AdaBoost

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for $t = 1, \ldots, T$
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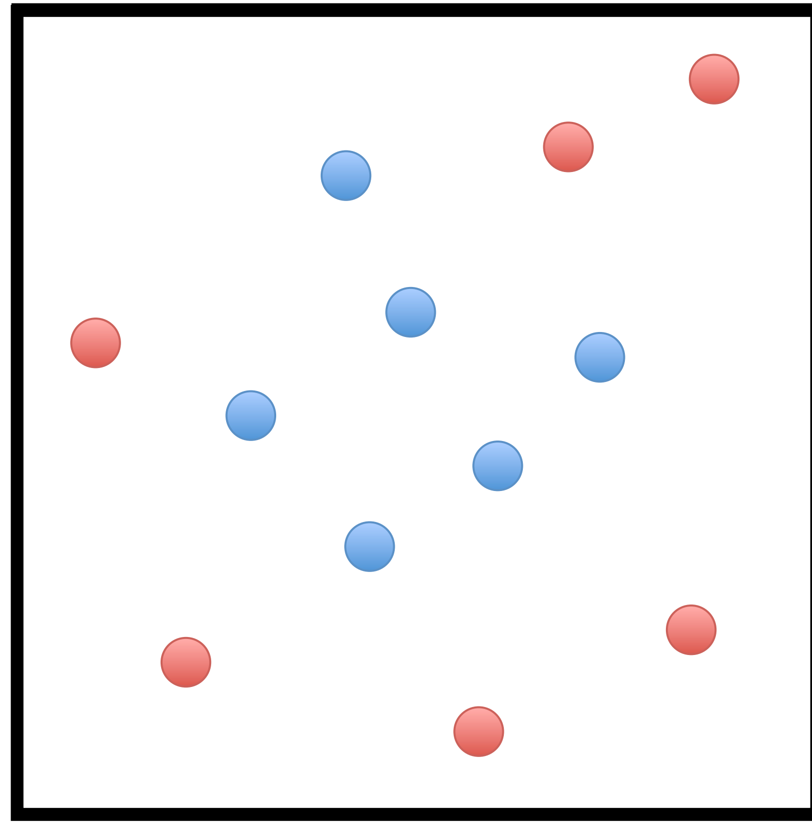
  Compute the weighted training error of h_t
  Choose $\beta_t = \frac{1}{2} \ln \left(\frac{1-\epsilon_t}{\epsilon_t} \right)$
  Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right)$$

  Normalize $\mathbf{w}_{t+1}$ to be a distribution
end for
Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$



- Size of point represents the instance's weight

Qui inizialmente abbiamo pallini (blu = vero, rosso = falso) della stessa dimensione.

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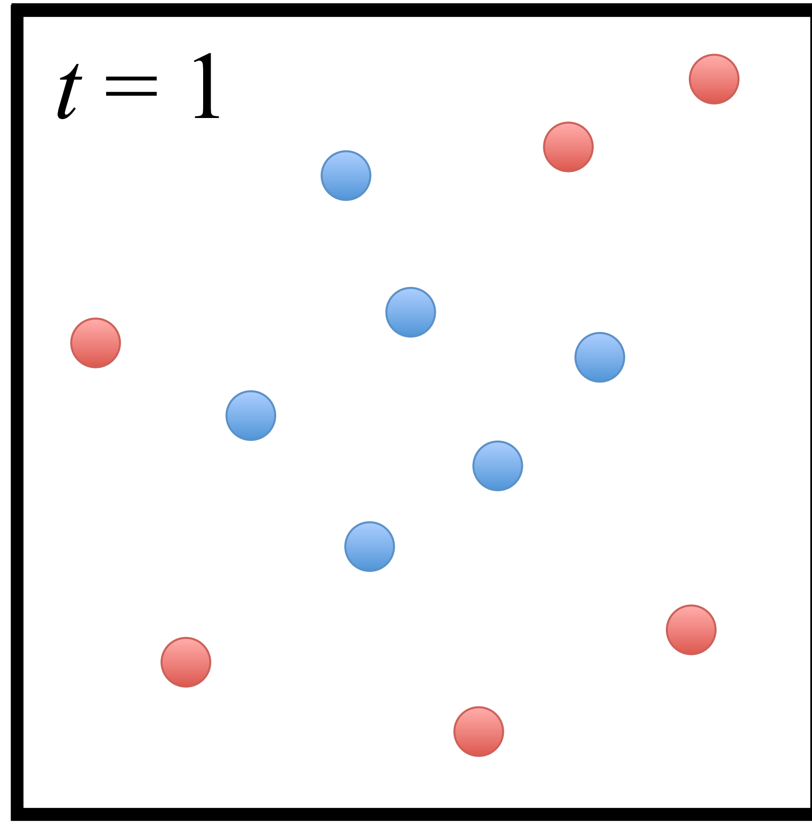

Questo è il primo step, facciamo una prima regressione logistica (cioè probabilità compresa tra 0 o 1), ovvero troveremo una retta (simile ad una soglia) che dividerà in due: da una parte "0" dall'altra "1".

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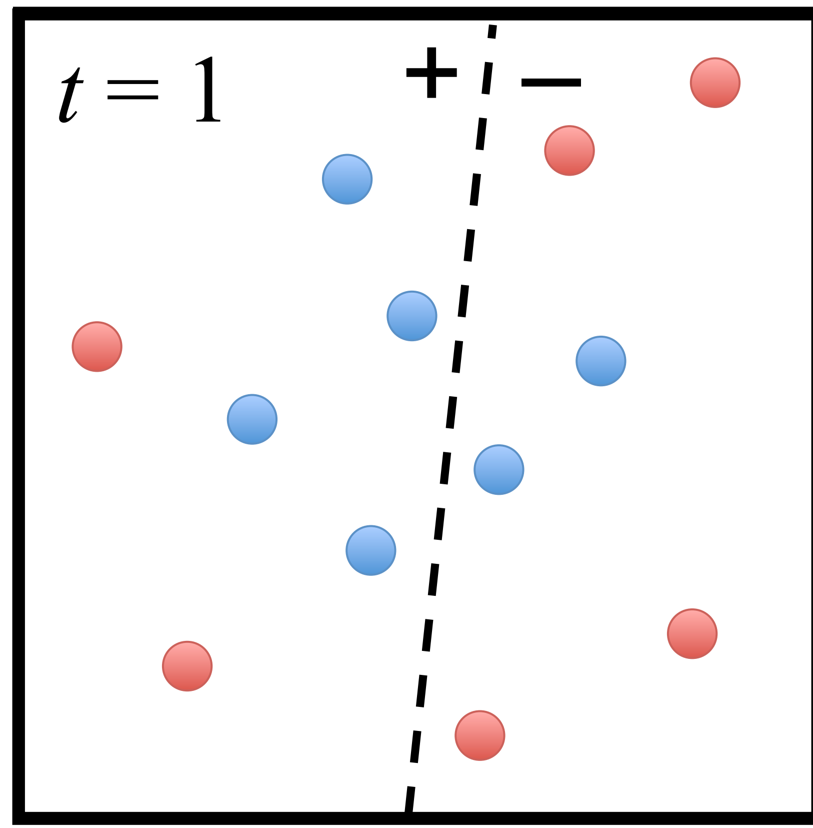

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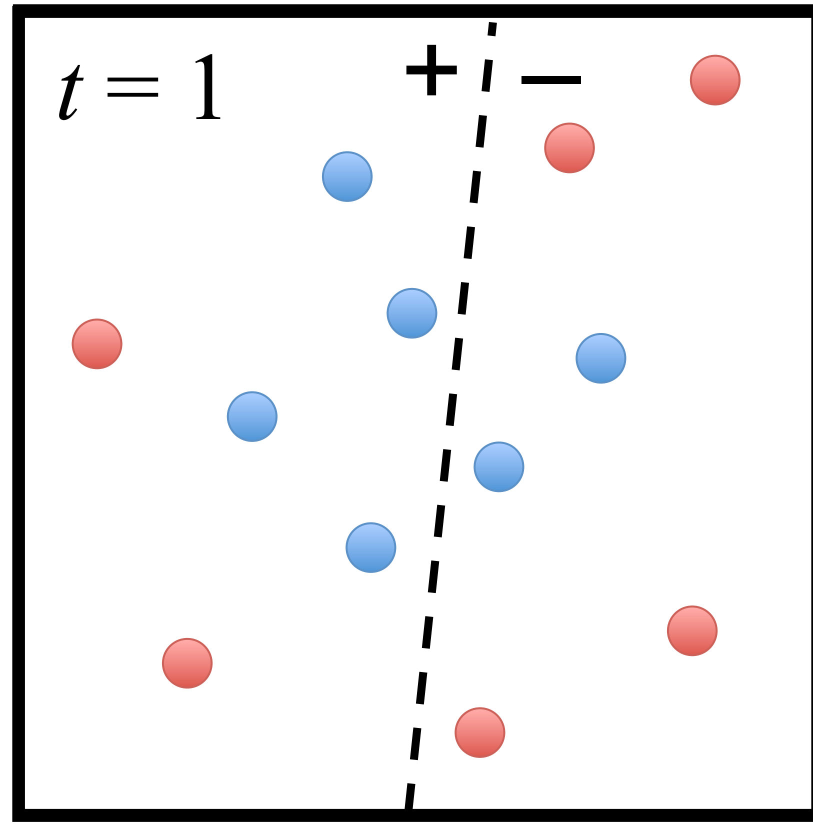

