Performance Modeling of Computer Systems and Networks

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Analytical models

Exercises

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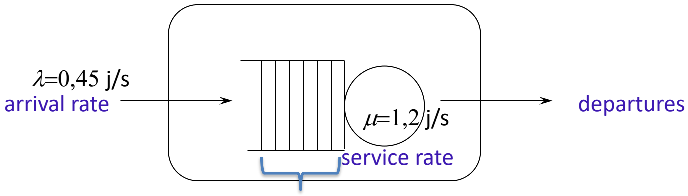
1

Consider a web server with a mean processing rate of 1.2 job/s.
If the server receives requests with a rate of 0.45 job/s and it has 0.225 enqueued jobs on average, determine:

a) the average utilization
b) the average response time.

During rush hours the arrival rate grows of 20% and the average number of enqueued jobs becomes 0.3681818.
Determine:
c) the performance metrics a) and b)
d) which further increasing in arrival rate makes the server collapsing
e) the performance metrics a) and b) for the limiting case d).

2



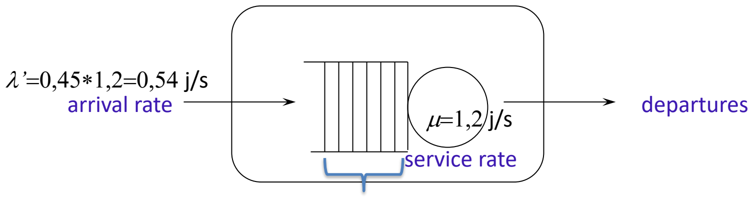
E(N_Q)=0,225

a) the average utilization
b) the average response time.

$$\rho = \lambda / \mu = 0{,}375$$

$$E(T_S) = \frac{E(N_S)}{\lambda} = 0{,}6/0{,}45 = 1{,}333333 \text{ s} \qquad E(N_S) = E(N_Q) + \rho = 0{,}225+0{,}375=0{,}6$$

3



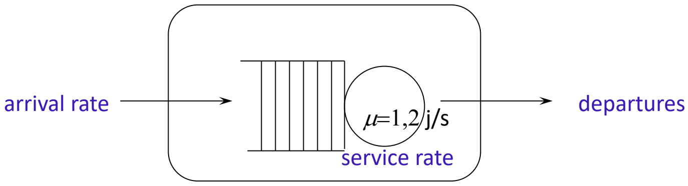
E(N_Q)=0,3681818

During rush hours the arrival rate grows of 20% and the average number of enqueued jobs becomes 0.3681818.
Determine:
c) the performance metrics a) and b)

$$\rho = \lambda' / \mu = 0{,}45$$

$$E(T_Q) = \frac{E(N_Q)}{\lambda'} = 0{,}681818 \text{ s} \qquad E(T_S) = E(T_Q) + E(S) = 1{,}515151$$

4



c) which further increasing in arrival rate makes the server collapsing
d) the performance metrics a) and b) for the limiting case d).

$\rho \rightarrow 1 \qquad \lambda' \rightarrow \mu$

$0{,}54 * x = 1{,}2 \qquad x = 2{,}222222$

$E(T_S) = \infty$

Let us consider a server that processes jobs with rate 0.8 jobs/s.
By assuming that the server receives jobs with a rate depending on the time slot as follows:

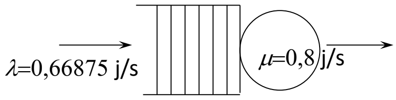
- 8.00 a.m. – 12.00 a.m. average arrival rate 1.5 jobs/s
- 12.00 a.m. – 2.00 p.m. average arrival rate 0.5 jobs/s
- 2.00 p.m. – 7.00 p.m. average arrival rate 1.5 jobs/s
- 7.00 p.m. – 9.00 p.m. average arrival rate 0.5 jobs/s
- 9.00 p.m. – 8.00 a.m. average arrival rate 0.05 jobs/s

Determine:

a) average arrival rate per day (24 hours)
b) average utilization per day
c) average throughput per day
d) average throughput for each time slot

Please, justify and comment the results by indicating the used laws.

a) average arrival rate per day (24 hours)
b) average utilization per day



I time slot:	4 h	1.5 jobs/s
II time slot:	2 h	0.5 jobs/s
III time slot:	5 h	1.5 jobs/s
IV time slot:	2 h	0.5 jobs/s
V time slot:	11 h	0.05 jobs/s

