cosa facciamo con i dati raccolti? vorrei statistiche utili.

Performance Modeling of Computer Systems and Networks

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Sample statistics

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parlo di statistica campionaria.

- Simulation involves *a lot* of data
- Must "compress" the data into meaningful statistics
- Collected data is a *sample* from a much larger *population*
- Two types of statistical analysis
  - "Within-the-run"
  - "Between-the-runs" (replication)
- Essence of statistics: analyze a sample and draw inferences

tempo interrarrivo, tempo servizio, numero coda, num. sistema tempo coda, tempo globale, tempo utilizzazione, sono valori che uso nella coda singola, figuarsi per 500 run.

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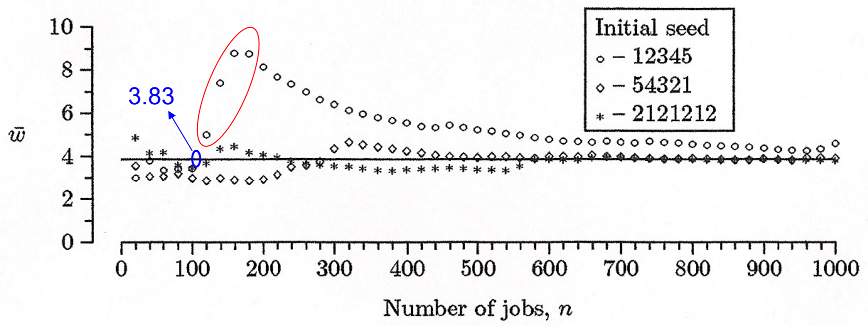
che cosa capisco da questi dati? la statistica, mediamente campione, ci fa capire se è rappresentativo, se lo è devo poter inferire qualcosa di utile. dati da interpretare.

in questo chart con 3 casi diversi, vedevo se il sistema convergeva o meno.

- The accumulated average wait was printed every 20 job



- The convergence of *w* is slow, erratic, and dependent on the initial seed

qui valori singoli, vedo su quegli stessi arrivi se tempi di servizio diversi quali cambiamenti apportano. questo non passa dal simulatore, è frutto del generatore dell'esponenziale.

arrival and service processes are *uncoupled*

stream 0 for arrivals, stream 1 for services

for 10025 jobs
average interarrival time = 1.99
average wait .... = 3.92
average delay .... = 2.41
average service time .... = 1.50
average # in the node ... = 1.96
average # in the queue .. = 1.21
utilization .... = 0.75

stream 0 for arrivals, stream 2 for services (or e.g. stream 128 to get more separation)

for 10025 jobs
average interarrival time = 1.99
average wait .... = 3.86
average delay .... = 2.36
average service time .... = 1.50
average # in the node ... = 1.93
average # in the queue .. = 1.18
utilization .... = 0.75

Theoretical values

$\bar{r}$	$\bar{w}$	$\bar{d}$	$\bar{s}$	$\bar{l}$	$\bar{q}$	$\bar{x}$
2.00	3.83	2.33	1.50	1.92	1.17	0.75

Sample Mean and Standard Deviation

Consider a sample $x_1, x_2, \ldots, x_n$ (continuous or discrete), let us define:

- *sample mean* $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ tendenza, non moda
- *sample variance* $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ discostamento media
- *sample standard deviation* $s = \sqrt{s^2}$ ≈ varianza
- *coefficient of variation* $s/\bar{x}$ senza misura $\left(\frac{[dati]}{[dati]}\right)$

- *mean*: a measure of central tendency
- *variance* and *deviation*: measures of dispersion about the mean

easier math
(no square root)

same units as data,
mean

- note that *coefficient of variation (CV)* is unit-less, but a common shift in data changes the CV

