Performance Modeling of Computer Systems and Networks

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Generating Continuous Random Variates

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Prerequisite

We assume the knowledge of continuous random variables (sect.7.1).
In particular:

- *Uniform*(*a*,*b*)
- *Exponential*(μ)
- *Normal*(μ,σ)
- *Lognormal*(*n*,*b*)
- *Erlang*(*n*,*b*)
- *Student*(*n*)

Nel caso continuo è tutto più facile, perchè c'è corrispondenza 1-1

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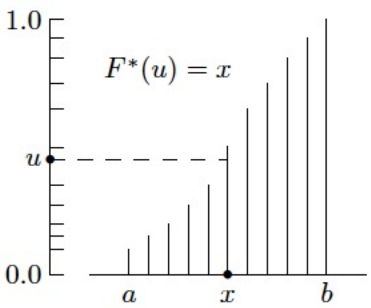
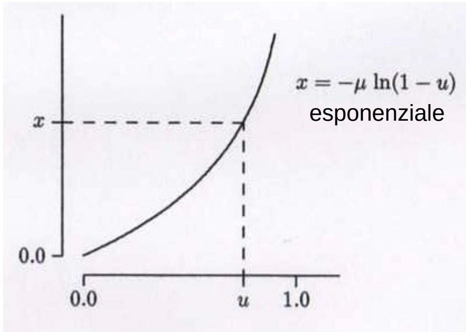
Discrete Random Variates

$$F^*(u) = \min_x \{x : u < F(x)\}$$



Continuous Random Variates



per un dato 'u' ho esattamente un unico e solo 'x'.
L'inversa è vera.

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In alcuni casi inversione facile, in altri casi esistono sempre dei metodi!

Preliminary Definitions

The *inverse distribution function* (idf) of X is the function

$F^{-1} : (0, 1) \rightarrow \chi$, $\forall\, u \in (0, 1)$ as

supporto,
da cui
estraggo 'u'

$$F^{-1}(u) = x$$

stesse definizioni,
qui essendo inversa vera
posso scrivere F^(-1).

where $x \in \chi$ is the unique possible value for $F(x) = u$

There is a one-to-one correspondence between possible values $x \in \chi$ and cdf values $u = F(x) \in (0, 1)$

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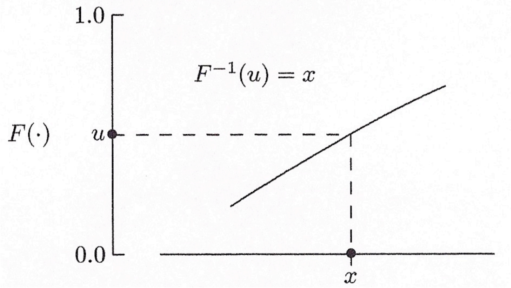
Continuous Random Variable idfs

- Unlike the a discrete random variable, the idf for a continuous random variable is a true inverse



- Can sometimes determine the idf in "closed form" by solving $F(x) = u$ for x

Examples

- if X is *Uniform(a,b)*, $F(x) = (x-a)/(b-a)$ for $a < x < b$

$$x = F^{-1}(u) = a + (b - a)u \qquad 0 < u < 1$$

- if X is *Exponential(μ)*, $F(x) = 1\text{-}\exp(-x/\mu)$ for $x > 0$

$$x = F^{-1}(u) = -\mu \ln(1 - u) \qquad 0 < u < 1$$

- if X is a continuous variable with possible value $0 < x < b$ and pdf $f(x) = 2x/b^2$, cdf $F(x) = (x/b)^2$

$$x = F^{-1}(u) = b\sqrt{u} \qquad 0 < u < 1$$

Random Variate Generation By Inversion

- X is a continuous random variable with idf $F^{-1}(\cdot)$
- Continuous random variable U is *Uniform*(0,1)
- Z is the continuous random variable defined by $Z = F^{-1}(U)$

Theorem
Z and X are identically distributed

Algorithm 1

```
u = Random();
return F^-1(u);
```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Inversion examples

- *Uniform*(*a*,*b*) Random Variate

```
u = Random();
return a + (b - a) * u;
```

- *Exponential*(μ) Random Variate

```
u = Random();
return - μ log(1-u);
```


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Inversion algorithms

- Algorithms in the previous two examples are:
  - portable, exact, robust, efficient, clear, synchronized and monotone

una chiamata alla random == una generazione di variata

- It is not always possible to solve for a continuous random variable idf explicitly by algebraic techniques
- Two other options may be available:
  1. Use a function that accurately *approximates* $F^{-1}(\cdot)$
  2. Determine the idf by solving $u = F(x)$ *numerically*

  (see section 7.2.2)

Se inversa difficile, o approssimo funzione inversa, oppure risolvendola numericamente.

Testing for correctness

per vedere se generazione è ok!

- generate a sample of n random variates where n is large
- evaluate sample mean and standard deviation
- compare them with the theoretical values,
  they should be *reasonably* close !!

This is not enough!! Different distributions can have the same mean and standard deviation !!!

dovrei anche costruire istogramma e confrontarlo con la distribuzione che sto approssimando (non lo vediamo).

- generate a sample of n random variates and construct a k-bin continuous-data histogram with bin width δ
- f' is the histogram density and $f(x)$ is the pdf

$$f' \to f(x) \quad \text{as } n \to \infty \quad \text{and} \quad \delta \to 0$$

- In practice, using a large but finite value of n and a small but non-zero value of δ, perfect agreement between f' and f will not be achieved

Discrete case: natural sampling variability !
Continuous case: variability+binning !!

Truncation

ovviamente esiste anche nel continuo!

- Let X be a continuous random variable with possible values χ and cdf $F(x)=\Pr(X \le x)$
- Suppose we wish to restrict the possible values of X to $(a, b) \subset \chi$

It is similar to, but simpler than truncation in the discrete-variable context

Sia che partendo dai due punti, che dalle due masse di probabilità che voglio eliminare, la trasformazione è esatta!

- X is $\le a$ with probability $\Pr(X \le a) = F(a)$
- X is $\ge b$ with probability $\Pr(X \ge b) = 1 - \Pr(X < b) = 1 - F(b)$
- X is between a and b with probability

$$\Pr(a < X < b) = \Pr(X < b) - \Pr(X \le a) = F(b) - F(a)$$

2 cases for truncation

Case 1 se conosco i "due punti"

if a and b are specified, the cdf of X can be used to determine the left-tail α, right-tail β truncation probabilities

$$\alpha = \Pr(X \le a) = F(a) \quad \text{and} \quad \beta = \Pr(X > b) = 1\text{-}F(b)$$

Case 2 parto dalle "masse di probabilità da escludere" e trovando i due punti associati.

if α and β are specified, the idf of X can be used to determine left and right truncation points

$$a = F^{-1}(\alpha) \quad \text{and} \quad b = F^{-1}(1\text{-}\beta)$$

$$F(b) = 1\text{-}\beta$$

Both transformations are exact !

Library `rvgs`

- Contains 7 continuous random variate generators
  - `double Chisquare(long` *n*`)`
  - `double Erlang(long` *n*`, double` *b*`)`
  - `double Exponential(double` μ`)`
  - `double Lognormal(double` *a*`, double` *b*`)`
  - `double Normal(double` μ`, double` σ`)`
  - `double Student(long` *n*`)`
  - `double Uniform(double` *a*`, double` *b*`)`