Strategic behaviour in auctions for substitutes: theory and experiment

[Preliminary draft]

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Abstract

We study strategic bidding behaviour in three first-price auctions for substitute goods: a Product-Mix auction, a sequential auction, and a simultaneous auction. Theory predicts that, in the unique risk-neutral Bayes-Nash equilibrium, the Product-Mix and the sequential format perform nearly identically with respect to bidder surplus, revenue, and welfare, and the simultaneous auction only slightly worse. We test these predictions in a virtual lab experiment, considering an asymmetric market with a flexible bidder and competitive fringes, and a symmetric market with three flexible bidders. The empirical results are in stark contrast with the theory: the Product-Mix auction outperforms both other formats in bidder surplus and welfare, while the simultaneous auction generates the highest revenue. With symmetric bidders, payoffs in the PMA are 90% (156%) higher than in the sequential (simultaneous) auction, and efficiency is 12% higher.

1 Introduction

Simultaneous and sequential sealed-bid auctions for multiple objects are common in electricity markets, for the allocation of online advertisement slots, for consumer goods, and many other products. In these auctions, buyers may often find it difficult to work out which objects to bid for or which auction to participate in. Suppose a buyer wants either of two distinct objects, but not more than one in total. In which of two simultaneous or sequential auctions should they participate? How should they bid? Product-mix auctions, and combinatorial auctions more generally, offer a simple solution: the buyer can submit a bid for either object but is guaranteed to win at most one of them. If objects are ordinary or strong substitutes for bidders, they can perfectly represent their preferences. In practice, this auction format has been used by the Bank of England to allocate central bank loans (Klemperer (2008, 2010)).

An important open question is how well a Product-Mix auction performs when bidders have informational advantages and strategically influence the market price and allocation. The crux in the analysis of strategic behaviour in multi-object auctions is that standard game-theoretical tools like the Bayes-Nash equilibrium cannot be easily applied. Even in very simple

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settings involving bidders with valuations for multiple, distinct objects, bid functions become multivariate functions which are much harder to work with. A long-standing tool to establish some understanding of more difficult multi-unit and multi-object markets are lab experiments Ledyard et al. (1997), Alsemgeest et al. (1998), Kagel & Levin (2001), Goeree et al. (2006). However, especially if not connected to a theory, it can be difficult to derive fundamental results and easy to run into issues with external validity.

My study is, to the best of my knowledge, the first to analyse strategic behaviour in a setting with multiple, distinct objects while simultaneously testing the theory with experimental results. I study the outcomes of a product-mix, a sequential, and a simultaneous auction under the *pay-as-bid pricing rule*, which has been widely used in practice for auctions of homogeneous goods,[1] and is also common in multi-unit markets that exhibit product differentiation.[2] Building on a theoretical model, I compare the performance of all three auction mechanisms with numerical simulations and I test these predictions in a virtual lab experiment. My theory predicts that, if bidders bid optimally, the product-mix auction performs nearly identically to the sequential auction with respect to the bidders' surplus, the auctioneer's revenue, and efficiency. Similarly, the simultaneous auction performs only slightly less well with a mere $\sim$5% (7%, 0.6%, 1%) difference in bidder surplus (flexible bidder payment, revenue, welfare). The experimental data, however, shows substantial and significant differences between all three auction formats. The Product-Mix auction generates the highest bidder surplus and welfare. This effect holds for both the computerised and the human environment, but is much stronger with symmetric human bidders: in the Product-Mix auction payoffs are 90% - 156% higher, and efficiency is 12% higher than in the other two auction formats. The simultaneous auction performs best in terms of revenue. All pairwise equivalence tests of outcomes (following the predictions of the theory) are non-significant, with the exception of the efficiency comparison between the sequential and the simultaneous auctions. I also discuss bidding behaviour and to analyse to what extent bidders shade their bids. I find that bidders overbid significantly in all three considered auction formats, a phenomenon that has been documented and studied in many other auction experiments (Goeree et al. 2002, Georganas et al. 2017, Dorsey & Razzolini 2003, Cooper & Fang 2008, Breitmoser 2019).