Relating the mean and standard deviation

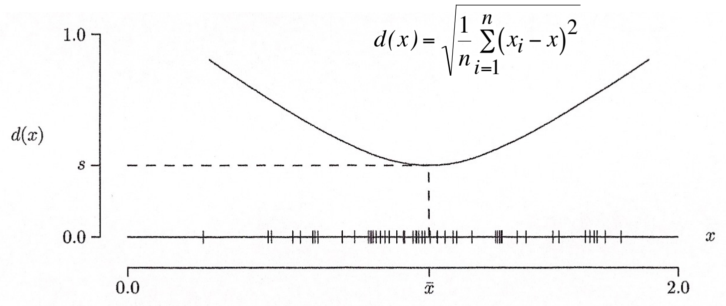
Consider the root-mean-square (rms) function

$$d(x) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x)^2}$$

- $d(x)$ measures dispersion about any value x
- the mean $\bar{x}$ gives the smallest possible value for $d(x)$ (Theorem 4.1.1)
- The standard deviation s is that smallest value

diagramma di dispersione:
come si vede, trovo il punto di minimo per la media.

50 samples from program buffon



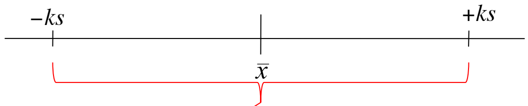
$$d(x) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x)^2}$$

$\bar{x} \cong 1.095, s \cong 0.354$

prendo la media, e prendo un intorno (k volte la deviazione standard). più lontani sono dalla media, più contribuiscono. tale insieme danno proprozione rispetto campione.

Chebyshev's inequality

Consider the number of points that lie within k standard deviations of the mean



- Points farthest from the mean make the most contribution to s

Define the set $S_k = \{x_i | \bar{x} - ks < x_i < \bar{x} + ks\}$

Let $p_k = |S_k|/n$ be the proportion of x_i within $\pm ks$ of $\bar{x}$

$$p_k \geq 1 - \frac{1}{k^2} \qquad (k > 1)$$

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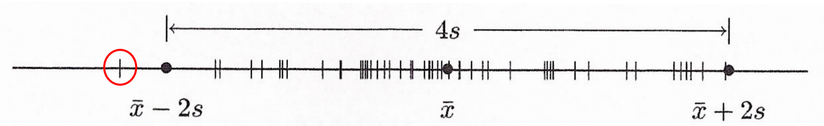
Chebyshev's inequality

- for *any* sample, at least 75% of the points lie within $\pm 2s$ of $\bar{x}$
- for k=2, the inequality is very conservative:
  typically 95% lie within $\pm 2s$ of $\bar{x}$
- $\bar{x} \pm 2s$ defines the "effective width" of a sample



- most (but not all) points will lie in this interval
- *outliers* should be viewed with suspicion

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nella pratica il 95% del campione cade in un intorno di 2*std dev. Se avessi troppi elementi fuori, allora il campione non sarebbe troppo corretto.

Linear data transformations

- Often need to convert to different units after data has been collected
- let x'_i be the "new data": $x'_i = ax_i + b$
- *sample mean*

$$\bar{x}' = \frac{1}{n}\sum_{i=1}^{n} x'_i = \frac{1}{n}\sum_{i=1}^{n}(ax_i + b) = \frac{a}{n}\left(\sum_{i=1}^{n} x_i\right) + b = a\bar{x} + b$$

- *sample variance* $(s')^2 = \frac{1}{n}\sum_{i=1}^{n}(x'_i - \bar{x}')^2 = \ldots = a^2 s^2$
- *sample standard deviation* $s' = |a| s$

Examples of Linear Data Transformations

- suppose $x_1, x_2, \ldots, x_n$ measured in seconds
  - to convert to minutes, let $x'_i = x_i/60$

  (a=1/60, b=0)

$$\bar{x}' = \frac{45}{60} = 0.75 \qquad s' = \frac{15}{60} = 0.25 \qquad \text{(minutes)}$$

- *standardize* data (a=1/s, b= $-\bar{x}/s$) $\quad x'_i = \frac{1}{s}x_i - \frac{\bar{x}}{s} \quad x'_i = \frac{x_i - \bar{x}}{s}$

  Then $\quad \bar{x}' = 0 \quad s' = 1$

Used to avoid problems with very large (or small) valued samples

se ho dati grandi, li standardizzo in val compresi tra 0 e 1 ad esempio.

le uso per sapere come si comportano i dati rispetto ad un qualcosa.
ad esempio con output booleano, voglio vedere se si comporta 'bene' o 'male'.

