} E(S^2)}{\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k+1} \rho_i\right)}$$

$$\frac{1}{\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k-1} \rho_i\right)} \leq \frac{1}{\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k+1} \rho_i\right)}$$

$$\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k-1} \rho_i\right) \geq \left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k+1} \rho_i\right)$$

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15

15

Analytical models
priority scheduling

Abstract priority without preemption

local performance measures

$$\cancel{\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k-1} \rho_i\right)} \geq \cancel{\left(1 - \sum_{i=1}^k \rho_i\right) \left(1 - \sum_{i=1}^{k+1} \rho_i\right)}$$

$$\cancel{1 - \sum_{i=1}^{k-1} \rho_i} \geq \cancel{1 - \sum_{i=1}^{k+1} \rho_i}$$

$$\sum_{i=1}^{k+1} \rho_i \geq \sum_{i=1}^{k-1} \rho_i \quad \rho_i \geq 0, \forall i$$

$$E(T_{Q_k}) \leq E(T_{Q_{k+1}})$$

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16

16

Analytical models
priority scheduling

Abstract priority without preemption

local performance measures

$$E(T_{S_k}) = E(T_{Q_k}) + E(S) \quad E(T_{S_k}) \leq E(T_{S_{k+1}})$$

$$E(N_{Q_k}) = \lambda_k E(T_{Q_k})$$

$$E(N_{S_k}) = \lambda_k E(T_{S_k}) \quad E(N_{S_k}) = E(N_{Q_k}) + \rho_k$$

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17

17

Analytical models
priority scheduling

Abstract priority without preemption

global performance measures

And the “global” performance?

$$E(T_Q)^{NP_priority} = E(E(T_{Q_k})) = \sum_{k=1}^r p_k E(T_{Q_k})$$

$$p_k = \frac{\lambda_k}{\lambda}$$

and similarly for $E(T_S)^{NP_priority}$

$$E(T_S)^{NP_priority} = E(T_Q)^{NP_priority} + E(S)$$

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18

18

attenzione: l'elemento cerchiato NON è rho, ma la probabilità di quella classe rispetto al flusso totale (es: due classi a cui do stesso Pk = 0.5, cioè equiprobabili al 50%)

Analytical models
priority scheduling

Abstract priority without preemption

$$\lambda_k = p_k \lambda$$

$$\rho_k = \lambda_k E(S) = p_k \lambda E(S) = p_k \rho$$

il rho della classe k è il rho totale per la probabilità della classe k.

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19

19

Analytical models
priority scheduling

priority vs no-priority

How are the performance improved in respect of a simple abstract scheduling not considering the priority classes?

Quanto è utile il priority?

$$E(T_{Q_k})^{NP_priority} = \frac{\frac{\lambda}{2} E(S^2)}{(1 - \sum_{i=1}^k \rho_i)(1 - \sum_{i=1}^{k-1} \rho_i)} \quad ? \quad E(T_Q)^{KP} = \frac{\frac{\lambda}{2} E(S^2)}{1 - \rho}$$

The highest priority class:

$$E(T_{Q_1})^{NP_priority} = \frac{\frac{\lambda}{2} E(S^2)}{(1 - \rho_1)} \leq E(T_Q)^{KP} \quad \smiley$$

La classe più importante è migliore di KP, ma era ovvio, hanno una importanza notevole!

20

20

Analytical models
priority scheduling

priority vs no-priority

How are the performance improved in respect of a simple abstract scheduling not considering the priority classes?

The lowest priority class: la classe meno importante è "snobbata", ma me lo aspettavo!

$$E(T_Q)^{NP_priority} = \frac{\frac{\lambda}{2} E(S^2)}{(1-\rho)(1 - \sum_{i=1}^{r-1} \rho_i)} \geq E(T_Q)^{KP}$$


And what about the "global" performance?

$$\begin{aligned} E(T_Q)^{NP_priority} &= E(T_Q)^{KP} \\ \downarrow & \\ E(T_S)^{NP_priority} &= E(T_S)^{KP} \end{aligned}$$

globalmente non ho guadagnato nulla!

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21

- 21 Ma può sempre essere utile per casi come QoS, dove magari mi interessano alcune classi, e non il globale.