In my setup, two differentiated objects A and B are sold, and two markets, or environments, are considered. In the first environment, a single bidder (called the 'flexible bidder') who wishes to buy either object, but not more than one overall, competes against two competitive fringes, represented by a computerised agent who bids truthfully. In the experiment, I also study the more general environment in which three flexible bidders compete against each other for the two objects. This environment is closely related: the flexible bidder may win either object or nothing, but, of course, the prior about the highest competing bids are different. While currently there does not exist any theoretical model that could predict bidding behaviour is the three-bidder environment, my study sheds some light on how it may connect to a simpler, but solvable model with competitive fringes. In the experiment, in the first environment (the

[1] In auctions for homogeneous goods, Pycia & Woodward (2021) show that pay-as-bid pricing is revenue dominant and possibly welfare dominant over uniform pricing.

[2] Pay-as-bid pricing is also common n electricity markets, e.g. in England&Wales, Mexico, Peru, Panama, and Chile. It is also used for online advertisement auctions, conducted on platforms such as Google's AdSense (Xandr and OpenX, e.g., use a mix of first-price and second-price auctions).

'computerised environment') each subject assumes the role of a flexible bidder and plays against computerised agents. In the second environment (the *'human environment'*), subjects compete against other human bidders with the same type of preference and symmetric priors in groups of three. My study is set up as a between-subject design: each participant is assigned to one auction mechanism in one given environment; hence, I consider six treatments. Each subject plays several auction rounds, bidding for one or both of the two objects, with round-independent induced values.

1.1 Related literature

Strategic behaviour in multi-unit markets has received much attention in the literature on auctions. For one, the stakes in many public and private sector auctions are extremely high: e.g., in wholesale electricity markets, in auctions for spectrum licensing, pollution permits, treasury bonds, government loans, and other financial markets. Moreover, market power often plays an important role in practice, as shown for examples for treasury auctions in various countries by Armantier & Sbaï (2006) (France), Malvey & Archibald (1998), Hortaçsu et al. (2018) (US), Hortaçsu & McAdams (2010) (Turkey). In electricity auctions, evidence of market power was found, e.g., by Borenstein et al. (2002) (California), Wolfram (1998) (UK), and Hortaçsu & Puller (2008) (Texas). Understanding and potentially improving the auction design in place was and still is in the interest of many decision-makers.

When the FCC began auctioning spectrum licences in 1994, first experiments with multi-object auctions were conducted not long after by Ledyard et al. (1997), testing the FCC's simultaneous procedure against sequential and combinatorial auctions. The auction formats, however, were all of ascending nature, and the considered market settings involved a greater number of players, rendering a theoretical analysis infeasible. In the more complex setups they study, the combinatorial auction outperforms the simultaneous. Similarly, Alsemgeest et al. (1998) study an ascending auction and a sealed-bid auction with uniform pricing, finding evidence of strategic demand reduction, and Gillen et al. (2016) and Katok & Roth (2004) investigate a descending-price auction for multiple, homogeneous units.[3] The question of uniform vs. discriminatory pricing has received great interest especially in the context of treasury auctions, as well as electricity auctions. In an experimental context, Abbink et al. (2006) find evidence that the uniform and a hybrid format used by the Bank of Spain raise higher revenue than discriminatory multi-unit auctions.

Other important experimental (and theoretical) papers set their focus, e.g. on anticipated loser regret in first-price auctions (Filiz-Ozbay & Ozbay 2007), collusion in ascending auctions (Sherstyuk & Dulatre 2008), closing rules (Sherstyuk 2009), resale (Filiz-Ozbay et al. 2015), or the comparison of supply function equilibrium to a standard multi-unit auction model (Brandts et al. 2013). Special attention has also been given to multi-unit auctions in which truth-telling is an equilibrium, namely the Vickrey and Ausubel's 'clinching auction' (Engelmann & Grimm 2009, Kagel & Levin 2009)[4] and mechanisms by Leonard (1991), Demange & Gale (1985) and

[3] Katok & Roth (2004) compare the Dutch auction also to an ascending uniform-price auction.