Nonlinear data transformations

- usually involves a Boolean (two-state) outcome
- the *value* of x_i is not as important as the *effect*
- let A be a fixed set; then

$$x'_i = \begin{cases} 1 & x_i \in A \\ 0 & \text{otherwise} \end{cases}$$

- let p be the proportion of x_i that fall in A:

$$p = \frac{\text{the number of } x_i \text{ in A}}{n}$$

then

$$\bar{x}' = p \qquad s' = \sqrt{p(1-p)}$$

Examples of Nonlinear Data Transformations

For the single server service queue

- let $x_i = d_i$ be the delay for job i

attesa numero reale >0

- let A = R$^+$, then x_i'=1 iff d_i>0

se = 1 c'è un ritardo, in attesa quindi.

- from exerc. 1.2.3 proportion of job delayed is $p = 0.723$
- then $\bar{x}' = 0.723$ and $s = \sqrt{(0.723)(0.277)} = 0.448$

voglio sapere quanti attendono, non quali attendono.

il calcolo della sample standard deviation può essere fatta in vari modi, anche se ognuno presenta delle limitazioni

DE simulation
Sample statistics

Computational considerations

Consider the sample standard deviation equation

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Requires two passes through the data:

1. Compute the mean $\bar{x}$
2. Compute the squared differences about $\bar{x}$

Must store or re-create the entire sample!
bad when *n* is large!

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DE simulation
Sample statistics

The conventional one-pass Algorithm

Consider the sample standard deviation equation

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i^2 - 2\bar{x}x_i + \bar{x}^2\right)$$

by separating and simplifying

$$= \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) - \bar{x}^2$$

round-off error, overflow

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L'algoritmo di Welford è il "migliore", in cui definiamo la media per passi 'i'.

DE simulation
Sample statistics

Welford's one-pass algorithm

- running sample mean until *i*

$$\bar{x}_i = \frac{1}{i}(x_1 + x_2 + \ldots + x_i)$$

- running sample sum of squared deviations until *i*

$$v_i = (x_1 - \bar{x}_i)^2 + (x_2 - \bar{x}_i)^2 + \ldots + (x_i - \bar{x}_i)^2$$

- $\bar{x}_i$ and v_i can be computed recursively $(\bar{x}_0 = 0, v_0 = 0)$:

$$\bar{x}_i = \bar{x}_{i-1} + \frac{1}{i}(x_i - \bar{x}_{i-1})$$

$$v_i = v_{i-1} + \left(\frac{i-1}{i}\right)(x_i - \bar{x}_{i-1})^2$$

- $\bar{x}_n$ is the sample mean, v_n / n is the variance

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DE simulation
Sample statistics

Welford's Algorithm
(program uvs)

- No *a priori* knowledge of the sample size *n* required
- Less prone to accumulated round-off error

$$\bar{x}_i = \bar{x}_{i-1} + \frac{1}{i}(x_i - \bar{x}_{i-1})$$

```
n = 0;
x̲ = 0.0;
v = 0.0;
while (more data ) {
    x = GetData();
    n++;
    d = x - x̲;
    v = v + d * d * (n - 1) / n;
    x̲ = x̲ + d / n;
}
s = sqrt(v / n);
return n, x̲, s;
```

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DE simulation
Sample statistics

Welford's Algorithm
(program `uvs`)

- No *a priori* knowledge of the sample size n required
- Less prone to accumulated round-off error

```
n = 0;
x = 0.0;
v = 0.0;
while (more data ) {
    x = GetData();
    n++;
    d = x - x;
    v = v + d * d * (n - 1) / n;
    x = x + d / n;
}
s = sqrt(v / n);
return n, x, s;
```

$$v_i = v_{i-1} + \left(\frac{i-1}{i}\right)(x_i - \bar{x}_{i-1})^2$$

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sopra si è parlato come esempio di coda singola, ma posso avere statistiche time average e job average (a seconda se cambino col tempo o con altro)

DE simulation
Sample statistics

Example

- let $x_1, x_2, \ldots, x_n$ be *Uniform*(a, b) random variates
- in the limit as $n \to \infty$

$$\bar{x} \to \frac{a+b}{2} \qquad s \to \frac{b-a}{\sqrt{12}} \qquad \left(s^2 = \frac{(b-a)^2}{12} \right)$$