Analytical models
priority scheduling

esempio con due classi per dimostrarlo: priority vs no-priority

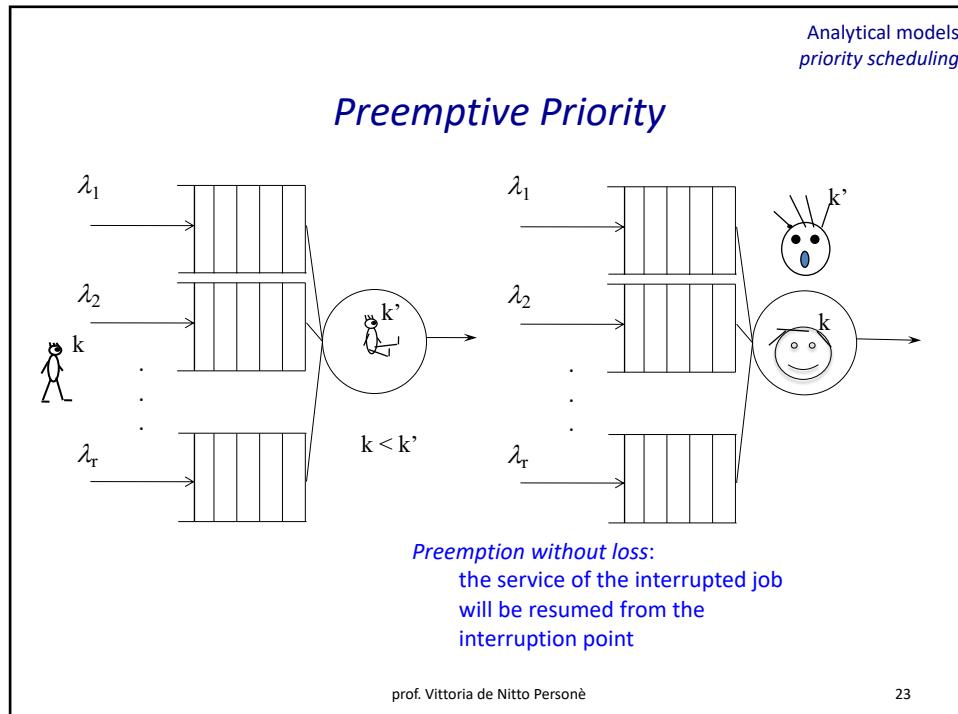
$$\begin{aligned} E(T_Q)^{NP_priority} &= E(E(T_{Q_k})) = \sum_{k=1}^r p_k E(T_{Q_k}) = E(T_Q)^{KP} \\ E(T_Q) &= p_1 E(T_{Q_1}) + p_2 E(T_{Q_2}) = p_1 \frac{\frac{\lambda}{2} E(S^2)}{(1-\rho_1)} + p_2 \frac{\frac{\lambda}{2} E(S^2)}{(1-\rho)(1-\rho_1)} \\ &= \frac{\lambda}{2} E(S^2) \left[\frac{p_1}{(1-\rho_1)} + \frac{p_2}{(1-\rho)(1-\rho_1)} \right] = \frac{\lambda}{2} E(S^2) \frac{p_1(1-\rho) + p_2}{(1-\rho)(1-\rho_1)} = \frac{\lambda}{2} E(S^2) \frac{1}{1-\rho} \end{aligned}$$

Abbiamo trovato lo stesso valore della KP.

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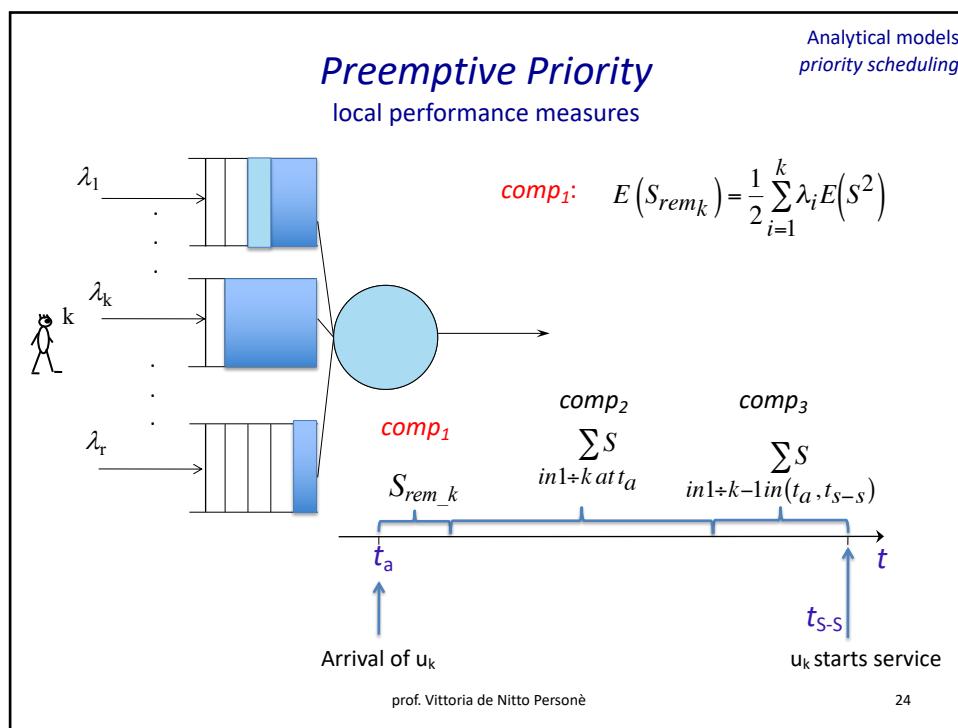
22