[4] Engelmann & Grimm (2009)test all of five different multi-unit auction format, an open, ascending-price and a sealed-bid uniform price auctions, a vickrey, an Ausubel, a discriminatory sealed-bid auction.

Demange et al. (1986) in experiments by Andersson et al. (2013), with respect to performance and truthful reporting.

An experimental study closely related to my research was conducted by Goeree et al. (2006), also comparing different first-price auctions for heterogeneous objects. However, in their setting every bidder is restricted to winning at most one object in all considered auction formats. This is only the case for the first-price Product-Mix auction I consider, a format also studied in Goeree et al. (2006). Importantly, however, I provide not only an experimental analysis but also a theoretical benchmark. Contrasting with Goeree et al. (2006)'s work, I am especially interested in the strategic incentives that arise if bidders are exposed to winning more than one object.

The remainder of the paper is organised as follows. In Section 2, I briefly present the results and predictions from the theoretical model (an in-depth analysis of the more general model is given in the companion paper Finster (2020)). The experimental design is detailed in Section 3. The empirical model and the hypotheses tested in this study are set out in Section 5. In Section 6, I present the results of the experiment and the outcomes of the hypothesis tests, and Section 7 concludes.

2 Theoretical results

For the first bidding environment, in which subjects play against computerised agents, theoretical and numerical results can be derived. The market model is of the type 'local-local-flexible' (LLF).[5] The 'flexible' bidder has substitutes preferences across different objects. She is competing against two groups of local bidders. Each subject in the experiment assumes the role of a flexible bidder, whereas a group of local bidders is represented by a computerised agent. In this section, I briefly describe the model and summarise the theoretical results. A more detailed analysis and discussion is given in Finster (2020).

Two indivisible objects A and B are for sale, at zero cost. There are two types of buyers in the market, a flexible bidder and two groups of local bidders. All bidders have quasi-linear utility. The *flexible bidder* wishes to buy object A or object B, but at most one. Her values for object A and B are given by $v_A > 0$ and $v_B > 0$, respectively, and at fixed prices p_A and p_B, her utility is

$$u(p_A, p_B) = \begin{cases} v_A - p_A & \text{if A is won} \\ v_B - p_B & \text{if B is won} \\ v_B - p_B - p_A & \text{if A and B are won} \\ 0 & \text{otherwise} \end{cases}$$

One group of at least two local bidders is interested only in object A, and another group of at least two local bidders is interested only in object B. Each local bidder of the first group values object A at a (and object B at zero), and each bidder of the second group values object B at b (and object A at zero). All values are privately known and the flexible bidder has a simple prior over the distribution of the local bidders' values: a and b are drawn from a uniform

[5] This is similar to LLG models, in which a global bidder competes against local bidders. A global bidders usually has weakly complementary preferences between objects, i.e. is interested in both and there may be synergies (cf. Krishna & Rosenthal (1996)).

distribution on $(0, \bar{v})$. For the flexible bidder's values I assume that $v_A, v_B \in (v_{min}, v_{max})$ for some $v_{min} > 0, v_{max} \leq \bar{v}$. For my theoretical results, I set w.l.o.g $\bar{v} = 1$. For simulations and in the experiment, I scale the distribution and values such that $v_{min} = 0$ and $\bar{v} = v_{max} = 100$.

Local bidders know that, within their group, bidders have identical values. Therefore, at least two local bidders in each group aim to outbid or undercut each other in bid prices, and hence bid their true value in any Bayes-Nash equilibrium (a formal proof is given in **?**). That is, the highest bids the flexible bidder is competing against, denoted by s_A and s_B, are also drawn uniformly from $(0, 1)$. As a point of reference, I note that if restricted to bidding in a standard first-price auction for only one object, say B, against a group of local bidders, the flexible bidder's optimal bid is $\frac{v_B}{2}$. In the following, I define the considered auction mechanisms. All auction mechanisms are pay-as-bid (first-price) mechanism, i.e. a winning bidder pays their own bid prices for the objects they won.

