Papers notes

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1 Grid pattern formation

Notes on [1]

1.1 Introduction

Mechanistic models account for hexagonal grids as the result of pattern-forming dynamics in a recurrent neural network with hand tuned center-surround connectivity
Normative models show the emergence of grid-cells from a neural architecture trained to solve a navigational task, due to the constraints of solving this task.
This work reproduces the results from different normative models and provides a theoretical unification of the two approaches.

1.2 1-layer NN

Unsupervised hebbian learning is performed on Np input units (number of place cells) with Ng output units.
The synaptic weigths W are updated according to the a generalized Hebbian with two terms: the first is the hebbian rule itself and the second implements a Gram-Schmidt like orthogonalization.
This is equivalent to perform PCA on P (input matrix). To force non-negativity just perform NNNF on P.

1.3 RNN

The RNN is trained to perform a task of path integration. The network has a vanilla RNN architecture: 2 linear input units for x and y velocity, a set of recurrently connected hidden units, and linear readout units.

The network update rule and output prediction equation are the following:

$$r^{t+1} = \sigma\left[Jr^t + M\vec{v}^t\right]$$
$$p^t = Wr^t$$

Where v is the velocity input, M is the network's input weights, r is the vector of neuron activities, J is the matrix of recurrent weights. σ is a pointwise tanh nonlinearity. p is the vector of estimated place cell activities and W is the place cell readout weights.
Network was trained using batches of 200 trajectories and RMSProp to minimize the cross-entropy beetween estimated and true place cell activities. To implement non-negativity the tanh is replaced by ReLU.

1.4 LSTM

The task and training protocol were identical to that of the RNN. The model architecture consist of of x and y velocity inputs to an LSTM with 128 units, followed by a linear layer of 512 units, followed by a final readout to the estimated place cell activities.

References

[1] Ben Sorscher, Gabriel Mel, Surya Ganguli, and Samuel Ocko. A unified theory for the origin of grid cells through the lens of pattern formation. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d'Alché-Buc, E. Fox, and R. Garnett, editors, *Advances in Neural Information Processing Systems 32*, pages 10003–10013. Curran Associates, Inc., 2019.