# Performance Modeling of Computer Systems and Networks

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Operational analysis:

Queueing Networks

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Da un punto di vista degli analisti di sistema si è creato dello scetticismo, perchè il modello era regolamentato da un approccio stazionario, le ipotesi richiedevano indipendenza stocastica tra i job, etc...



## Analytical models

*Queueing network (QN) modelling* is a particular approach to computer system (CS) modeling in which the CS is represented as a *network of queues* which is evaluated *analitically*

- Many analysts experienced doubts on its accuracy
- A series of assumptions:
  - ✓the system is modeled by a *stationary stochastic process*;
  - ✓jobs are *stochastically independent*;
  - ✓job steps from device to device follow a *Markov chain*;
  - ✓the system is in stochastic equilibrium;
  - ✓the service time requirements at each device conform to an *exponential distribution*;
  - ✓the system is *ergodic*, i.e. long-term time averages converge to the values computed for stochastic equilibrium



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Molte assunzioni erano "forzate" rispetto al caso in analisi.

# Analytical models

Some of these concepts are difficult and cannot be proved to hold by observing the system in a finite time period
Most can be disproved empirically

- ✓parameters change over time
- ✓jobs are dependent
- ✓systems are observable only for short periods
- ✓…



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# Analytical models

In applying or validating the results of Markovian QN theory, analists substituted *operational* values for stochastic parameters

directly measured

The Markovian QN equations are also valid among operational variables

They hold under different assumptions and apply to a large class of real systems[1]

[1] BUZEN, J.P. "Operational analysis: the key to the new generation of performance prediction tools," in Proc. IEEE COMPCON, 1976, IEEE, New York.
DENNING, P. J.; AND BUZEN, J. P. "Operational analysis of queueing networks," in Proc. Third Int. Symp. Computer Performance Modeling, Measurement, and Evaluatlon, 1977, North-Holland Publ. Co., Amsterdam, The Netherlands.



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## Operational Analysis

Three *operational principles*:

1. All quantities should be *precisely measurable* and all assumptions should be *directly testable*
2. The system must be *flow balanced*
3. The devices must be *homogeneous*,

   i.e., the routing must be independent of queue lengths (ql)
   the mean service time at a given device must not depend on ql of other devices

the same mathematical equations

but the operational assumptions can be tested

much more confidence and understanding of the QN technology



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## Operational Analysis

Def.

Hypotheses whose veracity can be established beyond doubt by measurement will be called *operationally testable*

Operational analysis provides a rigorous mathematical discipline for studying CS performance based solely on operationally testable hypotheses

two basic components:

a system (real or hypothetical)
a (finite) time period

*the observation period*



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## Operational Analysis

According to the *operational* approach, let us consider

*Basic quantities*

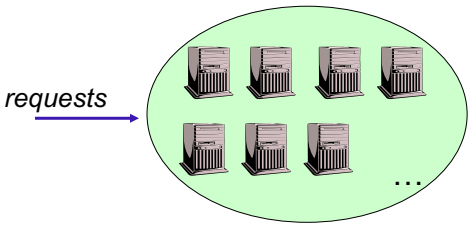
- $T$ the length of the observation period (op)
- $A$ the number of arrivals during op
- $B$ the total amount of time during which the system is busy $B \leq T$ during op
- $C$ the number of completions during op





## Multi Resources Systems 1

- a collection of servers, working together for incoming requests
- homogeneous servers



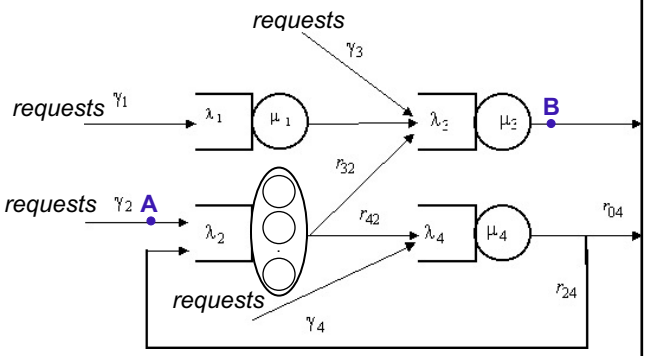

Data centers
Server farm

*What is the minimum number of servers needed to guarantee that only a small fraction of jobs are delayed?*

*Is a single central queue superior to having a queue at each server?*

## Multi Resources Systems 2

- a collection of different resources, connected, where incoming requests can require services more than once to each resources




