Machine Learning

Introduction to Reinforcement Learning

Gabriele Russo Russo   Francesco Lo Presti

Laurea Magistrale in Ingegneria Informatica – A.Y. 2023/24

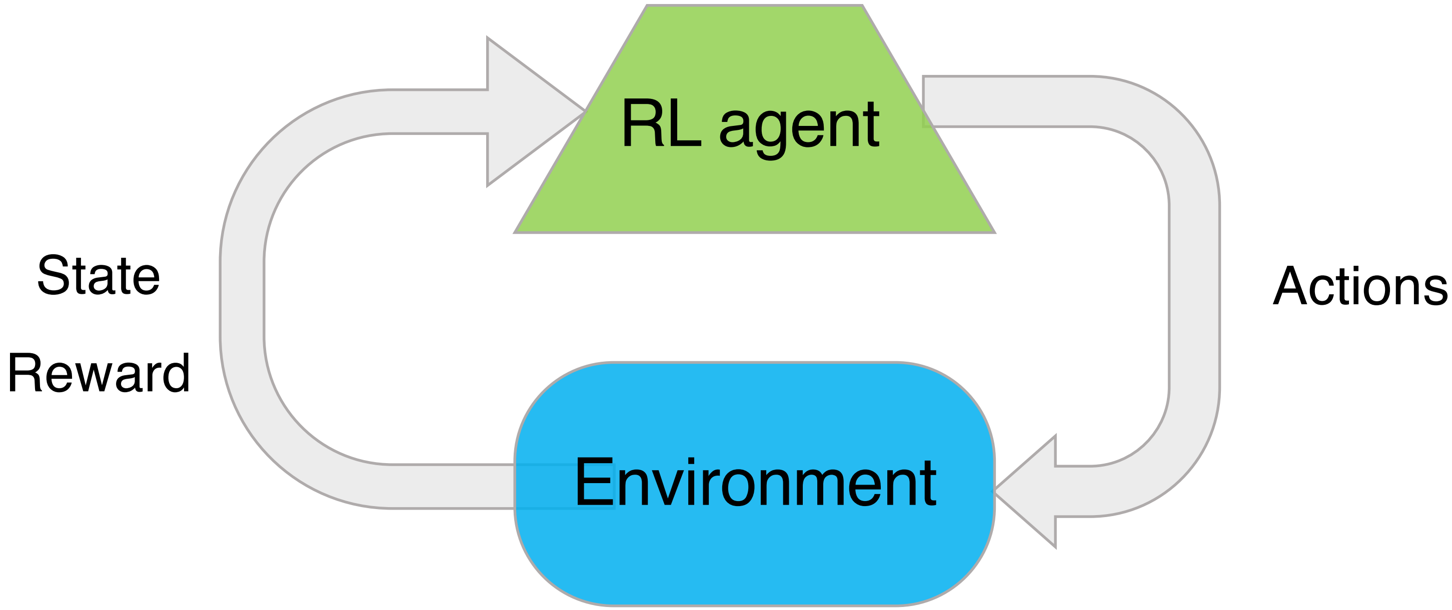
TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Dipartimento di Ingegneria Civile e Ingegneria Informatica

Reinforcement Learning

- Supervised learning
- Unsupervised learning
- Reinforcement learning
  - Branch of ML dealing with sequential decision-making

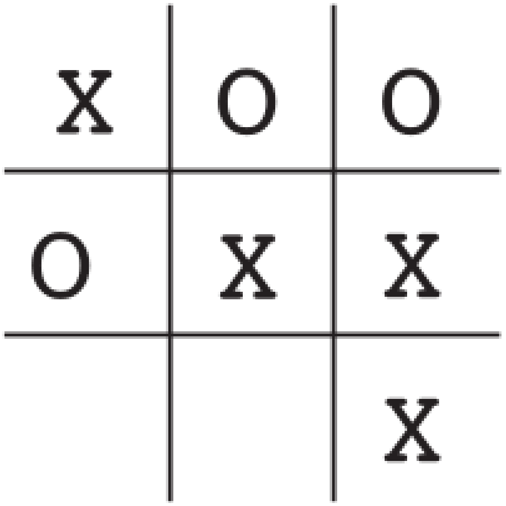
Reinforcement Learning



- Agent interacts with environment through actions
- Feedback in the form of reward (or paid cost)
- Goal: maximizing cumulated reward over the long run
- Trial-and-error experience (no complete knowledge of environment a priori)

Example: Tic-Tac-Toe

- State: representation of the board (3x3 matrix)
- Actions: available cells to mark
- Reward: 1 for a winning move, 0 otherwise



Example: AlphaZero by DeepMind

- Software able to play Go, Chess and Shogi [1]
  - Board games with huge number of legal positions (i.e., state space)
- Trained via self-play and advanced deep RL techniques
- Superhuman level of play with 24-hour training
- First presented in 2017; in 2019 MuZero, generalization to play Atari games and other board games without prior rule knowledge

[1] https://arxiv.org/abs/1712.01815

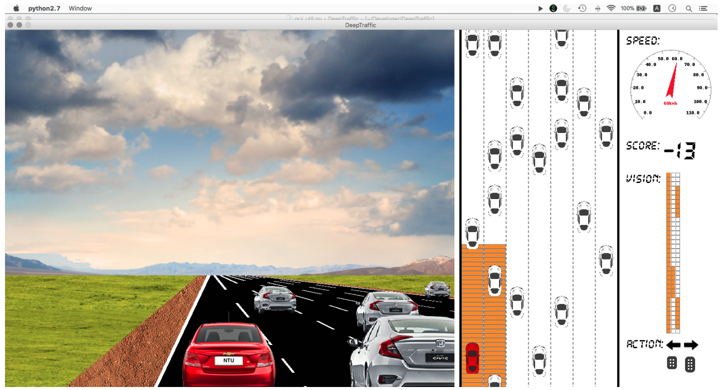
Example: AlphaDev by DeepMind

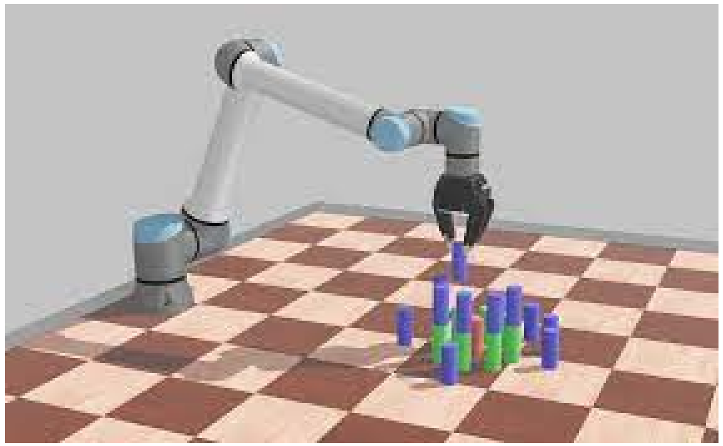
- Announced in 2023[2]
- RL used to develop new C++ sorting algorithm, now accepted in the standard library
- 70% faster on short sequences (2-3 items), 1.7% faster on long sequences
- State: instructions generated so far and state of the CPU
- Actions: assembly instructions to add
- Reward: based on sorting correctness and efficiency