Easy A/B



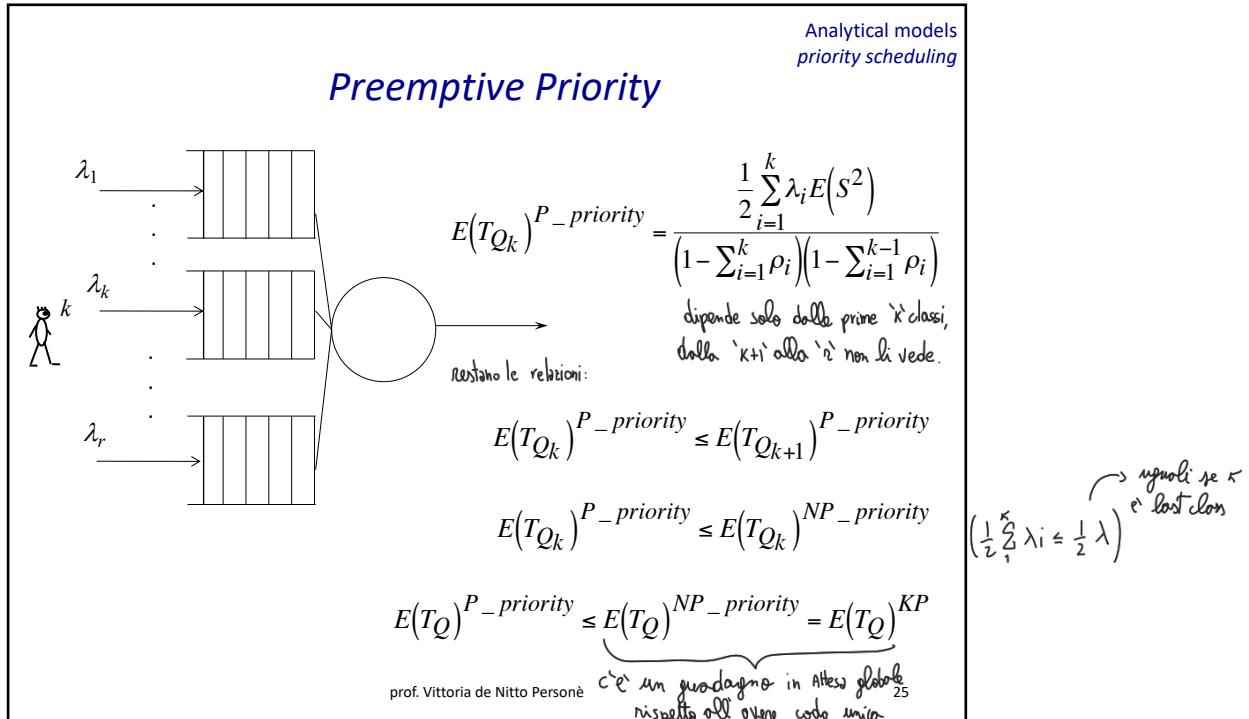
23

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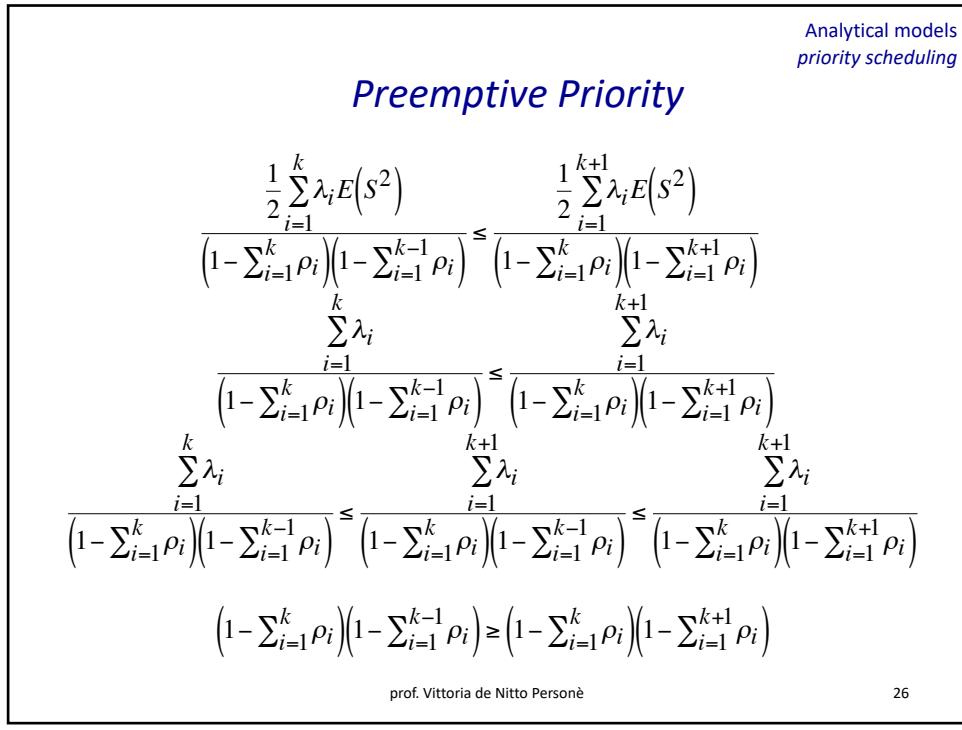
24

Con PRELAZIONE, se un job classe k è in servizio, e arriva un job $k' < k$, cioè più importante, il job classe k viene interrotto a favore del job k' . Lo stato di k viene salvato (altrimenti non avrebbe senso) e ripreso quando toccherà a lui. Solo il primo componente cambia, perché devo vedere gli elementi nelle prime k classi (più importanti di me). 12



25

La terza diseguaglianza è vera poichè $E(T_Q)^{P_priority} = \sum P_k \cdot E(T_{Q_k})^{P_priority}$
ma abbiamo appena dimostrato che: $E(T_{Q_k})^{P_priority} \leq E(T_{Q_k})^{NP_priority}$, a parità di P_k



26

Preemptive Priority

attesa globale :