- β_t measures the importance of h_t
- If $\epsilon_t \leq 0.5$, then $\beta_t \geq 0$ (can trivially guarantee)

(qui viene semplicemente segnato lo step corrente dell'algoritmo)

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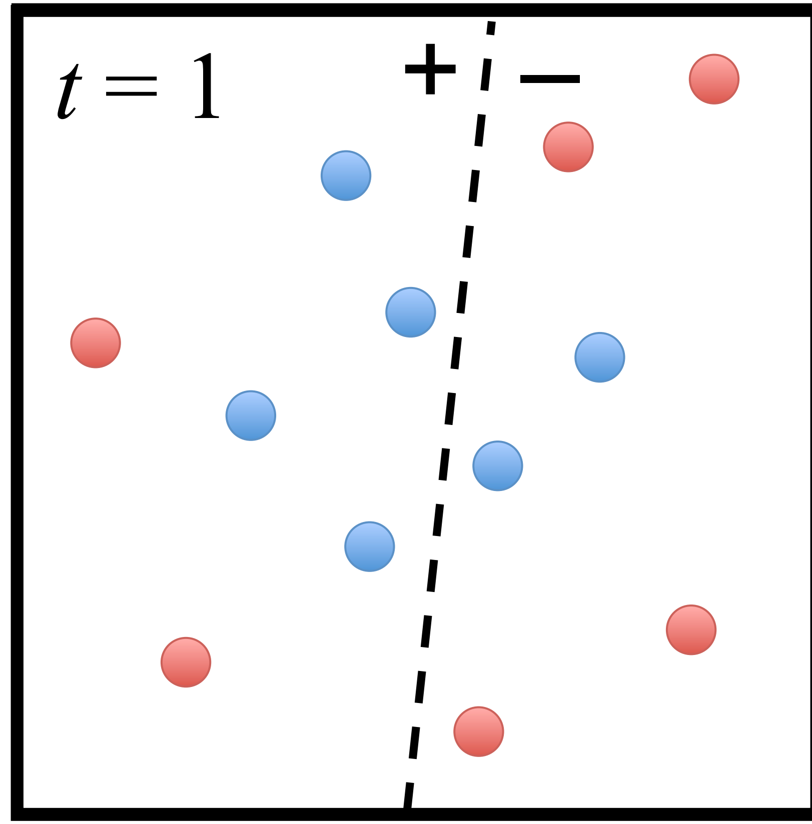

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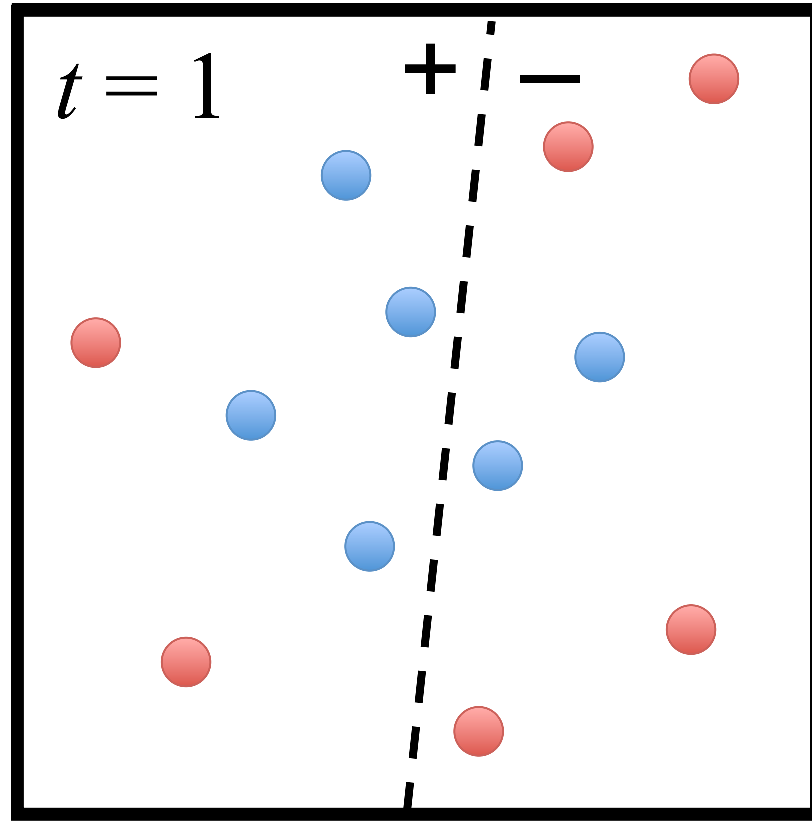

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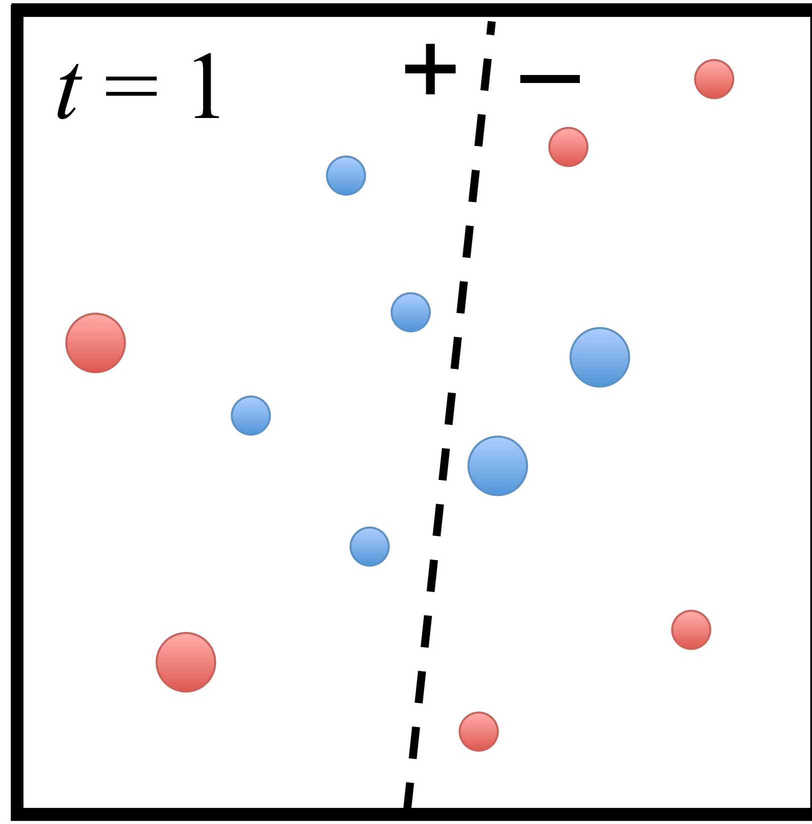

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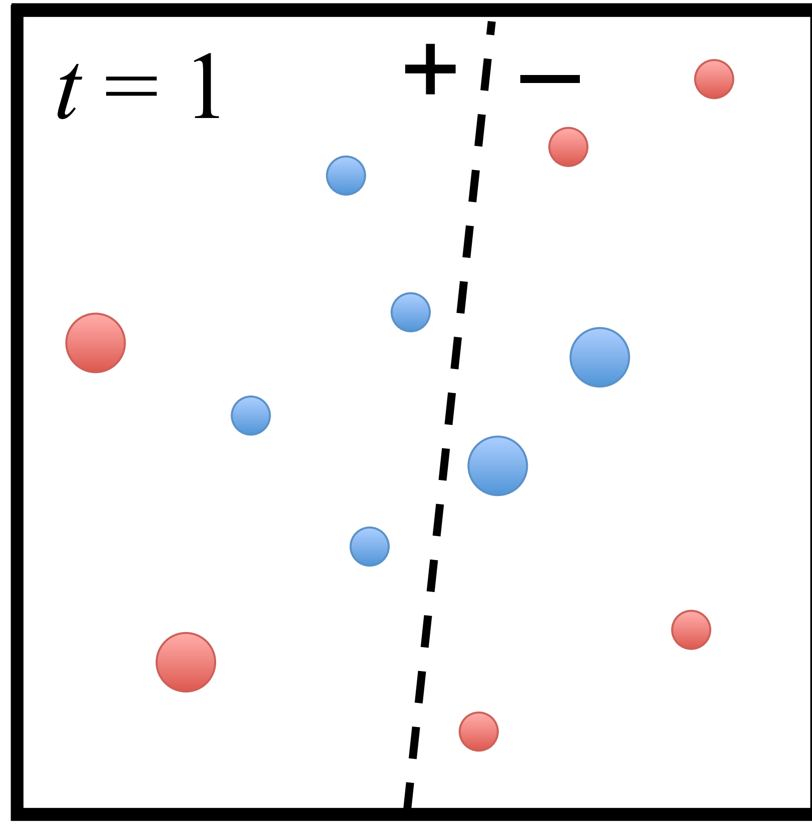