[2] `https://www.deepmind.com/blog/alphadev-discovers-faster-sorting-algorithms`

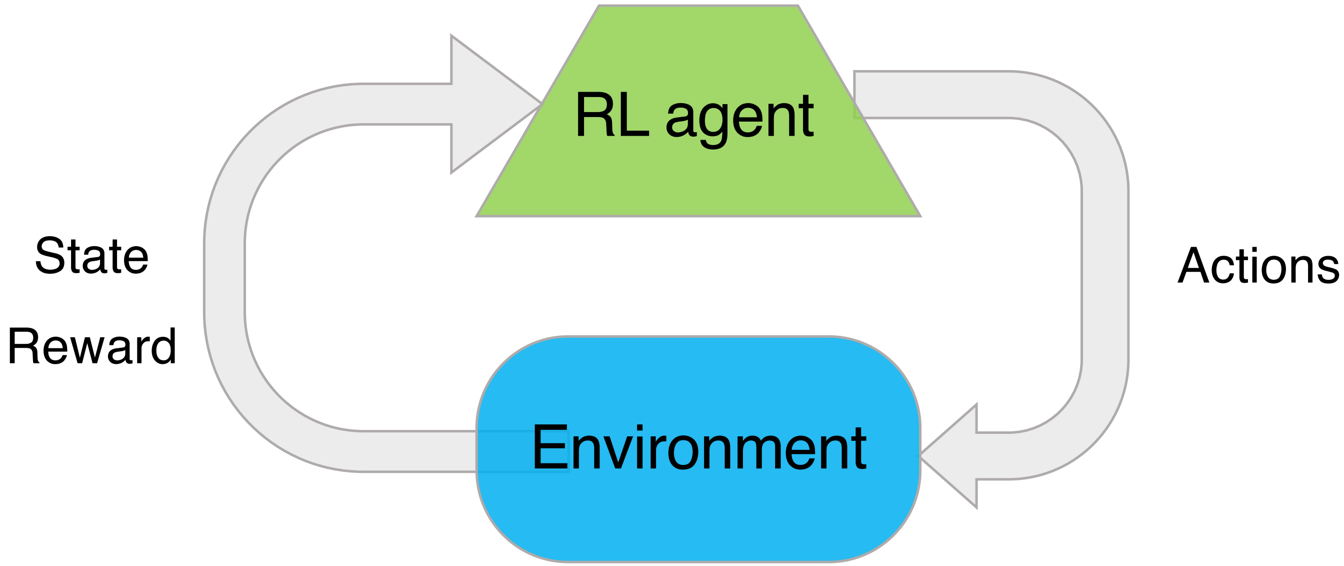
Other Examples

- Autonomous vehicles
- Robot control
- Trading
- Autonomous network and computer systems
- Videogames
- …





Reinforcement Learning

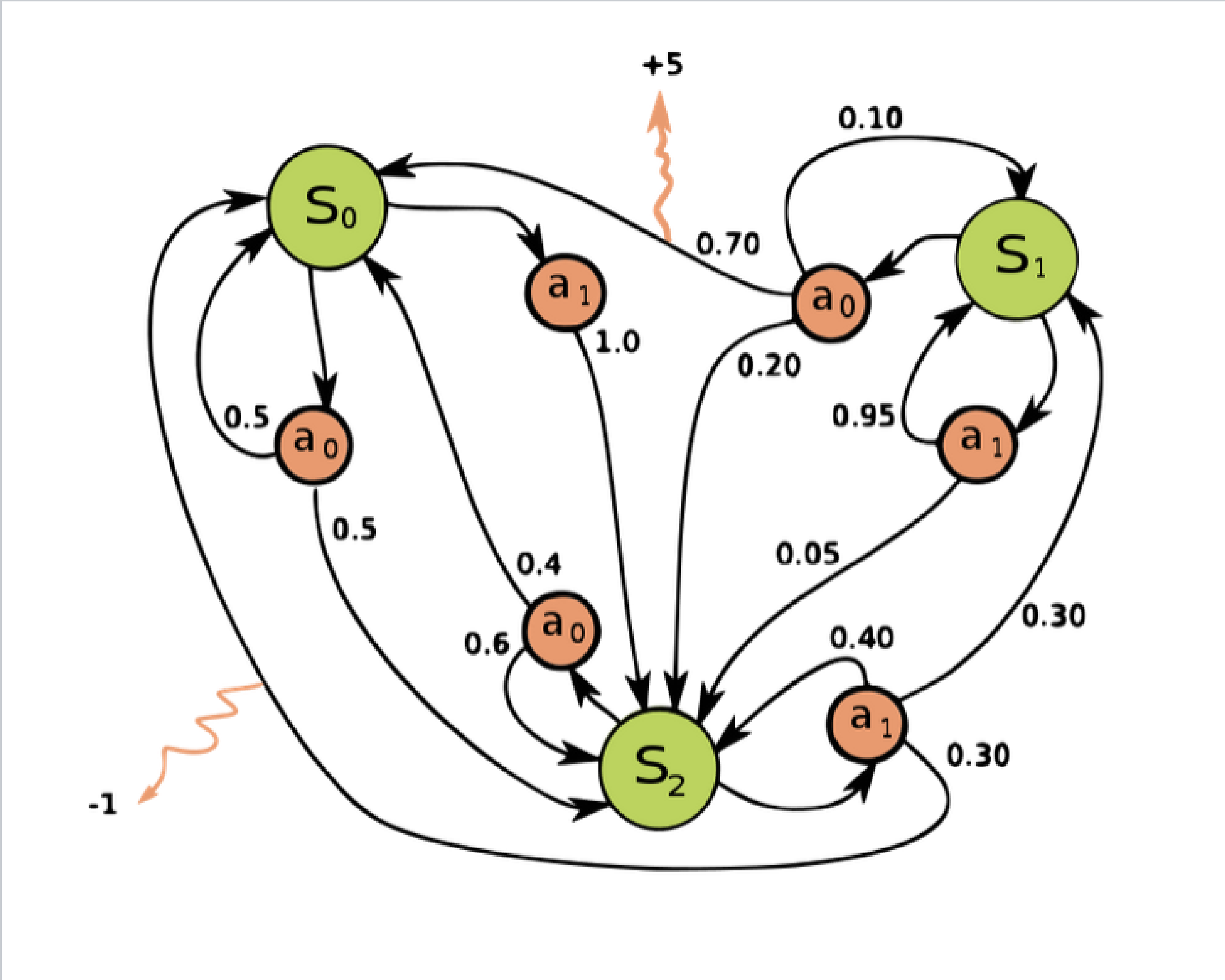


- Agent, environment, actions, state, rewards, …
- Modeled depending on the specific task
  - e.g., autonomous car uses different state information compared to chess player
- Formally defined as a Markov Decision Process (MDP)
- A framework to model decision making in situations where outcomes are partly random

