### Machine Learning

## Introduction to Reinforcement Learning

Gabriele Russo Russo Francesco Lo Presti

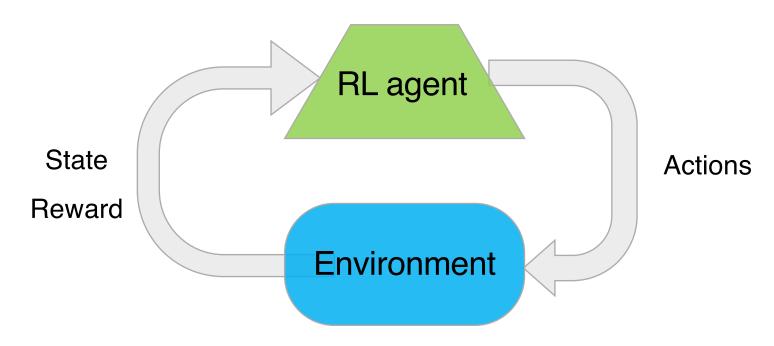
Laurea Magistrale in Ingegneria Informatica – A.Y. 2023/24



Dipartimento di Ingegneria Civile e Ingegneria Informatica

- Supervised learning
- Unsupervised learning
- **▶** Reinforcement learning

- Supervised learning
- Unsupervised learning
- Reinforcement learning
  - Branch of ML dealing with sequential decision-making



- Agent interacts with environment through actions
- Feedback in the form of reward (or paid cost)
- Goal: maximizing cumulated reward over the long run
- Trial-and-error experience (no complete knowledge of environment a priori)

## **Example: Tic-Tac-Toe**

- State: representation of the board (3x3 matrix)
- Actions: available cells to mark
- Reward: 1 for a winning move, 0 otherwise

Х	0	0
0	X	X
		Х

# Example: AlphaZero by DeepMind

- Software able to play Go, Chess and Shogi <sup>1</sup>
  - Board games with huge number of legal positions (i.e., state space)
- Trained via self-play and advanced deep RL techniques
- Superhuman level of play with 24-hour training
- First presented in 2017; in 2019 MuZero, generalization to play Atari games and other board games without prior rule knowledge

<sup>&</sup>lt;sup>1</sup>https://arxiv.org/abs/1712.01815

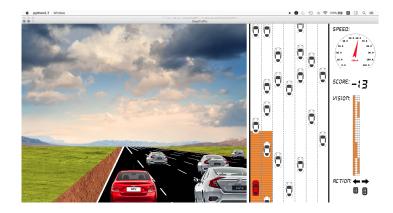
# Example: AlphaDev by DeepMind

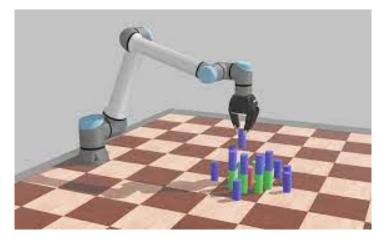
- ► Announced in 2023<sup>2</sup>
- RL used to develop new C++ sorting algorithm, now accepted in the standard library
- 70% faster on short sequences (2-3 items), 1.7% faster on long sequences
- State: instructions generated so far and state of the CPU
- Actions: assembly instructions to add
- Reward: based on sorting correctness and efficiency

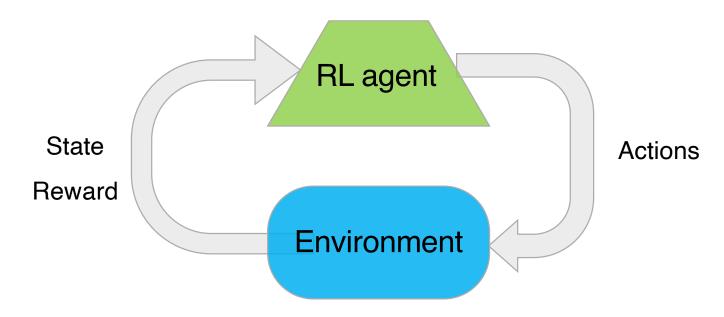
<sup>&</sup>lt;sup>2</sup>https://www.deepmind.com/blog/ alphadev-discovers-faster-sorting-algorithms

