Performance Modeling of Computer Systems and Networks

Prof. Vittoria de Nitto Personè

Operational analysis:

Queueing Networks

Università degli studi di Roma Tor Vergata
Department of Civil Engineering and Computer Science Engineering

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Da un punto di vista degli analisti di sistema si è creato dello scetticismo, perchè il modello era regolamentato da un approccio stazionario, le ipotesi richiedevano indipendenza stocastica tra i job, etc...

Analytical models

Queueing network (QN) modelling is a particular approach to computer system (CS) modeling in which the CS is represented as a *network of queues* which is evaluated *analitically*

- Many analists experienced doubts on its accuracy
- A series of assumptions:
  - ✓ the system is modeled by a *stationary stochastic process*;
  - ✓ jobs are *stochastically independent*;
  - ✓ job steps from device to device follow a *Markov chain*;
  - ✓ the system is in stochastic equilibrium;
  - ✓ the service time requirements at each device conform to an *exponential distribution*;
  - ✓ the system is *ergodic*, i.e. long-term time averages converge to the values computed for stochastic equilibrium

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Molte assunzioni erano "forzate" rispetto al caso in analisi.

Analytical models

Some of these concepts are difficult and cannot be proved to hold by observing the system in a finite time period
Most can be disproved empirically

- ✓parameters change over time
- ✓jobs are dependent
- ✓systems are observable only for short periods
- ✓...

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Analytical models

In applying or validating the results of Markovian QN theory, analists substituted *operational* values for stochastic parameters

directly measured

The Markovian QN equations are also valid among operational variables

They hold under different assumptions and apply to a large class of real systems[1]

[1] BUZEN, J.P. "Operational analysis: the key to the new generation of performance prediction tools," in Proc. IEEE COMPCON, 1976, IEEE, New York.
DENNING, P. J.; AND BUZEN, J. P. "Operational analysis of queueing networks," in Proc. Third Int. Symp. Computer Performance Modeling, Measurement, and Evaluatlon, 1977, North-Holland Publ. Co., Amsterdam, The Netherlands.

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Operational Analysis

Three *operational principles*:

1. All quantities should be *precisely measurable* and all assumptions should be *directly testable*
2. The system must be *flow balanced*
3. The devices must be *homogeneous*,

   i.e., the routing must be independent of queue lengths (ql)
   the mean service time at a given device must not depend on ql of other devices

the same mathematical equations

but the operational assumptions can be tested

↓

much more confidence and understanding of the QN technology

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Operational Analysis

Def.

Hypotheses whose veracity can be established beyond doubt by measurement will be called *operationally testable*

Operational analysis provides a rigorous mathematical discipline for studying CS performance based solely on operationally testable hypotheses

two basic components:

a system (real or hypothetical)
a (finite) time period → *the observation period*

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Operational Analysis

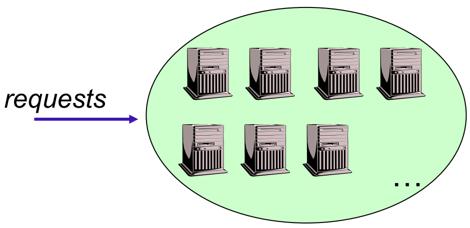
According to the *operational* approach, let us consider

Basic quantities

- T the length of the observation period (op)
- A the number of arrivals during op
- B the total amount of time during which the system is busy $B \leq T$ during op
- C the number of completions during op

Multi Resources Systems 1

- a collection of servers, working together for incoming requests
- homogeneous servers



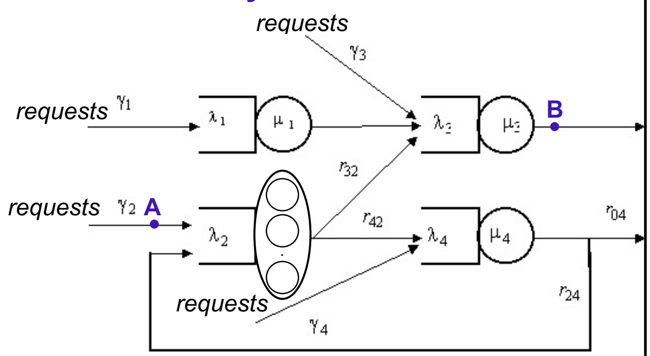
Data centers
Server farm

What is the minimum number of servers needed to guarantee that only a small fraction of jobs are delayed?

