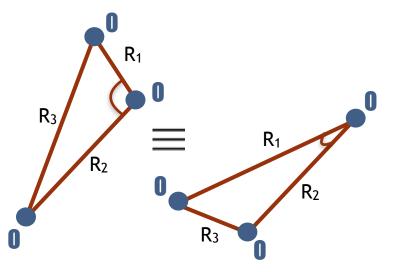






# **ANN for PESs: Methodology**

#### Multi-layer feed-forward Neural Networks (NN) have been adopted as fitting functional:



1. A Symmetrized Polynomial Vector (G) is constructed, in order to account for the permutation symmetries; for example, for a A3-type system: tunable parameters. where and

### Permutation Invariant Polynomials Neural Networks (PIP-NN):











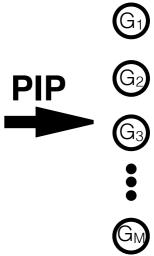












Easy to implement; Easy to train: Easy to generalize to new systems; Easy to differentiate in R;

Cost effective: Easy to be refined; Widely tested; Easy to be extended to the stochastic case.

$$G_1 = p_1 + p_2 + p_3$$
  
 $G_2 = p_1p_2 + p_2p_3 + p_1p_3$   
 $G_3 = p_1p_2p_3$ 

 $p_i = \exp(-\lambda_i(R_i - r_{e_i}))$ 

### theano



## ANN for PESs: Methodology

Multi-layer feed-forward Neural Networks (NN) have been adopted as fitting functional:

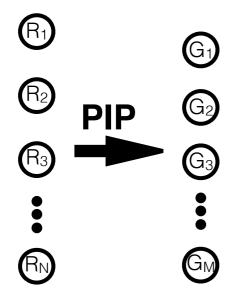
- ◆ Easy to implement;
- ◆ Easy to train;
- ◆ Easy to generalize to new systems;
- ◆ Easy to differentiate in R;

- ◆ Cost effective;
- ◆ Easy to be refined;
- ◆ Widely tested;
- ◆ Easy to be extended to the stochastic case.

#### Permutation Invariant Polynomials Neural Networks (PIP-NN):

## theano

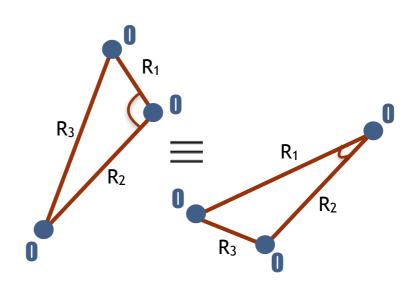
Lasagne



1. A **Symmetrized Polynomial Vector** (**G**) is constructed, in order to account for the permutation symmetries; for example, for a A3-type system:

$$G_1 = p_1 + p_2 + p_3$$
  
 $G_2 = p_1p_2 + p_2p_3 + p_1p_3$   
 $G_3 = p_1p_2p_3$ 

where  $p_i = \exp(-\lambda_i(R_i - r_{e_i}))$ , being  $\lambda_i$  and  $r_{e_i}$  tunable parameters.



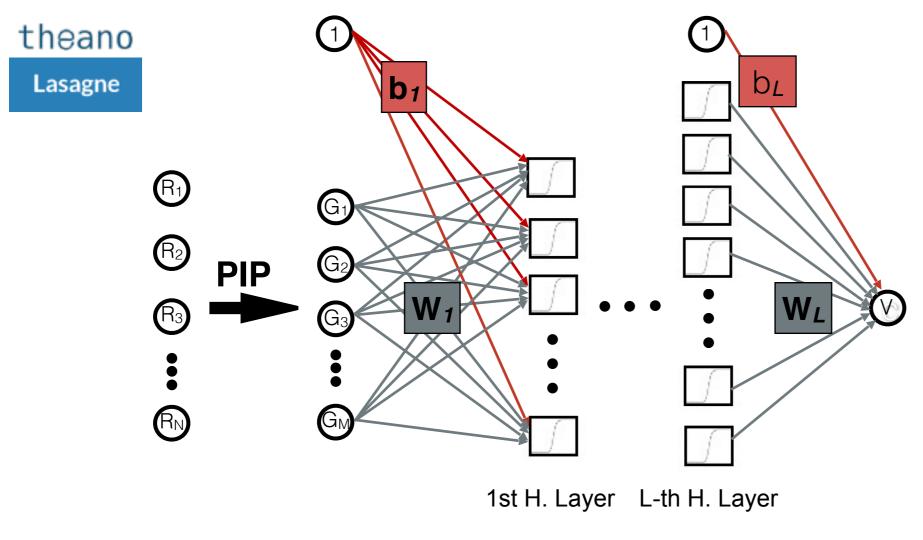
## ANN for PESs: Methodology

Multi-layer feed-forward Neural Networks (NN) have been adopted as fitting functional:

- ◆ Easy to implement;
- ◆ Easy to train;
- ◆ Easy to generalize to new systems;
- ◆ Easy to differentiate in R;

- **♦** Cost effective;
- ◆ Easy to be refined;
- ◆ Widely tested;
- ★ Easy to be extended to the stochastic case.

### Permutation Invariant Polynomials Neural Networks (PIP-NN):



**2. G** is fed to a feed-forward neural network, and it **flows through its layers** as a series of weighted linear combinations alternated to non-linear functions

Output from the k-th Neuron of the i-th Layer  $\begin{cases} z_i^k = \sum_{j=1}^{N_{i-1}} W_i^{jk} y_{i-1}^j + b_i^k \\ y_i^k = f_i \left( z_i^k \right) \end{cases}$