Disclaimer: Note that resized points in the illustration above are not necessarily to scale with β_t

Più beta cresce, peggio mi sono comportato! Rimpicciolisco i valori ben classificati, ingrandisco i valori mal classificati

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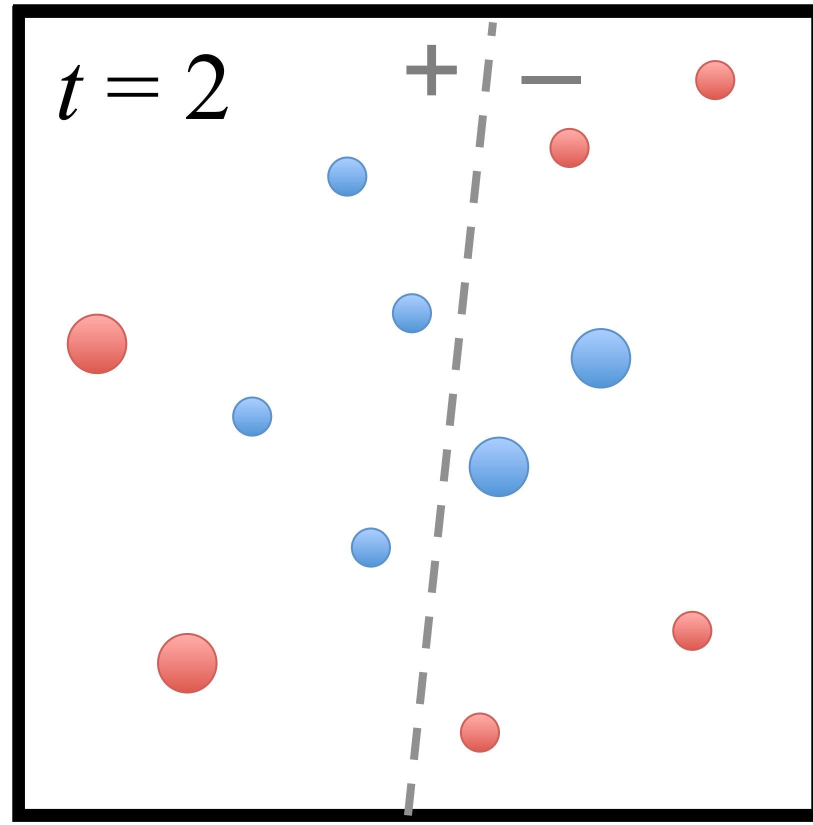

Al secondo passo, cambio i pesi ed eseguo una nuova classificazione.

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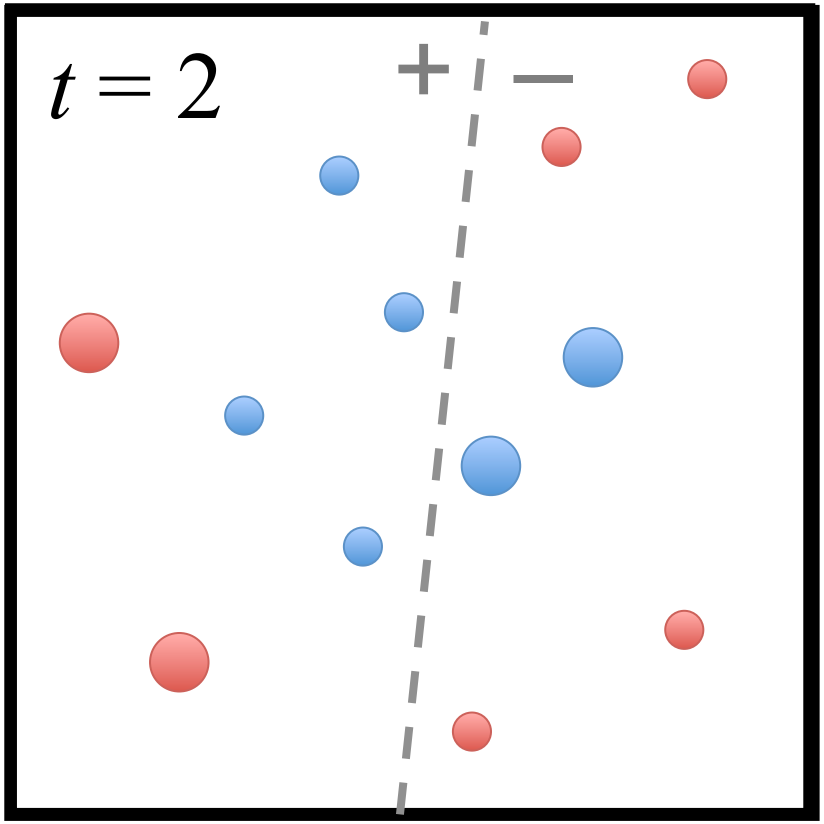

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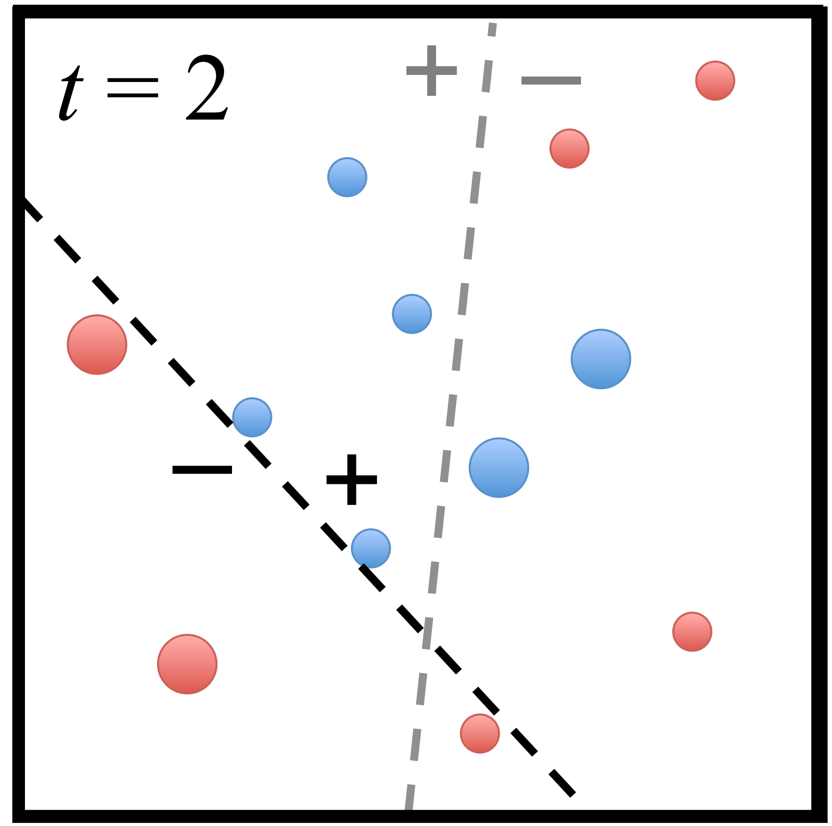

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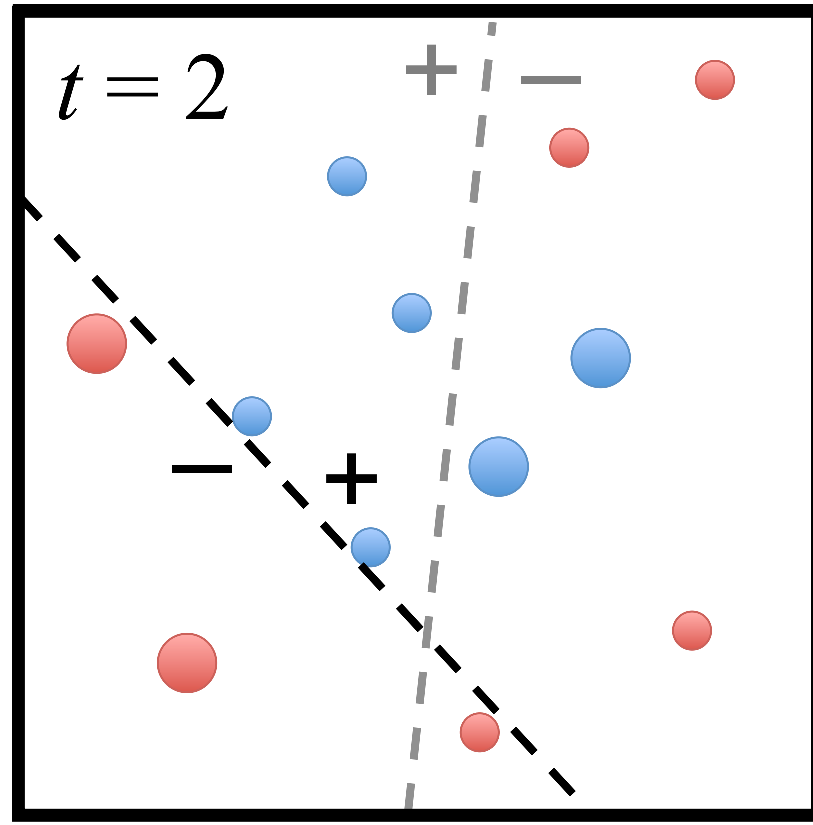

