Peergrade #3: Sequences and Sums, Graphs and Trees, Number Theory

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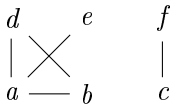
October 11, 2019

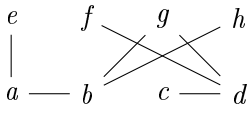
Exercise 1. *Consider the following graphs, then explain why each statement is true or false.*

1. 

   (a) *There exists a simple path from a to d*

   (b) *There exists a simple path from c to d*

   (c) *The graph is connected*

2. 

   (a) *The graph represents a tree*

   (b) *The graph represents a binary tree*

   (c) *The graph represents a full binary tree*

Exercise 2. *Compute the following values, showing the process for each computation:*

1. *Show the following sequences for* $1 < j < 10$*:*

   (a) $a_j = j^2$

   (b) $b_j = 1/j$

   (c) $c_j = \sum_{i=1}^{j} a_i \cdot b_i$

2. (a) $(763636 \cdot 437813 \cdot 936257) \bmod 43$

   (b) $(894461 \cdot (206193 + 83218)) \bmod 59$

   (Use the theorems shown in class to simplify each step of the computation.)

3. (a) *Convert* $(73217)_8$ *to its binary and decimal expansion.*

(b) Convert $(62290)_{10}$ *to its octal and hexadecimal expansion.*

4. *(a)* $gcd(2574, 1976)$

   (b) $lcm(1525, 4405)$

Exercise 3. *Let* x, y, z *and* m *be integers. Assume that* $x \equiv y \,(\mathbf{mod}\, m)$ *and that* $m \mid z$. *Prove that* $x + 2z \equiv y \,(\mathbf{mod}\, m)$.