

SAMPLE STATISTICS

#24

Cosa facciamo coi dati raccolti? → uso stat.

PRIME STATISTICHE da ottenere per campione ben definito

- media campionaria $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ (tendenza centrale)

- Varianza: $s^2 = \frac{1}{n} \sum_{i=1}^n x_i$ (discontinuo da media)

- etc...

- dispersione std $d(x) = \sqrt{\frac{1}{m} \sum_{i=1}^m (x_i - \bar{x})^2} = s$ (quanto m dispone)

se $x = \bar{x}$ ottengo d_{\min}

DISUGUAGLIANZA CHIOTICHE (le media e varianza)

dato campione di elementi, media \bar{x} e std dev. 's,
ho ret $S_k = \{x_i : \bar{x} - ks \leq x_i \leq \bar{x} + ks\}$,

di proporzione $P_k = \frac{|S_k|}{n} \Rightarrow P_k \geq 1 - \frac{1}{k^2}$ con $k \geq 1$

Potendo fare trasformazioni del tipo $x'_i = \alpha x_i + b$, trovando media \bar{x}' , varianza $\alpha^2 s^2$ e stdvar $|\alpha|s$ (simile a convertire sec in minuti $\bar{x}' = \frac{300 \text{ sec}}{60 \text{ sec}} = 5 \text{ min}$)

LINEARIZZAZIONE (dati gronda $\rightarrow \phi < x < 1$)

uso $\alpha = \frac{1}{s}$ e $b = -\frac{\bar{x}}{s} \rightarrow x'_i = \frac{x_i - \bar{x}}{s} \rightarrow \bar{x}' = \phi$ $\rightarrow s' = 1$

esistono anche NON-LINEAR, per vedere comportamento, no dati.

ESERCIZI

Consider a system with the following system characteristics:

- Single server with capacity 10^5 op.
- Potential mean service demand 4×10^4 op./job

System utilization 60%.

By knowing the job size, the service provider adopts a simple Size Based priority scheduling without preemption: jobs with size less (or equal) than the average will have the highest priority (class 1); jobs with size greater than the average have the lowest priority (class 2). Determine:

- the mean response time for both classes and the global mean response time.

The service provider wants to investigate if a dual core server would improve the service performance.

- Conjecture the behavior of the performance measures for both classes, by writing the mean waiting and response time definition for the dual core case.

→ Solo formula, non calcoli.

DATI

$$\bullet C = 10^5 \text{ op.} \quad \bullet Z = 4 \cdot 10^4 \text{ job} \quad \bullet P = 0.6 = \lambda E[S]$$

$$\bullet E[S] = \frac{Z}{C} = 0.4 \quad \text{[F_S]}$$

$$E[S] = 2.5 \quad \mu = \frac{\lambda}{E[S]} = 1.5$$

SizeB_NP

ho classe 1 se job $\leq E[S]$

$$E[S_i] = \int_0^\infty t \cdot f^n(t) dt \quad \text{dove } [x_{k-1}, x_k] \text{ sono le size delle classi, e } f^n(t) \text{ è la funzione di densità degli eventi normalizzata.}$$

$$\text{In questo caso } f^n(t) = \frac{\lambda e^{-\lambda t}}{F(1) - F(0)} \quad \text{per classe 1.}$$

$$P_1 = \left[1 - e^{-\mu t} \right]^{\frac{1}{\mu}} = 0,6963$$

$$P_1 + P_2 = 1$$

$$P_2 = \left[1 - e^{-\mu t} \right]^{\infty}_{\frac{1}{\mu}} = 0,363$$

Calcolo $E[S_1]$ $E[S_2]$:

$$E[S_1] = \int_{0,4}^{\infty} t \cdot 2,5 \cdot e^{-2,5t} dt = 0,167453$$

$$E[S_2] = \int_{\frac{1}{\mu} = 0,4}^{\infty} t \cdot 2,5 \cdot e^{-2,5t} dt = 0,29530$$

$$\text{controllo: } P_1 E[S_1] + P_2 E[S_2] = 0,6963 \cdot 0,167453 + 0,29530 \approx 0,4$$

Adesso calcolo $E[T_{a_1}]$ e $E[T_{a_2}]$

$$E[T_{a_k}] \stackrel{SB-NP}{=} \frac{\lambda}{2} E[S] = \frac{\lambda}{2} \left(\sum_{i=1}^{k-1} p_i \right) \left(1 - \sum_{i=1}^{k-1} p_i \right)$$

$$\text{avrm} E[T_{a_1}] = \frac{\lambda}{2} \frac{E[S]}{1 - P_1}, \quad E[T_{a_2}] = \frac{\lambda}{2} \frac{E[S^2]}{(1 - P)(1 - P_1)}$$

sto in contesto esponenziale + unico servente

$$\frac{\lambda}{2} E(S^2) \rightarrow LS$$

$$P_K = \lambda \int_0^{x_K} t \cdot f(t) dt$$

↳ densità esponenziale normalizzata

$$P_{K=1} = P_1 = \lambda \int_0^{\mu} t \cdot \mu e^{-\mu t} dt = 0,158545 = \lambda \cdot P_1 \cdot E[S_1]$$

PROBABILITÀ

$$P_2 = \lambda \int_{\mu}^{\infty} t \cdot \mu e^{-\mu t} dt = 0,441455 = \lambda \cdot P_2 \cdot E[S_2]$$

VERIFICA: $\sum_{i=1}^5 p_i = 0,6 = \lambda \int_0^{\infty} t \cdot f(t) dt$

Trovavo anche λ_1 e λ_2

$$\lambda_1 = \lambda \cdot P_1 = 0,9468 \quad \frac{\text{OP}}{\text{job}}$$

$$\lambda_2 = \lambda \cdot P_2 = 0,5532 \quad \frac{\text{OP}}{\text{job}}$$

$$\lambda_1 + \lambda_2 = \lambda \quad \checkmark$$

$$E[T_{Q_1}] = \frac{P E(S)}{(1 - P)^2}$$

0,28522

$$E[T_{Q_2}] = \frac{P E(S)}{(1 - P_1)(1 - P_2)} = 0,71305$$

OK

allora

$$E[T_{Q_1}]$$

$$E[S_1]$$

$$E[T_{S_1}] = 0,28522 + 0,167453 = 0,452673 \quad \checkmark$$

$$E[T_{S_2}] = 0,71305 + 0,79999 = 1,51304 \quad \checkmark$$

$$E[T_Q] = P_1 E[T_{Q_1}] + P_2 E[T_{Q_2}] = 0,443063 \quad \checkmark$$

$$E[T_S] = E[T_Q] + E[S] = 0,843003 \quad \checkmark$$

b) Passo ad un dual, mantenendo le due classi.
 Poiché sto in un contesto di scheduling estremo (le classi interamente sono FIFO) posso usare ERLANG.

