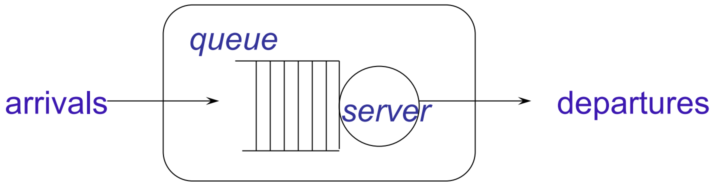
Single server queue



Consider n=10 job and given arrival and service times:

Arrival times:

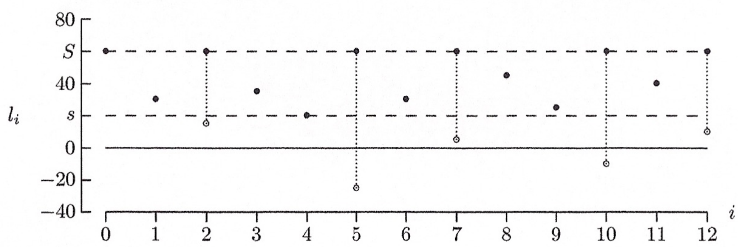
15 47 71 111 123 152 166 226 310 320

Service times:

43 36 34 30 38 40 31 29 36 30

A simple inventory system

i	:	1	2	3	4	5	6	7	8	9	10	11	12
d_i	:	30	15	25	15	45	30	25	15	20	35	20	30



- ssq1 and sis1 require input data from an outside source
- The usefulness of these programs is limited by amount of available data:
  - What if more data needed?
  - What if the model changed?
  - What if the input data set is small or unavailable?

↓

Random number generator

- It produces real values between 0.0 and 1.0
- The output can be converted to random variate via mathematical transformations

Performance Modeling of Computer Systems and Networks

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Random Number Generators

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Historically there are three types of generators
- table look-up generators (1950)
- hardware generators
- algorithmic (software) generators

Algorithmic generators are widely accepted because they meet all of the following criteria:
- *randomness* - output passes all reasonable statistical tests of randomness
- *controllability* - able to reproduce output, if desired
- *portability* - able to produce the same output on a wide variety of computer systems
- *efficiency* - fast, minimal computer resource requirements
- *documentation* - theoretically analyzed and extensively tested

Algorithmic Generators

- An *ideal* random number generator produces output such that *each* value in the interval 0.0 < u < 1.0 is *equally likely* to occur
- A *good* random number generator produces output that is (almost) statistically indistinguishable from an ideal generator

Conceptual Model

- Choose a *large* positive integer $m>0$. This defines the set
  $\chi_m = \{1,2,\ldots m-1\}$
- Fill a (conceptual) urn with the elements of χ_m
- Each time a random number u is needed, draw an integer x "at random" from the urn and let $u = x/m$

- Each draw *simulates* a sample of an independent identically distributed sequence of *Uniform*(0, 1)

- The possible values are $1/m$, $2/m$, … $(m-1)/m$

- It is important that m be large so that the possible values are densely distributed between 0.0 and 1.0

Conceptual Model

- 0.0 and 1.0 are impossible
  This is important for some random variates

- the same probability for each draw→ replacement of the drawn element

- for practical reasons, we will draw without replacement
  If m is large and the number of draws is small relative to m, then the distinction is largely irrelevant

Lehmer Generator

- is defined in terms of two fixed parameters:
  - *modulus m*, a fixed large prime integer
  - *multiplier a*, a fixed integer in χ_m
- the possible values are 1/*m*, 2/*m*, … (*m*-1)/*m*

The integer sequence x_0, x_1, … is defined by the iterative equation

$$x_{i+1} = g(x_i)$$

with

$$g(x) = ax \bmod m$$

$x_0 \in \chi_m$ is called *initial seed*

- Because of the mod operator, $0 \leq g(x) < m$
- 0 must not occur
  - since *m* is prime, $g(x) \neq 0$ if $x \in \chi_m$
  - if $x_0 \in \chi_m$, then $x_i \in \chi_m$ for all $i \geq 0$
- IF the multiplier and prime modulus are chosen properly, a Lehmer generator is statistically indistinguishable from drawing from χ_m with replacement
- NOTE, there is nothing random about a Lehmer generator

→ pseudo-random generator

Parameter Considerations

- the choice of *m* is dictated, in part, by system considerations
  - on a system with 32-bit 2's complement integer arithmetic, 2^{31}-1 is a natural choice (it is prime!)
  - with 16-bit or 64-bit integer representation, the choice is not obvious (the maxes are not prime)
  - in general, we want to choose *m* to be the largest representable prime integer
- Given *m*, the choice of *a* must be made with great care

- For a chosen (a, *m*) pair, does the function g(·) generate a **full-period** sequence?
- If a full period sequence is generated, how random does the sequence appear to be?
- Can *ax* mod *m* be evaluated efficiently and correctly?
  - Integer overflow can occur when computing *ax*