$$\begin{aligned} E(T_Q)^{X_priority} &= E(E(T_{Q_k})) = \sum_{k=1}^r p_k E(T_{Q_k}) \\ &= p_1 E(T_{Q_1}) + p_2 E(T_{Q_2}) + \dots + p_r E(T_{Q_r}) \end{aligned}$$

$$\begin{aligned} E(T_Q)^{NP_priority} &= p_1 E(T_{Q_1}) + p_2 E(T_{Q_2}) + \dots + p_r E(T_{Q_r}) \\ &\quad \text{vi} \quad \text{vi} \quad \text{vi} \quad \text{stesso schedule} \\ E(T_Q)^{P_priority} &= p_1 E(T_{Q_1}) + p_2 E(T_{Q_2}) + \dots + p_r E(T_{Q_r}) \end{aligned}$$

$$E(T_Q)^{P_priority} \leq E(T_Q)^{NP_priority} = E(T_Q)^{KP}$$

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27

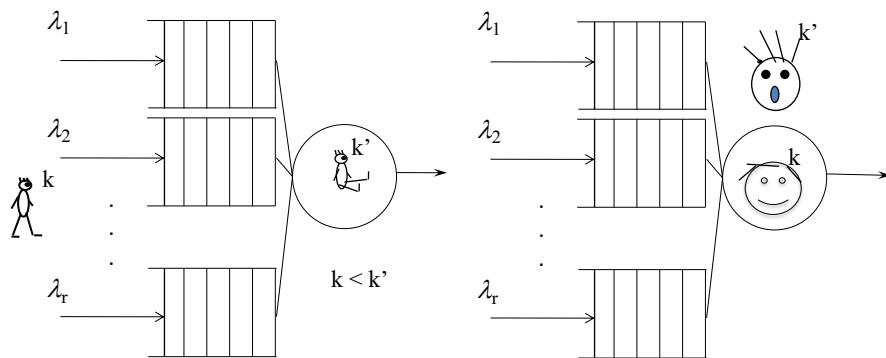
27

Analytical models
priority scheduling

Preemptive Priority

Due to preemption, the service time of a job of class k may increase for the arrivals of higher priority classes

Soprattutto se sono un job con bassa priorità(k alto).