Product-Mix auction. The flexible bidder submits a 'paired bid' (w_A, w_B) to state demand for object A *or* object B, but not both, at prices of up to w_A or w_B respectively.[6] The local bidders' highest bid on object A is $(s_A, 0)$ and $(0, s_B)$ on object B. Given the bids $(w_A, w_B), (s_A, 0), (0, s_B)$, the auctioneer allocates objects A and B in order to maximise efficiency with respect to the reported preferences.[7] The nature of the flexible bidder's bid does not allow her winning object A and B. Hence the 'valid combinations' of potential winning bids are $C := \{(w_A, s_B), (s_A, w_B), (s_A, s_B)\}$. The winning bid x_A on A and the winning bid x_B on B are those that maximise the sum of the valid combinations of submitted bids, i.e. $(x_A, x_B) \in \arg\max_{(x_A, x_B) \in C} \{x_A + x_B\}$.[8,9]

Simultaneous auction. Two single-unit first-price auctions, one for object A and one for object B, are held simultaneously. Bidders can participate in either one or both of the two auctions. The flexible bidder submits a bid w_A for object A and a bid w_B for object B (either of which may be zero). The local bidders' highest bids are s_A and s_B for the two objects, respectively. In each of the two simultaneous auctions, the respective highest bid wins.

Sequential auction. Two single-unit first-price auctions, one for object A and one for object B are held sequentially. The (for the flexible bidder) less valuable object A is sold first. After the first auction, the winner is announced. Bidders then choose their bid in the second auction. The flexible bidder submits a bid w_A for object A and a bid w_B for object B in case she won A, and a bid $\overline{w}_B$ for object B in case she did not win A. The local bidders' highest bids are s_A and s_B. In each of the two sequential auctions, the respective highest bid wins.

The abbreviations '*pma*', '*seq*', and '*sim*' indicate the Product-Mix, the sequential, and the simultaneous mechanism and $\mathcal{M} := \{pma, seq, sim\}$ denotes the set of all considered mecha-

[6] Paired bids are an instance of XOR bids (see, e.g., Sandholm (2002)) where each component of an XOR bid specifies the quantity and price of exactly one object. Let p_A and p_B denote the auction prices. A paired bid (w_A, w_B) for object A and object B respectively expresses the following preferences: if $w_A - p_A > w_B - p_B$ and $w_A > p_A$, then the bidder wants up to one unit of object A. If the first inequality is reversed and $w_B > p_B$, she wants up to one unit of object B (equalities corresponds to indifference).

[7] In the Product-Mix auction with uniform pricing, assuming all bidders bid truthfully, actual efficiency is maximised. However, with pay-as-bid pricing, truthful bidding cannot be expected.

[8] Ties are broken by randomly (with equal probability) choosing one of the (sets of) winners, but this is not important for the theoretical analysis.

[9] The allocation and pricing rule are equivalent to choosing the revenue-maximising allocation, as defined for the 'menu auction' by Bernheim & Whinston (1986).

nisms.

2.1 Equilibrium bidding

In all three auction mechanisms, I uniquely characterise and, in some cases numerically, compute the optimal bids of the flexible bidder. As noted above, the local bidders' highest, optimal bids truthfully reveal their values and therefore equilibrium bidding boils down to the task of determining the flexible bidder's optimal bids. Fix an auction mechanism $m \in \mathcal{M}$. Then I let w_A^m and w_B^m denote the flexible bidder's bid for object A and B. In the sequential auction, let w_B^{seq} denote the flexible bidder's bid for object B conditional on winning object A, and let $\overline{w}_B^{seq}$ denote the flexible bidder's bid for object B conditional on *not* winning object A. When the context is clear, I omit the superscript m for legibility. The theoretical results in this section also appear in greater detail in the companion paper Finster (2020).