- using *Uniform*(0, 1) $\bar{x}$ and s should converge to

$$\frac{0+1}{2} = 0.5 \qquad \frac{1-0}{\sqrt{12}} \cong 0.2887$$

- The convergence to theoretical values is not necessarily monotone

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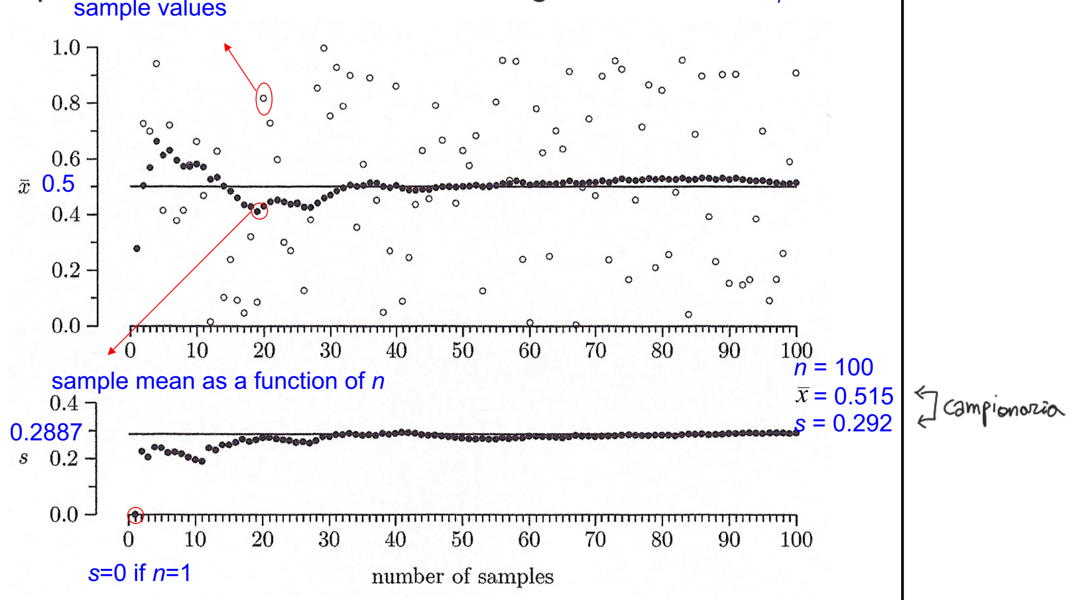
i 100 elementi del campione sono pallini bianchi, media teorica (barra 0.5), pallini neri è la media calcolata con quel numero di elementi. Convergenza non monotona.

DE simulation
Sample statistics



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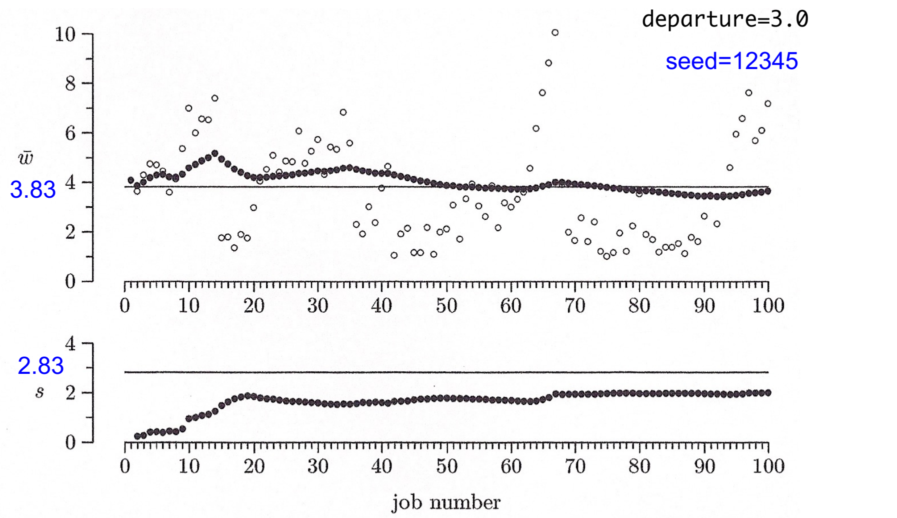
nella deviazione standard si parte con sottostima e poi converge.
si è partiti da caso stazionario.
C'è sempre una sottostima perchè c'è sempre un pò di correlazione lineare, che porta sottostima da correggere.

