Introduction

Example: Mass-Spring-Damper, Approximated System

Reliable Physical Theory

$$\mathcal{R}(\mathbf{u}, \tau_{\mathbf{m}}; \mathbf{r}) = \mathbf{0}$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$$

$$\mathbf{u} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \qquad \tau = \begin{bmatrix} c \\ k \end{bmatrix} \qquad \mathbf{r} = \begin{bmatrix} m \\ x(0) \\ \dot{x}(0) \end{bmatrix}$$

Approximated Embedded Model

$$\bar{\tau}_m = (\cancel{\mathbf{u}}, \cancel{\mathbf{v}}, \cancel{\mathbf{s}}, \theta)$$

$$\boxed{k = \text{const}}$$

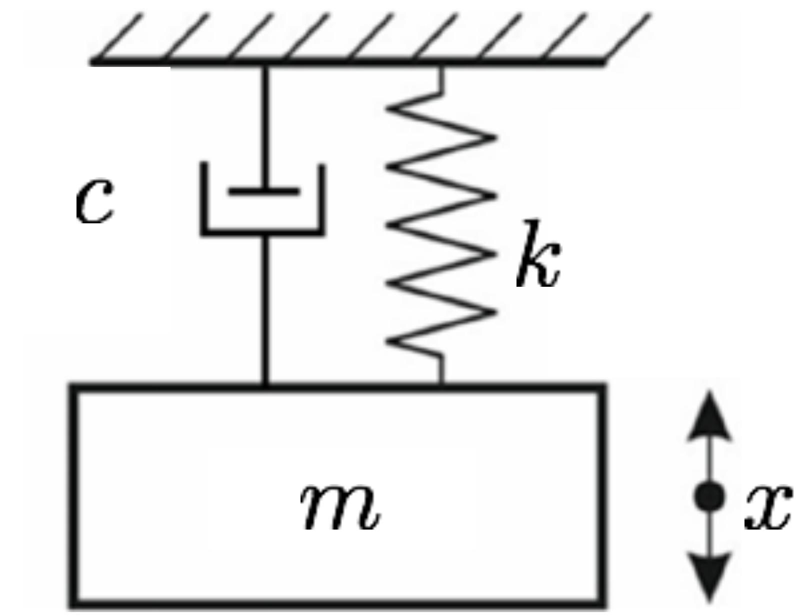
$$\theta = k$$

Inadequacy Model

$$\epsilon_m(\cancel{\mathbf{u}}, \cancel{\mathbf{s}}; \alpha)$$

$$\boxed{c \sim \mathcal{N}(\mu_c, \sigma_c^2)}$$

$$\alpha = \begin{bmatrix} \mu_C \\ \sigma_C \end{bmatrix}$$



$$\boxed{\begin{aligned} \mathbf{y} &= x \\ \mathbf{q} &= \max(\dot{x}) \end{aligned}}$$

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Note: It is critical that parameters be **identifiable** (i.e., parameters can be uniquely determined from observations). For:

$$m\ddot{x} + c\dot{x} + kx = 0$$

the parameter sets:

$$\theta = [m, c, k] \quad \text{and} \quad \theta = [1, c/m, k/m]$$

yield the same state values.

x

Approximated Embedded Mode

$$\bar{\tau}_m = (\cancel{u}, \cancel{v}, \cancel{s}, \theta)$$

$$\boxed{k = \text{const}}$$

$$\theta = k$$

Inadequacy Model

$$\epsilon_m(\cancel{u}, \cancel{s}; \alpha)$$

$$\boxed{c \sim \mathcal{N}(\mu_c, \sigma_c^2)}$$

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$$\boxed{\begin{aligned} \mathbf{y} &= x \\ \mathbf{q} &= \max(\dot{x}) \end{aligned}}$$