In the Product-Mix auction and the simultaneous auction, i.e. for $m \in \{pma, sim\}$, the flexible bidder's expected utility is given by

$$\mathcal{U}^m(w_A, w_B) = P_A^m(w_A, w_B)[v_A - w_A] + P_B^m(w_A, w_B)[v_B - w_B] + P_{AB}^m(w_A, w_B)[v_B - w_A - w_B]$$

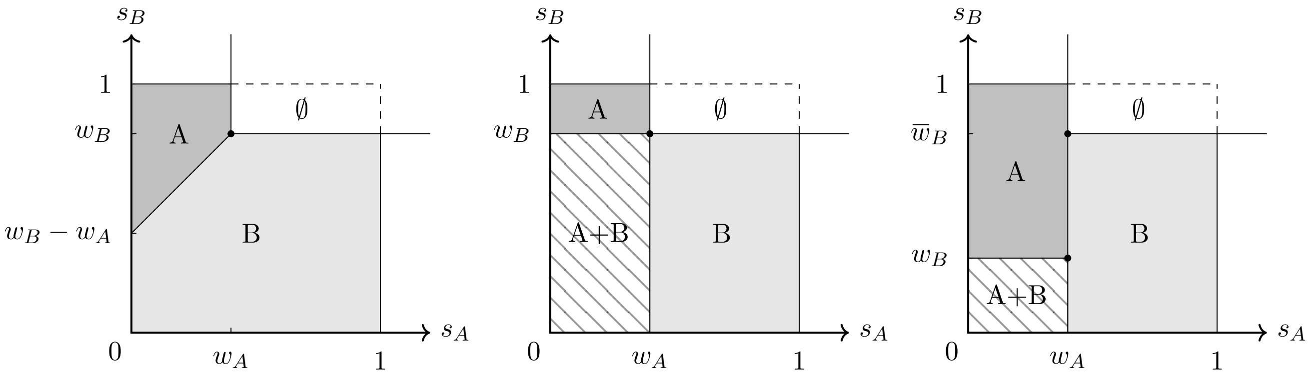
where P_A^m, P_B^m, and P_{AB}^m denote the flexible bidder's probability of winning object A only, object B only, and objects A and B together in mechanism m. In the sequential auction, the flexible bidder's expected utility is given by

$$\mathcal{U}^{seq}(w_A, w_B) = P_A^{seq}(w_A, w_B)[v_A - w_A] + P_B^{seq}(w_A, \overline{w}_B)[v_B - \overline{w}_B] + P_{AB}^{seq}(w_A, w_B)[v_B - w_A - w_B]$$

To determine the probabilities of winning in the individual auction mechanisms, I illustrate the allocation to the flexible bidder as a function of the local bidders' highest bids s_A and s_B in Fig. 1. Note that in the Product-Mix auction, the allocation rule implies that the flexible bidder obtains object A if and only if $w_A - s_A > w_B - s_B$ and $w_A > s_A$, and she wins object B if and only if $w_B - s_B > w_A - s_A$ and $w_B > s_B$.

The dark shaded region marked with 'A' indicates the flexible bidder winning object A, the light shaded region marked with 'B' indicates her winning object B, and the striped region marked with 'A+B' indicates her winning both objects. The figures are drawn assuming w_A and w_B ($\overline{w}_B$ in the sequential auction) to be the same across auction mechanisms, and assuming $w_A \leq w_B$. The optimal bids will differ between mechanisms. Recall that s_A and s_B are distributed uniformly on $(0, 1)$. Hence, the marked regions immediately correspond to the probabilities of winning object A, object B, and objects A and B, respectively.

First, I make trivial observation: in any auction mechanism, bidding more than the true value v_A or v_B on the corresponding object can never bid optimal. Now consider the Product-Mix auction. To write down the flexible bidder's payoff function, I first ascertain that, in equilibrium, bids are such that $w_A \leq w_B$. A simple proof is given in Finster (2020): suppose $w_A > w_B$ in equilibrium, then simply switching the bids between the two objects would make the flexible bidder strictly better off. From the allocation rule (and illustrated in Fig. 1(a)), it is also clear