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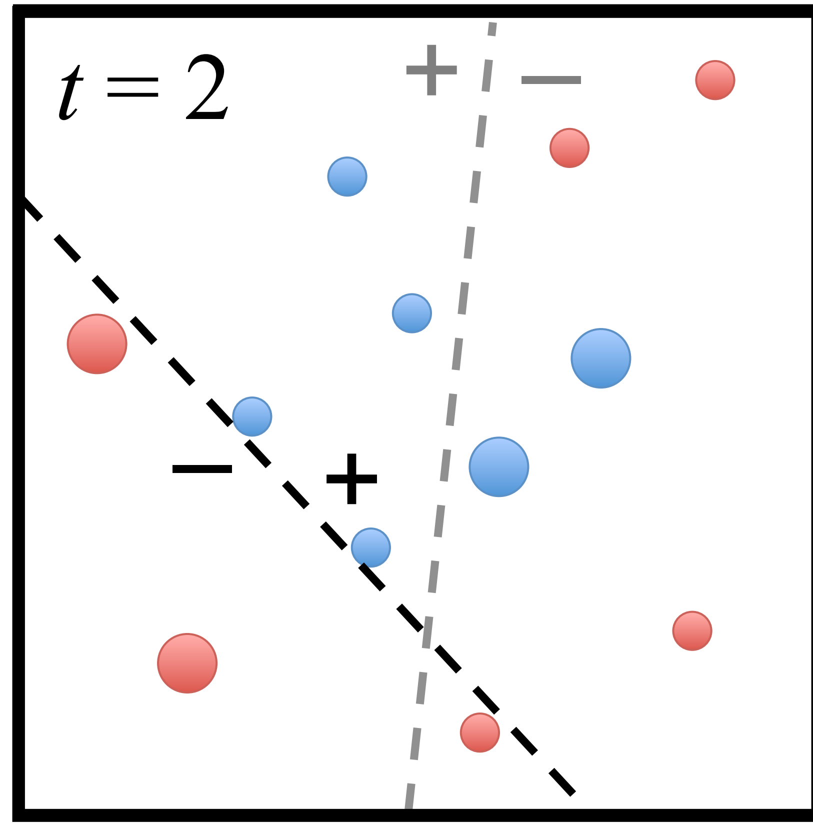

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AdaBoost

```
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    Train model h_t on X, y with weights w_t
    Compute the weighted training error of h_t
    Choose β_t = (1/2) ln((1 − ε_t) / ε_t)
    Update all instance weights:
        w_{t+1,i} = w_{t,i} exp(−β_t y_i h_t(x_i))
    Normalize w_{t+1} to be a distribution
end for
Return the hypothesis
        H(x) = sign( Σ_{t=1}^{T} β_t h_t(x) )
```

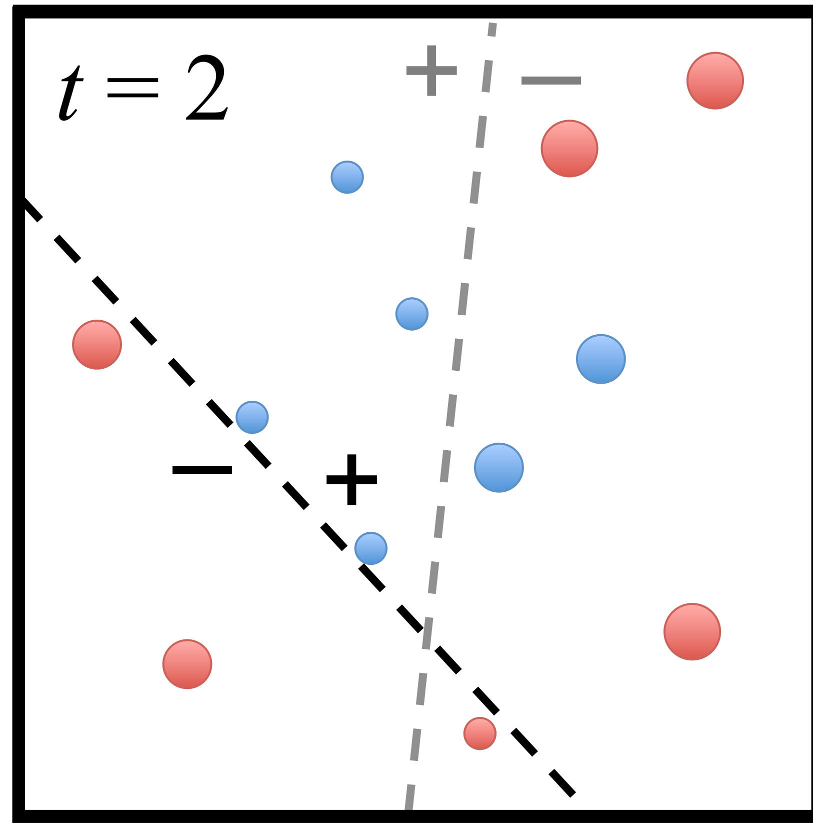

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sempre uguale anche aumentando l'iterazione

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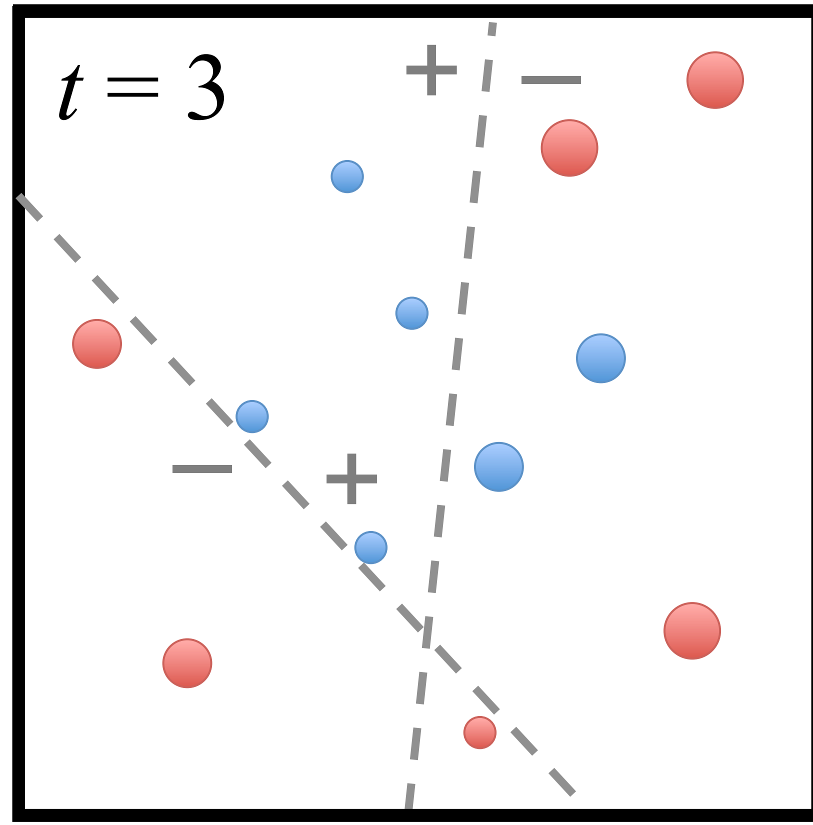