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28

28

Analytical models
priority scheduling

Preemptive Priority

Due to preemption, the service time of a job of class k may increase for the arrivals of higher priority classes

$$\text{Virtual service time } E(S_{virt_k}) = \left(\text{servizio originale } E(S) + \text{altri interruzioni} \right)$$

$$E(S_{virt_k}) = \frac{E(S)}{1 - \sum_{i=1}^{k-1} \rho_i}$$

per classe i
non interrompibile
che $E(S)$ e non S_{VIRT}

$$E(T_{S_k})^{P_priority} = E(T_{Q_k})^{P_priority} + E(S_{virt_k})$$

$$E(T_{S_k})^{NP_priority} = E(T_{Q_k})^{NP_priority} + E(S)$$

? ?

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29

$E(S_{Virtuale})$ è il tempo totale per il servizio, con PRIORITY migliori i tempi di attesa, ma si allungano quelli di servizio.

Analytical models
priority scheduling

Preemptive Priority

global response time

And what about the “global” performance?

$$E(T_S)^{P_priority} = E(E(T_{S_k})) = \sum_{k=1}^r p_k E(T_{S_k})$$

$$E(T_S)^{P_priority} = \sum_{k=1}^r p_k [E(T_{Q_k}) + E(S_{virt_k})]$$

$$= \sum_{k=1}^r p_k E(T_{Q_k}) + \sum_{k=1}^r p_k E(S_{virt_k})$$

$$= E(T_Q)^{P_priority} + \sum_{k=1}^r p_k E(S_{virt_k})$$

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30

Analytical models
priority scheduling

preemption vs no-preemption

~~tempo resp. glob.~~ $E(T_S)^{P_priority} = E(T_Q)^{P_priority} + \sum_{k=1}^r p_k E(S_{virt_k})$

$\wedge \quad \quad \quad \vee \quad \quad \quad ?$

~~fiscale NO priorità (=KP)~~ $E(T_S)^{NP_priority} = E(T_Q)^{NP_priority} + E(S) = E(T_S)^{KP}$

In general $E(T_S)^{P_priority} ? E(T_S)^{KP}$

For exponential service time (la memoria si "fregà")

$E(T_S)^{P_priority} = E(T_S)^{KP}$!!!

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31

31

Analytical models
priority scheduling

preemption vs no-preemption

$r=2$ ~~ossi~~

$$\begin{aligned} E(T_S)^{P_priority} &= p_1 E(T_{S_1}) + p_2 E(T_{S_2}) \\ &= p_1 \left[\frac{\lambda_1}{2} E(S^2) + E(S) \right] + p_2 \left[\frac{\lambda}{2} E(S^2) + E(S) \right] \\ &\quad \boxed{\text{classe 1}} \quad \text{allungato dalle priorità} \end{aligned}$$

from the expo assumption

$$\begin{aligned} &= p_1 \left[\frac{\rho_1 E(S)}{(1-\rho_1)} + E(S) \right] + p_2 \left[\frac{\rho E(S)}{(1-\rho)(1-\rho_1)} + \frac{E(S)}{1-\rho_1} \right] \\ &= E(S) \left\{ p_1 \left[\frac{\rho_1 + 1 - \rho_1}{(1-\rho_1)} \right] + p_2 \left[\frac{\rho + (1-\rho)}{(1-\rho)(1-\rho_1)} \right] \right\} \end{aligned}$$

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32

32

Analytical models
priority scheduling

preemption vs no-preemption

$r = 2$

$$\begin{aligned} E(T_S)^{P_priority} &= p_1 E(T_{S1}) + p_2 E(T_{S2}) \\ &= E(S) \left[\frac{p_1}{(1-\rho_1)} + \frac{p_2}{(1-\rho)(1-\rho_1)} \right] \\ &= E(S) \frac{p_1(1-\rho) + p_2}{(1-\rho)(1-\rho_1)} = \frac{E(S)}{1-\rho} = E(T_S)^{KP} \end{aligned}$$

tempo risposta esponenziale = $K\rho$, vale anche con $r > 2$, ma più difficile

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33

33