Performance Modeling of Computer Systems and Networks

Prof. Vittoria de Nitto Personè

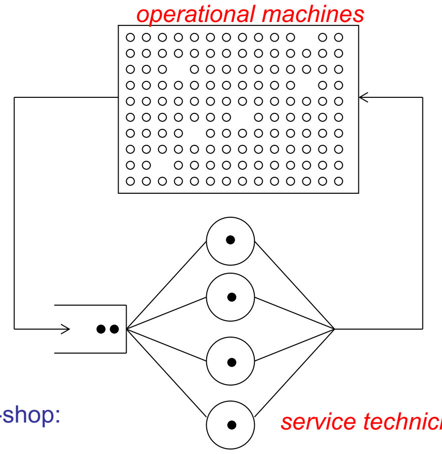
Trace-driven simulation
Case study 1

Università degli studi di Roma Tor Vergata
Department of Civil Engineering and Computer Science Engineering

1

Discrete-event simulation
conceptual model

operational machines



failed machines

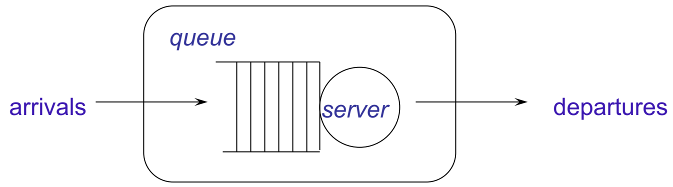
service technicians

in the machine-shop:

- The "jobs" are the failed machines and the "servers" are the technies
- If there is only one service technician, the conceptual model includes a single-server queue (CASO LIMITE)

2

Single server queue



Service node

• *def. 1* a *single server service node* consists of a *server* plus its *queue*

terminology

synonymous

- queue/center/node
- job/user/request

usual		in the book
• waiting time	→	delay
• response/sojourn time	→	wait

P.23 discrete

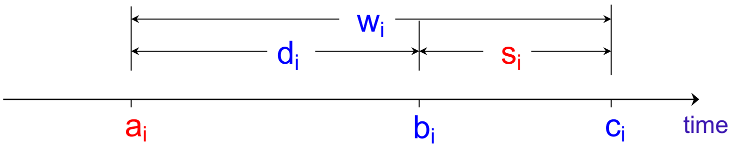
For a job *i*:

- The *arrival time* is a_i
- The *service time* is s_i

} input variables

- The *delay* in the queue is d_i (delay, usually this is known as "waiting time")
- The time that service begins is $b_i = a_i + d_i$ (begin)
- The *wait* in the node is $w_i = d_i + s_i$ (wait, aka response time)
- The departure time is $c_i = a_i + w_i$ (completion)

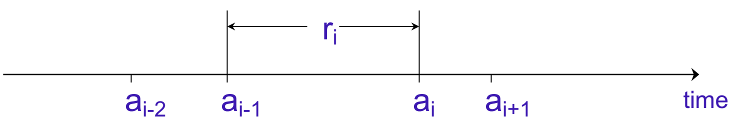
} output variables



The *interarrival time* between jobs *i − 1* and *i* is

$$r_i = a_i - a_{i-1}$$

where, by definition, $a_0 = 0$



Assume only one arrival per time instant
$r_i > 0, \forall i$

NO *bulk* (arrivi multipli in a_i)

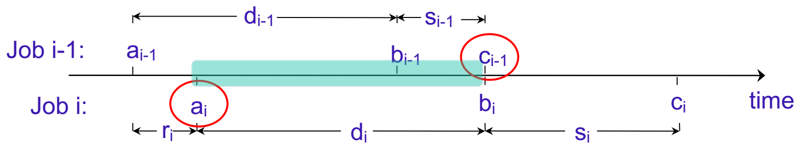
trace-driven simulation

- The model is driven by external data:
  Given the arrival times a_i and service times s_i, can the delay times d_i be computed? (avere r_i è equivalente! $a_i = r_1 + r_2 + \dots + r_i$)
- For some queue disciplines, this question is difficult to answer
- If the queue discipline is FIFO, d_i is determined by when a_i (the arrival) occurs relative to c_{i-1} (the previous departure)

P.24)

