

message integrity  
II

## **Message Authentication:**

# user authentication

## **Secure hash-based constructions**

# **Confidentiality ≠ Integrity**

## **→ Confidentiality**

→ Message hiding

⇒ Only the legitimate destination should be able to see the content of a message

## **→ Integrity**

→ Message authenticity

⇒ Nobody should be able to modify a message while in transmission

⇒ Only the legitimate source should be able to forge a message

## **→ Integrity-only: often is the only thing you want**

⇒ Example: your CV

→ Everybody should be able to read it! (no confidentiality)

→ But only →YOU← should be able to modify it

# Encryption does NOT guarantee integrity!

→ Example: let's take the best possible cipher one time pad – unconditionally secure...



PT

1000	1000	1000	1111
------	------	------	------

KEY:

0100	1101	0011	0101
------	------	------	------

⊕



Conclusion:  
Best possible  
Cipher has  
zero integrity!!



(cipher) CT

⊕

Δ

1100	0101	1011	1010
------	------	------	------

0000	1001	0000	0000
------	------	------	------

CT'

1100	1100	1011	1010
------	------	------	------

⊕

KEY:

0100	1101	0011	0101
------	------	------	------

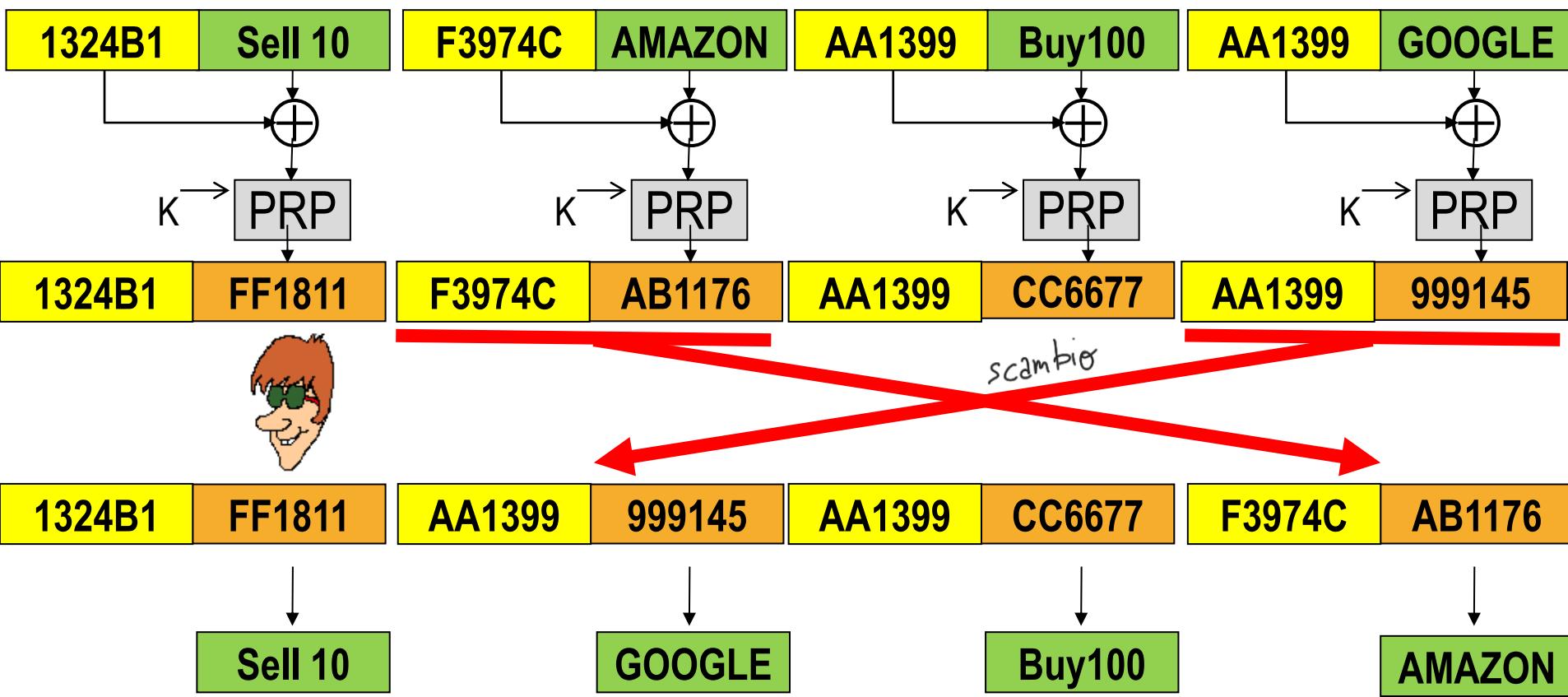
PT'

1000	0001	1000	1111
------	------	------	------



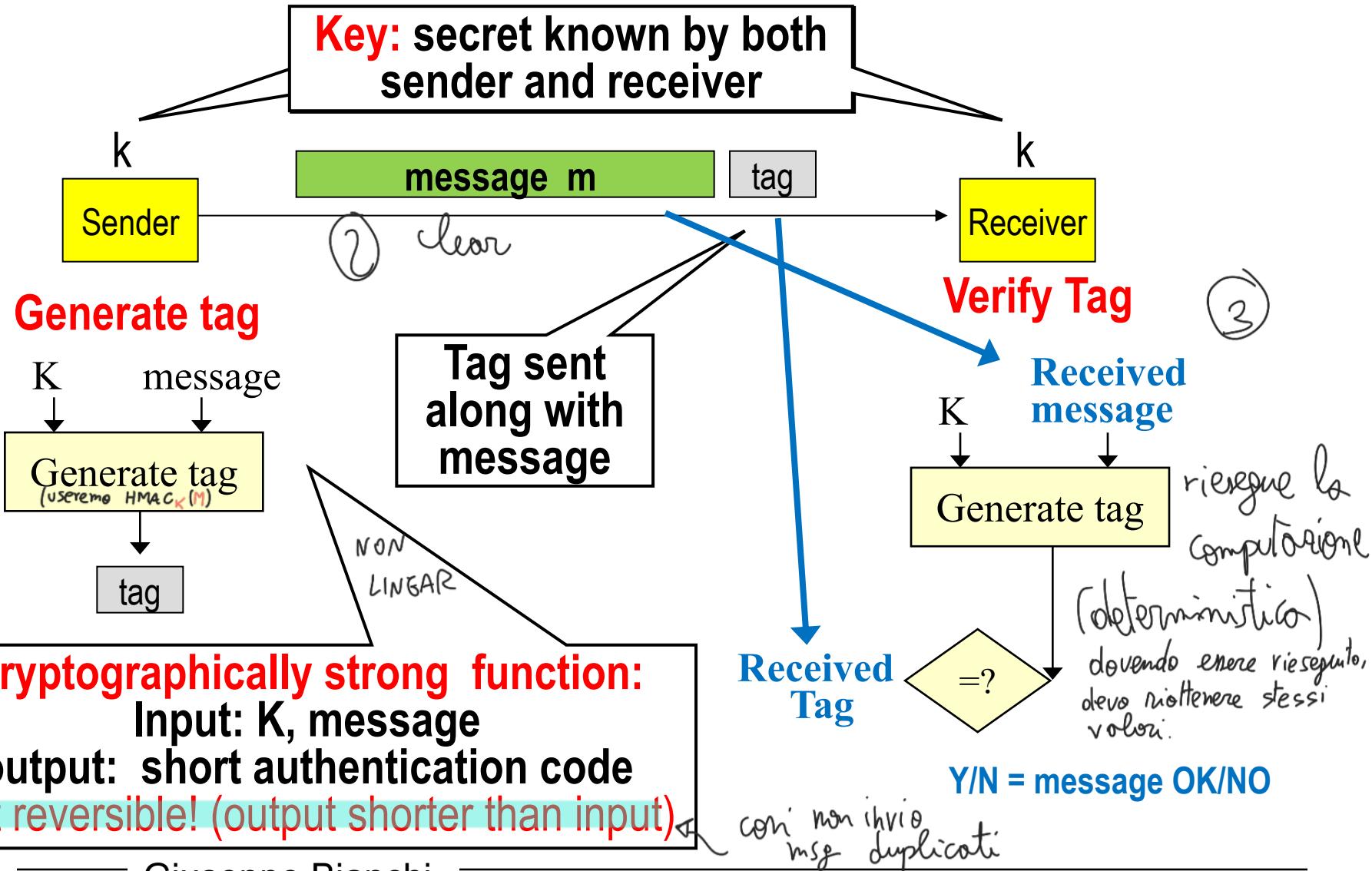
# Encryption does NOT guarantee integrity!