# **Other Examples**

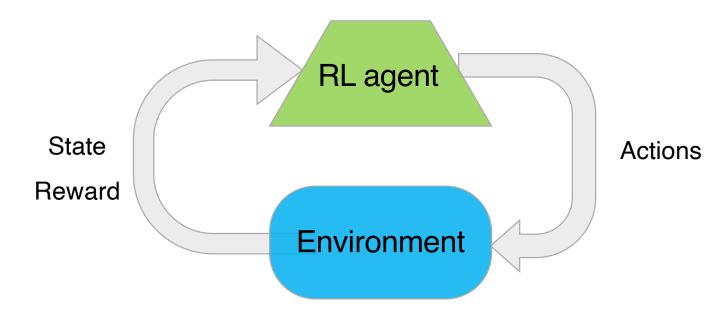
- Autonomous vehicles
- Robot control
- Trading
- Autonomous network and computer systems
- Videogames
- ...







- Agent, environment, actions, state, rewards, ...
- Modeled depending on the specific task
  - e.g., autonomous car uses different state information compared to chess player



- Agent, environment, actions, state, rewards, ...
- Modeled depending on the specific task
  - e.g., autonomous car uses different state information compared to chess player
- ► Formally defined as a Markov Decision Process (MDP)
- A framework to model decision making in situations where outcomes are partly random

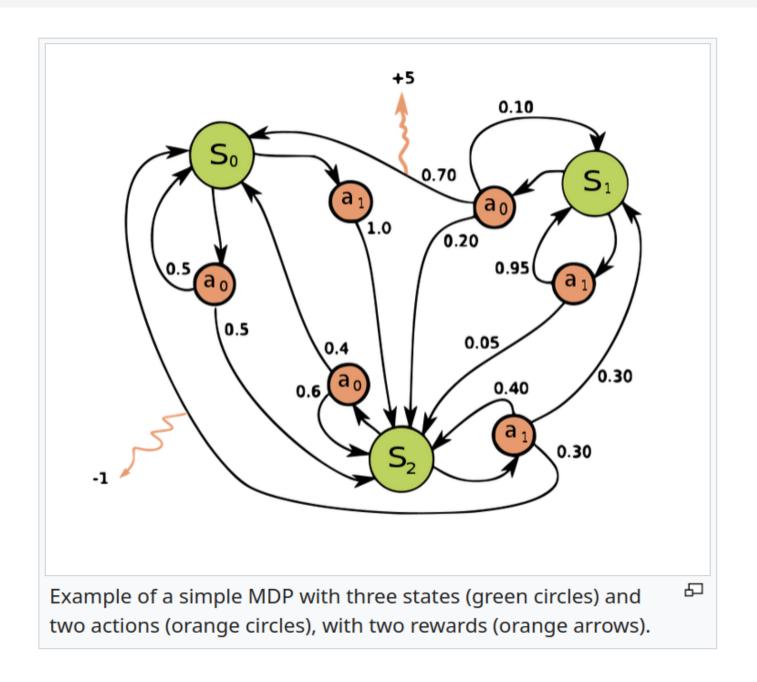
- Extension of discrete-time Markov chains
- ightharpoonup At each time step t, the process is in some state  $s_t$

- Extension of discrete-time Markov chains
- ightharpoonup At each time step t, the process is in some state  $s_t$
- The decision maker (the agent) chooses an action  $a_t$  among those available in state  $s_t$ 
  - e.g., robot observes current position and decides direction to move; some directions might be blocked by obstacles

- Extension of discrete-time Markov chains
- $\triangleright$  At each time step t, the process is in some state  $s_t$
- The decision maker (the agent) chooses an action  $a_t$  among those available in state  $s_t$ 
  - e.g., robot observes current position and decides direction to move; some directions might be blocked by obstacles
- Following  $a_t$ , the process moves to (random) state  $s_{t+1}$ 
  - e.g., autonomous drone chooses an action to reduce altitude;
     actual outcome may depend on (unpredictable) wind speed

- Extension of discrete-time Markov chains
- ightharpoonup At each time step t, the process is in some state  $s_t$
- The decision maker (the agent) chooses an action  $a_t$  among those available in state  $s_t$ 
  - e.g., robot observes current position and decides direction to move; some directions might be blocked by obstacles
- Following  $a_t$ , the process moves to (random) state  $s_{t+1}$ 
  - e.g., autonomous drone chooses an action to reduce altitude;
     actual outcome may depend on (unpredictable) wind speed
- Agent receives a reward (or, equivalently, pays a cost)
  - e.g., robot may get a reward for reaching its final destination
  - e.g., chess player rewarded at the end of a match