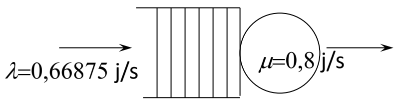
$$\frac{9}{24}\times 1{,}5+\frac{4}{24}\times 0{,}5+\frac{11}{24}\times 0{,}05 = 0{,}66875\ j/s$$

$\rho = \lambda/\mu = 0{,}835937$

nelle 24 ore, tutto ciò che è entrato
è stato servito, in quanto lambda < mu

7

c) average throughput per day



I time slot:	4 h	1.5 jobs/s
II time slot:	2 h	0.5 jobs/s
III time slot:	5 h	1.5 jobs/s
IV time slot:	2 h	0.5 jobs/s
V time slot:	11 h	0.05 jobs/s

$$\cancel{\begin{array}{l} \lambda < \mu \rightarrow X = \lambda \\ \lambda > \mu \rightarrow X = \mu \end{array}}$$

$$\frac{9}{24}\times 0{,}8+\frac{4}{24}\times 0{,}5+\frac{11}{24}\times 0{,}05 = 0{,}4062496\ j/s < 0{,}66875\ \text{j/s}$$

???

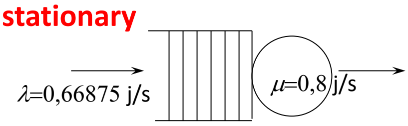
The system is not stationary!!!

avrei dovuto avere due valori uguali, allora sarebbe stato stazionario.

8

The system is not stationary

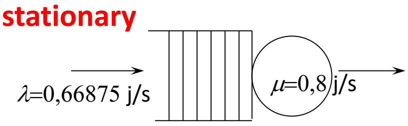
d) average throughput for each time slot



I time slot:	4 h = 14.400 x1,5=	21.600	jobs arrived
	14.400x0,8=	<u>11.520</u>	jobs served
		10.080	jobs in queue!!
II time slot:		10.080	job in queue
	2 h =7.200x0.5=	3.600	job arrived
	7.200x0.8=	<u>5.760</u>	job served
		7.920	jobs in queue
III time slot:		7.920	jobs in queue
	5 h =18.400x1.5=	27.000	jobs arrived
	18.400x0.8=	<u>14.400</u>	jobs served
		20.520	jobs in queue

The system is not stationary

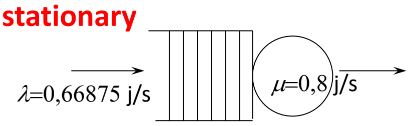
d) average throughput for each time slot



IV time slot:		20.520	jobs in queue
	2 h =7.200x0.5=	3.600	job arrived
	7.200x0.8=	<u>5.760</u>	job served
		18.360	jobs in queue
V time slot:		18.360	jobs in queue
	11 h =39.600x0.05=	<u>1.980</u>	jobs arrived
		20.340	all served!!!

The system is not stationary

d) average throughput for each time slot



server never empty!!

I time slot:	4 h = 14.400 x1,5=	21.600	jobs arrived
	14.400x0,8=	11.520	jobs served
		10.080	jobs in queue!!

X=0,8 j/s

II time slot:		10.080	job in queue
	2 h =7.200x0.5=	3.600	job arrived
	7.200x0.8=	5.760	job served
		7.920	jobs in queue

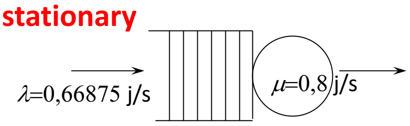
X=0,8 j/s

III time slot:		7.920	jobs in queue
	5 h =18.400x1.5=	27.000	jobs arrived
	18.400x0.8=	14.400	jobs served
		20.520	jobs in queue

X=0,8 j/s

The system is not stationary

d) average throughput for each time slot



IV time slot:		20.520	jobs in queue
	2 h =7.200x0.5=	3.600	job arrived
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		18.360	jobs in queue

X=0,8 j/s

V time slot:		18.360	jobs in queue
	11 h =39.600x0.05=	1.980	jobs arrived
		20.340	all served!!!

X= 20.340/39600= 0,513636 j/s

$$\frac{13}{24}\times 0,8+\frac{11}{24}\times 0,5136363=0,6687499\ j/s$$