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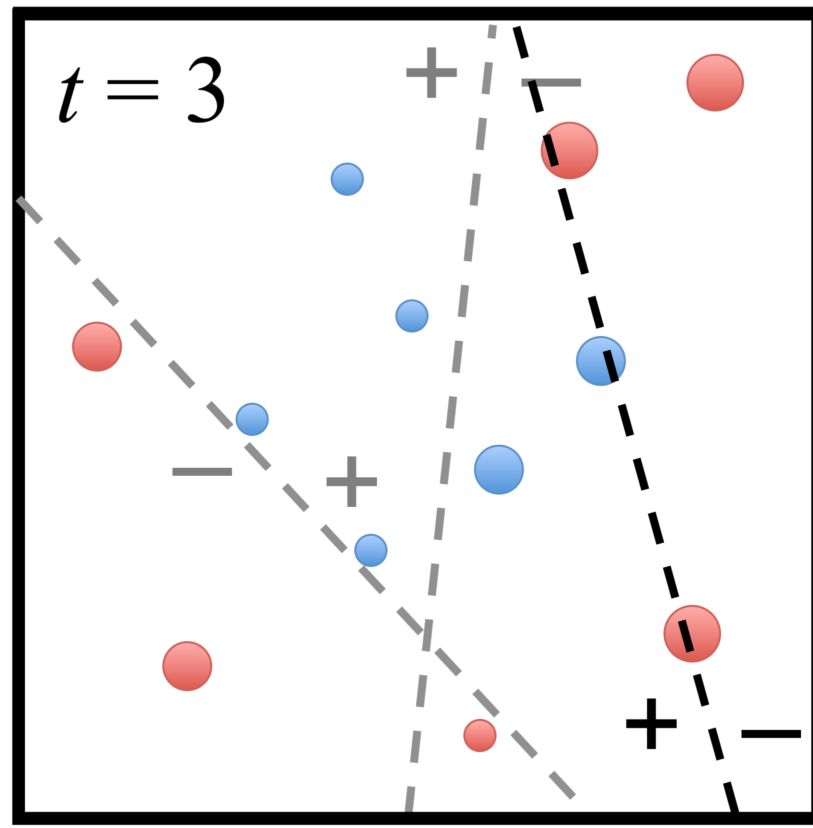

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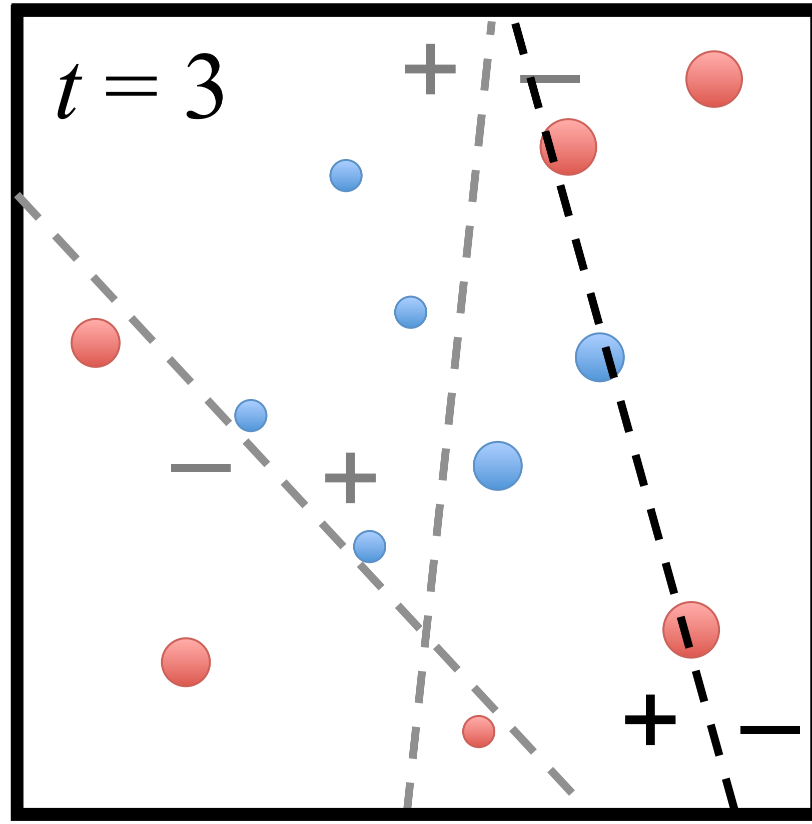

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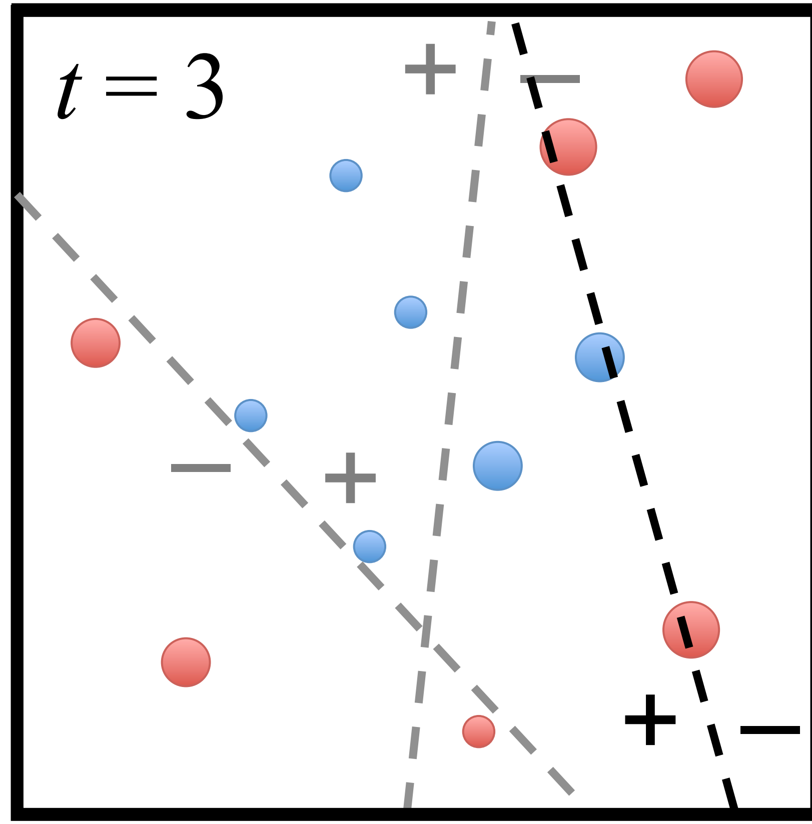

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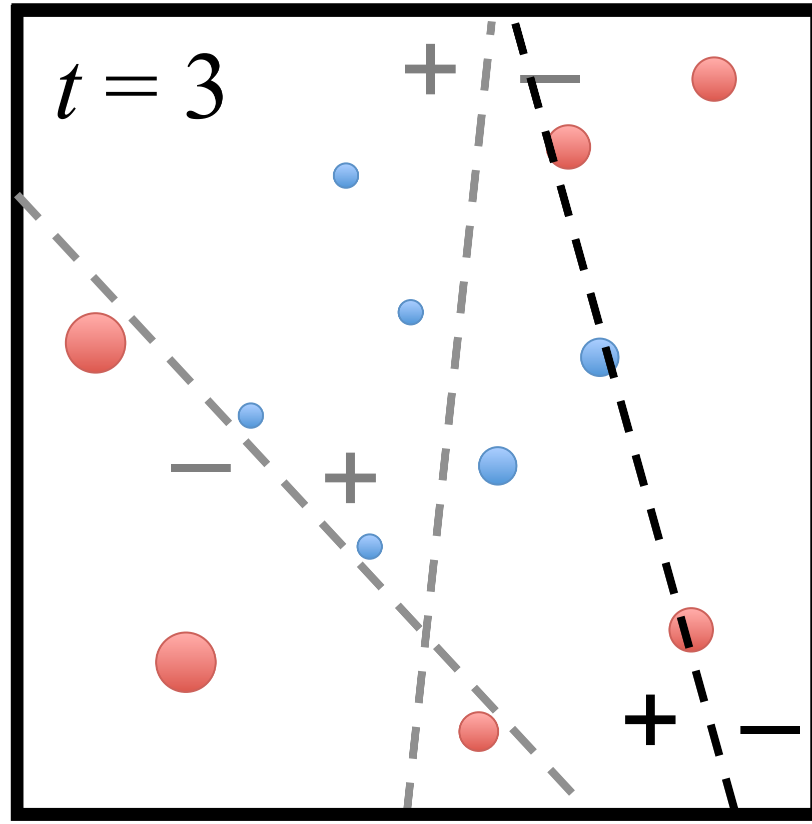