Ci sono alcune osservazioni: NB: Ese invece che P_i del "job" dovrebbero essere $P_i = (1 - P_i)E[S_i]$, non è stato ben spiegato.

- NON Ho PRELAZIONE

$$E[T_a] = P_1 \cdot P_a E[S_1] + P_2 \cdot P_a E[S_2]$$

(1 - P_1 - P_2)

NON i job vengono interrotti

DB-NP

attesa di un job
di classe 1 in

ovvero ha $E[T_{a1}]$ che $E[T_{a2}]$ hanno P_a , poiché i job non vengono interrotti. Come visto Preemptive, $E[T_{a1}]$ avrebbe avuto P_{a1} (in funzione di P_1)

- Poiché ho 2 classi, $E[S_1] = E[S_2]$, ovvero ogni job è dimezzato, con tempi servizio moltiplicati per 2, $E[S_1] = E[S_2] = 2E[S] = 0.8$

- In $E[T_a]$, $E[S] = 0.4$, NON uso $E[S_i]$, perché mi interessa tempo di servizio generale (uno job in S, e magari un altro), mentre con $E[S_i]$ considero entrambi.

$$P_a = \frac{m!}{m^m} (1-P)^m$$

$$P(0) = \frac{(2 \cdot 0.6)^2}{2! (1-0.6)} \left[\sum_{i=0}^{m-1} \frac{(mp)^i}{i!} + \frac{(mp)^m}{m! (1-m)} \right]^{-1}$$

$$= \frac{(2 \cdot 0.6)^2}{2 (1-0.6)} \left[1 + 2p + \frac{(2p)^2}{2!} \right]^{-1}$$

, 15 ; $P_a \cdot E[S] = 0,18$
 $1 - 0,6312 \quad 0,45$

$$\rightarrow E[T_a] = \frac{P_a \cdot E[S]}{(1 - P_1)} \left[P_1 + \frac{P_2}{P_1} \right]$$

, 3176

0,1887 / 0,6312 = 0,2948 E[S]

SB → 1. Il responsabile di uno sportello comunale per il rilascio di certificati anagrafici vuole investigare le prestazioni del servizio. Analizzando lo storico dell'attività, si desume che una distribuzione uniforme $Uniform(2, 15)$ può ben caratterizzare il tempo di servizio (espresso in min). Gli utenti, identificati con la propria richiesta, arrivano in modo random con frequenza 0.112 req/min. Si assuma che sia possibile conoscere il tempo di servizio della pratica all'istante di arrivo. Si calcolino i seguenti indici:

- 1.a. tempi di attesa e risposta per una pratica qualsiasi;
 - 1.b. i tempi di attesa e risposta per classi e globali assumendo di usare un meccanismo prioritario opportunamente scelto (senza prelazione);
 - 1.c. lo slowdown condizionato, per richieste di 5 min o di 10 min nel caso 1.a;
 - 1.d. lo slowdown condizionato, per richieste di 5 min o di 10 min nel caso 1.b;
- Segnare della soluzione al punto 1.b. In quali assunzioni utilizzate per la soluzione.

$$Uniform(2, 15) \quad \lambda = 0.112 \text{ req/MIN}$$

\downarrow
 $a = 2$
 $b = 15$

Abbiamo a che fare con una UNIFORME!

$$f(x) = \frac{1}{b-a} ; E[S] = \frac{x-a}{b-a} ; \text{media} = \frac{a+b}{2} ; \sigma^2 = \frac{(b-a)^2}{12}$$

Svolgimento

$$E[T_q] = \frac{\lambda}{2} E[S^2]$$

NON è $E[S^2]$!

→ formula M/G/1

$$\sigma^2 = E[S^2] - (E[S])^2 = 14,0833 \quad E[S] = \frac{15+2}{2} = 8,5 \text{ min}$$

$$\text{allora } E[S^2] = \sigma^2 + (E[S])^2 = 14,0833 + (8,5)^2 = 86,3333$$

$$\text{Da Little } P = \frac{E[S]}{1 - E[S]} = 0,952$$

$$E[T_q] = \frac{0,112 \times 22222}{1 - 0,952} = 200,7 \text{ min}$$

$$E[T_s] = E[T_q] + E[S] = 109,2222 \text{ min}$$

1.b) Il meccanismo prioritario NP da considerare è SIZE BASED. Posso dividere le classi partendo da $E[S] = 8.5$ ovvero classe 1 (2, 8.5) ; classe 2 (8.5, 15) | è il tempo che potevo usare altrimenti schedule o più classi

Sappiamo che $E[S_i] = \int_{x_{k-1}}^{x_k} t \cdot f_i^n(t) \frac{P_i}{P_i}$

$$P_1 = F(8.5) - F(2) = \frac{8.5 - 2}{15 - 2} - \frac{2 - 2}{15 - 2} = 0.5 = P_2$$

NB: F ed f sono relative all' UNIFORME !