# **Example**



#### What defines an MDP?

- $\triangleright$  S: a (finite) set of states
- $\triangleright$   $\mathcal{A}$ : a (finite) set of actions
- p: state transition probabilities

$$p(s'|s,a) = P[S_{t+1} = s'|S_t = s, A_t = a]$$

r: reward function (or, c: cost function)

1. 
$$r(s, a) = E[R_t | S_t = s, A_t = a]$$

2. 
$$r(s, a, s') = E[R_t | S_t = s, A_t = a, S_{t+1} = s'] \longrightarrow r(s, a) = \sum_{s'} p(s' | s, a) r(s, a, s')$$

#### What defines an MDP?

- $\triangleright$  S: a (finite) set of states
- $\triangleright$   $\mathcal{A}$ : a (finite) set of actions
- p: state transition probabilities

$$p(s'|s,a) = P[S_{t+1} = s'|S_t = s, A_t = a]$$

c: cost function (Alternative)

1. 
$$c(s, a) = E[C_t | S_t = s, A_t = a]$$

2. 
$$c(s, a, s') = E[C_t | S_t = s, A_t = a, S_{t+1} = s'] \longrightarrow c(s, a) = \sum_{s'} p(s' | s, a) c(s, a, s')$$

# **Markov Property**

"The future is independent of the past given the present"

#### **Definition**

A state  $S_t$  is Markov if and only if

$$P[S_{t+1}|S_1,...,S_t] = P[S_{t+1}|S_t]$$

- The state captures all relevant information from the history
- i.e., the state is a sufficient statistic of the future

## **Objective: Episodic Tasks**

- Informally, we said that the agent aims to maximize the collected reward over time
- Let's consider an episodic task, where the agent-environment interaction naturally terminates at some final time step T
  - e.g., the end of a chess match
  - e.g., the time a robot reaches its destination or runs out of battery
- ightharpoonup At time t, we aim to maximize the expected return  $G_t$

$$G_t = R_t + R_{t+1} + \ldots + R_T$$

# **Objective: Continuing Tasks**

- In many cases the agent-environment interaction does not break naturally into identifiable episodes, but goes on continually without limit
  - e.g., an agent managing VM migration in a Cloud datacenter
  - e.g., the control system of RL-based traffic lights
- In this scenario, the goal of the agent is maximizing the expected cumulative discounted reward

$$G_t = R_t + \gamma R_{t+1} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k}$$

where  $\gamma \in [0, 1)$  is the discount factor

### **Reward vs Cost**

You can either maximimize the expected reward or minimize the expected cost

$$G_t = C_t + \gamma C_{t+1} + \ldots = \sum_{k=0}^{\infty} \gamma^k C_{t+k}$$

The two formulations are equivalent; you can easily switch between them by setting

$$r(s,a) = -c(s,a)$$

In the following, we will mostly refer to costs; keep in mind this equivalence

# **Policy**

#### **Definition**

A policy  $\pi$  is a distribution over actions given a state s

$$\pi(a|s) = p(A_t = a|S_t = s)$$

- A policy fully defines agent's behavior
- MDP policies depend on the current state only

# **Policy**

#### **Definition**

A policy  $\pi$  is a distribution over actions given a state s

$$\pi(a|s) = p(A_t = a|S_t = s)$$

- A policy fully defines agent's behavior
- MDP policies depend on the current state only
- Special case: deterministic policy

$$\pi: \mathcal{S} \to \mathcal{A}$$

# **Example: Deterministic Policy**

Action
$a_1$
$a_1$
<i>a</i> <sub>2</sub>
$a_1$

### **Value Function**

### Value function is a prediction of future costs

- can be used to evaluate how good/bad states and/or actions are
- and therefore to select actions e.g.

State	$a_1$	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>
<i>s</i> <sub>1</sub>	10	5	3
<i>s</i> <sub>2</sub>	8	6	4
<i>S</i> 3	6	5	6
<i>S</i> <sub>4</sub>	5	4	6
<i>S</i> 5	4	3	7
<i>S</i> <sub>6</sub>	1	5	9
<i>S</i> 7	0	9	15

### **Value Function**

### Value function is a prediction of future costs

- can be used to evaluate how good/bad states and/or actions are
- and therefore to select actions e.g.