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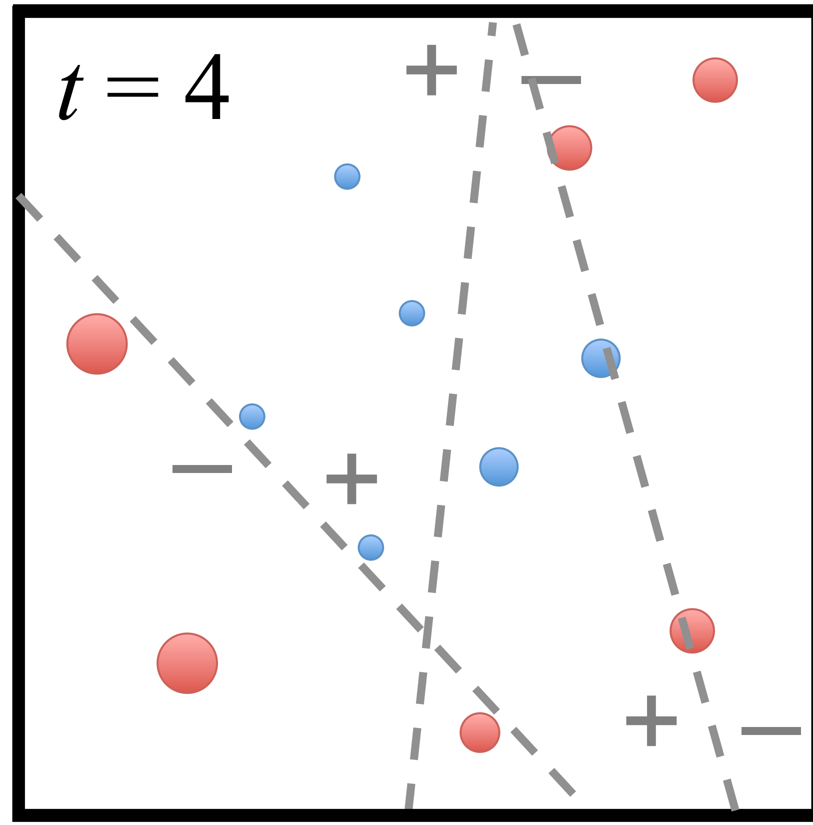

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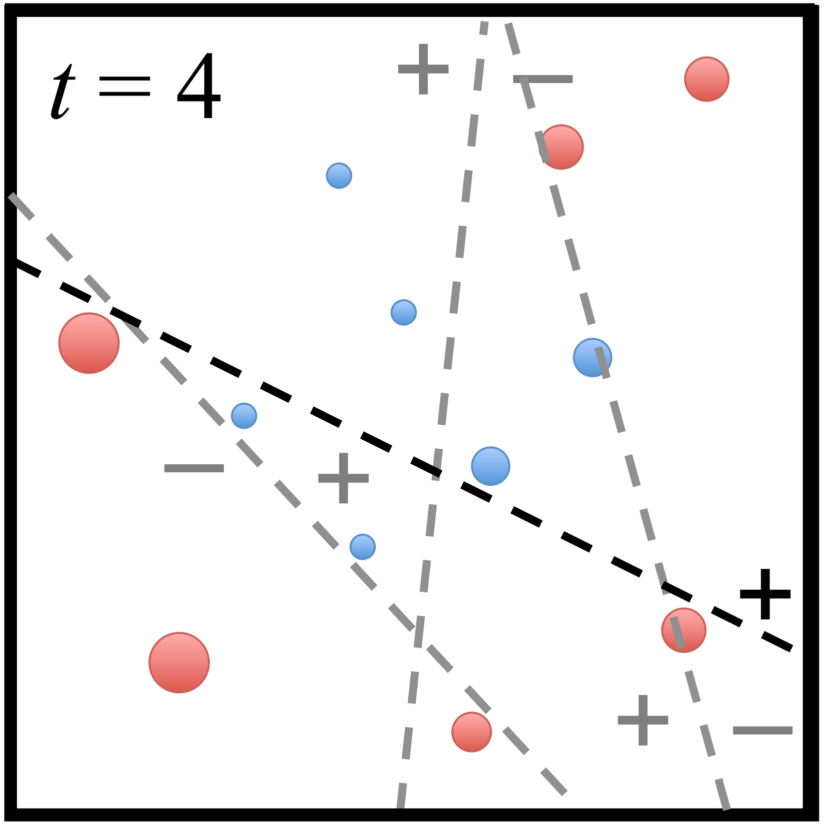

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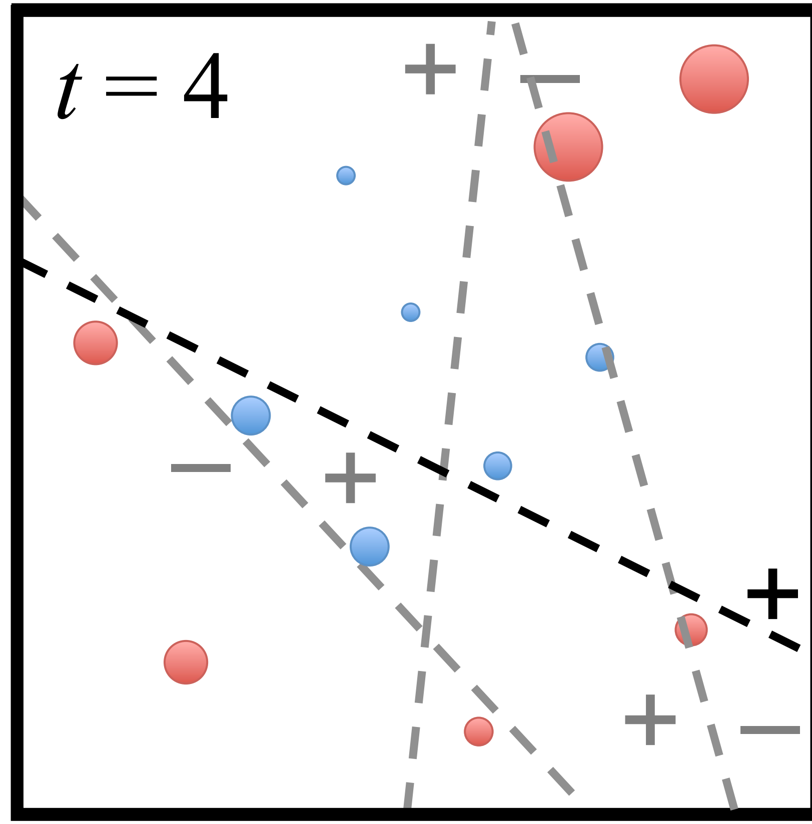

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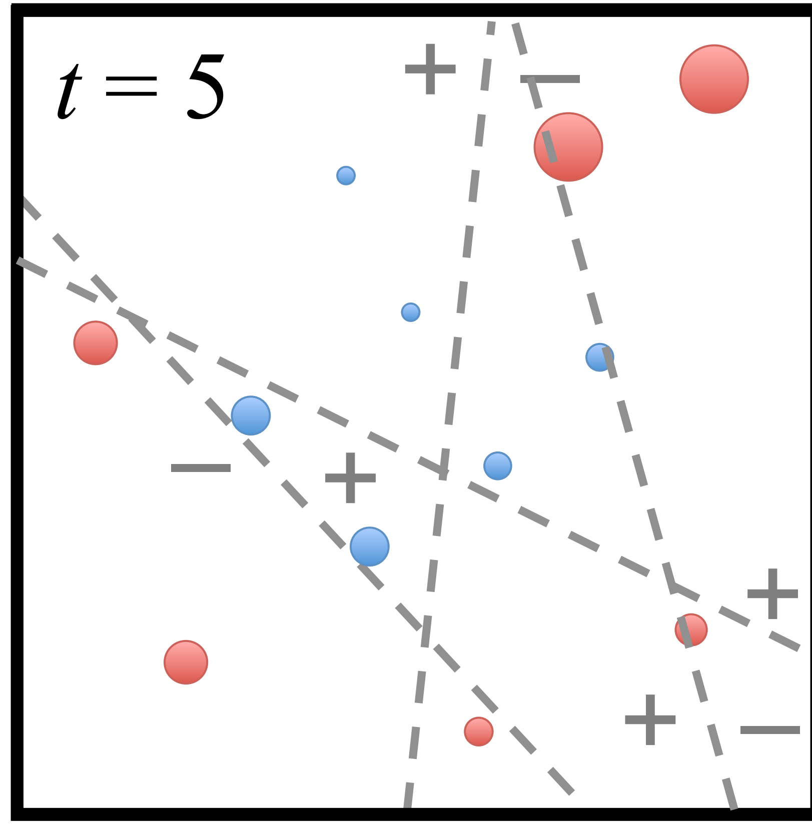

AdaBoost

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    Compute the weighted training error of h_t
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$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right)$$