$$f(t) = \frac{1}{13} \text{ (è costante)}$$

$$E[S_1] = \int_2^{8.5} t \cdot \frac{1}{13} \cdot \frac{1}{0.5} = \frac{1}{6.5} t^2 \Big|_2^{8.5} = 5.25 \text{ min}$$

$$E[S_2] = \int_{8.5}^{15} \frac{1}{6.5} \cdot t = 11.75 \text{ min}$$

$$\lambda_1 = \lambda \cdot P_1 = 0.056 = \lambda_2 \quad (P_1 = P_2)$$

$$\rho_1 = \lambda_1 E[S_1] = 0.294$$

$$\rho_2 = \lambda_2 E[S_2] = 0.658 \quad \Rightarrow \quad \rho_1 + \rho_2 = 0.952 = \rho$$

$$E[T_{Q_1}] = \frac{\frac{0.112}{2} \cdot 36.333}{1 - 0.294} = 6.84 \text{ min}$$

$$E[T_{Q_2}] = \frac{\frac{0.112}{2} \cdot 86.333}{(1 - 0.294)(1 - 0.952)} = 142.6604 \text{ min}$$

$$E[T_{\alpha}] = 0.5(6.84) + 0.5(192.6604) = 74.75 \text{ min} < \text{NO SB}$$

$$E[T_{S_1}] = E[T_{\alpha_1}] + E[S_1] = 12,09797$$

$$E[T_{S_2}] = E[T_{\alpha_2}] + E[S_2] = 154.41604$$

$$E[T_S]^{glob} = 0.5 E[T_{S_1}] + 0.5 \cdot E[T_{S_2}] = 83.257005$$

1.c)

$$sd = \frac{E[T_\alpha] + x}{x}$$

indip da x

$x=5 \rightarrow sd(s) = 21.1444$

$x=10 \rightarrow sd(10) = 10,0722$

2.d) qui lavoro sulle due classi ($E[T_{\alpha_1}]$ ed $E[T_{\alpha_2}]$)

~~$$sd(x) = \frac{E[T_\alpha] + x}{x}$$~~

$x=5 \rightarrow sd(5) = 2.369594$

~~$$sd(x) = \frac{E[T_{\alpha_2}] + x}{x}$$~~

$x=10 \rightarrow sd(10) = 15.266$

N.B.: se $x=5$ va con $E[T_{\alpha_1}]$, poiché comprende niente da 2 a 8.5, $x=10$ va nella seconda classe (8.5, 15)

Non posso fare $x=10$ con $E[T_{\alpha_2}]$

Consider a web server with processing capacity $C = 10^5$ op/sec. The server receives requests with a mean rate 2 req/sec. The requests have different demand Z . Consider the following intervals:

- ◆ $Z < 20.000$ op
- ◆ $20.000 \text{ op} \leq Z < 40.000$ op
- ◆ $Z \geq 40.000$ op

$$Z = \text{avg demand}$$

By assuming that:

- i. the mean size is 40.000 op, characterized by an exponential distribution;
- j. the arrival rate is characterized by a Poisson process;

Define a management mechanism of the server to satisfy the following QoS requirements:

1. Mean response time ≤ 1.5 s for all requests
2. Mean waiting time ≤ 0.5 s, for $Z < 40.000$ op.ni.

Evaluate

- a. The mean throughput for the server with the chosen management mechanism;
- b. The mean conditional slowdown for jobs with size $x=0.1$ s, 0.3 s
- c. Compare the mean slowdown obtained in b. with the corresponding mean slowdown for FIFO and PS scheduling.

Please comment all the obtained results.

Svolgimento

$$E[S] = \frac{\bar{Z}}{C} = \frac{4 \cdot 10^4}{10^5} = 0.4 \text{ s} \rightarrow \rho = \lambda \cdot E[S] = 0.8$$

diviso in 3 domi (basandosi sugli intervalli): Size Based

$$C_1 (0, 0.2) \\ \frac{1}{2} \\ \frac{2 \cdot 10^4}{10^5}$$

$$C_2 (0.2, 0.4)$$

$$C_3 (0.4, \infty)$$

$$\text{calcolo } \mu = \frac{1}{E[S]} = 2.5$$

$$P_1 = [1 - e^{-\mu t}]^{0.2} = 0.393469$$

$$P_2 = [1 - e^{-\mu t}]^{0.4}_{0.2} = 0.23865$$

$$\sum_{i=1}^3 P_i = 1 \quad \checkmark$$

$$P_3 = [1 - e^{-\mu t}]^{\infty}_{0.4} = 0.367873$$

$$E[S_1] = \int_0^{0.2} t \cdot \frac{\mu e^{-\mu t}}{P_1} = 0.0917013 \text{ s}$$

$$E[S_2] = \int_{0.2}^{0.4} t \cdot \frac{\mu e^{-\mu t}}{P_2} = 0.291701 \text{ s}$$

$$E[S_3] = \int_{0.4}^{\infty} t \cdot \frac{\mu e^{-\mu t}}{P_3} = 0.8 \text{ s}$$

• calcolo P

$$P_1 = \lambda \int_0^{0.2} t \mu e^{-\mu t} dt = 0.0721632$$

$$P_2 = 2 \int_{0.2}^{0.4} t \cdot \mu e^{-\mu t} = 0.13923 \quad \sum_{i=1}^3 P_i = P = 0.8 \checkmark$$

$$P_3 = 2 \int_{0.4}^{\infty} t \mu e^{-\mu t} = 0.588607$$

• calcolo $E[T_{\alpha_i}]$

$$E[T_{\alpha_1}] = \frac{P E[S]}{1 - P_1} = 0.34888 \text{ s}$$

vincolo

rispettato

$$E[T_{\alpha_2}] = \frac{\lambda E[S]}{(1 - P_1 - P_2)(1 - P)} = 0.437338 \text{ s}$$

$$E[T_{\alpha_3}] = \frac{P E[S]}{(1 - P)(1 - P_1 - P_2)} = 2.0288945 \text{ s}$$

non era
richiesto

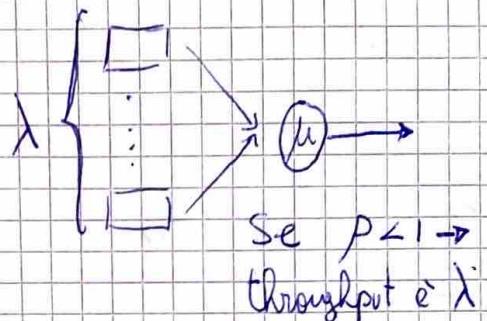
$$E[T_a] = \sum p_i E[T_{a,i}] = 0.988032295$$

$$E[S] = \sum p_i E[S_i] = 0.4$$

$$E[T_s] = E[T_a] + E[S] = 1.38803 < 1.5 \checkmark$$

a) throughput medio

$\mu > \lambda$ e $p < 1$, stabile.