→ Example with block cipher (posso combinare la semantică)  
(ECB with explicit IV per block to make it semantically secure, more later)



VERY different semantics than what meant by transmitter!

# Message authentication (with symmetric key)



# **Message authentication: what it does NOT protect?**

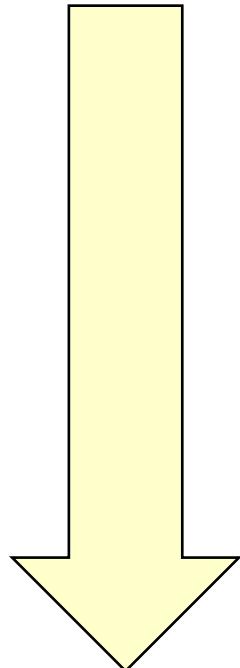
# Protects from MITM attacker



message  $m$

TAG

$$\text{TAG} = F(K, m)$$



Modify  
 $m \rightarrow m^*$



man in  
the middle

But (assumptions on  $F$  = cryptographically strong):

- 1) Sees TAG &  $m$ : must NOT be able to «invert»  $F$  and get  $K$
- 2) Cannot change  $\text{TAG} \rightarrow \text{TAG}^* = F(K, m^*)$  without knowing  $K$
- 3) Cannot change  $m \rightarrow m^*$  such that  $F(K, m) == F(K, m^*)$

message  $m^*$

TAG

$$F(K, m^*) = \text{TAG}^* \neq \text{TAG}$$

Someone modified  $m!!$



# Protects from message spoofing

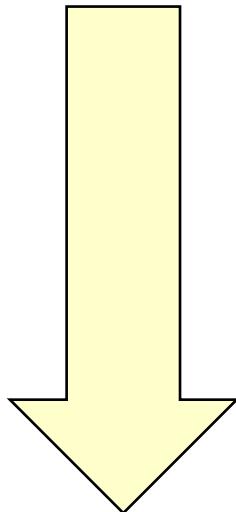
(metto NUOVO msg nello scambio)



Spoofed message X

TAG = F(K,X)

?????



(assumptions on  $F$  = cryptographically strong):  
Cannot compute  $F(K, X)$  without knowing  $K$

tiro a caso!

Spoofed message X

Invalid TAG

$F(K, X) \neq \text{TAG}$   
**Spoofed message!**



# Does NOT protect from replay attacks



I'm sending the same message twice  
as I really want to pay 2000\$

Pay 1000 \$

TAG

Pay 1000 \$

TAG



Let's stop. I just want  
to pay \$1000 once.

Non  
DISTINGUIBILI!

Ops! This is a  
valid message!

Pay 1000 \$

TAG

Pay 1000 \$

TAG



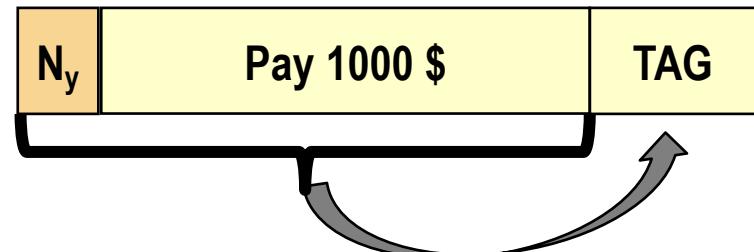
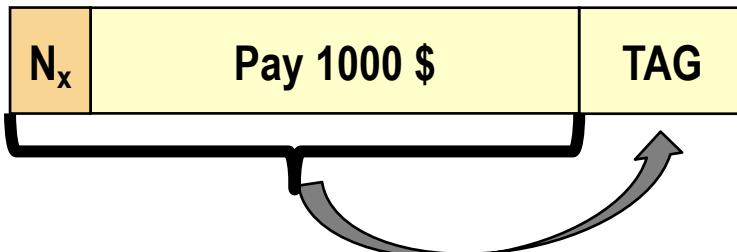
Nah! I'll make  
you pay twice!!

# Preventing replay attacks: Nonces

→ Replay attacks prevented if ALL messages are different each other!

→ How to?

⇒ Application MAY legitimately send two identical messages!



→ Add «nonce» to each packet

⇒ Nonce=«fresh» information, guaranteed to differ in each message!

⇒ Does not have to remain secret → can be added to the msg in clear

- Sequence number
- Random number
- Time stamp

$$F(N_x, MSG, K)$$

→ and include it in the TAG computation!

# Test your understanding

→ Which pros and cons of each type of nonce?

→ Sequence number?

⇒ How to manage «reboots» ☺ ?

→ Random numbers?

(come capisco l'ordine?, dovei more DB e fare check)

⇒ How RX can control that packet is a fresh one?

→ Time stamps

(es: Cambio orario al telefono)

⇒ Who guarantees time? «now» is really «now»?