Is a single central queue superior to having a queue at each server?

Multi Resources Systems 2

- a collection of different resources, connected, where incoming requests can require services more than once to each resources



Which is the "load" of each resource?

Which is the time spent on each resource?

Which is the time to go from point A to point B? (end-to-end response time)

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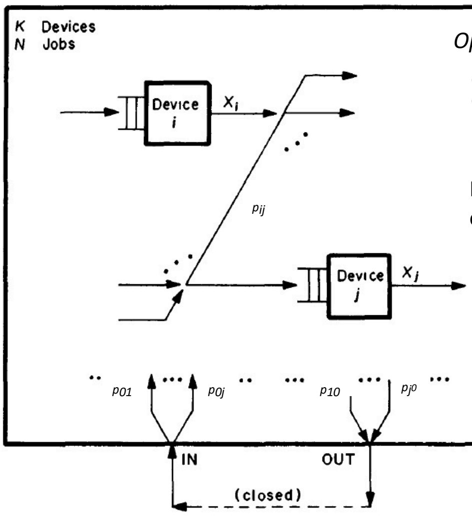
Operational Analysis

Queueing Networks



Operationally connected:

each device is visited at least once by some job during op

No job overlaps its use of different devices

Work-conserving

No part of the system can block progress in another part

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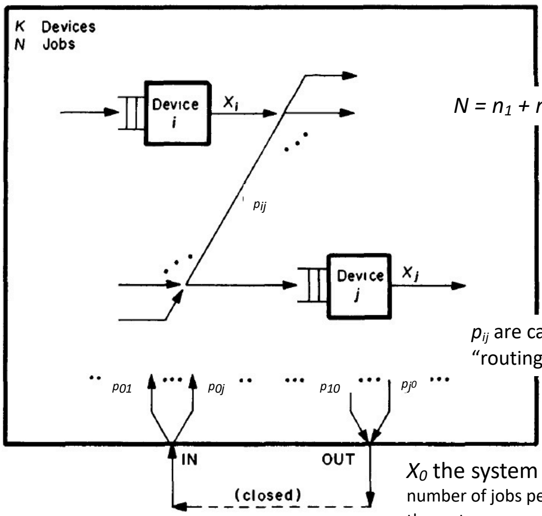
Queueing Networks

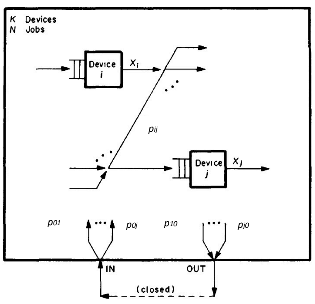


$N = n_1 + n_2 + \dots n_K$

p_{ij} are called the "routing frequencies"

X_0 the system output rate, the number of jobs per second leaving the system

Queueing Networks



If the system is *open*, X_0 is known and N varies as jobs enter or leave the system

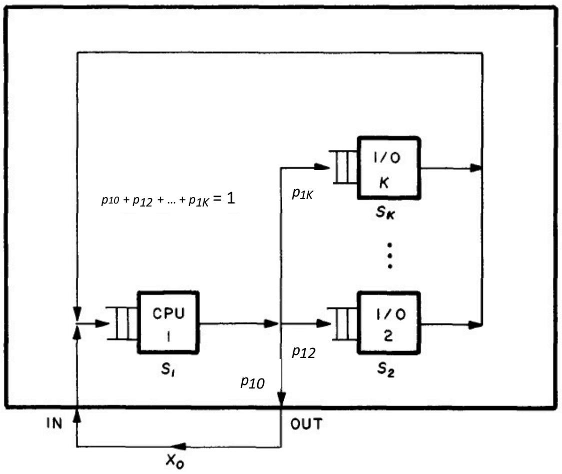
An analysis of an open system assumes that X_0 is known and seeks to characterize the distribution of N

If the system is *closed* the number of jobs N is fixed

An analysis of a closed system begins with N given and seeks to determine the resulting X_0 along the OUT/IN path