State	$a_1$	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>
<i>s</i> <sub>1</sub>	10	5	3
<i>s</i> <sub>2</sub>	8	6	4
<i>S</i> 3	6	5	6
<i>S</i> <sub>4</sub>	5	4	6
<i>S</i> 5	4	3	7
<i>S</i> <sub>6</sub>	1	5	9
<i>S</i> 7	0	9	15

### **Value Function**

### Value function is a prediction of future costs

- can be used to evaluate how good/bad states and/or actions are
- and therefore to select actions e.g.

State	$a_1$	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>
<i>s</i> <sub>1</sub>	10	5	3
<i>s</i> <sub>2</sub>	8	6	4
<i>S</i> 3	6	5	6
<i>S</i> <sub>4</sub>	5	4	6
<i>S</i> 5	4	3	7
<i>S</i> <sub>6</sub>	1	5	9
<i>S</i> 7	0	9	15

S	$\pi(s)$
<i>s</i> <sub>1</sub>	<i>a</i> <sub>3</sub>
<i>s</i> <sub>2</sub>	<i>a</i> 3
<i>S</i> 3	<i>a</i> <sub>2</sub>
<i>S</i> <sub>4</sub>	$a_2$
<i>S</i> 5	<i>a</i> <sub>2</sub>
<i>s</i> <sub>6</sub>	<i>a</i> <sub>1</sub>
<i>S</i> 7	$a_1$

## **Value Functions**

#### Action value function (or, Q function)

Expected cost starting from state s, taking action a and then following policy  $\pi$ 

$$Q_{\pi}(s, a) = E_{\pi}[G_t | S_t = s, A_t = a]$$

#### State value function

Expected cost starting from state s and then following policy  $\pi$ 

$$V_{\pi}(s) = E_{\pi}[G_t|S_t = s]$$

- immediate cost
- $\triangleright$  discounted costs from successor state  $S_{t+1}$

$$Q_{\pi}(s,a) = E_{\pi}[G_t|S_t=s,A_t=a]$$

- immediate cost
- ightharpoonup discounted costs from successor state  $S_{t+1}$

$$Q_{\pi}(s, a) = E_{\pi}[G_t | S_t = s, A_t = a]$$
  
=  $E_{\pi}[C_t + \gamma C_{t+1} + \gamma^2 C_{t+2} \dots | S_t = s, A_t = a]$ 

- immediate cost
- $\triangleright$  discounted costs from successor state  $S_{t+1}$

$$Q_{\pi}(s, a) = E_{\pi}[G_{t}|S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma C_{t+1} + \gamma^{2} C_{t+2} \dots | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma (C_{t+1} + \gamma C_{t+2} \dots) | S_{t} = s, A_{t} = a]$$

- immediate cost
- $\triangleright$  discounted costs from successor state  $S_{t+1}$

$$Q_{\pi}(s, a) = E_{\pi}[G_{t}|S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma C_{t+1} + \gamma^{2} C_{t+2} \dots | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma (C_{t+1} + \gamma C_{t+2} \dots) | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma G_{t+1}|S_{t} = s, A_{t} = a]$$

- immediate cost
- ightharpoonup discounted costs from successor state  $S_{t+1}$

$$Q_{\pi}(s, a) = E_{\pi}[G_{t}|S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma C_{t+1} + \gamma^{2} C_{t+2} \dots | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma (C_{t+1} + \gamma C_{t+2} \dots) | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma G_{t+1}|S_{t} = s, A_{t} = a]$$

$$= c(s, a) + \gamma E_{\pi}[G_{t+1}|S_{t} = s, A_{t} = a]$$

The action value function can be decomposed into two parts:

- immediate cost
- $\triangleright$  discounted costs from successor state  $S_{t+1}$

$$Q_{\pi}(s, a) = E_{\pi}[G_{t}|S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma C_{t+1} + \gamma^{2} C_{t+2} \dots | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma (C_{t+1} + \gamma C_{t+2} \dots) | S_{t} = s, A_{t} = a]$$

$$= E_{\pi}[C_{t} + \gamma G_{t+1}|S_{t} = s, A_{t} = a]$$

$$= c(s, a) + \gamma E_{\pi}[G_{t+1}|S_{t} = s, A_{t} = a]$$

### Bellman equation:

$$Q_{\pi}(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) Q_{\pi}(s', \pi(s'))$$

### **State Value Functions**

The value function can be similarly decomposed into two parts:

- ightharpoonup immediate cost  $C_t$
- ightharpoonup discounted cost from successor state  $V(S_{t+1})$

$$V_{\pi}(s) = E_{\pi}[G_{t}|S_{t} = s]$$

$$= E_{\pi}[C_{t} + \gamma C_{t+1} + \gamma^{2} C_{t+2} \dots |S_{t} = s]$$

$$= E_{\pi}[C_{t} + \gamma (C_{t+1} + \gamma C_{t+2} \dots) |S_{t} = s]$$

$$= E_{\pi}[C_{t} + \gamma G_{t+1}|S_{t} = s]$$

### Bellman equation:

$$V_{\pi}(s) = c(s, \pi(s)) + \gamma \sum_{s'} p(s'|s, \pi(s)) V_{\pi}(s')$$

# **Optimal Value Function**

#### Optimal action value function

 $Q^*(s;a)$  is the minimum action-value function over all policies

$$Q^*(s,a) = min_{\pi}Q_{\pi}(s,a)$$

#### Optimal state value function

 $V^*(s)$  is the minimum value function over all policies

$$V^*(s) = min_{\pi}V_{\pi}(s)$$

## **Bellman Optimality Equations**

$$Q_{\pi}(s,a) = c(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) Q_{\pi}(s',\pi(a'))$$

$$\downarrow \downarrow$$

$$Q^{*}(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^{*}(s', a')$$

$$V^{*}(s) = \min_{a} Q^{*}(s, a)$$

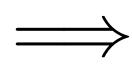
$$Q^{*}(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) V^{*}(s')$$

# **Optimal Policy**

Given  $Q^*(s, a)$  the optimal action when the system is in state s is:

$$\pi^*(s) = a^*(s) = \arg\min_{a \in \mathcal{A}} Q^*(s, a)$$

State	$a_1$	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>
s <sub>1</sub>	10	5	3
<i>S</i> <sub>2</sub>	8	6	4
<i>5</i> 3	6	5	6
<i>S</i> <sub>4</sub>	5	4	6
<i>S</i> 5	4	3	7
<i>S</i> <sub>6</sub>	1	5	9
<i>S</i> 7	0	9	15



Optimal Action
<b>a</b> 3
<b>a</b> 3
$a_2$
$a_2$
$a_2$
$a_1$
$a_1$

## How to compute $V^*$ ?

- If we know the optimal value function, we have an optimal policy!
- But... how do we compute the optimal value function??

### Value Iteration

### **Bellman Equation**

$$Q^*(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

### Value Iteration

### **Bellman Equation**

$$Q^*(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

- ▶ Suppose we know the solution to subproblems  $Q^*(s', a')$
- $\triangleright$   $Q^*(s, a)$  can be computed by one-step lookahead

$$Q^*(s, a) \leftarrow c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

## Value Iteration

### **Bellman Equation**

$$Q^*(s, a) = c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

- ▶ Suppose we know the solution to subproblems  $Q^*(s', a')$
- $ightharpoonup Q^*(s, a)$  can be computed by one-step lookahead

$$Q^*(s, a) \leftarrow c(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \min_{a'} Q^*(s', a')$$

- The idea is to apply these updates iteratively
- Proven to converge (see, Contraction Mapping Theorem in Sutton's book)

## Value Iteration: Algorithm

#### Value Iteration

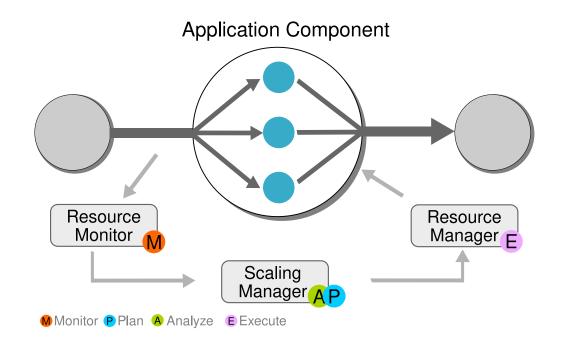
```
1 i \leftarrow 0
 2 Q_i(s,a) \leftarrow 0, \forall s \in \mathcal{S}, \forall a \in \mathcal{A}(s)
 3 repeat
          forall s \in \mathcal{S} do
               forall a \in \mathcal{A}(s) do
 5
                      Q_{i+1}(s,a) \leftarrow
 6
                       c(s, a) + \gamma \sum_{s' \in \mathcal{S}} p(s'|s, a) \min_{a' \in \mathcal{A}(s')} Q_i(s', a')
                end
 7
          end
        i \leftarrow i + 1
10 until \max_{s,a} |Q_i(s,a) - Q_{i-1}(s,a)| < \epsilon
11 \pi^*(s) = \operatorname{arg\,min}_a Q_i(s, a), \forall s \in \mathcal{S}
```