DE simulation
Sample statistics



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sono proposte nello stesso modo, sono 'realizzazioni del processo', perchè ho sample path (non variabile) variabile nel tempo t.

time-averaged sample statistics

- Let $x(t)$ be the sample path of a stochastic process for $0 < t < \tau$
- *Sample-path mean* $\bar{x} = \frac{1}{\tau}\int_0^\tau x(t)dt$
- *Sample-path variance* $s^2 = \frac{1}{\tau}\int_0^\tau (x(t)-\bar{x})^2 dt$
- *Sample-path standard deviation* $s = \sqrt{s^2}$
- One-pass equation for variance $s^2 = \left(\frac{1}{\tau}\int_0^\tau x^2(t)dt\right) - \bar{x}^2$

$$s^2 = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) - \bar{x}^2$$

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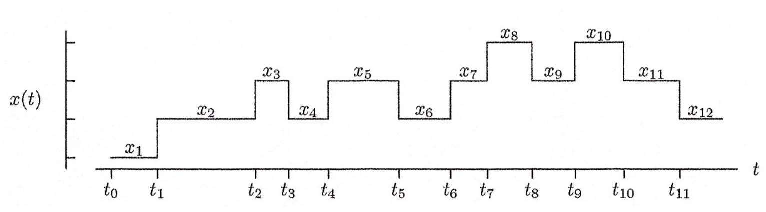
Computational considerations

- For DES, a sample path is *piecewise constant*
- Changes in the sample path occur at *event times* $t_0, t_1, \ldots$



- For computing statistics, integrals reduce to summations

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piuttosto che l'integrale, l'area sotto la funzione è valore*tempo per cui vale. questo quando la funzione è costante a gradino.

Computational sample-path formulas

Consider a piecewise constant sample path

è una scrittura in termini più formali:

$$x(t) = \begin{cases} x_1 & t_0 < t \le t_1 \\ x_2 & t_1 < t \le t_2 \\ . & . \\ x_n & t_{n-1} < t \le t_n \end{cases}$$

- *Sample-path mean* $\bar{x} = \frac{1}{\tau} \int_0^\tau x(t)dt = \frac{1}{t_n} \sum_{i=1}^{n} x_i \delta_i$ with $\delta_i = t_i - t_{i-1}$ inter-event time
- *Sample-path variance*

$$s^2 = \frac{1}{\tau} \int_0^\tau (x(t) - \bar{x})^2 dt = \frac{1}{t_n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \delta_i = \left(\frac{1}{t_n} \sum_{i=1}^{n} x_i^2 \delta_i \right) - \bar{x}^2$$

Welford's sample path Algorithm

- based on the definitions

$$\bar{x}_i = \frac{1}{i}(x_1\delta_1 + x_2\delta_2 + \ldots + x_i\delta_i)$$
$$v_i = (x_1 - \bar{x}_i)^2 \delta_1 + (x_2 - \bar{x}_i)^2 \delta_2 + \ldots + (x_i - \bar{x}_i)^2 \delta_i$$

- $\bar{x}_i$ is the sample-path mean of $x(t)$ for $t_0 \le t \le t_i$
- v_i / t_i is the sample-path variance
- $\bar{x}_i$ and v_i can be computed recursively $(\bar{x}_0 = 0, v_0 = 0)$

$$\bar{x}_i = \bar{x}_{i-1} + \frac{\delta_i}{t_i}(x_i - \bar{x}_{i-1})$$
$$v_i = v_{i-1} + \frac{\delta_i t_{i-1}}{t_i}(x_i - \bar{x}_{i-1})^2$$

Exercises

- Exercises: 4.1.7, 4.1.8

il file uvs.c calcola media e dev standars secondo welford.