b) sd (0.1) in $E[T_{a,1}] = \frac{0.344888 + 1}{0.1} = 4,4488$

sd (0.3) in $E[T_{a,2}] = \frac{0.437338}{0.3} + 1 = 2,4577$

c) sd FIFO = $\frac{\frac{1}{2} E[S^2]}{x(1-p)} + 1 = \frac{p E[S]}{x(1-p)} + 1 = \frac{0.8 \cdot 0.4}{0.1(0.2)} + 1 = 17$

da $x = 0.3$ ho $5.\overline{3} + 1 = 6.\overline{3}$

d) PS (fair, riguale 1 job) = $\frac{1}{1-p} = \frac{1}{1-0.8} = 5$

- 1.2. Consider a single-core server hosting a web service. Requests arrive to the server according to a Poisson, with an average inter-arrival time of 200 ms.
- Knowing that the maximum buffer size is $N = 4$ (including the jobs in service) and that each request requires on average 200 ms of processing time, compute the throughput of the system.
 - Consider a CPU upgrade to a faster single-core processor which can process a request in 150 ms. Compute the throughput of the upgraded system.
 - Consider a CPU upgrade to a slower quad-core processor, which can process a request in 300 ms using one of its processor cores. Compute the throughput of the upgraded system.

Svolgimento

$$\text{interarrivo} = 200 \text{ ms} \rightarrow \lambda = \frac{1}{200} = 0,005 \left[\frac{\text{J}}{\text{ms}} \right]$$

$$N = 4 \quad E[S] = 200 \text{ ms} \rightarrow \mu = \frac{1}{E[S]} = 0,005 \left[\frac{\text{J}}{\text{ms}} \right]$$

1) $\lambda' = \lambda(1 - P_{\text{loss}})$ è throughput (capacità finita, non è solo λ)

avendo SINGLE CORE) con $C = 4$ ($\lambda = \mu$)

$$\pi_4 = \left(\frac{\lambda}{\mu}\right)^4 \pi_0 \quad \text{con } \pi_0 = \frac{1}{\sum_{i=0}^4 \left(\frac{\lambda}{\mu}\right)^i} = \frac{1}{5}$$

$$\text{allora } \pi_4 = 1^4 \cdot \frac{1}{5} = \frac{1}{5} = P_{\text{loss}}$$

$$\text{e } \lambda' = 0,005 \left(1 - \frac{1}{5}\right) = 0,004 \text{ J/ms} = 4 \text{ J/s}$$

2) request in 150 ms $\rightarrow \mu = 0.00666666$

tranne che per μ , lo calcolo come prima:

$$\Pi_0 = \frac{1}{\sum_{i=0}^4 \left(\frac{0.005}{0.006}\right)^i} = 0.327785$$

$$\Pi_4 = P_{Loss} = \Pi_0 \cdot \left(\frac{0.005}{0.006}\right)^4 = 0.103713$$

$$\lambda = 0.005(1 - 0.103713) = 0.00448143 \text{ J/ms} = 4.48 \text{ J/s}$$

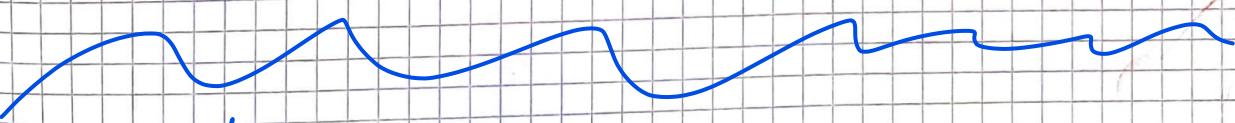
3) qui ho 4 core (= 4 serventi), ciascuno ha req. 300 ms

$$-\mu = 0.003333 \left[\frac{\text{J}}{\text{ms}} \right] \quad P = \frac{\lambda}{m\mu} = 0.375$$

non ho 'coda' (1 job A core). Ma solo erlong - B !

$$P_{Loss} = \Pi_4 = \frac{\left(\frac{\lambda}{\mu}\right)^4 / 4!}{\sum_{j=0}^4 \left(\frac{\lambda}{\mu}\right)^j / j!} = 0.048$$

$$\text{e quindi } \lambda' = \lambda(1 - P_{Loss}) = 4,76 \text{ J/s}$$



Sotto c'è

Monezza !!

$$P = \frac{\lambda}{\mu} = 0.4$$

$$\frac{1}{\mu} = E[S] = 0,00001$$

Esercizio 1

- single processor capacity $10^5 \text{ op/s} = \mu$
- domanda media $\exp 9 \cdot 10^5 \text{ op/job} = \lambda$
- syst. utilization $60\% = P$? ?
- Size based without preemption 2 classi
 - $c_1 \rightarrow \text{job} \leq E[S]$
 - $c_2 \rightarrow \text{job} > E[S]$
- a) mean response time per c_1 e c_2 , global response time.

Svolgimento

$$E(T_{S_1})^{SB-NP} = E(T_{a_1})^{SB-NP} + E[S_1]$$

$$\cdot E(S_1) = \int_{X_{K-1}}^{X_K} t \cdot f^n(t) dt = \int_{X_{K-1}}^{X_K} t \cdot \frac{\mu e^{-\mu t}}{F(X_K) - F(X_{K-1})}$$

$$= \int_0^{\frac{1}{\mu}} t \cdot \frac{\mu e^{-\mu t}}{F(\frac{1}{\mu}) - F(0)} = \int_0^{\frac{1}{\mu}} t \cdot \frac{\mu e^{-\mu t}}{P_1} dt$$