Markov Decision Process (MDP)

- Extension of discrete-time Markov chains
- At each time step t, the process is in some state s_t
- The decision maker (the agent) chooses an action a_t among those available in state s_t
  - e.g., robot observes current position and decides direction to move; some directions might be blocked by obstacles
- Following a_t, the process moves to (random) state s_{t+1}
  - e.g., autonomous drone chooses an action to reduce altitude; actual outcome may depend on (unpredictable) wind speed
- Agent receives a reward (or, equivalently, pays a cost)
  - e.g., robot may get a reward for reaching its final destination
  - e.g., chess player rewarded at the end of a match

Example



Example of a simple MDP with three states (green circles) and two actions (orange circles), with two rewards (orange arrows).

Markov Decision Process (2)

What defines an MDP?

- $\mathcal{S}$: a (finite) set of states
- $\mathcal{A}$: a (finite) set of actions
- p: state transition probabilities

$$p(s'|s, a) = P[S_{t+1} = s'|S_t = s, A_t = a]$$

- r: reward function (or, c: cost function)
  1. $r(s, a) = E[R_t|S_t = s, A_t = a]$
  2. $r(s, a, s') = E[R_t|S_t = s, A_t = a, S_{t+1} = s'] \longrightarrow$
     $r(s, a) = \sum_{s'} p(s'|s, a) r(s, a, s')$

Markov Property

"The future is independent of the past given the present"

Definition

A state S_t is Markov if and only if

$$P[S_{t+1} | S_1, \ldots, S_t] = P[S_{t+1} | S_t]$$

- The state captures all relevant information from the history
- i.e., the state is a sufficient statistic of the future

Objective: Episodic Tasks

- Informally, we said that the agent aims to maximize the collected reward over time
- Let's consider an episodic task, where the agent-environment interaction naturally terminates at some final time step T
  - e.g., the end of a chess match
  - e.g., the time a robot reaches its destination or runs out of battery
- At time t, we aim to maximize the expected return G_t

$$G_t = R_t + R_{t+1} + \ldots + R_T$$

Objective: Continuing Tasks

- In many cases the agent–environment interaction does not break naturally into identifiable episodes, but goes on continually without limit
  - e.g., an agent managing VM migration in a Cloud datacenter
  - e.g., the control system of RL-based traffic lights
- In this scenario, the goal of the agent is maximizing the expected cumulative discounted reward

$$G_t = R_t + \gamma R_{t+1} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k}$$

where $\gamma \in [0, 1)$ is the discount factor

Reward vs Cost

- You can either maximimize the expected reward or minimize the expected cost

$$G_t = C_t + \gamma C_{t+1} + \ldots = \sum_{k=0}^{\infty} \gamma^k C_{t+k}$$

- The two formulations are equivalent; you can easily switch between them by setting

$$r(s, a) = -c(s, a)$$

- In the following, we will mostly refer to costs; keep in mind this equivalence

Policy

Definition

A policy π is a distribution over actions given a state s

$$\pi(a|s) = p(A_t = a|S_t = s)$$

- A policy fully defines agent's behavior
- MDP policies depend on the current state only
- Special case: deterministic policy

$$\pi : \mathcal{S} \to \mathcal{A}$$

Example: Deterministic Policy

State	*Action*
s_1	a_1
s_2	a_1
s_3	a_2
s_4	a_1

Value Function

Value function is a prediction of future costs

- can be used to evaluate how good/bad states and/or actions are
- and therefore to select actions e.g.

State	a_1	a_2	a_3
s_1	10	5	3
s_2	8	6	4
s_3	6	5	6
s_4	5	4	6
s_5	4	3	7
s_6	1	5	9
s_7	0	9	15

s	$\pi(s)$
s_1	a_3
s_2	a_3
s_3	a_2
s_4	a_2
s_5	a_2
s_6	a_1
s_7	a_1

Value Functions

Action value function (or, Q function)

Expected cost starting from state s, taking action a and then following policy π

$$Q_\pi(s, a) = E_\pi[G_t | S_t = s, A_t = a]$$

State value function

Expected cost starting from state s and then following policy π

$$V_\pi(s) = E_\pi[G_t | S_t = s]$$

Action Value Functions

The action value function can be decomposed into two parts:

- immediate cost
- discounted costs from successor state S_{t+1}

$$
\begin{aligned}
Q_\pi(s,a) &= E_\pi[G_t | S_t = s, A_t = a] \\
&= E_\pi[C_t + \gamma C_{t+1} + \gamma^2 C_{t+2} \ldots | S_t = s, A_t = a] \\
&= E_\pi[C_t + \gamma (C_{t+1} + \gamma C_{t+2} \ldots) | S_t = s, A_t = a] \\
&= E_\pi[C_t + \gamma G_{t+1} | S_t = s, A_t = a] \\
&= c(s,a) + \gamma E_\pi[G_{t+1} | S_t = s, A_t = a]
\end{aligned}
$$

Bellman equation:

$$
Q_\pi(s,a) = c(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) Q_\pi(s', \pi(s'))
$$

State Value Functions

The value function can be similarly decomposed into two parts:

- immediate cost C_t
- discounted cost from successor state $V(S_{t+1})$

$$
\begin{aligned}
V_\pi(s) &= E_\pi[G_t | S_t = s] \\
&= E_\pi[C_t + \gamma C_{t+1} + \gamma^2 C_{t+2} \ldots | S_t = s] \\
&= E_\pi[C_t + \gamma (C_{t+1} + \gamma C_{t+2} \ldots) | S_t = s] \\
&= E_\pi[C_t + \gamma G_{t+1} | S_t = s]
\end{aligned}
$$

