



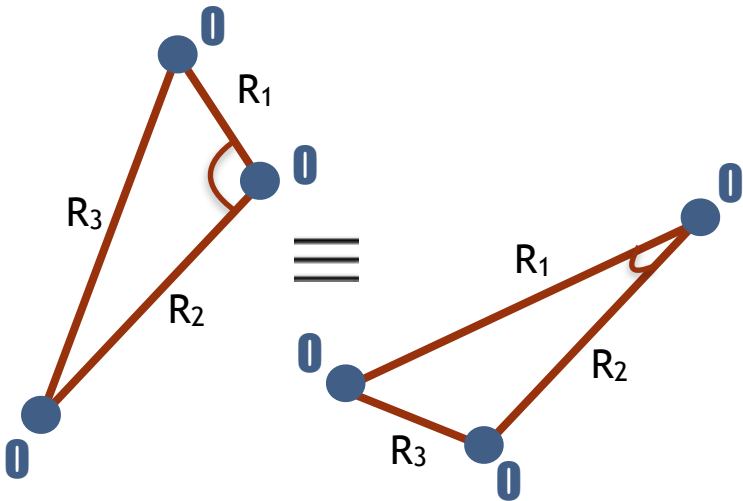






# ANNforPREs: Methodology

# Multi-layer feed-forward Neural Networks (NN) have been adopted as fitting function:



1. A **Symmetrized Polynomial Vector (G)** is constructed, in order to account for the permutation symmetries; for example, for a A3-type system:

where

, being and tunable parameters.



# Permutation-Invariant Polynomials Neural Networks (PIP-NN):



















R

2

R

3





PIP



- ✦ Easy to implement;
- ✦ Easy to train;
- ✦ Easy to generalize to new systems;
- ✦ Easy to differentiate in R;

- ✦ Cost effective;
- ✦ Easy to be refined;
- ✦ Widely tested;
- ✦ Easy to be extended to the stochastic case.

1

3



$$G_1 = p_1 + p_2 + p_3$$

$$G_2 = p_1p_2 + p_2p_3 + p_1p_3$$

$$G_3 = p_1p_2p_3$$

$$p_i = \exp(-\lambda_i(R_i - r_{e_i}))$$







Lasagne

# ANN for PESs: Methodology

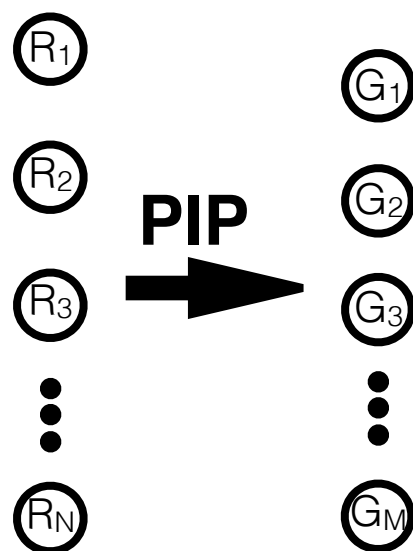
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**Permutation Invariant Polynomials Neural Networks (PIP-NN):**

theano

Lasagne



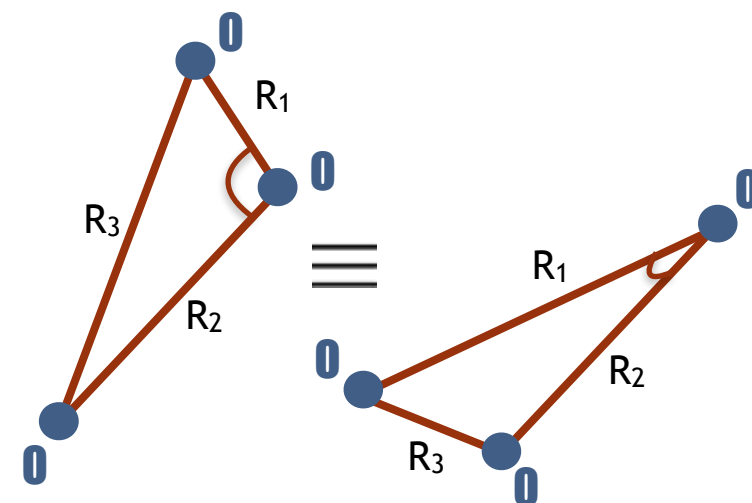
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where  $p_i = \exp(-\lambda_i(R_i - r_{e_i}))$ , being  $\lambda_i$  and  $r_{e_i}$  tunable parameters.