*Which is the "load" of each resource?*

*Which is the time spent on each resource?*

*Which is the time to go from point A to point B? (end-to-end response time)*



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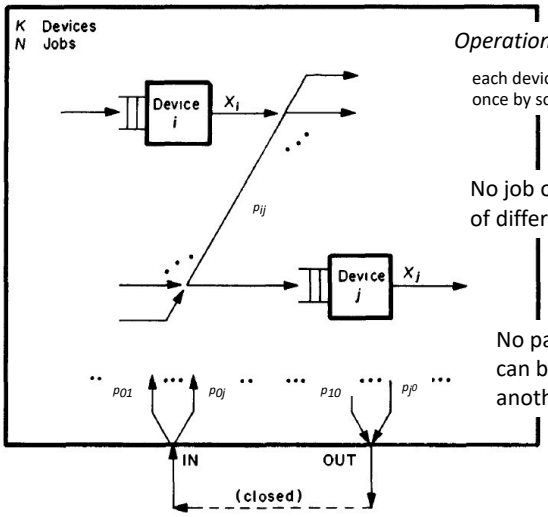
Operational Analysis

## Queueing Networks




*Operationally connected:*

each device is visited at least once by some job during op

No job overlaps its use of different devices

Work-conserving

No part of the system can block progress in another part



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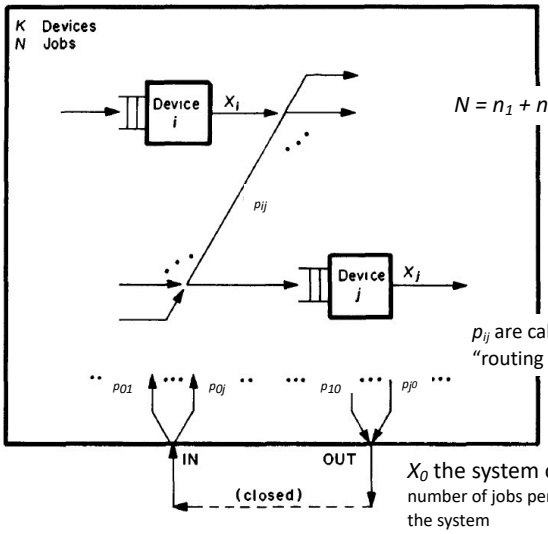
# Queueing Networks




$N = n_1 + n_2 + \ldots n_K$

$p_{ij}$ are called the "routing frequencies"

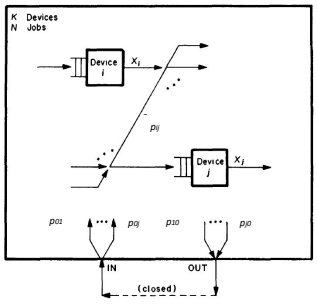
$X_0$ the system output rate, the number of jobs per second leaving the system







# Queueing Networks




If the system is *open*, $X_0$ is known and $N$ varies as jobs enter or leave the system

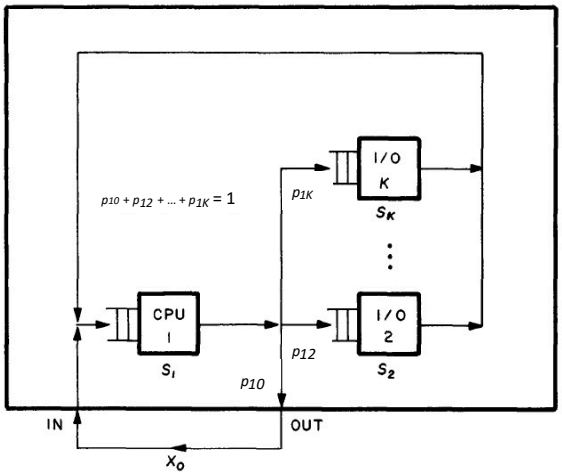
An analysis of an open system assumes that $X_0$ is known and seeks to characterize the distribution of $N$

If the system is *closed* the number of jobs $N$ is fixed

An analysis of a closed system begins with $N$ given and seeks to determine the resulting $X_0$ along the OUT/IN path

Queue lengths and response times at the devices may be sought in both cases

## Central Server Network




A batch processing system under a backlog:

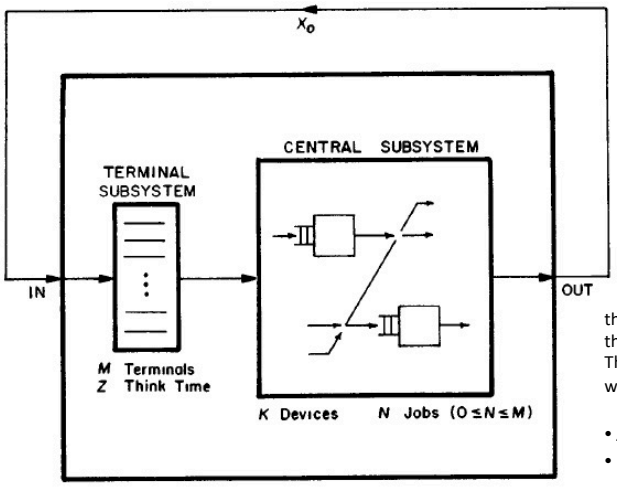
- a job begins with a CPU service interval and continues with zero or more I/O intervals which alternate with further CPU bursts
- a new job enters the system as soon as an active job terminates







## Terminal Driven System




Time sharing systems, driven by interactive terminals:

a user alternates between *thinking* and *waiting*

the mean time a user spends in a thinking interval: *Z think time*
The mean time a user spends in a waiting interval: *R response time*

- *Z* is independent of *M*
- *R* is a function of *M*:

  jobs delay each other while contending for resources

## Queueing Networks

The system is measured for an op of $T$ seconds, the following data are collected for each device $i=1, 2, \ldots K$

- $A_i$ the number of arrivals;
- $B_i$ total busy time, during which $n_i > 0$
- $C_{ij}$ number of times a job requests service at device $j$ immediately after completing a service request at device $i$; note that it is possible $C_{ii} > 0$.

If we treat the "outside world" as device "0", we can define also

- $A_{0j}$ number of jobs whose first service request is for device $j$;
- $C_{i0}$ number of jobs whose last service request is for device $i$.



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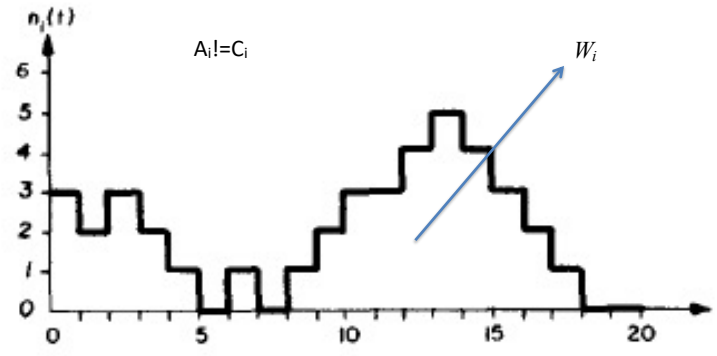
## Queueing Networks



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## Operational Equations

$$X_j = \sum_{i=0}^{K} X_i p_{ij}$$ Job Flow Balance equations

$$\begin{cases} V_0 = 1 \\ V_j = p_{0j} + \sum_{i=1}^{K} V_i p_{ij} \end{cases}$$ Visit Ratio equations



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## Operational Equations

$U_i = X_i S_i$ *Utilization Law*

$\bar{n}_i = X_i R_i$ *Little's Law*

$X_0 = \sum_{i=0}^{K} X_i p_{i0}$ *Output Flow Law*

$R = \sum_{i=1}^{K} V_i R_i$ *General Response Time Law*

$R = M/X_0 - Z$ *Interactive Response Time Formula* (Assumes flow balance)



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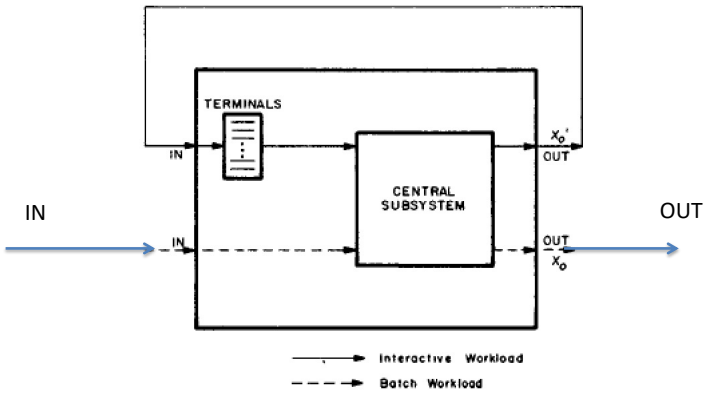
TERMINALS
IN
$X_0'$
OUT
CENTRAL SUBSYSTEM
IN
OUT
$X_0$
IN
OUT
Interactive Workload
Batch Workload



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