Full Period Multipliers

- If we pick any initial seed $x_0 \in \chi_m$ and generate the sequence x_0, x_1, x_2, … then x_0 will occur again
- Further x_0 will reappear at index p that is either $m - 1$ or a divisor of $m - 1$

We are interested in choosing full-period (FP) multipliers where p = m-1

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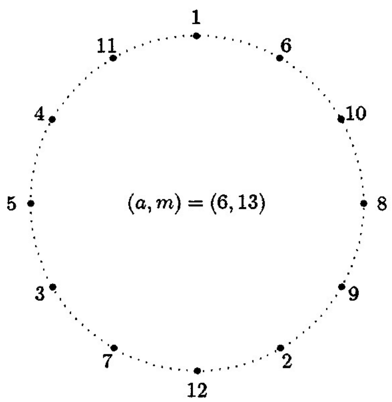
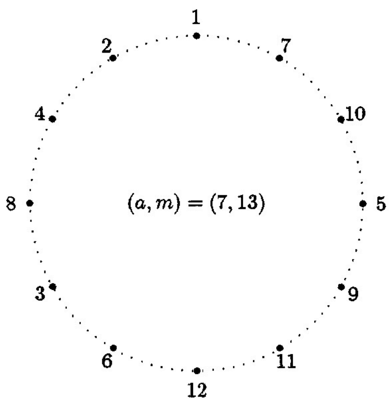
Full-period multipliers generate a virtual circular list with m-1 distinct elements.





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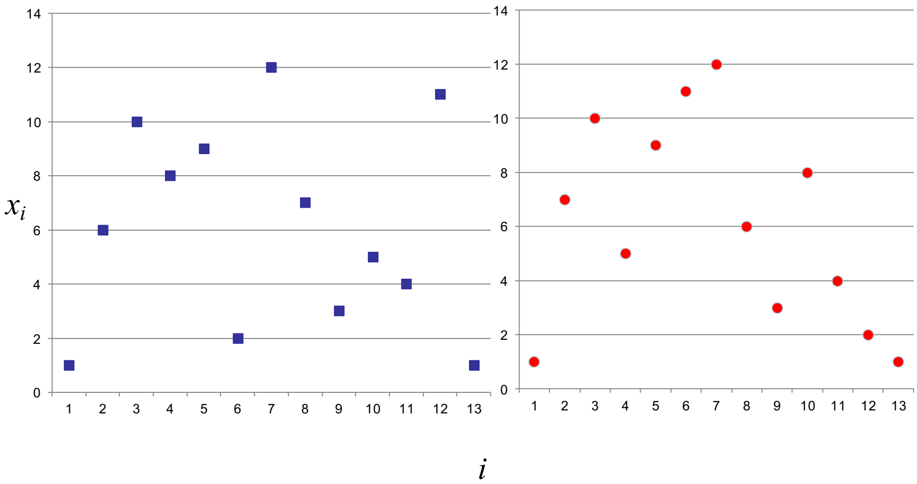
m=13

a=6, x_0=1 a=7, x_0=1



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m=13

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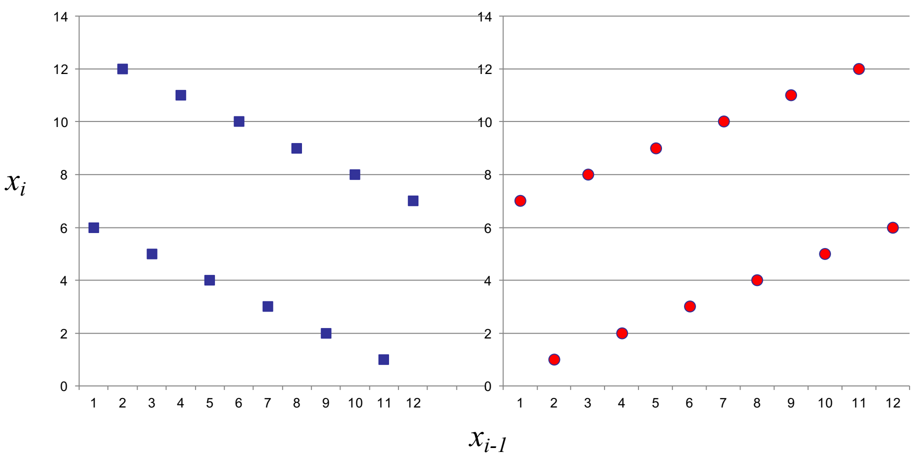

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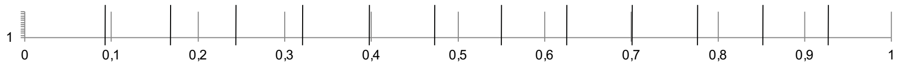

0,076923077	0,461538462	0,769230769	0,615384615	0,692307692	0,153846154
0,923076923	0,538461538	0,230769231	0,384615385	0,307692308	0,846153846

0,076923077	0,538461538	0,769230769	0,384615385	0,692307692	0,846153846
0,923076923	0,461538462	0,230769231	0,615384615	0,307692308	0,153846154
12	**6**	**3**	**8**	**4**	**2**