Bellman equation:

$$
V_\pi(s) = c(s, \pi(s)) + \gamma \sum_{s'} p(s' | s, \pi(s)) V_\pi(s')
$$

Optimal Value Function

Optimal action value function

$Q^*(s; a)$ is the minimum action-value function over all policies

$$Q^*(s, a) = \max_{\pi} Q_{\pi}(s, a)$$

Optimal state value function

$V^*(s)$ is the minimum value function over all policies

$$V^*(s) = \max_{\pi} V_{\pi}(s)$$

Bellman Optimality Equations

$$Q_\pi(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) Q_\pi(s', \pi(a'))$$

$$\Downarrow$$

$$\boxed{\begin{aligned} Q^*(s, a) &= c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a') \\ V^*(s) &= \min_{a} Q^*(s, a) \\ Q^*(s, a) &= c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) V^*(s') \end{aligned}}$$

Optimal Policy

Given $Q^*(s, a)$ the optimal action when the system is in state s is:

$$\pi^*(s) = a^*(s) = \arg\min_{a \in \mathcal{A}} Q^*(s, a)$$

State	a_1	a_2	a_3
s_1	10	5	3
s_2	8	6	4
s_3	6	5	6
s_4	5	4	6
s_5	4	3	7
s_6	1	5	9
s_7	0	9	15

$$\Longrightarrow$$

Optimal Action
a_3
a_3
a_2
a_2
a_2
a_1
a_1

How to compute V^*?

- If we know the optimal value function, we have an optimal policy!
- But… how do we compute the optimal value function??

Value Iteration

Bellman Equation

$$Q^*(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

- Suppose we know the solution to subproblems $Q^*(s', a')$
- $Q^*(s, a)$ can be computed by one-step lookahead

$$Q^*(s, a) \leftarrow c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

- The idea is to apply these updates iteratively
- Proven to converge (see, Contraction Mapping Theorem in Sutton's book)

Value Iteration: Algorithm

Value Iteration

$i \leftarrow 0$
$Q_i(s, a) \leftarrow 0, \forall s \in \mathcal{S}, \forall a \in \mathcal{A}(s)$
repeat
  forall $s \in \mathcal{S}$ **do**
    forall $a \in \mathcal{A}(s)$ **do**
      $Q_{i+1}(s, a) \leftarrow$
        $c(s, a) + \gamma \sum_{s' \in \mathcal{S}} p(s'|s, a) \min_{a' \in \mathcal{A}(s')} Q_i(s', a')$
    end
  end
  $i \leftarrow i + 1$
until $\max_{s,a} |Q_i(s, a) - Q_{i-1}(s, a)| < \epsilon$
$\pi^*(s) = \arg\min_a Q_i(s, a), \forall s \in \mathcal{S}$

Value Iteration: Alternative Algorithm

Value Iteration - Alternative

$i \leftarrow 0$

$Q_i(s, a) \leftarrow 0, \forall s \in \mathcal{S}, \forall a \in \mathcal{A}(s)$

$V_i(s) \leftarrow 0, \forall s \in \mathcal{S}$

repeat

    forall $s \in \mathcal{S}$ **do**

        forall $a \in \mathcal{A}(s)$ **do**

            $Q_{i+1}(s, a) \leftarrow c(s, a) + \gamma \sum_{s' \in \mathcal{S}} p(s' | s, a) V_i(s)$

        end

        $V_{i+1}(s) = \min_{a' \in \mathcal{A}(s)} Q_{i+1}(s, a')$

    end

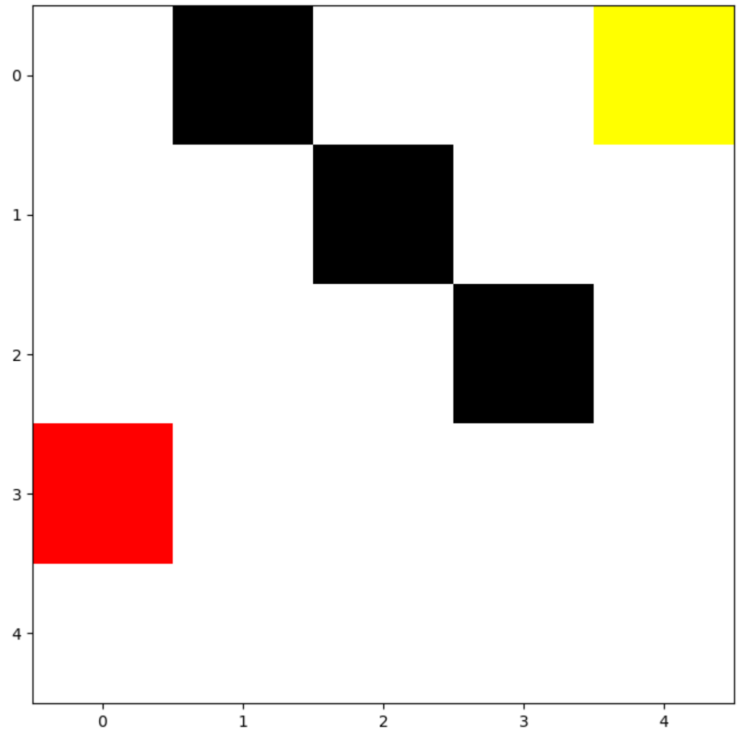
    $i \leftarrow i + 1$

until $\max_{s,a} |Q_i(s, a) - Q_{i-1}(s, a)| < \epsilon$

$\pi^*(s) = \arg\min_a Q_i(s, a), \forall s \in \mathcal{S}$

Example: Maze

- Consider a $S \times S$ grid
- Episodes start with agent randomly located in a cell in the first column
- Goal: reaching target cell $(1, S)$
- Some cells are blocked
- Some cells are slippery: when entering, the agent has a probablity p_{slip} of slipping one cell ahead along her current direction