# **MAC = Digital Signature?**

→ **Digital signature (DS):**

⇒ **Nobody** can modify a digitally signed message (except creator)

→ **Message Authentication Code (MAC):**

⇒ **Nobody except TX and RX (which have the key)** can modify a MAC-authenticated message

→ **Both have an identical purpose!**

⇒ Message/data Integrity

→ **But unlike MAC, DS further guarantees non repudiation (source authentication)**

⇒ DS = **stronger** form of authentication than MAC

→ But requires different crypto → asymmetric, more later

⇒ Why this is useful? Legal scenarios, Multicast scenarios

# Defining Security for Message Authentication Codes



→ **Security = Unforgeability**

⇒ Attacker should not be able to create or modify a message

⇒ Implies that attacker should not be able to extract key from pair {M, TAG(K,M)}

→ **Formally: a game against an attacker model**

⇒ From weaker to stronger: Known Message Attack →

→ Chosen Message Attack → Adaptively Chosen Message Attack

→ **Attacker is given (may choose) a number of past message/tag pairs**

⇒  $(m_1, t_1), (m_2, t_2), (m_3, t_3), \dots$

→ Now sees (chooses) message m: must be unable to forge tag

→ More specifically:

**probability to forge valid pair must be NEGIGIBLE**

# Test your understanding

definiisce 8 bit

→ 1 byte tags

$$\text{dim } [F(\text{msg grande}, K 128 \text{ bit})] = 8 \text{ bit}$$

da agg  
ol msg

→ No way for attacker to guess tag from msg, beyond pure random choice

→ Is this secure?

Se modifico msg, con random tag:  $\frac{1}{256} \rightarrow 2^8$

→ NO! Probability of guessing = 1/256

→ Not nearly negligible!!

→ Note the crucial difference with encryption security definitions!

**Which (highly non linear) function can we use?**

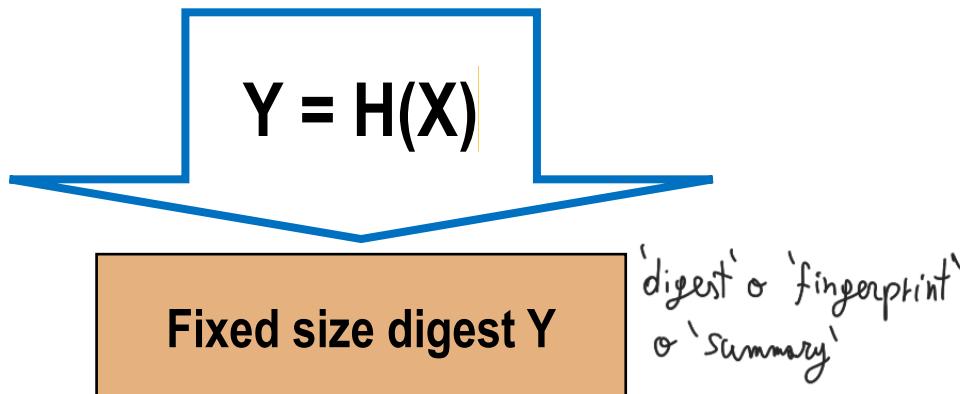
**Hash function? What's this?**



# What is a Hash function?

(non crittografico)

Any length message X



**Arbitrary size X → fixed size Y=H(X)**

(from as short as 1 single bit to... any size) → e.g. exactly 256 bits for SHA256

**Y=H(X): “fingerprint” (short summary) of X**

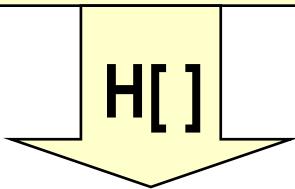
$H(X)$  should be relatively easy to compute for any given  $X$ , making both hardware and software implementations practical

Se modifico N volte il msg, non potrò avere msg diversi e stesso hash  
dell'originale

# (collisioNE) Cryptographic hash functions

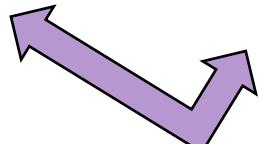


Hic ego: laudare igitur eloquentiam et quanta vis sit eius expromere quantamque eis, qui sint eam consecuti, dignitatem afferat, neque propositum nobis est hoc loco neque necessarium. hoc vero sine ulla dubitatione confirmaverim, sive illa arte pariatur aliqua sive exercitatione quadam sive natura, rem unam esse omnium difficultumam. quibus enim ex quinque rebus constare dicitur, earum una quaeque est ars ipsa magna per se. quare quinque artium concursus maxumarum quantam vim quantamque difficultatem habeat existimari potest.

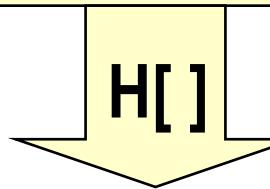


c6c8258947bffe06ea4a0c8132af337a3c74ec  
81d754a96d5a29e3ca7d8ce49d

Fixed size digest  
(e.g. SHA-256: 64 hex)



Hic ego: laudare igitur eloquentiam et quanta vis sit eius expromere quantamque eis, qui sint eam consecuti, dignitatem afferat, neque propositum nobis est hoc loco neque necessarius. hoc vero sine ulla dubitatione confirmaverim, sive illa arte pariatur aliqua sive exercitatione quadam sive natura, rem unam esse omnium difficultumam. quibus enim ex quinque rebus constare dicitur, earum una quaeque est ars ipsa magna per se. quare quinque artium concursus maxumarum quantam vim quantamque difficultatem habeat existimari potest.



3238ead7fb611463703c47adc4215aa245a1f  
1a4a0cea4c11296b466a76bbac4

No way for an attacker to either **create** or  
**purposedly modify/extend/replace** initial text  
so as to obtain original digest!!

# Property #1 of a cryptographic hash function

→ **Preimage resistance (one way)** (reverse difficile)

⇒ Given Y = result of a hash, it is hard to find ANY X such as H(X)=Y

?

?

?

?

**One way:** much more than  
«non-invertible»

Infinite (!! ) messages can generate this digest  
but you should NOT be able  
(in a reasonable time)  
to find ANYONE of such messages!!

$2^{256}$   
Digest:

c6c8258947bffe06ea4a0c8132af337a3c74ec  
81d754a96d5a29e3ca7d8ce49d

La funzione HASH deve resistere a 'forza bruta', il digest deve avere dimensioni elevate.  
Se 16 bit  $\rightarrow 2^{16}$  output = 65536 valori. UN PC attuale computa 66 Milioni di Hash/s. NON BASTANO GRANDI DIMENSIONI, conta anche l'implementazione. Se ad esempio 2 bit sono dipendenti, mi serve ancora meno tempo, la metà:  $2^{16}/2 = 2^{15}$   
Esempio: SHA1 usava 160 bit, ma per 'romperlo' bastano  $2^{10}$  tentativi. Ogni 10 bit, ho un fattore di '1024'. Da 40 bit a 60 bit, in caso forza bruta, diff è 20 bit, quindi se ogni 10 bit = 1024  $\rightarrow 1024 \cdot 1024 \approx 1 \text{ mln}$ .  $\frac{2^{160}}{2^{61}}$  ha diff 100 bit, quindi  $(1024)^{10}$ .