## Value Iteration: Alternative Algorithm

#### Value Iteration - Alternative

```
1 i \leftarrow 0
 2 Q_i(s,a) \leftarrow 0, \forall s \in \mathcal{S}, \forall a \in \mathcal{A}(s)
 3 V_i(s) \leftarrow 0, \forall s \in \mathcal{S}
 4 repeat
           forall s \in \mathcal{S} do
 5
                forall a \in \mathcal{A}(s) do
 6
                 Q_{i+1}(s,a) \leftarrow c(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V_i(s)
 7
               end
 8
                V_{i+1}(s) = \min_{a' \in \mathcal{A}(s)} Q_{i+1}(s, a')
 9
          end
10
        i \leftarrow i + 1
11
12 until \max_{s,a} |Q_i(s,a) - Q_{i-1}(s,a)| < \epsilon
13 \pi^*(s) = \operatorname{arg\,min}_a Q_i(s, a), \forall s \in S
```

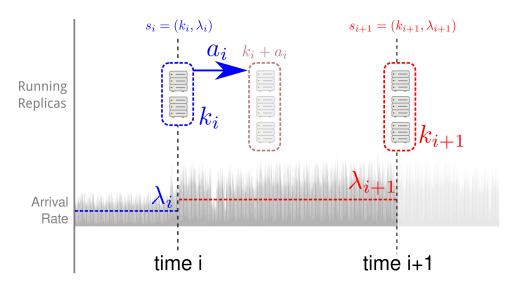
## Horizontal Auto-scaling: MDP formulation



- We periodically make a decision about scaling in/out an app component
- ► We are concerned with 3 objectives:
  - Monetary resource cost (or, resource usage in general)
  - Performance req. satisfaction (e.g., max response time)
  - Scaling overhead

## Horizontal Auto-scaling: MDP formulation

We model the problem as a Markov Decision Process (MDP)



- ▶ State at time slot i:  $s_i = (k_i, \lambda_i)$ 
  - $\triangleright$   $k_i$  component parallelism
  - $\triangleright$   $\lambda_i$  avg. arrival rate (of requests, jobs, data, ...)
- Action at time slot  $i: a_i \in \{0, +1, -1\}$

### **MDP Model: Transition Probabilities**

- ▶ State of the system  $s = (k, \lambda)$ 
  - ▶  $1 \le k \le K^{max}$  Component parallelism
  - $\triangleright \lambda$  avg. input rate
    - $\lambda$  is discretized, i.e.,  $\lambda_i \in \{0, \Delta\lambda, 2\Delta\lambda, (L-1)\Delta\lambda\}$
    - $ightharpoonup \Delta \lambda$  quantization step size, L number of discrete values
- ► Available actions  $A = \{-1, 0, +1\}$
- ► Transition probabilities  $p(s'|s, a) = p((k', \lambda')|(k, \lambda), a)$

$$p(s'|s,a) = P[s_{t+1} = (k',\lambda')|s_t = (k,\lambda), a_t = a] =$$

$$= \begin{cases} P[\lambda_{t+1} = \lambda'|\lambda_t = \lambda] & k' = k+a \\ 0 & \text{otherwise} \end{cases} =$$

$$= \mathbb{1}_{\{k'=k+a\}} P[\lambda_{t+1} = \lambda'|\lambda_t = \lambda]$$

### MDP Model: Cost Function

- Simple Additive Weighting method
- Cost associated with action execution and state transition  $(s,a) \rightarrow s'$

$$c(s, a, s') = w_{res} \frac{k+a}{K^{max}} + w_{perf} \mathbb{1}_{\{R(s, a, s') > R^{max}\}} + w_{rcf} \mathbb{1}_{\{a \neq 0\}}$$