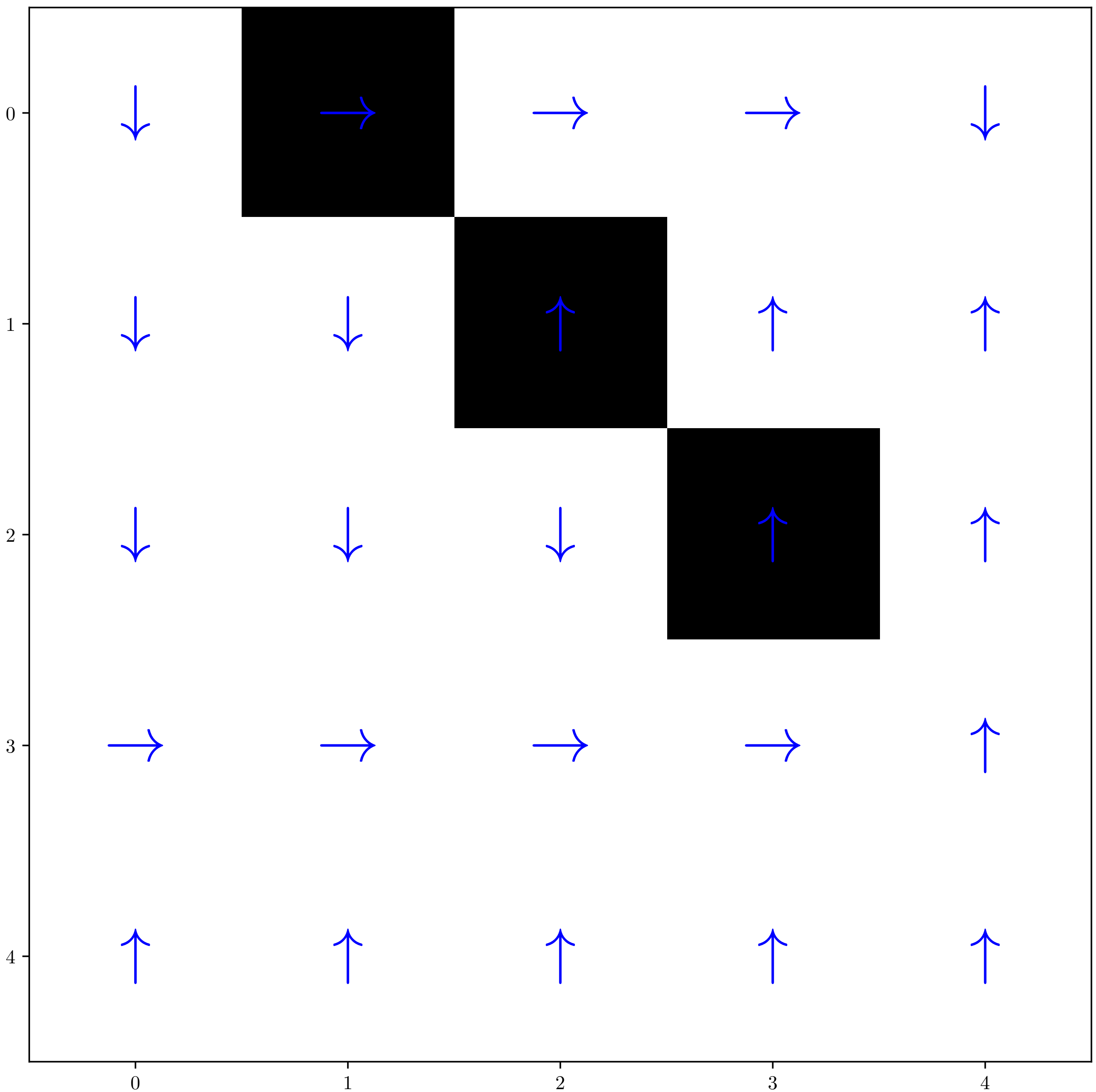


Example: Maze (2)

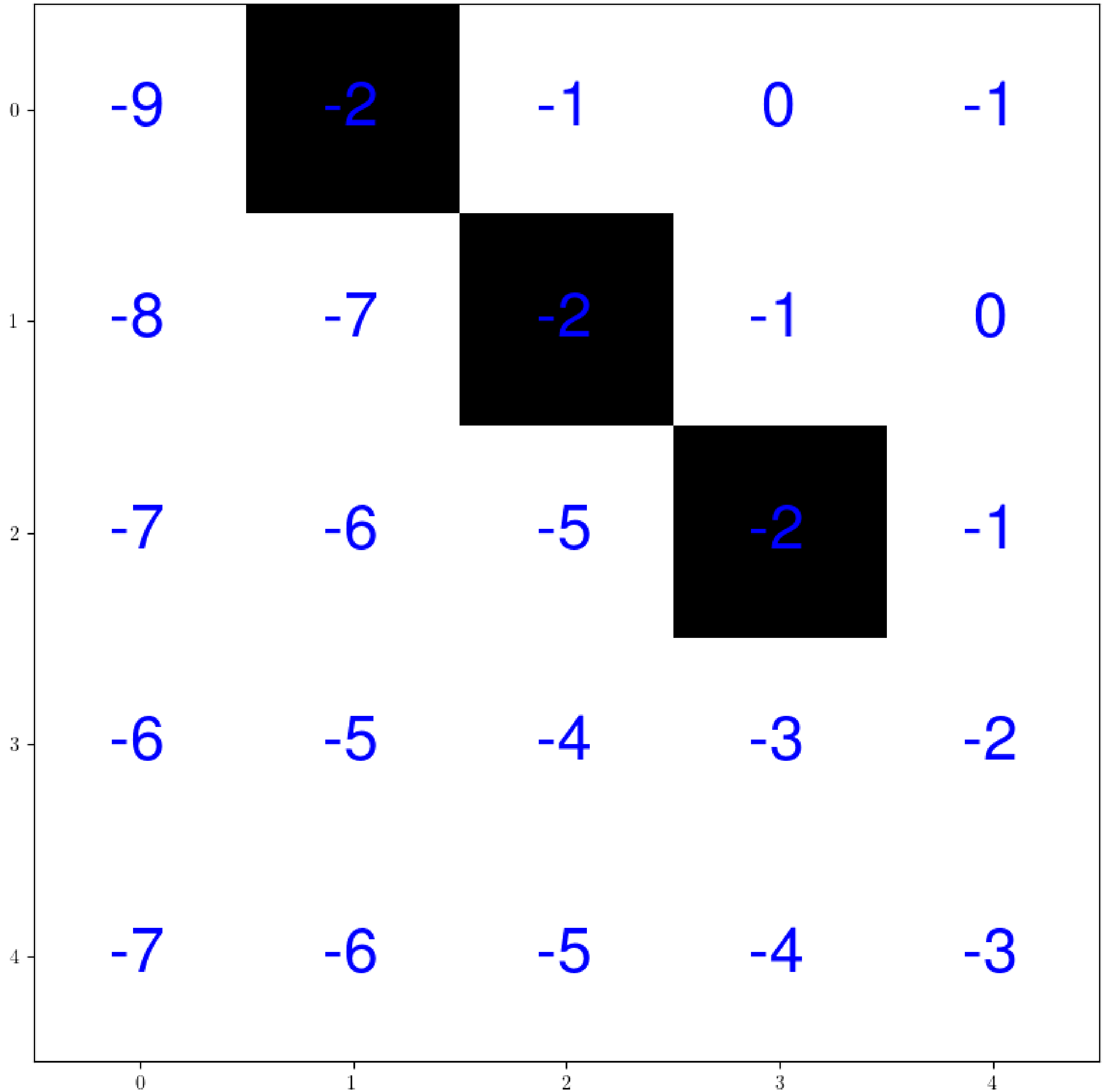
- Let the goal cell coordinates (x^*, y^*)
- State: $s = (x, y)$
- Actions: $a \in \{(0, 1), (0, -1), (1, 0), (-1, 0)\}$
- Reward:
  - 0 for entering the goal cell
  - -M for exiting the grid or crashing into a blocked cell ($M \gg 1$)
  - -1 otherwise

maze.py (`--agent mdp`)