# Property #2 of a cryptographic hash function

## → Second preimage resistance (weak collision resistance)

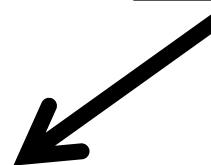
⇒ Given X, it is hard to find another X' such that H(X) = H(X')

Good Message M:

Cari ragazzi non potete modificare questo messaggio se, come prova di quanto sto scrivendo, conservo la sua hash in una mia penna usb

???

Attacker cannot find ANY other message which has the same hash



Digest:

920d7e8cf22725391c2415ade26358f8c099e  
d37837e1a8911786b0e3684f23c

Giuseppe Bianchi

Stronger property than one-wayness!

there are functions which satisfy property #1 but fail property #2 (raro)

MODULAR EXPONENTIATION

LIMITATO COME SERVE (INPUT  $\infty$ , OUT FINITO)  
 $y = F(x) = g^x \text{ mod } p$ ,  $1 < \text{digest} < p-1$ ,  $p$  large prime number,  $g$  not

Graficamente  , se  $2^x \text{ mod}(10!) = 60$  e' possibile trovare 'x', ma complessita' esponenziale.

RISPETTA priorita' #1 ( $y \xrightarrow{\text{hard}} x$ )  
NON priorita' #2 ( $\text{periodo} = p-1$ , quindi:  $g^{x+k(p-1)} \text{ mod } p = g^x \text{ mod } p$ )  
(collision resistant)

# Property #3 of a cryptographic hash function

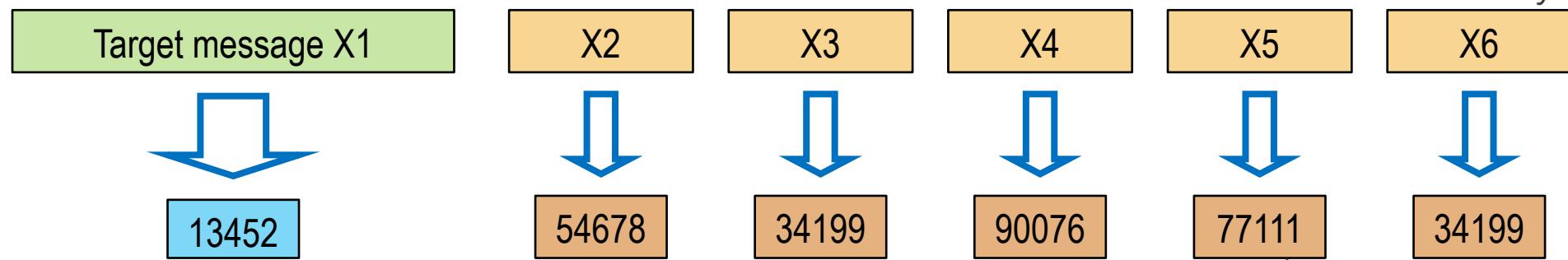
più hard da garantire

## → Collision resistance (strong collision resistance)

⇒ It is hard to find two generic  $X_1$  and  $X_2$  such that  $H(X_1) = H(X_2)$

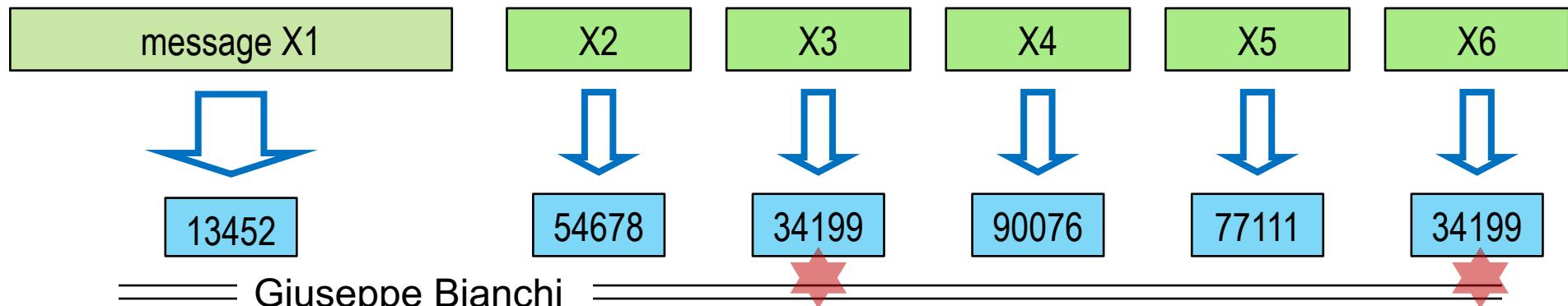
Weak collision resistance: target message

(provo finché ho collisione con  $X_1$ )



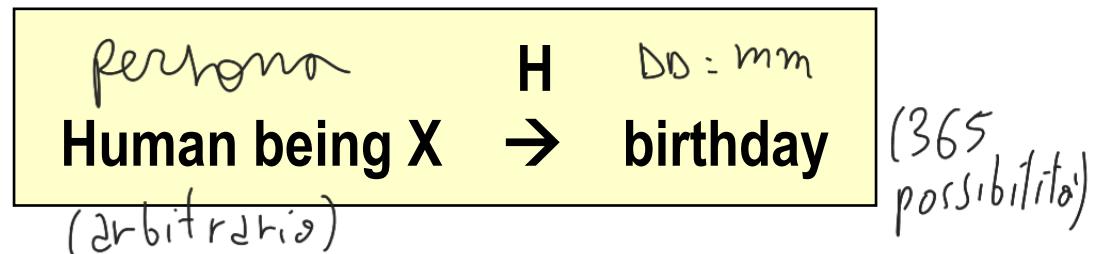
Strong collision resistance: ANY message

(tra ogni msg !!)



# Birthday paradox

23 people are in a class  
(22 + myself)



1. What is the probability that none of you N=22 is born in my same day?
2. What is the probability that no two+ of us N=23 are born the same day?

$$\left(1 - \frac{1}{365}\right)^{22} = \left(\frac{364}{365}\right)^{22} = \dots \quad 94.1\%$$

(Non nato giorno "3")

= you have my same birthday in less than 6% cases

giorno x           giorno y ≠ x           giorno λ ≠ x ≠ λ ≠ ...

$$1 \cdot \left(1 - \frac{1}{365}\right) \cdot \left(1 - \frac{2}{365}\right) \cdots \left(1 - \frac{22}{365}\right) =$$
$$= \frac{365 \cdot (365 - 1) \cdots (365 - 22)}{365^{23}} =$$
$$= \frac{365! / (365 - 23)!}{365^{23}} = \dots \quad 49.3\%$$

In more than 50% cases two of us have the same birthday!!! Not so obvious...