Case 1. The job arrives *before* the previous job completes

$a_i < c_{i-1}$

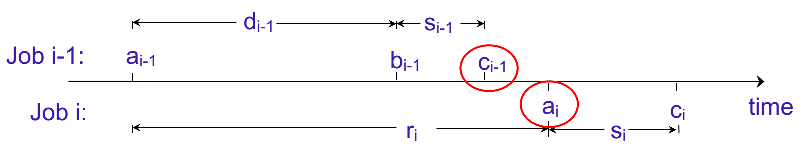


$\longrightarrow$ $d_i = c_{i-1} - a_i$ → i arriva

i-1 finisce

Case 2. The job arrives *after* the completion of the previous job

$a_i \geq c_{i-1}$



$\longrightarrow$ $d_i=0$

Output statistics

(intuitiva)

- The purpose of simulation is insight — gained by looking at statistics
- The importance of various statistics varies on perspective:
  - User perspective (job): wait time is most important
  - Manager perspective: utilization is critical
- Statistics are broken down into two categories
  - Job-averaged
  - Time-averaged

P.27)

Job-averaged statistics

- *Average interarrival time* $\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i = \frac{a_n}{n}$ $= \frac{1}{n} \cdot \left[a_0 + (a_1 - \cancel{a_0}) + (a_2 - \cancel{a_1}) \ldots \right]$

  Arrival rate $\frac{1}{\bar{r}}$

- *Average service time* $\bar{s} = \frac{1}{n}\sum_{i=1}^{n} s_i$

  Service rate $\frac{1}{\bar{s}}$

esempio: $\bar{r}$ = average interarrival è 32 sec/Job ; $\frac{1}{\bar{r}} \rightarrow$ [Job]
$\bar{s}$ = average service time è 34.7 sec/Job ; $\frac{1}{\bar{s}} \rightarrow$ [sec]

Job-averaged statistics

- The *average delay* and *average wait* are defined as

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i \qquad \bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i$$ (response time)

$w_i = d_i + s_i$ — d_i: attesa prima di essere servito; s_i: tempo servizio

Recall $w_i=d_i+s_i$ $\forall$ i, hence

$$\bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i = \frac{1}{n}\sum_{i=1}^{n}(d_i + s_i) = \frac{1}{n}\sum_{i=1}^{n} d_i + \frac{1}{n}\sum_{i=1}^{n} s_i = \bar{d} + \bar{s} = \bar{w}$$

Sufficient to compute any two of $\bar{w}, \bar{d}, \bar{s}$

con 2 val. trovo il 3°, ma è bene anche calcolarlo da ø e poi confrontarli! (VERIFICO)

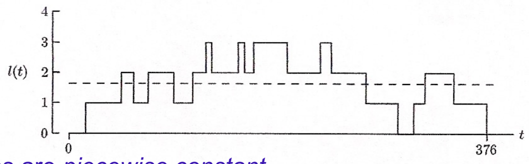
time-averaged statistics

For SSQ, need three additional functions

- l(t): number of jobs in the service node at time t (tutti quelli nel sist)
- q(t): number of jobs in the queue at time t (in coda)
- x(t): number of jobs in service at time t (in servizio)

By definition l(t)=q(t)+x(t) ∀ t

l(t) = 0, 1, 2, …
q(t) = 0, 1, 2, …
x(t) = 0, 1



The three functions are *piecewise constant*
(a tratti)

SINGLE SERVER QUEUE

Over the time interval (0, τ):

time-averaged number in the node: $\bar{l} = \frac{1}{\tau}\int_0^\tau l(t)dt$

time-averaged number in the queue: $\bar{q} = \frac{1}{\tau}\int_0^\tau q(t)dt$

time-averaged number in service: $\bar{x} = \frac{1}{\tau}\int_0^\tau x(t)dt$
(≐ server utilization)

Def. *Utilization*
The proportion of time that the server is busy

Since l(t)=q(t)+x(t) ∀ t $\bar{l} = \bar{q} + \bar{x}$

$\bar{x} \in [0,1]$ poiché singolo servente

P.30

How are job-averaged and time-average statistics related?

in particolare: legame $\langle \bar{w}, \bar{d}, \bar{s} \rangle$ e $\langle \bar{l}, \bar{q}, \bar{x} \rangle$?