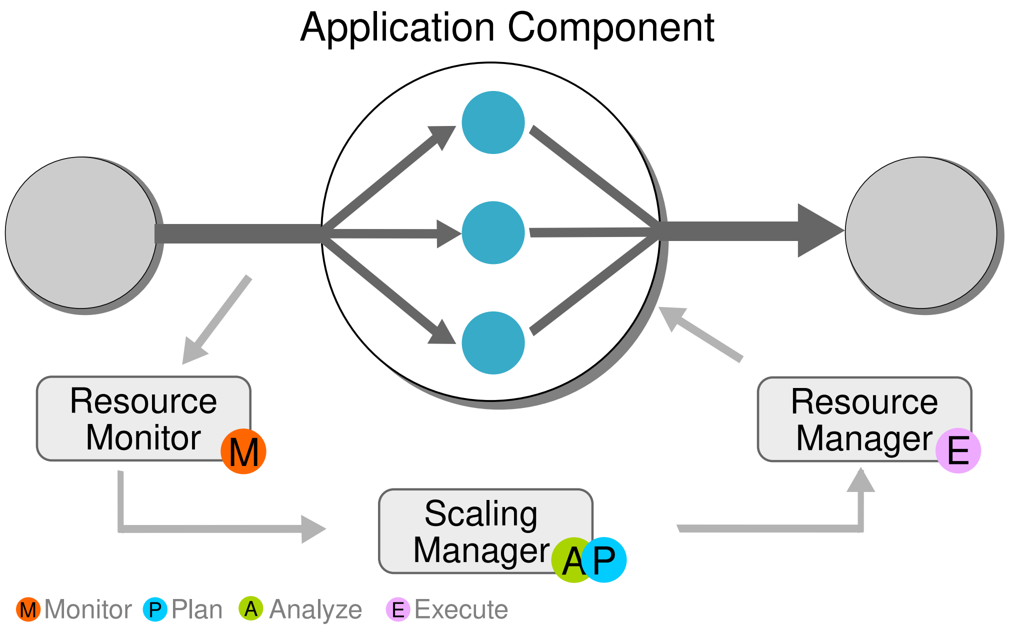
Maze: Optimal Policy



Maze: Optimal Value Function



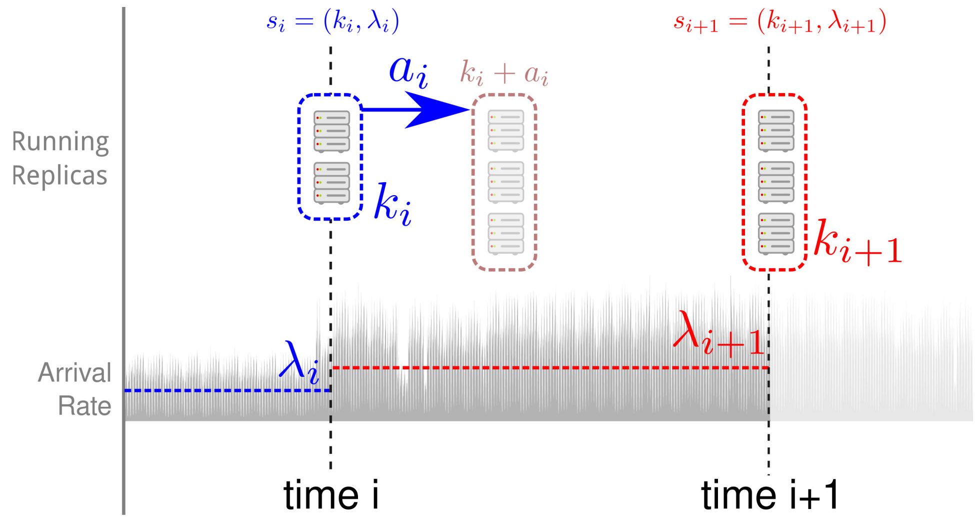
Example: Cloud Auto-scaling



- We periodically make a decision about scaling in/out an app component
- We are concerned with 3 objectives:
  - Monetary resource cost (or, resource usage in general)
  - Performance req. satisfaction (e.g., max response time)
  - Scaling overhead

Auto-scaling: MDP formulation

- We model the problem as a Markov Decision Process (MDP)



- State at time slot i: $s_i = (k_i, \lambda_i)$
  - k_i component parallelism
  - λ_i avg. arrival rate (of requests, jobs, data, ...)
- Action at time slot i: $a_i \in \{0, +1, -1\}$

MDP Model: Transition Probabilities

- State of the system $s = (k, \lambda)$
  - $1 \leq k \leq K^{max}$ Component parallelism
  - λ avg. input rate
    - λ is discretized , i.e., $\lambda_i \in \{0, \Delta\lambda, 2\Delta\lambda, (L-1)\Delta\lambda\}$
    - $\Delta\lambda$ quantization step size, L number of discrete values
- Available actions $\mathcal{A} = \{-1, 0, +1\}$
- Transition probabilities $p(s'|s, a) = p\left((k', \lambda')|(k, \lambda), a\right)$

$$
\begin{aligned}
p(s'|s, a) &= P[s_{t+1} = (k', \lambda')|s_t = (k, \lambda), a_t = a] = \\
&= \begin{cases} P[\lambda_{t+1} = \lambda'|\lambda_t = \lambda] & k' = k + a \\ 0 & \text{otherwise} \end{cases} = \\
&= \mathbb{1}_{\{k'=k+a\}} P[\lambda_{t+1} = \lambda'|\lambda_t = \lambda]
\end{aligned}
$$

MDP Model: Cost Function

- Simple Additive Weighting method
- Cost associated with action execution and state transition $(s, a) \rightarrow s'$

$$c(s, a, s') = \boxed{w_{res} \frac{k + a}{K^{max}}} + \boxed{w_{perf} \mathbb{1}_{\{R(s,a,s') > R^{max}\}}} + \boxed{w_{rcf} \mathbb{1}_{\{a \neq 0\}}}$$