[chi è "F"? è f.distr. arrivo $\rightarrow \exp$ "1 - $e^{-\mu t}$ "]

$$P_1 = [1 - e^{-\mu t}] \Big|_0^{\frac{1}{\mu}} = 1 - e^{-\mu \frac{1}{\mu}} - [1 - 1] = 0,63212$$

$$P_2 = [1 - e^{-\mu t}] \Big|_0^\infty = 0,36788 \rightarrow \text{check: } P_1 + P_2 = 1$$

$$\rightarrow E[S_1] = \int_0^{10^5} t \cdot \frac{(10^5) \cdot e^{-(10^5 \cdot t)}}{0,63212} = 4,18024 \cdot 10^{-6}$$

$$E[S_2] = \int_{\frac{1}{10^5}}^{\infty} t \cdot \frac{10^5 e^{-10^5 t}}{0,36788} dt = \frac{10^5}{0,36788} \int_{\frac{1}{10^5}}^{\infty} t \cdot e^{-10^5 t} dt$$

for parts

~~$$= -\frac{1}{10^5} e^{-10^5 t} + C$$~~

~~constant~~ = 221

$$\frac{10^5}{0,36788} \left[-\frac{1}{10^5} e^{-10^5 t} + \int 1 \cdot e^{-10^5 t} dt \right] = \frac{10^5}{0,36788} \left[-\frac{1}{10^5} e^{-10^5 t} + \frac{1}{10^5} e^{-10^5 t} \right]$$

$$= 271828 \cdot \int_{\frac{1}{10^5}}^{\infty} e^{-10^5 t} \cdot t dt \quad \text{per parti} \quad \begin{cases} f = t \\ g' = e^{-10^5 t} \end{cases}$$

\downarrow

$$f = t dt \quad g = -\frac{1}{10^5} e^{-10^5 t}$$

$$= 271828 \left[t \cdot -\frac{1}{10^5} e^{-10^5 t} - \int_{\frac{1}{10^5}}^{\infty} 1 \cdot -\frac{1}{10^5} e^{-10^5 t} dt \right] = 0,00002$$

EXERCISE check

$$E[S] = p_1 E(S_1) + p_2 E(S_2) \stackrel{?}{=} 10^{-5} \quad \text{OK}$$

$$= [0,63212 \cdot 4 \cdot 18024 \cdot 10^{-6}] + [0,36788 \cdot 0,00002]$$

$$= 0,00001 \quad \text{OK}$$

1.c) slowdown condiz. nel caso 1 per richieste di 5 min e 10 min indip da size

$$Sd = \frac{(\overbrace{E[T_a]}^{\text{INDIP da size}} + 5)}{5} \times (\text{quanto chiede})$$

$$Sd = \frac{E[T_a] + x}{x} \quad | E[T_a] = 100.722, x = 10 \\ = 10.0722$$

1.d) slowdown con nel caso 2b.

$$Sd = \frac{\overbrace{E[T_{a_1}]}^{6.84} + 5}{5} = 2.369594 \quad] Q_1$$

$$\cancel{Sd = \frac{6.84 + 10}{10} = 1.684}$$

~~$$Sd = \frac{142.6604 + 10}{10} = 15.26604 \quad] Q_2$$~~

$$Sd = \frac{142.6604 + 10}{10} = 15.26604 \quad] Q_2$$

es. 3)

$$\mu = 10^5 \text{ op/s} = C \quad \lambda = 2 \text{ req/s}, \text{ tali richieste}$$

sono vere

$$\begin{cases} z < 20.000 \text{ op} \\ 20.000 \leq z \leq 40.000 \text{ op} \\ z \geq 40.000 \text{ op} \end{cases}$$

ottimizzare:

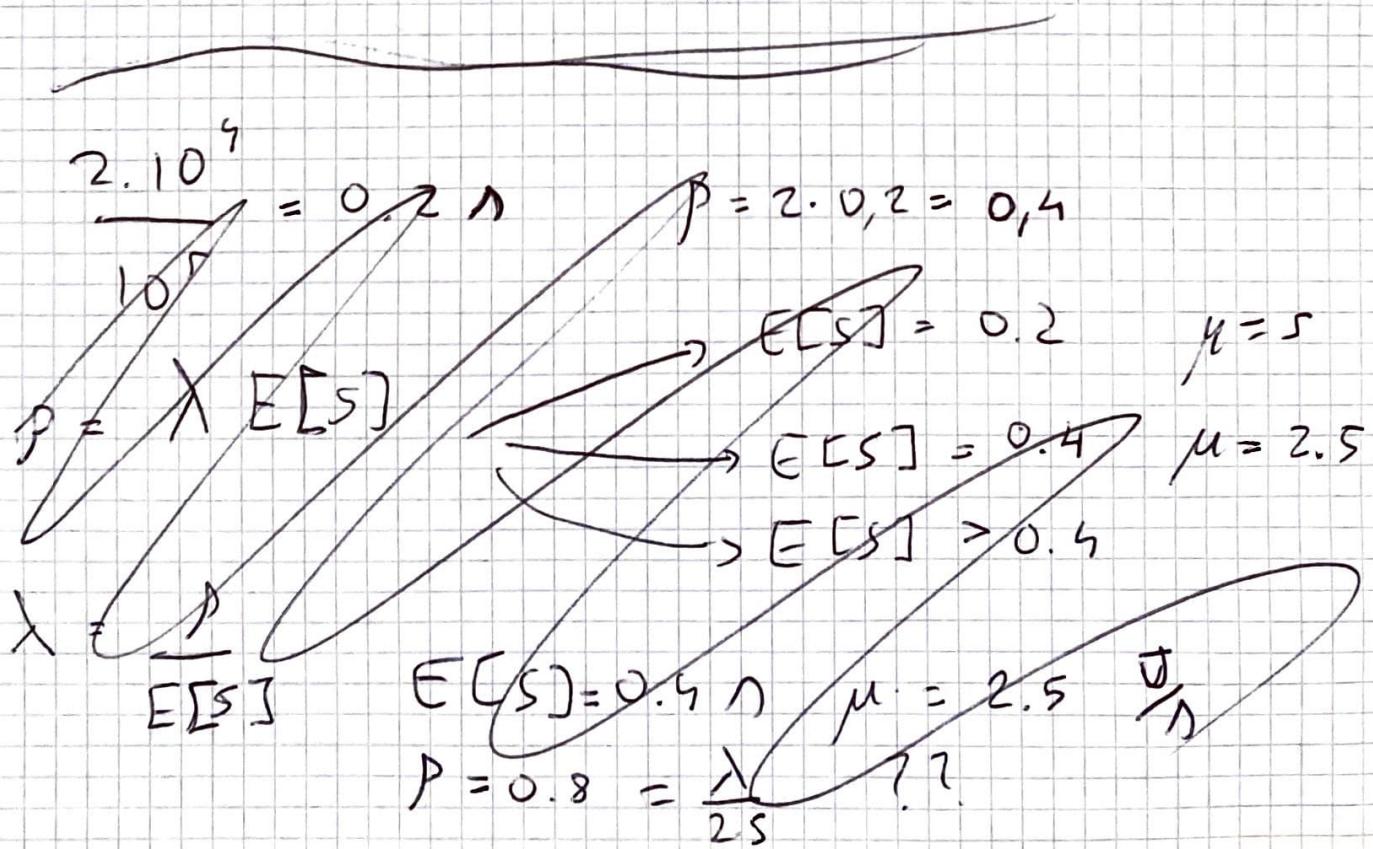
- arrival rate è Poisson
- Mean size è 40000 op (exp) = z (avg demand)