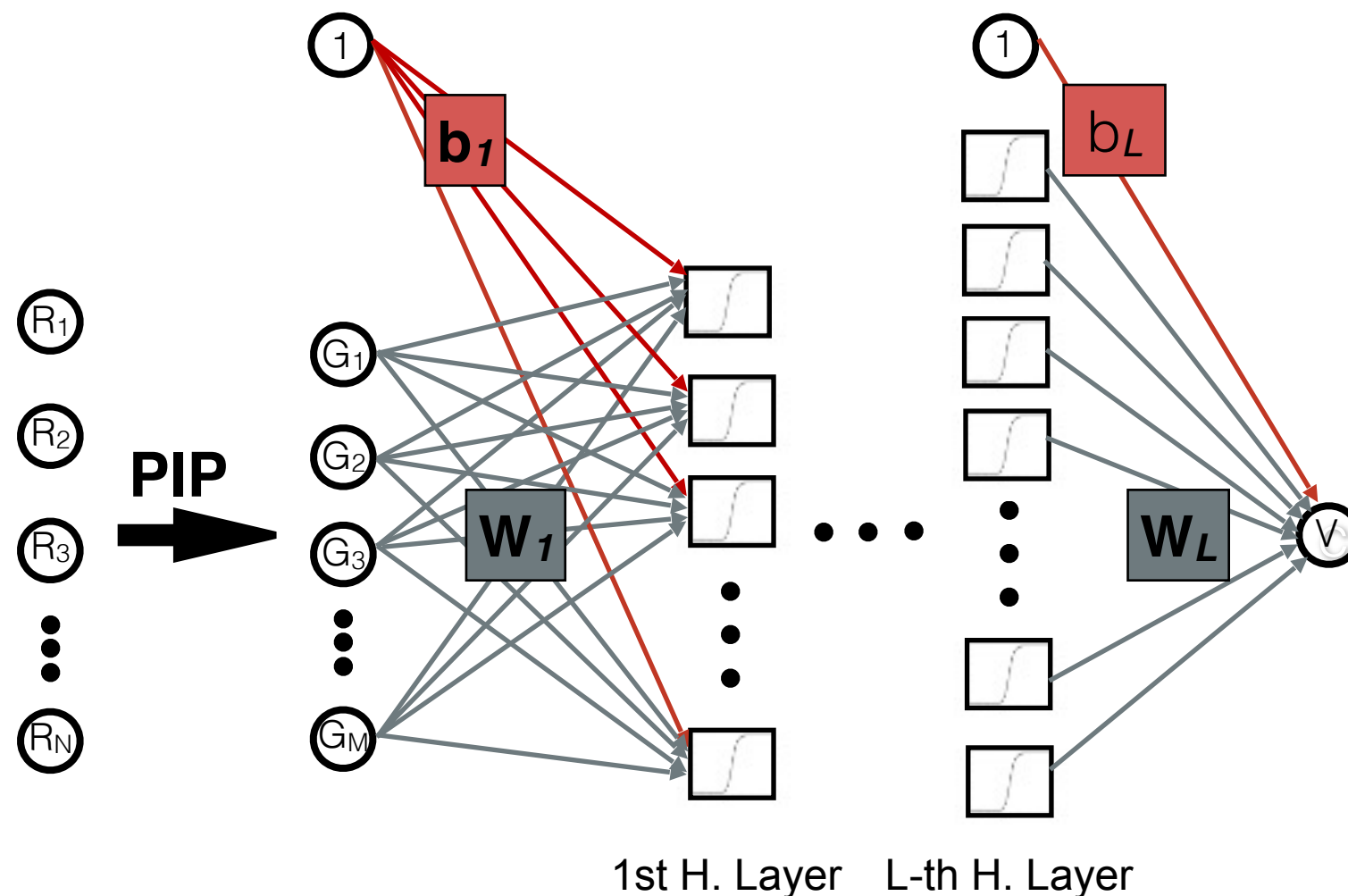
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2.  $G$  is fed to a feed-forward neural network, and it flows through its layers as a series of weighted linear combinations alternated to non-linear functions

Output from the  $k$ -th Neuron of the  $i$ -th Layer

$$\begin{cases} z_i^k = \sum_{j=1}^{N_{i-1}} W_i^{jk} y_{i-1}^j + b_i^k \\ y_i^k = f_i(z_i^k) \end{cases}$$