(64 bits)

# Birthday paradox math

n bit digest  $\rightarrow N = 2^n$  $K = \overset{\circ}{n}.$  messages

p = collision probability

$$P(\text{no collision}) = 1 - p = \frac{N! / (N-K)!}{N^K} =$$

$$= \frac{N}{N} \cdot \frac{N-1}{N} \cdot \frac{N-2}{N} \cdot \dots \cdot \frac{N-(K-1)}{N} =$$

$$= 1 \cdot \left(1 - \frac{1}{N}\right) \cdot \left(1 - \frac{2}{N}\right) \cdot \dots \cdot \left(1 - \frac{K-1}{N}\right) =$$

$$= \prod_{i=1}^{K-1} \left(1 - \frac{i}{N}\right) \approx \prod_{i=1}^{K-1} e^{-i/N} = e^{-\sum_{i=1}^{K-1} i/N}$$

$$= e^{-\frac{K(K-1)/2}{N}} \approx e^{-\frac{K^2}{2N}} = 1 - p$$

Remember: for x small,  
1-x approx  $e^{-x}$

# Birthday paradox math

So, we concluded that:  $1 - p \approx e^{-\frac{K^2}{2N}}$

*quanti msg servono per avere una probabilità p prefissata.*

Let's solve in K:  $\log(1 - p) \approx -\frac{K^2}{2N} \Rightarrow K = \sqrt{2N} \cdot \sqrt{\log \frac{1}{1-p}}$

50% (p=1/2) collision probability:  $K \approx \sqrt{2} \cdot \sqrt{\log 2} \sqrt{N} \approx 1.177 \sqrt{N}$

And since  $N=2^n$ , using n bits:  $K \approx 1.177 \sqrt{2^n} \approx 2^{n/2}$

**Conclusion: n-bit digest → security level is NOT  $2^n$  but  $2^{n/2}$**

# Hash digest size

(truly Random)

## → Must be assessed against birthday paradox!

⇒ 32 bits (RAND) → 4.3 billion outputs

→ But 50% collision after  $2^{16} \sim 60.000$  msg (very little!)

(da "collezionare" per avere stesso output Hash)

⇒ 128 bits (MD5) → 50% collision after  $2^{64} = 1.8 \times 10^{19}$  (weakish today)

» but MUCH worse than this: MD5 broken via cryptoanalysis since 2005

⇒ 256 bit (SHA256) → 50% collision after  $2^{128} = 3.4 \times 10^{38}$ , OK today

»  $3.4 \times 10^{38} \rightarrow \sim 2000 \times 4$  superenalotto in a row

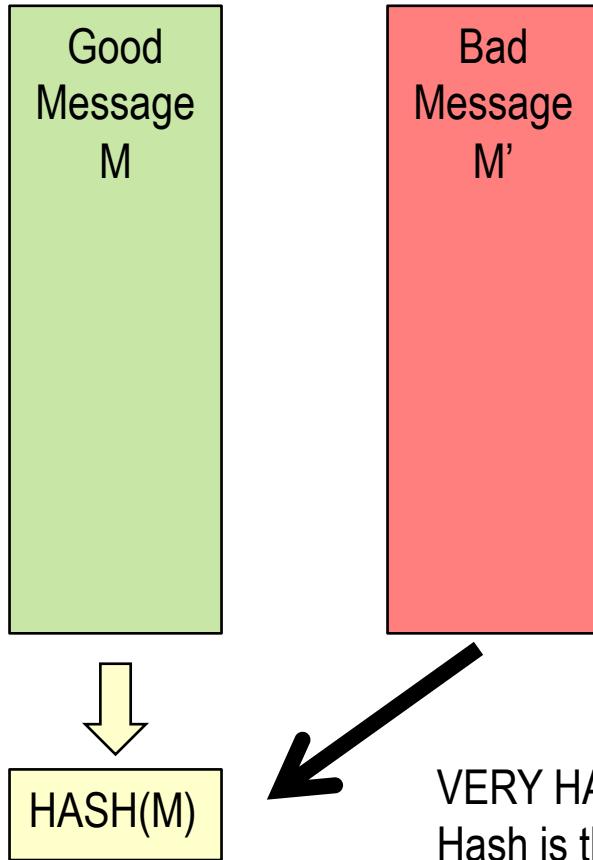
~ 1 billion centuries of work by whole

worldwide bitcoin network (march 2020 ~  $10^{20}$  H/s)

**Back to Message  
authentication!**

# Message Authentication Code using hash functions

## ingredient 1: good hash



### Good crypto HASH:

Today: SHA-2 family (**SHA256**, SHA224, SHA384, SHA512)

Past: **SHA1, MD5 (both broken)**

next: SHA-3 family (still 224, 256, 384, 512, but new construction)

Most common (used in TLS & Ipsec)

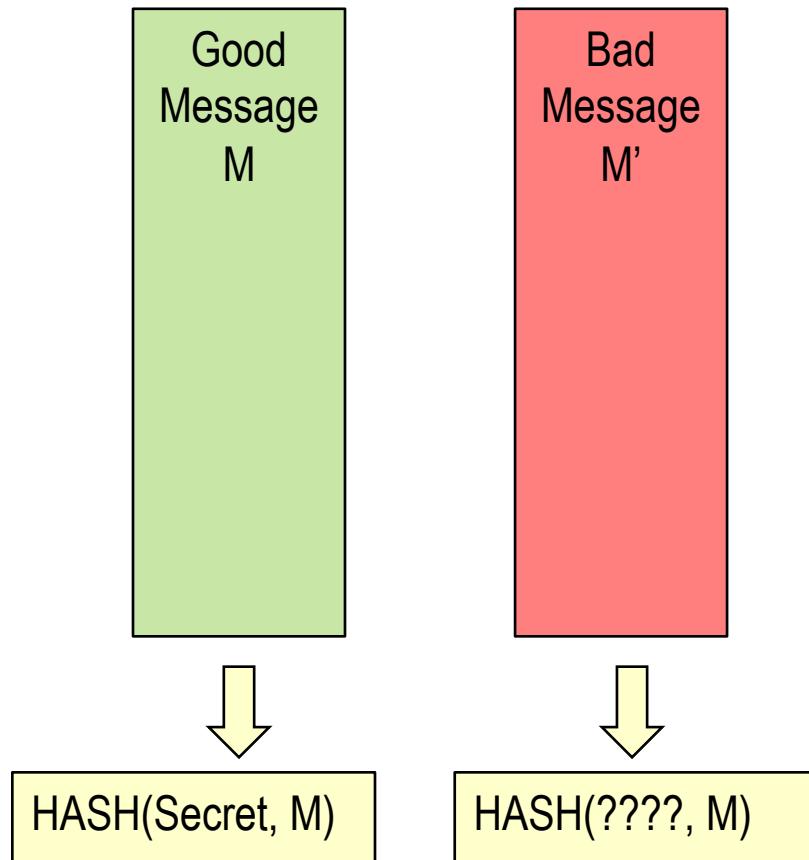
trivial to use (e.g. linux sha256sum command)

(*well, apparently ☺ - devil is in details, more later!*)

VERY HARD to find message M' whose  
Hash is the same of M (collision resistant hash)

# Message Authentication Code using hash functions

**ingredient 2: include secret in the hash**



**secret = authentication key**

Since it is not known by attacker, computationally hard to construct a valid message authentication code

# Where to put authentication key in hash?

$H(M \parallel S)$ ?