dove richiedere:

$$E(T_s) \leq 1.5 \text{ s} \wedge \text{req}$$
$$E(T_q) \leq 0.5 \text{ s}, \text{ se } z \leq 40000$$

• Voluto throughput

- slowdown $\times = 0.15, 0.35$ (mean)
- confronto con mean slowdown FIFO e PS.



Ora ricento di $E[T_{Q_1}]$ ed $E[T_{Q_2}]$

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2} E[S^2]}{(1 - \sum_{i=1}^K p_i) \left(1 - \sum_{i=1}^{K-1} p_i\right)}$$

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2} E[S^2]}{1 - p_1}$$

$$E[T_{Q_2}] = \frac{\frac{\lambda}{2} E[S^2]}{(1-p)(1-\lambda)}$$

$\frac{\lambda}{2} E(S^2) = \rho E(S)$ se EXP, ma con priority classes

ho in realtà $p_{Q_1} E[S]$ e $p_{Q_2} E[S]$ perché c'è priorità, devo calcolare

$$\bullet P_1 \bullet P_{Q_1} \bullet P_Q$$

$$D_{K=1} = \lambda \int_{x_{K-1}}^{x_K} t \cdot f(t) dt \quad \text{e' RHO}$$

~~NON NORMALIZZ.~~

$$D_K = \lambda \int_0^{\infty} t \cdot \mu e^{-\mu t} dt = (4 \cdot 10^4) \int_0^{\infty} t \cdot 10^5 \cdot e^{-10^5 t} dt$$

$$0.1657 = 1,0570 \cdot 10^{-6} = \lambda \cdot 0.63212 \cdot E[S] \checkmark$$

$$P_2 = 2,9430 \cdot 10^{-6}$$

$$\sum_{i=1}^K p_i = \lambda \int_0^{\infty} t f(t) dt = 0.4 \quad \text{Consistente}$$

$$\lambda_1 = \lambda \cdot 0,63212 = 25284,8 \quad] = 4 \cdot 10^4 \text{ OK}$$

$$\lambda_2 = 14715,2$$

$$E[T_{Q_1}] = \frac{\rho E(S)}{(1 - 1.057 \cdot 10^{-6})} = 47076 = 4,727 \cdot 10^{-6}$$

$$E[T_{Q_2}] = \frac{\rho E(S)}{(1 - 1.057 \cdot 10^{-6})(1 - 0.4)} = 7,4576 \cdot 10^{-6}$$

$$E[T_S]^{SB-NP} = 4,727 \cdot 10^{-6} + 4,18024 \cdot 10^{-6} = 8,6529 \cdot 10^{-6}$$

$$E[T_S] = 7,4576 \cdot 10^{-6} + 0,00002 = 77,4576 \cdot 10^{-6}$$

$$E(T_Q) = p_1 E(T_{Q_1}) + p_2 E(T_{Q_2}) = 5,5707 \cdot 10^{-6}$$

$$E(T_S) = E(T_Q) + E(S) = 0.400005570785012$$

4
0,4

Can dual core? has 2 core \rightarrow no erlang

$$E[T_a] = p_1 \underbrace{\frac{P_{a1} E(S)}{1-p_1}}_{\text{done 1}} + p_2 \underbrace{\frac{P_a E[S]}{(1-p)(1-p_1)}}_{\text{done 2}} \quad M/M/m \quad m=2$$

$$P_a = \frac{(m \cdot p)^m}{m! (1-p)} \quad p(0) = 0.22857$$

$$P(0) = \sum_{k=0}^1 = \left[1 + 2p + \frac{(2p)^2}{2(1-p)} \right]^{-1} \quad \begin{matrix} p=0.4 \\ m=2 \\ p=0.4 \end{matrix} = 0.42857$$

↓
P_{a1} can viene OK 0.22857

$$P_{a1} = \frac{(m p_1)^m}{m! (1-p_1)} \quad p(0) = 0.03430$$

↓
 $p_1 = 0.14$

$$p(0) = 0.80880$$

↓
 $p = 0.10570$

$$P_{a1} = 0.02020$$

$$E[T_a] = 0.05134$$

$$E[T_s] = E[T_a] + 0.4 = 0.45134$$

FINE

Q1.2

• tempo servizio $\sim \text{Unif}(\frac{a}{2}, \frac{b}{2})$ (in m)

• arrivi random $\lambda = 0.112 \text{ req/min.}$

• Posso sapere tempo servizio all'istante di arrivo.