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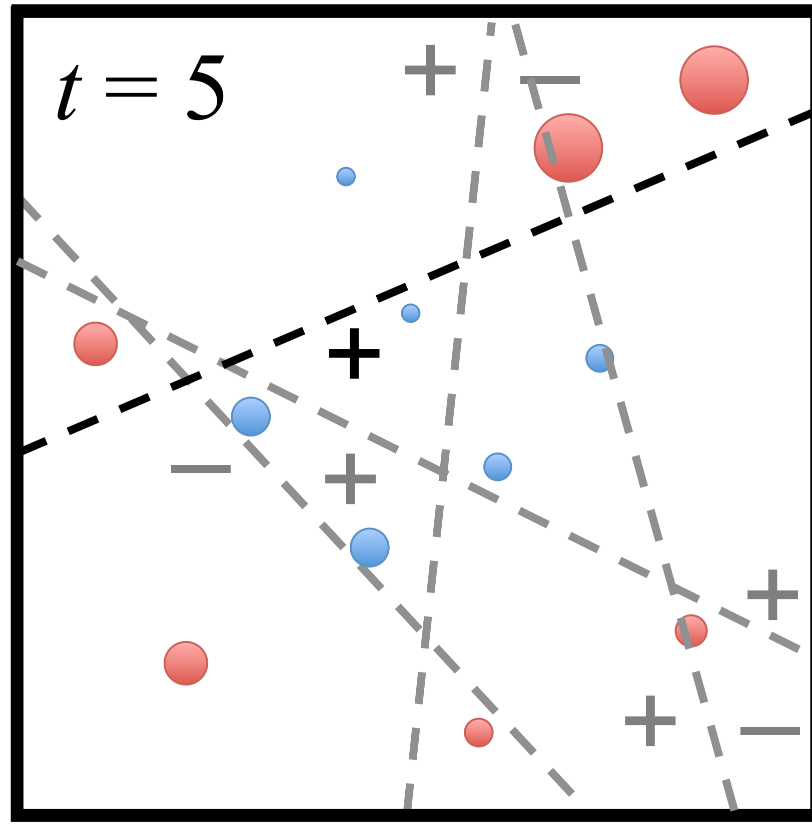

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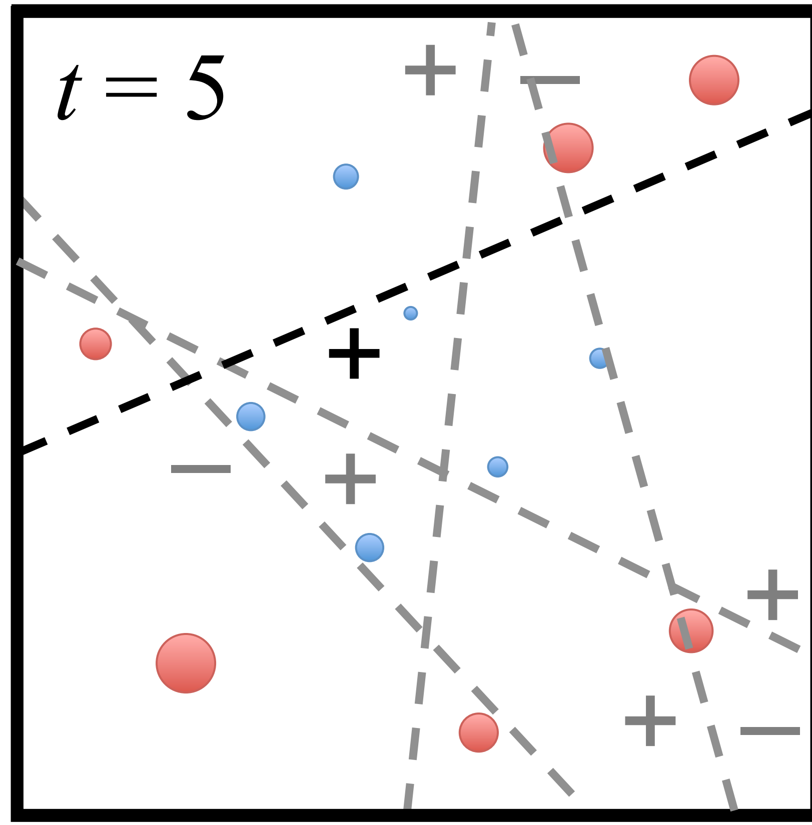

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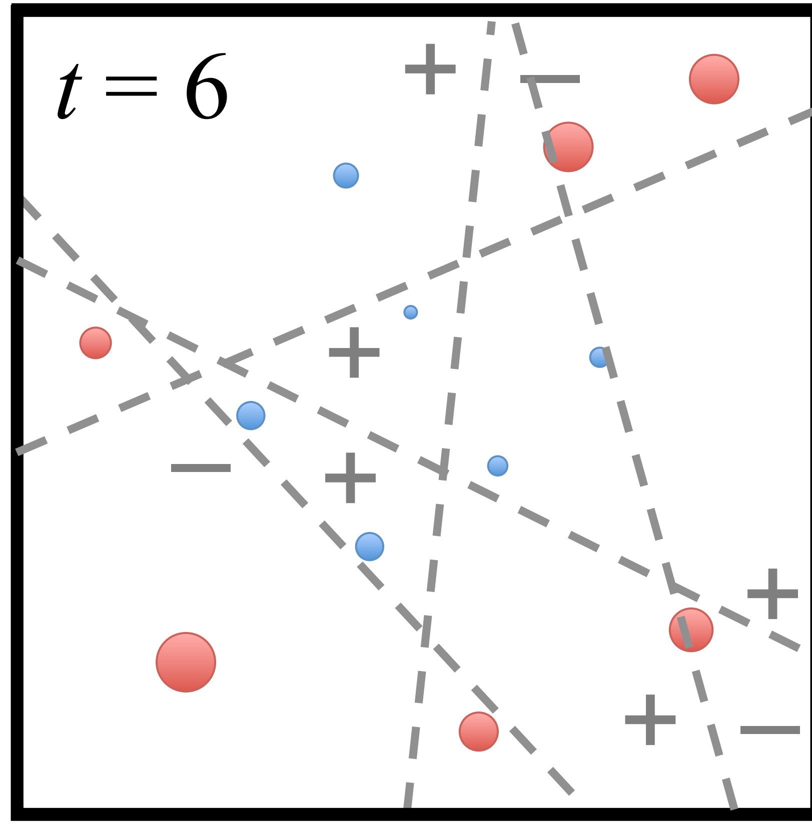

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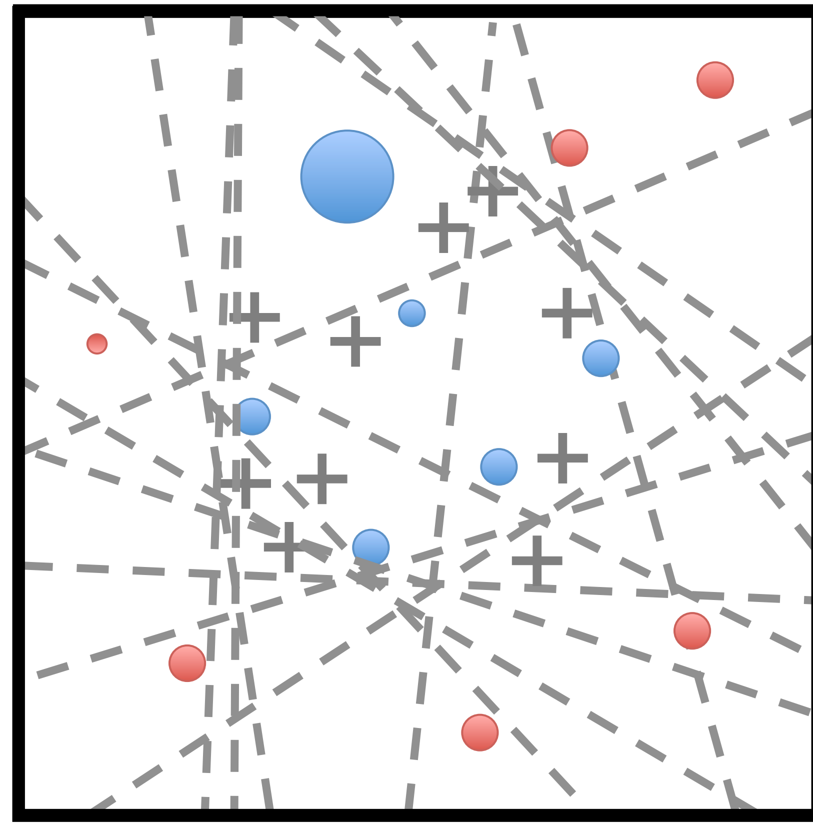
$t = \mathrm{T}$



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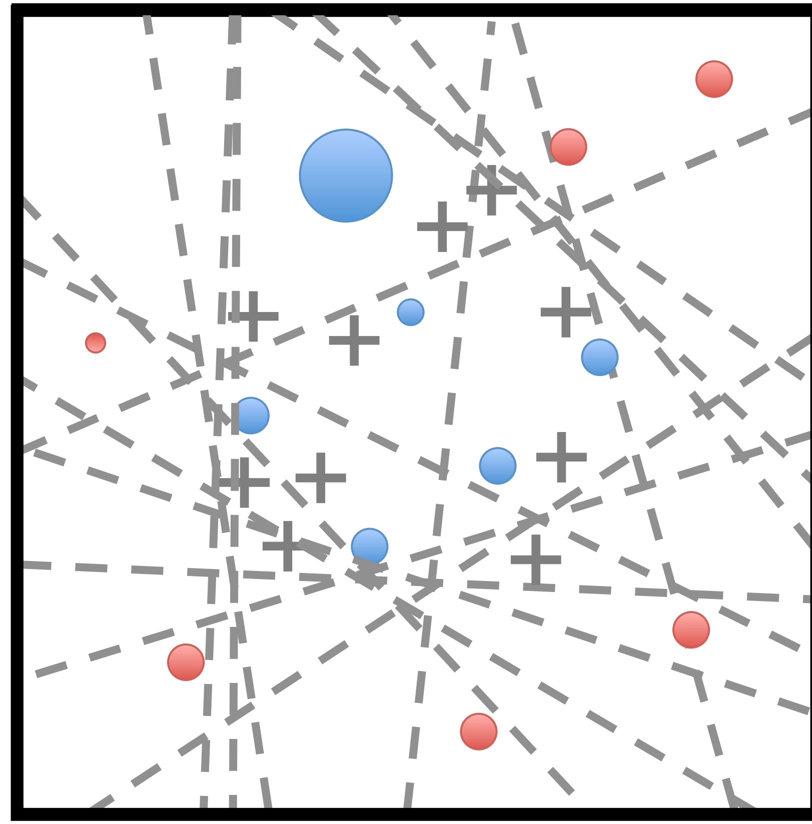

a t = 1 ho underfitting, a t = T posso avere Overfitting. Ogni classificatore ha un peso Beta e li metto insieme, tramite media pesata.