Message M  
Dear all, I'd wish to authenticate this message. Let me hash this along with a secret.

Auth key K  
0c8132af337a3c74  
ec81d754a96d5a29

$H(S \parallel M)$ ?

Auth key K  
0c8132af337a3c74  
ec81d754a96d5a29

Message M  
Dear all, I'd wish to authenticate this message. Let me hash this along with a secret.

Tag

3596c18d7c6f7b5df983b1edb5f7a0d3  
ddb8b1d041d202a1fc9b5fb4916d20ed

DIGEST

Tag  
12d6aeade831f55eb5b0556a0f7e52c  
e241451ad2556aa70c04e7f32ee4b19

Some other fancy way? E.g. «envelope»:  $H(K \parallel M \parallel K)$ ?

But why you ask? Silly question, why should we care?

# We would NOT care IF hash function were a «perfect» one: Random Oracle

→ «oracle» = black box model

→ Random oracle: *= funzione hash perfetta*

⇒ Given any distinct input  $X$ ,  $H(X)$  = truly random value

→ But with repeatability property:  
same input  $X$ , same output  $H(X)$

→ Fact: NO practical hash function can be a random oracle

⇒ Digest cannot be a «truly» random value

# **But WE HAVE TO BE CAREFUL!**

## **devil is in details...**

→ Matter is: Hash functions are not black boxes!

# Hash constructions: iterative

## Merkle-Damgård construction – example: SHA-256

but all practical hash functions of yesterday (e.g. MD5, SHA1) and today use this –only SHA-3 differs ☺

da msg lungo  $\infty \sim$  tab fibito.

( $\forall$  msg)  
SHA256 IV: sempre

$H_0^{(0)}$ : 6a09e667

$H_0^{(1)}$ : bb67ae85

$H_0^{(2)}$ : 3c6ef372

$H_0^{(3)}$ : a54ff53a

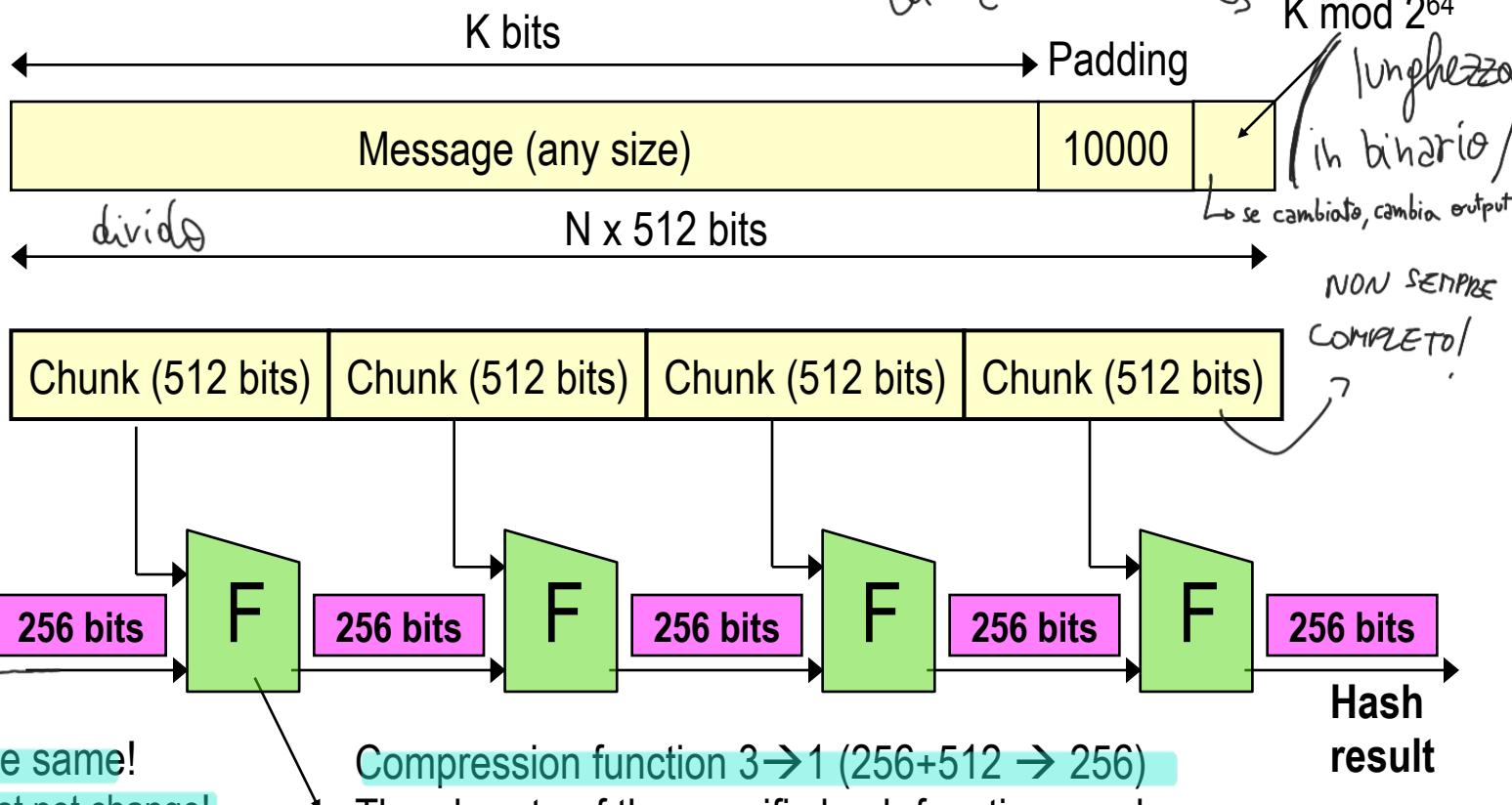
$H_0^{(4)}$ : 510e527f

$H_0^{(5)}$ : 9b05688c

$H_0^{(6)}$ : 1f83d9ab

$H_0^{(7)}$ : 5be0cd19

NOME  
NON  
ABATTO  
Initialization  
Vector



Constant: always the same!

For same  $M$ ,  $H(M)$  must not change!