Little's Law (1961)

If (a) queue discipline is FIFO,

(b) service node capacity is infinite, and

(c) server is idle both at the beginning and end of the observation interval ($t = 0$, $t = c_n$)

se $t_0 = \emptyset$, $t_{finale} = 320$, in questi tempi NON devono esserci Job nel Server

then (allora):

Job nel service node in t

$$\int_0^{c_n} l(t)\,dt = \sum_{i=1}^{n} w_i$$

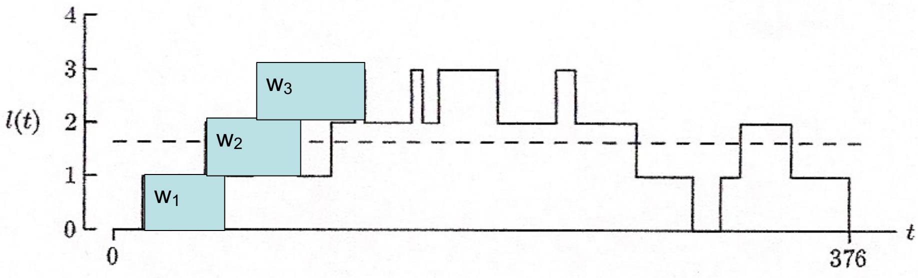
Job service node (coda + server)

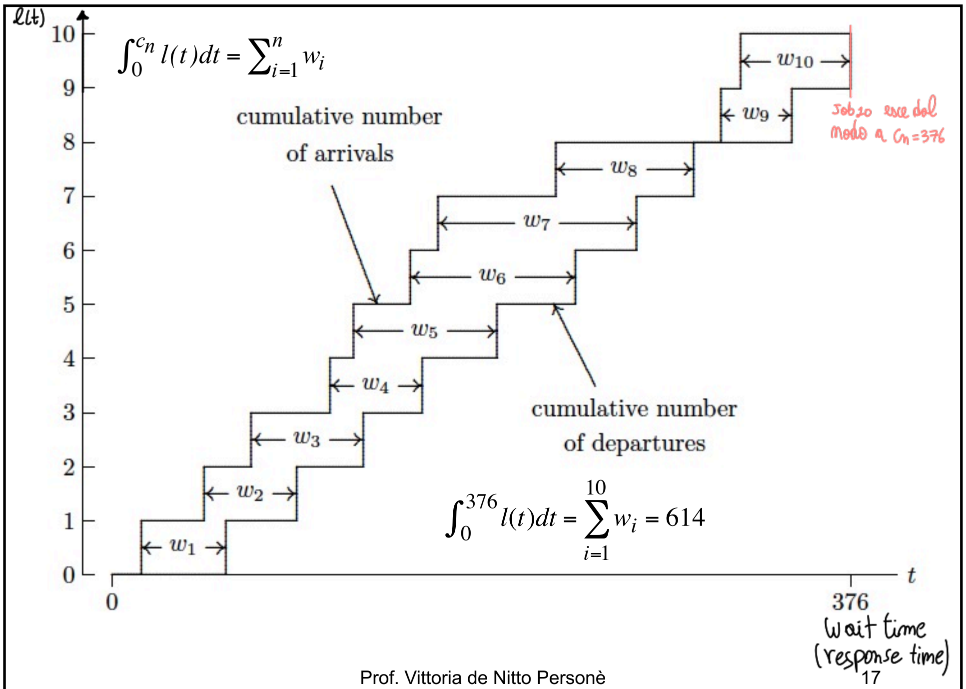
Job coda in t

$$\int_0^{c_n} q(t)\,dt = \sum_{i=1}^{n} d_i$$

Job servizio in t

$$\int_0^{c_n} x(t)\,dt = \sum_{i=1}^{n} s_i$$





Using $\tau = c_n$ in $\bar{l} = \frac{1}{\tau}\int_0^{\tau} l(t)dt$ (media)

along with Little's Theorem, we have:

$$c_n \bar{l} = \int_0^{c_n} l(t)dt = \underbrace{\sum_{i=1}^{n} w_i}_{} = n\bar{w} \qquad \bar{w} = \frac{1}{n} \cdot \sum_{i=1}^{n} w_i$$

As a consequence: $\bar{l} = \frac{n}{c_n}\bar{w}$

Same holds for: $\bar{q} = \frac{n}{c_n}\bar{d}$ $\quad \bar{x} = \frac{n}{c_n}\bar{s}$; $C_n \cdot \bar{q} = n \cdot \bar{d}$ $\quad C_n \cdot \bar{x} = n \cdot \bar{s}$

$\frac{n}{c_n}$ represents the *average system throughput* in c_n
Note that, for infinite queue, this corresponds to the average arrival rate

P.32)

Def. Traffic intensity

The ratio of the arrival rate to the service rate

$$\frac{1/\bar{r}}{1/\bar{s}} = \frac{\bar{s}}{\bar{r}} = \frac{\bar{s}}{a_n/n} = \left(\frac{c_n}{a_n}\right)\bar{x}$$

$$\bar{x} = \frac{n}{c_n}\bar{s}$$

When c_n/a_n is close to 1.0, the traffic intensity and utilization will be nearly equal