AdaBoost

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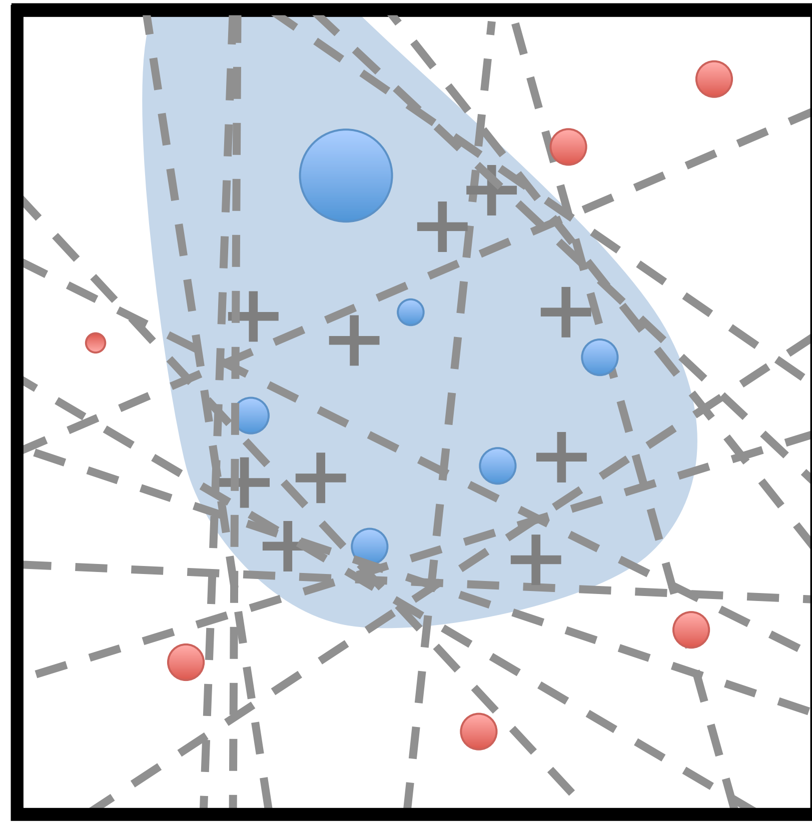

- Final model is a weighted combination of members
  - Each member weighted by its importance

Ciò che ottengo non è un classificatore lineare!
(simile a ciò che avveniva nel Bagging con lo spazio delle ipotesi)

AdaBoost

[Freund & Schapire, 1997]

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n}$,
the number of iterations T

1: Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$
2: **for** $t = 1, \ldots, T$
3: Train model h_t on X, y with instance weights $\mathbf{w}_t$
4: Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i : y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

5: Choose $\beta_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$
6: Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1, \ldots, n$$

7: Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

8: **end for**
9: **Return** the hypothesis

$$H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x})\right)$$

AdaBoost

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the number of iterations T

Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$
for $t = 1, \ldots, T$
  Train model h_t on X, y with instance weights $\mathbf{w}_t$
  Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i : y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

  Choose $\beta_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right)$
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  Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

end for
Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$

$\mathbf{w}_t$ is a vector of weights over the instances at iteration t

All points start with equal weight

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$,
the number of iterations T

1: Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$
2: **for** $t = 1, \ldots, T$
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8: **end for**
9: **Return** the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^T \beta_t h_t(\mathbf{x}) \right)$$

We need a way to weight instances differently when learning the model…

Training a Model with Weighted Instances

- For algorithms like logistic regression, can simply incorporate weights $\mathbf{w}$ into the cost function
  - Essentially, weigh the cost of misclassification differently for each instance

$$J_{\text{reg}}(\boldsymbol{\theta}) = -\sum_{i=1}^{n} w_i \left[y_i \log h_{\boldsymbol{\theta}}(\mathbf{x}_i) + (1 - y_i) \log \left(1 - h_{\boldsymbol{\theta}}(\mathbf{x}_i)\right)\right] + \lambda \|\boldsymbol{\theta}_{[1:d]}\|_2^2$$

aggiungo peso per ogni istanza.
nb: non c'è conformità di notazione con le vecchie slide.

- For algorithms that don’t directly support instance weights (e.g., ID3 decision trees, etc.), use weighted bootstrap sampling
  - Form training set by resampling instances with replacement according to $\mathbf{w}$

i pesi li metto sul boostrap sampling

Base Learner Requirements

- AdaBoost works best with "weak" learners
  - Should not be complex
  - Typically high bias classifiers
  - Works even when weak learner has an error rate just slightly under 0.5 (i.e., just slightly better than random)
    - Can prove training error goes to 0 in O(log n) iterations

- Examples:
  - Decision stumps (1 level decision trees)
  - Depth-limited decision trees
  - Linear classifiers

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n}$,
the number of iterations T

Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$

for $t = 1, \ldots, T$

Train model h_t on X, y with instance weights $\mathbf{w}_t$

Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i : y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

l'errore è la somma dei pesi classificati mali, SOLO PESI

Error is the sum the weights of all misclassified instances

Choose $\beta_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$

Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1, \ldots, n$$

Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

end for

Return the hypothesis

$$H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x})\right)$$

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n}$,
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Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$

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Train model h_t on X, y with instance weights $\mathbf{w}_t$

Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i: y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

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Update all instance weights

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\right.$$

Normalize $\mathbf{w}_{t+1}$ to be a dist

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall$$

end for

Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x})\right)$$

- β_t measures the importance of h_t !!
- If $\epsilon_t \leq 0.5$, then $\beta_t \geq 0$
  - Trivial, otherwise flip h_t's predictions
- β_t grows as error h_t's shrinks

AdaBoost

INPUT: traini
the n

Initialize a vecto

for $t = 1, \ldots, T$

Train model

Compute the

$$\epsilon_t = \sum_{i: y_i \neq h_t(\mathbf{x}}$$

Choose $\beta_t = \frac{1}{2} \ln \left(\quad \frac{}{\epsilon_t} \right)$

This is the same as:

will be ≤ 1

$$w_{t+1,i} = w_{t,i} \times \begin{cases} \boxed{e^{-\beta_t}} & \text{if } h_t(\mathbf{x}_i) = y_i \\ \boxed{e^{\beta_t}} & \text{if } h_t(\mathbf{x}_i) \neq y_i \end{cases}$$

will be ≥ 1

y_i vale +1 o -1

Essentially this emphasizes misclassified instances.

Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1, \ldots, n$$

Se y_i e h_t sono concordi (quindi +1 entrambi, o -1 entrambi -> il prodotto fa 1 e la classificazione è corretta.

Se y_i e h_t sono discordi, il prodotto fa -1 e la classificazione è errata.

Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

end for

Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$

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Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$
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Train model h_t on X, y with instance weights $\mathbf{w}_t$
Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i : y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

Choose $\beta_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right)$
Update all instance weights:

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Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

Make $\mathbf{w}_{t+1}$ sum to 1

end for
Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$

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$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1,$$

  Normalize $\mathbf{w}_{t+1}$ to be a distribution:

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end for
Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$

Member classifiers with less error are given more weight in the final ensemble hypothesis

Final prediction is a weighted combination of each member's prediction

Dynamic Behavior of AdaBoost

- If a point is repeatedly misclassified...
  - Each time, its weight is increased
  - Eventually it will be emphasized enough to generate a hypothesis that correctly predicts it

- Successive member hypotheses focus on the hardest parts of the instance space
  - Instances with highest weight are often outliers

AdaBoost è sequenziale, ovvero:
- il 2° ciclo pesa di più gli errori fatti nel 1° ciclo, il 3° pesa di più gli errori del 2° etc...

Ogni iterazione baserà la predizione sui valori PIU' PESANTI (spesso outliers), magari andando a sbagliarne altri. Dovrò infatti COMBINARE queste linee ottenute per approssimare al meglio l'insieme in cui la classificazione è errata.