Compression function  $3 \rightarrow 1$  ( $256+512 \rightarrow 256$ )

The «heart» of the specific hash function used

Merkle-Damgård theorem: if  $F$  resistant, also iteration is

DOVE LO METTO?

# Secret Suffix: H(M,S)?

olla fine?

NO !!

State precomputation, once!!

Brute-force attack to secret

Message (any size)

$N \times 512$  bits

Secret | pad+len

Forza bruta costa:  
 $(\text{HASH} \cdot \text{medio tentativi}) / N_{\text{blocchi}}$

Chunk (512 bits) | Chunk (512 bits) | Chunk (512 bits)

Chunk (512 bits)

(se Hash costa 10 ms, e ho 10 blocchi,  
1 block = 1 ms)

IV  
256 bits

F

256 bits

F

256 bits

F

256 bits

F

256 bits

NON mi serve ricompilare, m'interessa solo secret

Hash result

Known msg  $\rightarrow$  secret  $\rightarrow$  TAG

Se  $s=64 \rightarrow \mathcal{O}(X \cdot 2^{64})$   
 $H(MSG, x)$

$n^{\circ}$  tentativi  $\approx \frac{2^{64}}{2}$

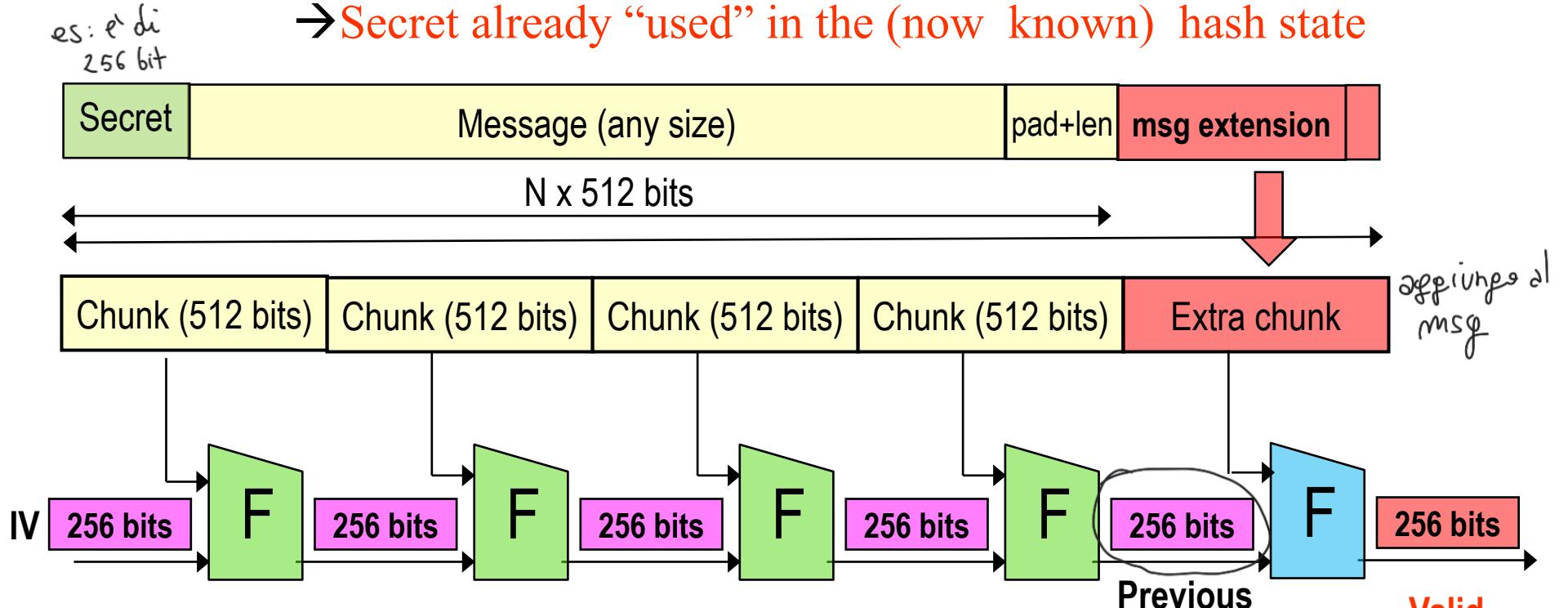
Weakened security: from  $N$  compression blocks per trial, to just 1!

all'inizio, con' devo computerare il secret tutto quanto! Ho dei controlli? Si  
**Secret Prefix: H(S,M)?** *Peggio!*

## → Expansion attack!

⇒ Message extension does not require knowledge of secret

→ Secret already “used” in the (now known) hash state



Practical attacker might need to still overcome pad/len-strengthening....

But in terms of security, we have forged a new message without knowing the secret key → security broken!

Valid MAC for extended message !!!

# **Expansion attack!**

- **Trivial to “extend” the message!**
- **Especially critical if secret at the beginning**
  - ⇒ Example: start from  $\text{SHA256}(k \mid x)$ , k unknown secret
  - ⇒ Append y
  - ⇒ To compute  $\text{SHA256}(k \mid x \mid y)$  use iterative Merkle construction!

→ No need to know k!!!

- **Length (Damgard) strengthening:  
helps but does not solve the problem**

⇒ A strong reason to use different constructs (HMAC)

→ [http://csrc.nist.gov/pki/HashWorkshop/2005/Nov1\\_Presentations/Puniya\\_hashDesign.pdf](http://csrc.nist.gov/pki/HashWorkshop/2005/Nov1_Presentations/Puniya_hashDesign.pdf)

# HMAC to the rescue!

→ Lesson learned so far: A secure hash is not enough!

→ Actually, a secure hash may yield a completely insecure MAC  
(e.g. expansion attack). Serve anche 'Segret' ed un modo per combinarli

→ Question: is there a «secure» way to include a secret in a hash?

- ⇒ Irrespective of the specific hash construction
- ⇒ hash should be considered as a black box!

→ Yes: problem solved in 1996: HMAC

- Original paper: Bellare, Canetti, Krawczyk, Keying hash functions for message authentication, crypto 96
- standard: IETF RFC 2104
- ⇒ Provably secure construction
- ⇒ «pluggable» hash (you choose/replace it)
- ⇒ (more or less) same performance of used hash

Applico hash 2 volte.