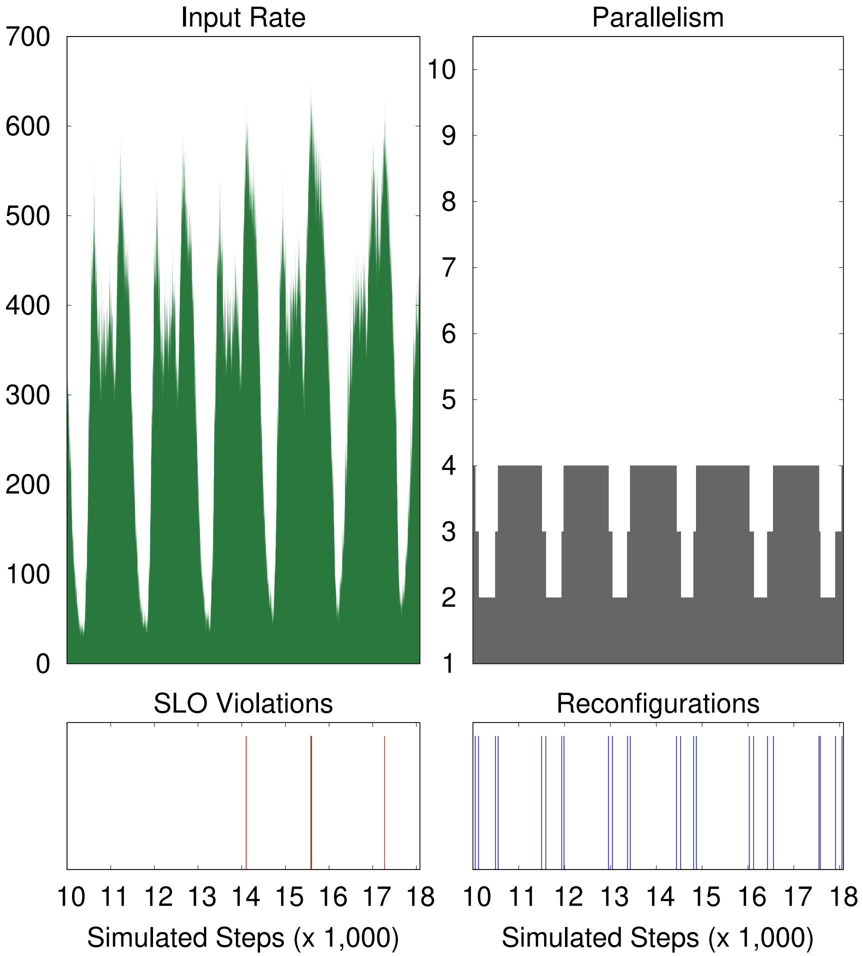
Resource Cost   Performance   Reconfiguratio

- $w_{res} + w_{perf} + w_{rcf} = 1$, $w_x \geq 0$, $x \in \{res, perf, rcf\}$
- $R(s, a, s')$ is the component performance index, e.g, response time,
- R^{max} is the component reference performance value

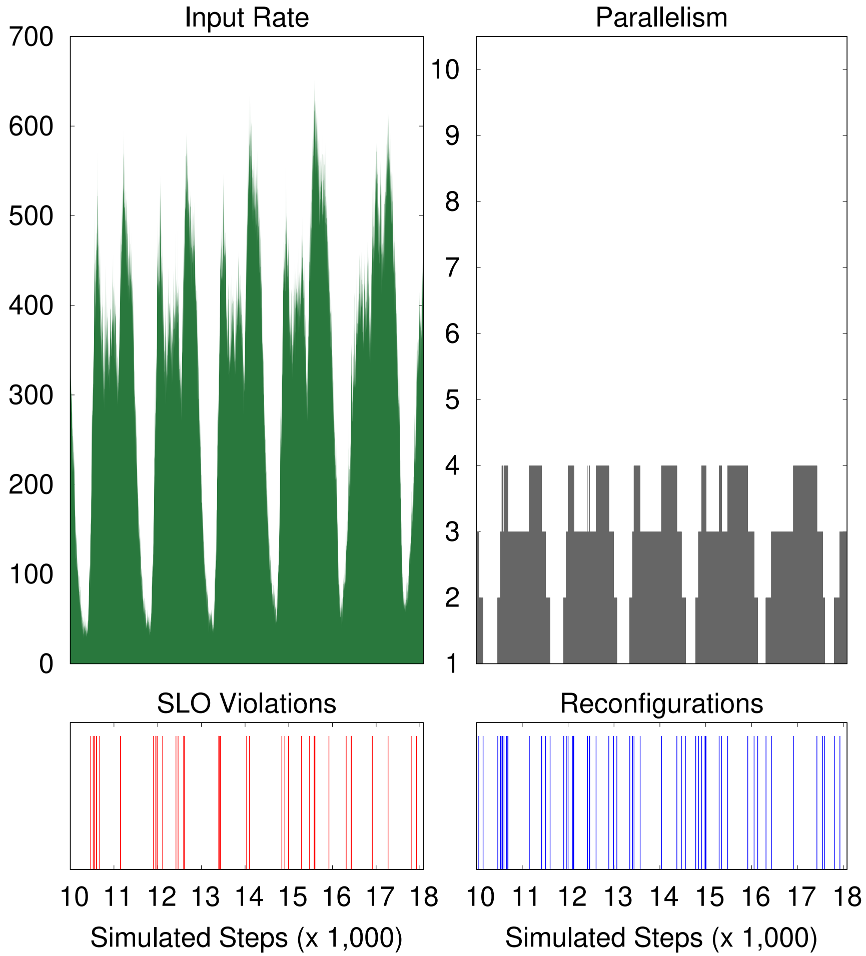
We want to minimize $\sum_{t=0}^{\infty} \gamma^t c(s_t, a_t, s_{t+1}), \quad \gamma \in [0, 1)$