Queue lengths and response times at the devices may be sought in both cases

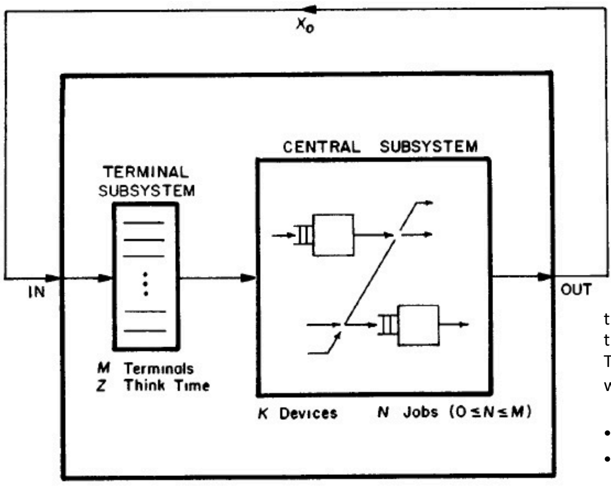
Central Server Network



A batch processing system under a backlog:

- a job begins with a CPU service interval and continues with zero or more I/O intervals which alternate with further CPU bursts
- a new job enters the system as soon as an active job terminates

Terminal Driven System



Time sharing systems, driven by interactive terminals:

a user alternates between *thinking* and *waiting*

the mean time a user spends in a thinking interval: *Z think time*
The mean time a user spends in a waiting interval: *R response time*

- *Z* is independent of *M*
- *R* is a function of *M*:

  jobs delay each other while contending for resources

Queueing Networks

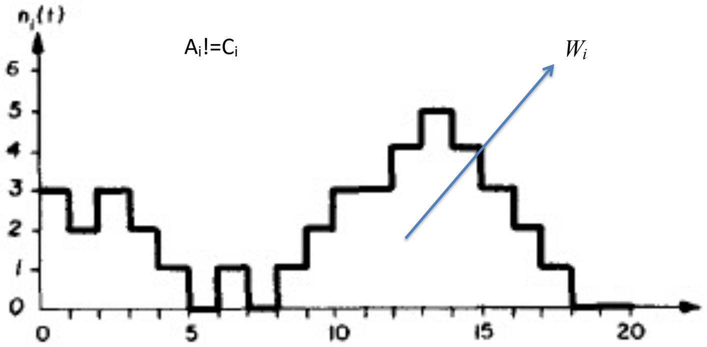
The system is measured for an op of T seconds, the following data are collected for each device $i=1, 2, \ldots K$

- A_i the number of arrivals;
- B_i total busy time, during which $n_i > 0$
- C_{ij} number of times a job requests service at device j immediately after completing a service request at device i; note that it is possible $C_{ii} > 0$.

If we treat the "outside world" as device "0", we can define also

- A_{0j} number of jobs whose first service request is for device j;
- C_{i0} number of jobs whose last service request is for device i.

Queueing Networks



Operational Equations

$$X_j = \sum_{i=0}^{K} X_i p_{ij}$$ Job Flow Balance equations

$$\begin{cases} V_0 = 1 \\ V_j = p_{0j} + \sum_{i=1}^{K} V_i p_{ij} \end{cases}$$ Visit Ratio equations

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Operational Equations

$U_i = X_i S_i$ *Utilization Law*

$\bar{n}_i = X_i R_i$ *Little's Law*

$X_0 = \sum_{i=0}^{K} X_i p_{i0}$ *Output Flow Law*

$R = \sum_{i=1}^{K} V_i R_i$ *General Response Time Law*

$R = M / X_0 - Z$ *Interactive Response Time Formula* (Assumes flow balance)

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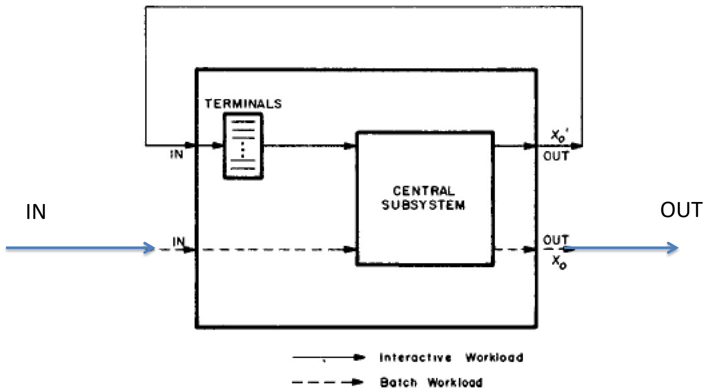

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