

HarmonicFlow:

For each vertex P_i , we check if P_i is a boundary point, if not we find all the adjacent vertex add the to \vec{H} and divide by their number a_A .

$$\vec{H} = \frac{1}{n_A} \sum_{j=1}^{n_A} \vec{P}_j$$

HarmonicAreaFlow:

For each vertex P_i , we check if P_i is a boundary point, if not we find all the adjacent points and save them in the array "neighbours". First, we calculate the harmonic flow \vec{H} then we calculate the area in the following way:

- Take P_i as the first element in the array
- We take every two consecutive elements in the array

The area is :

$$A = \frac{1}{2} \sum_{j=1}^{n_A-1} (\vec{P}_j P_i * \vec{P}_{j+1} P_i)$$

Finally the flow will be:

$$\vec{H} = \frac{1}{A} \vec{H}$$

VolumeConservationFlow:

Step 1: (Gradient)

For each vertex P_i , we loop over the faces and for the incident face we do the following steps:

- Find the position of P_i in this face
- We loop over all the vertices other than P_i two by two and calculate their cross product :

$$\det P = \vec{P}_1 * \vec{P}_2$$

The gradient will be array of all these $\det P$.

Step 2: (Renormalization of the flow)

For each vertex P_i , the flow will be :

$$\vec{F} = \vec{H} - \frac{\det P \cdot \vec{H}}{\det P \cdot \det P} \det P$$

MeanCurvatureFlow:

For each vertex,

We used the following notations :

Q is the point we calculate the mcf for in each iteration

P_i is the predecessor of Q on face j, the successor of Q on face (j+1) and

therefore the shared vertex of faces (j, j+1)

P_{i-1} is the successor of Q on face j

P_{i+1} is the predecessor of Q on face (j+1)

\vec{M}_i is the edge (P_i , Q)

We applied the following steps:

- We find the first incident face f and the predecessor and the successor of Q in f
- We iterate over the face in the right order by finding the face show contain Q and its predecessor
- The flow will be:

$$mcf[i] = \frac{-1}{2} \sum_f \frac{Q\vec{P}_{i-1} * Q\vec{P}_{i+1}}{\|Q\vec{P}_{i-1} * Q\vec{P}_{i+1}\|} * P_i\vec{P}_{i-1}$$

MeanCurvatureFlowCotan:

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$angelbefore = (Q\vec{P}_{i-1}, P_{i-1}\vec{P}_i)$

$angelafter = (P_i\vec{P}_{i+1}, P_{i+1}\vec{Q})$

We applied the following steps:

- We find the first incident face f and the predecessor and the successor of Q in f
- We iterate over the face in the right order by finding the face show contain Q and its predecessor
- The flow will be:

$$mcf[i] = \frac{1}{2} \sum_f \left(\frac{1}{\tan angelbefore} + \frac{1}{\tan after} \right) * M_i$$