Calcolare

• tempi attesa e risposta per pratica qualcosa

NB: $f(x) = \frac{1}{b-a}$, $F(x) = \frac{x-a}{b-a}$, medie $= \frac{a+b}{2}$

$$\text{Varianza} = \frac{(b-a)^2}{12}$$

M/U/1

$$E[T_a] = \frac{\frac{\lambda}{2} E[S^2]}{1-\rho} \quad \text{per M/G/1}$$

$$\lambda = 0.112 ; E[S] = \frac{a+b}{2} = 8.5 \text{ min}$$

$$\sigma^2 = E[X^2] - E[X]^2 = \frac{(b-a)^2}{12} = 14.0833$$

$$\rightarrow E[X^2] = 14.0833 + (8.5)^2 \cdot 86.3333 = E[S^2]$$

Ds Little: ~~$\lambda = 0.112 \cdot E[S] = 0.952$~~

$$\rightarrow E[T_a] = \frac{0.112 \cdot 86.3333}{1 - 0.952} = 100.722 \text{ min}$$

$$\rightarrow E[T_s] = E[T_a] + E[S] = 109.2222$$

1.6) tempi attesi / risposte clam - globali con meccanismi preventivi a netto NO prelievi.

size based, 2 clam $\rightarrow (2, 8.5)$
 $\rightarrow (8.5, 15)$

$$E(S_1) = \int_2^{8.5} t \cdot \frac{\mu e^{-\mu t}}{F(8.5) - F(2)} dt$$



$$\frac{F(8.5) - F(2)}{F(8.5) - F(2)} = 1 - e^{-\mu t} \Big|_2 = P_1$$

$$F(8.5) - F(2) = \frac{8.5 - 2}{15 - 2} - \frac{2 - 2}{18 - 2} = \frac{6.5}{13} = 0.5$$

$$f(t) = \frac{1}{6-\alpha} = \frac{1}{13}$$

$$E(S_1) = \int_2^{8.5} t \cdot \frac{f(t)}{F(8.5) - F(2)} dt = \frac{1}{13} \cdot 0.5 \int_2^{8.5} t dt = \frac{0.5}{13} \frac{t^2}{2} \Big|_2^{8.5} = 5.25$$

$$E(S_2) = \int_{8.5}^{15} \frac{1}{13} \cdot t \cdot \frac{1}{0.5} dt = \frac{1}{6.5} \cdot \frac{t^2}{2} \Big|_{8.5}^{15} = 11.75$$

$$\lambda_1 = \lambda \cdot p_1 = 0.056 = \lambda \cdot p_2 = \lambda_2$$

$$P_1 = \lambda_1 E[S_1] = 0.294 \quad \boxed{P = 0.952}$$

$$P_2 = \lambda_2 E[S_2] = 0.658$$

$$E[T_{Q_1}] = \frac{E[S]}{1 - 0.294} \rightarrow E(S) = 0.5(5.25 + 11.75)$$

No

No late exp

$$E[T_{Q_1}] = \frac{\frac{\lambda}{2} E(S^2)}{1 - 0.294} =$$

$$\frac{\frac{0.112}{2} (86.333)}{1 - 0.294} = 6.84 \text{ min}$$

$$E[T_{Q_2}] = \frac{\frac{\lambda}{2} E(S^2)}{(1 - 0.294)(1 - \rho)} = \frac{E[T_{Q_1}]}{1 - 0.952} = 142,6604 \text{ min}$$

$$E[T_Q] = 0.5(6.84 + 142,6604) = 74,75 \text{ min} < \text{No SB}$$

$\hookrightarrow P_1 = P_2 \text{ (equiprof)}$

~~E(TS) = E(TQ) + E[S]~~

$$E[T_{S_1}] = E[T_{Q_1}] + E[S_1] = 12,09797$$

$$E[T_{S_2}] = E[T_{Q_2}] + E[S_2] = 154.41604$$

$$\hookrightarrow E[T_S]^{\text{glob}} = \cancel{E[T_S]} + 0.5[12.09797 + 154.41604] \\ = 83.257005 \text{ ridotta}$$

SVOLGIMENTO

$$E(S) = \frac{c}{\lambda} = \frac{4 \cdot 10^4}{10^5} = 0.4 \text{ A} \rightarrow \rho = \lambda \cdot E(S) = 2 \cdot 0.4 = 0.8$$

L
→ costante

nei punti di 3 classi (ha 3 divisioni)

$$C_1(0, 0.2) \quad C_2(0.2, 0.4) \quad C_3(0.4, \infty)$$

$$\mu = \frac{1}{E(S)} = 2.5$$

~~E(S)~~

$$P_1 = [1 - e^{-\mu t}]_0^{0.2} = 0.393469$$

$$P_2 = [1 - e^{-\mu t}]_{0.2}^{0.4} = 0.238651 \quad \sum = 1$$

$$\mu = \frac{1}{E(S)}$$

$$P_3 = [1 - e^{-\mu t}]_{0.4}^{\infty} = 1 - 0 - [1 - e^{-2.5 \cdot 0.4}] = 0.367879$$

$$E(S_1) = \int_0^{0.2} t \cdot \frac{\mu e^{-\mu t}}{P_1} = 0.0917013 \text{ A}$$

$$E(S_2) = \int_{0.2}^{0.4} \frac{t \cdot \mu e^{-\mu t}}{P_2} = 0.2917013 \text{ A}$$

$$E(S_3) = \int_{0.4}^{\infty} \frac{t \cdot \mu e^{-\mu t}}{P_3} = 0.8 \text{ A}$$

$$P_1 = \lambda \int_0^{0.2} t \cdot \mu e^{-\mu t} dt = 0.0721632$$

$$P_2 = \lambda \int_{0.2}^{0.4} t \cdot \mu e^{-\mu t} dt = 0.13923$$

$$P_3 = \lambda \int_{0.4}^{\infty} t \cdot \mu e^{-\mu t} dt = 0.588607$$

OK

$$E(T_{a_1}) = \frac{P E[S]}{(1 - P_1)} = 0.344838 \wedge$$

$$E(T_{a_2}) = \frac{P E[S]}{(1 - P_1 - P_2)(1 - P_1)} = 0.437338 \wedge$$

$$E(T_{a_3}) = \frac{P E[S]}{(1 - P)(1 - P_1 - P_2)} = 2.0288895 \wedge$$

rispettato

$$E[T_a] = \sum p_i E(T_{a_i}) = 0.988032295$$

$$E[S] = \sum p_i E[S_i] = 0.4$$

$$E[T_S] = E[T_a] + E[S] = 1.38803 \quad \checkmark \quad \text{rispettato}$$

a) throughput

$\mu > \lambda$ stable, $P < 1$, throughput = λ 2 req/s ?