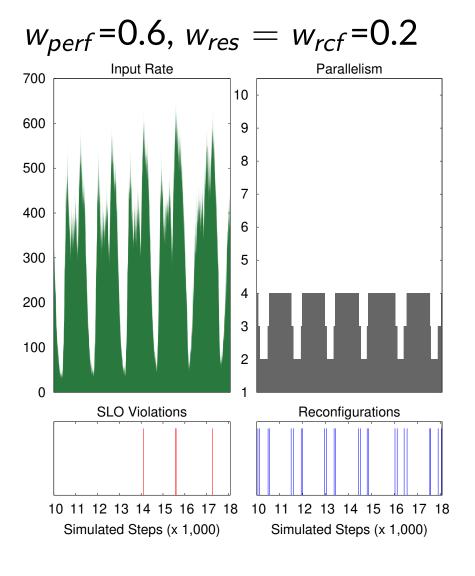
Resource Cost Performance

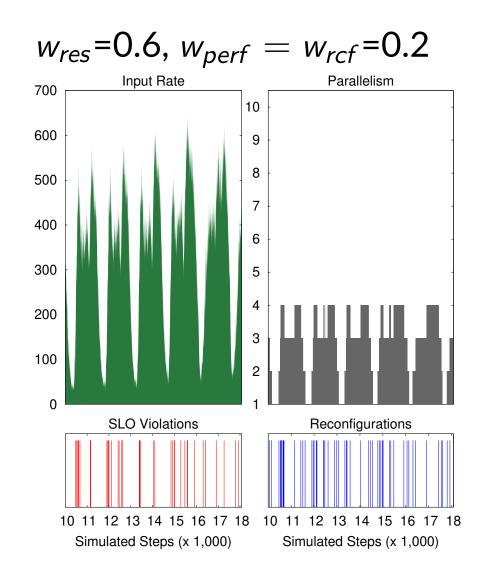
Reconfiguratio

- $w_{res} + w_{perf} + w_{rcf} = 1$ ,  $w_x \ge 0$ ,  $x \in \{res, perf, rcf\}$
- ightharpoonup R(s, a, s') is the component performance index, e.g, response time,
- $ightharpoonup R^{max}$  is the component reference performance value

We want to minimize 
$$\sum_{t=0}^{\infty} \gamma^t c(s_t, a_t, s_{t+1}), \quad \gamma \in [0, 1)$$

## **Trading-off Objectives**





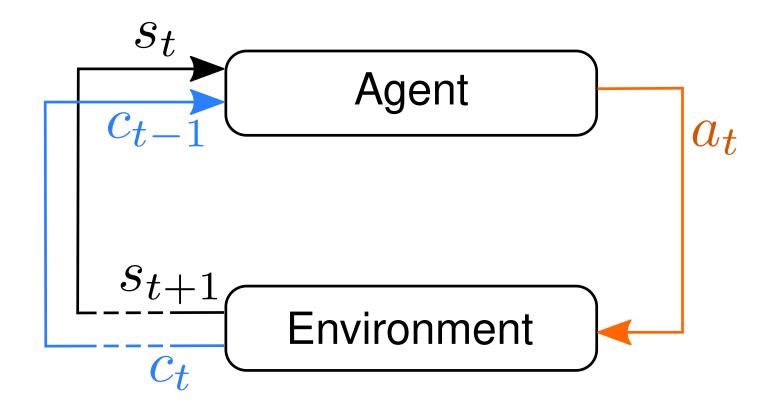
### **MDP** Resolution

- We can use the Value Iteration algorithm to solve the MDP
  - i.e., finding the optimal policy
- Is this enough?

### **MDP** Resolution

- We can use the Value Iteration algorithm to solve the MDP
  - i.e., finding the optimal policy
- Is this enough?
- Unfortunately, solving the MDP requires exact and complete knowledge of the underlying model
  - state transition probabilities
  - cost function
- In practice, we don't have such information!

# Reinforcement Learning



RL aims to learn the optimal policy through interaction and evaluative feedback

## Model-free vs Model-based RL

- Model-free RL: no model of the environment is available or used; the optimal policy is learned through experience only
- Model-based RL: a (possibly partial) model of the environment is available and used to derive the optimal policy
  - a partial model can boost learning speed
  - RL may also be used in presence of a complete model instead of VI; e.g., with a large number of rarely visited states VI would unnecessarily run for a long time!
  - You may also try to learn the model online and use it to compute a policy

# Value-based vs Policy-based RL

- Value-based RL: aims to learn the optimal value function through experience; the policy is derived from it
  - Simplest RL algorithms belong to this group
  - We will mainly focus on this group in the following
- Policy-based RL: aims to directly learn the optimal policy through experience; no explicit computation/learning of the value function
- Hybrid approaches: e.g., the Actor-Critic framework