Trading-off Objectives

w_{perf}=0.6, $w_{res} = w_{rcf}$=0.2



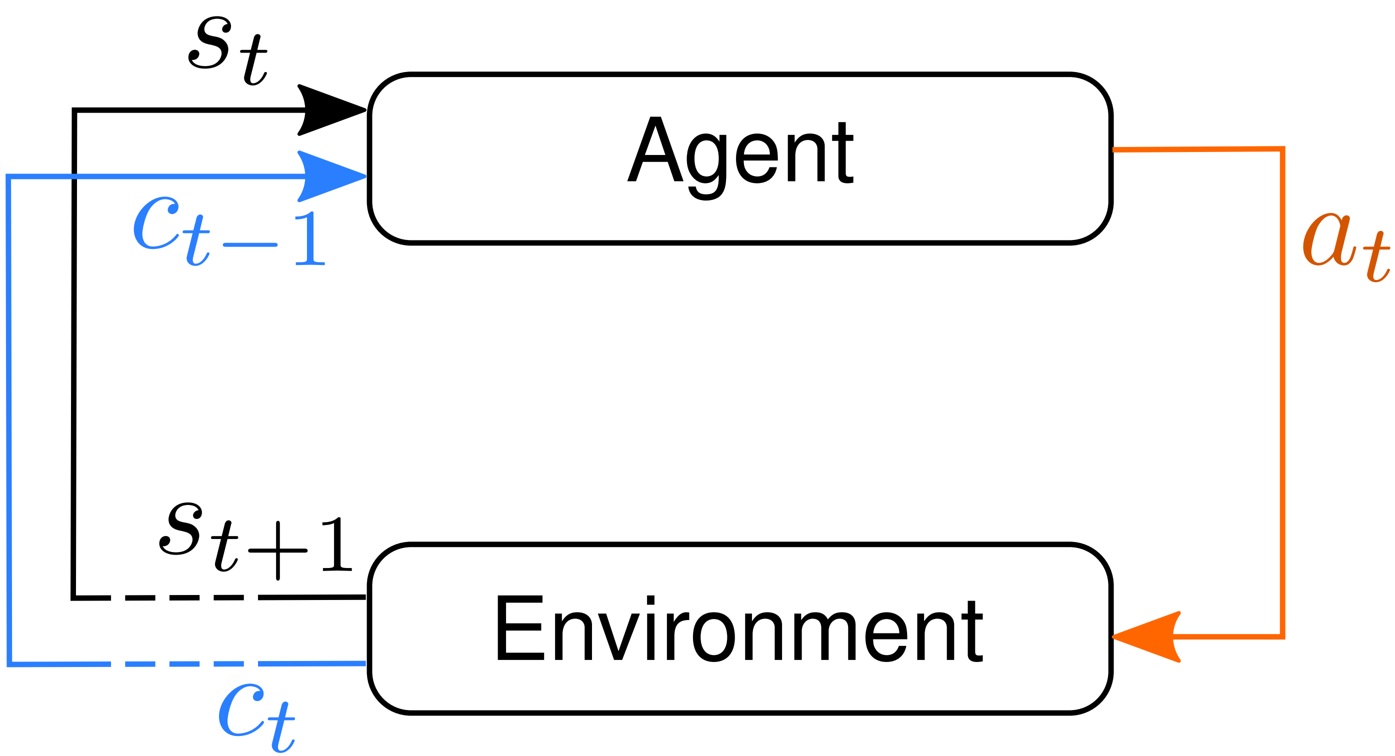
w_{res}=0.6, $w_{perf} = w_{rcf}$=0.2



MDP Resolution

- We can use the Value Iteration algorithm to solve the MDP
  - i.e., finding the optimal policy
- Is this enough?
- Unfortunately, solving the MDP requires exact and complete knowledge of the underlying model
  - state transition probabilities
  - cost function
- In practice, we don't have such information!

Reinforcement Learning



- RL aims to learn the optimal policy through interaction and evaluative feedback

Model-free vs Model-based RL

- **Model-free** RL: no model of the environment is available or used; the optimal policy is learned through experience only
- **Model-based** RL: a (possibly partial) model of the environment is available and used to derive the optimal policy
  - a partial model can boost learning speed
  - RL may also be used in presence of a complete model instead of VI; e.g., with a large number of rarely visited states VI would unnecessarily run for a long time!
  - You may also try to learn the model online and use it to compute a policy

Value-based vs Policy-based RL

- **Value-based** RL: aims to learn the optimal value function through experience; the policy is derived from it
  - Simplest RL algorithms belong to this group
  - We will mainly focus on this group in the following
- **Policy-based** RL: aims to directly learn the optimal policy through experience; no explicit computation/learning of the value function
- **Hybrid** approaches: e.g., the Actor-Critic framework

Simple Value-based RL Algorithm

A simple RL algorithm

$t \leftarrow 0$
Initialize Q
Loop
    $t \leftarrow t + 1$
EndLoop

Q-learning

- Proposed by Chris Watkins in 1989
- One of the most known (and simplest) RL algorithms
- Proven to converge to the optimal policy under mild assumptions
  - …after n steps, with $n \to \infty$

Q-learning: Action Selection

- How to choose an action at every time step?
- Exploration vs Exploitation dilemma

- Exploitation: using available knowledge to maximize reward
  - choose the "best" action, i.e., $a_t = \arg\max_a Q(s_t, a)$
- Exploration: discovering more information about the environment
  - choose other actions to learn more about the environment

Q-learning converges only if all state-action pairs are visited an infinite number of times as $t \to \infty$

- you can't exploit all the time
- you can't explore all the time

ϵ-Greedy Exploration

- Popular approach for the exploration-exploitation dilemma
- With probability $1 - \epsilon$ choose the greedy action $a^* = \arg\max_{a \in \mathcal{A}} Q(s, a)$
- With probability ϵ choose an action at random

- Improvement: ϵ-greedy with decaying ϵ (similar to decaying learning rate in SGD)

Softmax Action Selection

- Alternative to the ϵ-greedy strategy
- All actions assigned non-zero probability of being chosen
- Action $a \in \mathcal{A}$ is selected with probability

$$\pi(a|s) = \frac{exp(Q(s,a)/\tau)}{\sum_{a' \in \mathcal{A}} exp(Q(s,a')/\tau)}$$

- τ is the "temperature"
  - Small τ leads to greedy behavior
  - Large τ leads to random action selection
  - You usually start with a large temperature value and let it decay

Q-learning: Updating Q

With known model, we can compute Q iteratively using:

$$Q(s,a) \leftarrow c(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) \max_{a'} Q(s',a')$$

Q-learning uses *point estimates* on experience $\{s_t, a_t, c_t, s_{t+1}\}$:

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \underbrace{\alpha_t}_{\text{Learning Rate}} \Big[\underbrace{r_t + \gamma \max_{a' \in \mathcal{A}} Q(s_{t+1},a')}_{\text{Target}} - Q(s_t,a_t) \Big]$$

Q-learning: Algorithm

Q-learning

$t \rightarrow 0$
Initialize Q (e.g., zero-initialized)
Loop
  choose a_t (e.g., ϵ-greedy or softmax selection)
  observe next state s_{t+1} and reward r_t
  $Q(s_t, a_t) \leftarrow$
    $Q(s_t, a_t) + \alpha_t \left[r_t + \gamma \max_{a' \in \mathcal{A}} Q(s_{t+1}, a') - Q(s_t, a_t)\right]$
  $t \leftarrow t + 1$
EndLoop

Example: Maze

- `python maze.py --agent qlearning --episodes N [-- plot_reward]`
- Compare different reward models

maze.py