b) $S_d(0.1) = \frac{0.34488}{0.1} + 1 = 4,4488$

$$S_d(0.3) = \frac{0.437338}{0.3} + 1 = 2.4577$$

FIFO

$$0.1 \left(\frac{\frac{\lambda}{2} [E[S^2]]}{0.1(1-P)} + 1 \right) = \frac{P E[S]}{0.1(1-0.8)} + 1 = \frac{0.8 \cdot 0.9}{0.1(0.2)} + 1 = 17$$

$$0.3 \left(\frac{5}{3} + 1 \right) = 6.3$$

PS

$$\frac{1}{1-P} = 5 \text{ A job}$$

OK

4)

• arriv Poisson, λ = intervallo = 200 ms

$$\rightarrow \frac{1 \text{ job}}{200 \text{ ms}} = 0,005 \text{ j/ms}$$

• SIZE Buff = 4 (incluso Job servizio)

• $E[S] = 200 \text{ ms} \rightarrow \mu = \frac{1}{E[S]} = 0,005 \text{ j/ms}$

con coda (3 job in coda) \rightarrow erlang c

$$\lambda' = \lambda(1 - P_{\text{loss}})$$

$$P_{\text{loss}} = \frac{(m p)^m}{m! (1-p)^{m+1}}$$

$p = 1/N$

$(C=3)(\gamma-1)$ $\lambda = \mu = 0,005$ $C = 4$

$$\pi_i = \frac{(\Delta)^i}{\mu^i} \pi_0 = \pi_0 = \frac{1}{\sum_{i=0}^3 \frac{(0,005)^i}{(0,005)^i}} = \frac{1}{4}$$

$$\lambda' = 3 \left(1 - \frac{1}{3}\right) = 3 \left(\frac{2}{3}\right) = 2 \text{ j/s}$$

$$\lambda' = 0,005 \left(1 - \frac{1}{4}\right) = 0,004 \text{ j/ms} = 4 \text{ j/s}$$

b) Request in 150 ms ~~edge~~ $\Rightarrow \mu = 0.006$

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \pi_0 =$$

$$\rightarrow \pi_3 = \left(\frac{0.005}{0.006}\right)^3 \pi_0 = 0.5787 \cdot \pi_0 = 0.186283$$

$$\pi_0 = \frac{1}{1 + 0.933 + 0.6944 + 0.5787 + 0.4822} = 0.3219$$

$$\rightarrow \lambda' = 0.005 \cdot (1 - 0.186283) =$$

$$\pi_0 = \frac{1}{1 + 0.933 + 0.6944 + 0.5787 + 0.4822} = 0.27864$$

$$\pi_4 = p_{loss} = \left(\frac{0.005}{0.006}\right)^4 \cdot \pi_0 = 0.134375$$

$$\lambda' = 0.005 \cdot (1 - 0.134375) = 0.00432 \text{ s/ms} = 4,328 \text{ 1/s}$$

c) 4 core, request 300 ms words un core.

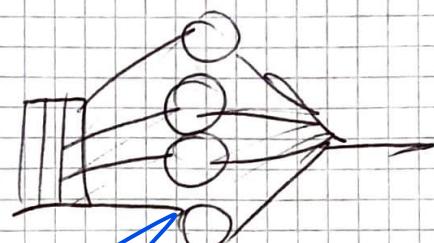
throughput?

$$\mu' = 0.0033 \text{ ms}$$

$$\lambda = 0.005 \text{ J/ms}$$

$$P_d = 4 \left(\frac{\lambda}{\mu'} \right)^4 P(0)$$

$$\frac{4!}{(1 - \frac{\lambda}{\mu'})^4}$$



$$P = \frac{\lambda}{\mu'} = 0.3750 \text{ OK}$$

$$P(0) = \sum_{i=0}^3 \left[\frac{(4P)^i}{i!} + \frac{(4P)^{i+1}}{m!(1-P)} \right]^{-1}$$

$$= \left[\frac{1 + (4P)^4}{4!(1-P)} \right] + \left[\frac{4P + (4P)^2}{4!(1-P)} \right] + \left[\frac{(4P)^2}{2!} + \frac{(4P)^3}{4!(1-P)} \right]$$

$$+ \left[\frac{(4P)^3}{3!} + \frac{4P}{4!(1-P)} \right]^{-1}$$

$$= \left[1 + 4P + \frac{16P^2}{2} + \frac{64P^3}{6} + 4 \left(\frac{4P}{4!(1-P)} \right)^4 \right]^{-1} = 0.180587$$

$$P_{dx} = 0.00095 \rightarrow \lambda = \lambda (1 - 0.0095) = 0.0049525$$

$$\rightarrow 1.95 \text{ J/ms}$$

erlang B

$$\pi_4 = \left(0.3750\right)^4 \frac{1}{4!} \pi_0 = 0$$

$$\pi_0 = \frac{1}{\sum_{i=0}^{\infty} \left(\frac{0.3750}{m} \right)^i} = \frac{1}{0.629669} = 0.07375$$

$$\pi_4 = \left(0.3750\right)^4 / 4!$$

$$\left(\frac{\lambda}{m}\right)^m / m!$$

$$\sum_{i=0}^m \left(\frac{\lambda}{m}\right)^i / i!$$

OK

0.00512

0.00375

Ü.1 NOTG

$$C = 20^5 \text{ op/h}$$

$$Z = 4 \cdot 10^4$$

$$\rho = 0.6 = \lambda E[S]$$

$$E[S] = \frac{Z}{C} = 0.4$$

$$\mu = \frac{1}{E[S]} = 2.5$$

$$\rho = 0.6 = \lambda \cdot 0.4 \rightarrow \lambda = \frac{0.6}{0.4} = 1.5$$

clane 2

$$\leq 0.4 = E[S]$$

$$P_1 = 0.6312$$

$$E[S_1] = 0.1674531$$

$$\frac{1}{\mu} = E[S]$$

$$P_1 E[S_1] = 0,1056963$$

clane 2

$$> 0.4 = E[S]$$

$$P_2 = 0.36788$$

$$E[S_2] = 0.799989$$

$$P_2 E[S_2] = 0,29430$$