**HMAC construction**

$$\text{HMAC}_K(M) = H(K^+ \oplus \text{opad} \parallel H(K^+ \oplus \text{ipad} \parallel M))$$

secret su  $H(S, M)$

message → key

$H(S \parallel H(S, M))$

due segreti  
diverri

Più precisamente, parto  
da una SOLO chiave e la  
metto in XOR con opad e ipad

→  $K^+$  = shared key, but “extended” to block size

» E.g. SHA256 → padded with zeros up to 512 bits

» *Makes sure secret uses the entire first hash chunk* (1° chunk tutto per secret)

così disposto soffre  
expansion attack

128 bit secret	000
512 bit Chunk (SHA256 example)	

⊕

00110110.00110110.00110110.00110110.00110110.....
---

→ opad = 0x36 = 00110110 repeated as needed

→ ipad = 0x5C = 01011100 repeated as needed

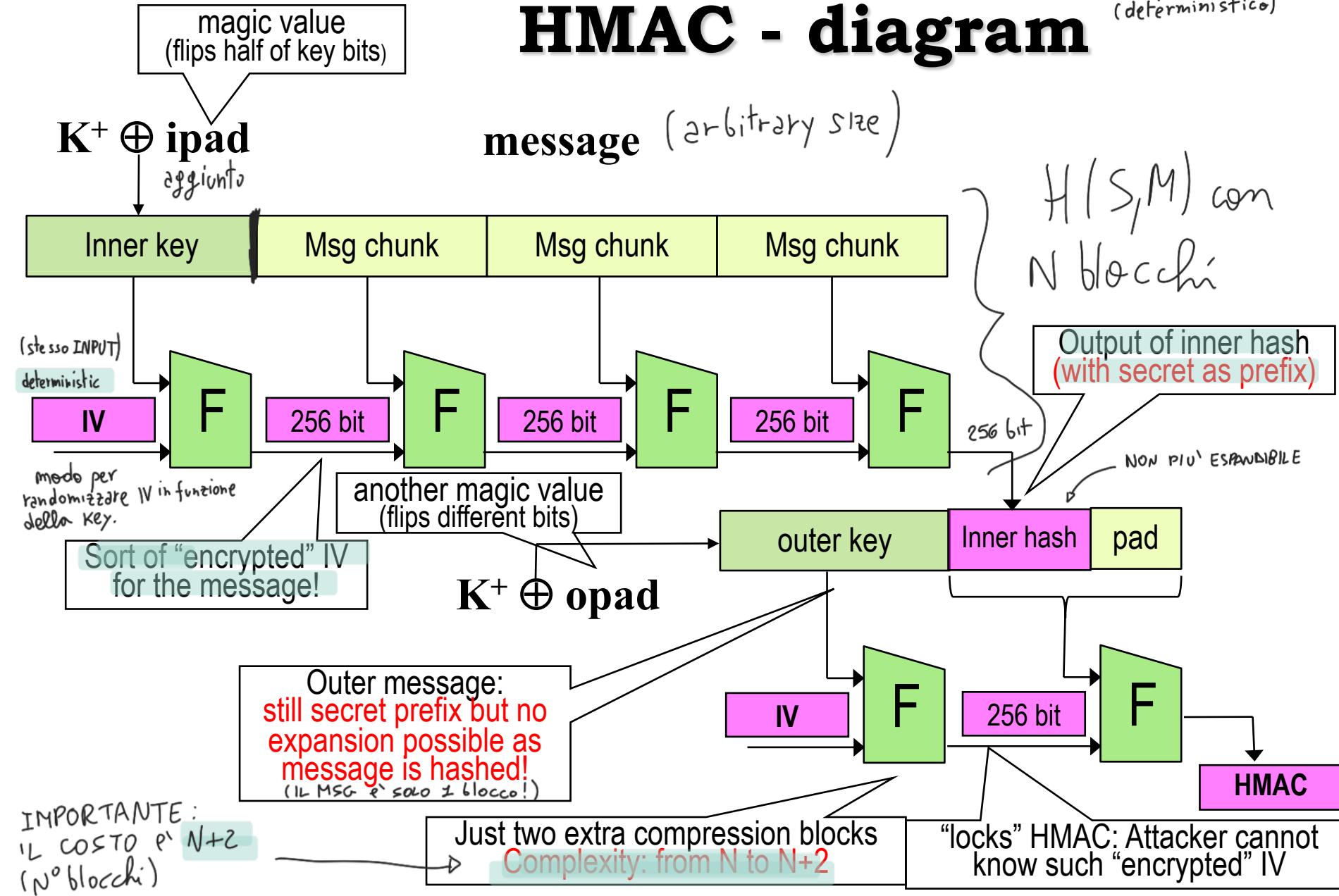
» *We would need TWO different (inner and outer) secrets, but since we have only one, let's “pseudo-randomly” derive inner and outer keys from the single key (deterministic)*

(roughly) equivalent to **NMAC construction**:  $H(K_o \parallel H(K_i \parallel M))$

↳ Nasty (uno dentro l'altro)

- Se NOU cambio la chiave  
l'output non cambia  
(deterministico)

# HMAC - diagram



IMPORTANTE:  
IL COSTO È  $N+2$   
( $N$  blocchi)

- AUTHENTICATION CODE → NO Hash
- Devo usare Hash funz? USO HMAC!

# HMAC security

IPOTESI: pseudo-random output (scorrelati tra loro)  
• collision resistant  $\Rightarrow$  HASH secure

→ **quantitatively proven robustness:  
as secure as its underlying hash is**

- ⇒ see Bellare, Canetti, Krawczyk paper for math proof
- ⇒ we will not dig into this, further

→ **Actually more secure (surprise!) than HASH**

- ⇒ Bellare 2006: collision resistance NOT necessary
  - Pseudorandomness is the only requirement
- ⇒ You might even still use HMAC with MD5 or SHA-1 even if they are both broken - there are algorithms to compute collision

# Collisions without HMAC

→ Collision in hash = collision in MAC

→ H(M,S) construction:

⇒ Obvious

→ find collision on first part of the message, then expand

→ H(S,M) construction:

⇒ less obvious, but same problem

→ Start from  $H(S,x) \rightarrow IV$

→ Find collision  $H^*(IV, X_1) = H^*(IV, X_2)$

→  $M_i = x \mid pad(x) \mid X_i \mid pad(X_i)$

→ Hence  $H(S, M_1) = H(S, M_2)$

# Take-home

→ In crypto, NEVER use a function which is «more or less» close to your needs.

HASH NON VA BENE per tutto, è funz. A scopo!  
computare M<sub>essage</sub> A<sub>uthentication</sub> C<sub>ode</sub> ≠ computare fingerprint (HASH)

→ Rather, ONLY use functions SPECIFICALLY designed for your PRECISE need!

→ Hash: seemed OK, but was NOT!

⇒ We needed to create a new function, HMAC

# **Summary**

- **Confidentiality ≠ integrity**
- **Message authentication (symmetric key)**
  - ⇒ And formal definition of a secure MAC scheme
- **Replay attacks → MAC not enough → nonces**
- **Ideal vs real world hash functions:**
  - ⇒ Perfect hash: Random Oracle
  - ⇒ Real world hash: Iterative (merkle-damgard) constructions
- **How to include key in hash? Non trivial!**
  - ⇒ Solution: HMAC construction