(a) Product-Mix (b) Simultaneous (c) Sequential

Figure 1: Allocations to the flexible bidder

that the flexible bidder cannot win both objects at once, i.e. $P_{AB}^{pma} = 0$. I obtain

$$P_A^{pma}(w_A, w_B) = \int_0^{w_A} \int_{w_B - w_A + s_A}^{1} \mathrm{d}s_B \, \mathrm{d}s_A \tag{1}$$

$$P_B^{pma}(w_A, w_B) = \int_0^{w_A} \int_0^{w_B - w_A + s_A} \mathrm{d}s_B \, \mathrm{d}s_A + (1 - w_A)w_B \tag{2}$$

In the simultaneous auction, if the flexible bidder submits strictly positive bids w_A and w_B, there is a non-zero probability of winning both objects. The probabilities of winning are easily determined as $P_A^{sim}(w_A, w_B) = (1 - w_B)w_A$, $P_B^{sim}(w_A, w_B) = (1 - w_A)w_B$, and $P_{AB}^{sim}(w_A, w_B) = w_A w_B$.

In the sequential auction, the flexible bidder can condition her bid on object B on (not) having won object A. The probabilities of winning are $P_A^{seq}(w_A, w_B) = (1 - w_B)w_A$, $P_B^{seq}(w_A, \overline{w}_B) = (1 - w_A)\overline{w}_B$, and $P_{AB}^{seq}(w_A, w_B) = w_A w_B$. Given these characterisations of winning probabilities, I can provide the following characterisation of the flexible bidder's optimal bids:

Proposition 1. *In the first-price Product-Mix auction, the flexible bidder's optimal (equilibrium) bids are uniquely characterised on* $(0, v_A) \times (0, v_B)$ *by equations*

$$-\frac{3}{2}{w_A}^2 + w_A\left(-2 + 3w_B + v_A - v_B\right) + v_A(1 - w_B) = 0 \text{ and} \tag{3}$$

$$\frac{3}{2}{w_A}^2 + v_B - 2w_B - v_A w_A = 0. \tag{4}$$

In the proof I show that the optimal bid must be interior, i.e. on $(0, v_A)\times(0, v_B)$, and therefore must be characterised by first-order conditions. Furthermore, Eqs. (3) and (4) admit a unique, real solution on this interval. The flexible bidder making a strictly positive bid on object A and object B (as opposed to one zero bid) is not entirely obvious, as increasing the bid on one object, while increasing the probability of winning that object, also decreases the probability of winning the respective other object. However, at an arbitrarily small (but positive) price, winning the less valuable object with some positive probability is always profitable in expectation.

Proposition 2. *In the first-price sequential auction, the flexible bidder's optimal bid is uniquely characterised by* $w_A = \frac{1}{4}\left(\frac{{v_A}^2}{2} + v_A(2 - v_B)\right)$, $w_B = \frac{1}{2}(v_B - v_A)$, *and* $\overline{w}_B = \frac{v_B}{2}$.

The flexible bidder's optimal bid for object B, in case she won object A, adjusts for the potential incremental gain in value. Because at most $v_B - v_A$ can be gained by winning object B in addition to object A, the bidder's willingness to pay exactly half of this potential gain. The price paid for A is treated as a sunk cost and not relevant for the subsequent bid. Trivially, in case she did not win object A, the flexible bidder's optimal bid is that of a standard first-price auction in my setup. Her optimal bid in the first auction takes the trade-offs of subsequently winning/not winning object B into consideration.[10]

Proposition 3. *In the first-price simultaneous auction, the flexible bidder's optimal bid is uniquely characterised by* $w_A = \frac{v_A(2-v_B)}{4-{v_A}^2}$ *and* $w_B = \frac{2v_B - {v_A}^2}{4-{v_A}^2}$.

The simultaneous auction is intuitively the most disadvantageous auction mechanism for the flexible bidder. She has to balance the trade-off between not winning two objects at a loss too often, but bidding high enough so as to win often enough, in expectation. It is indeed optimal to bid such that she might win both objects only at prices that still guarantee a positive surplus, i.e. such that $w_A + w_B < v_B$. Just as in the Product-Mix auction, it is not obvious that two non-zero bids are optimal, but, similarly, it can be shown that the solution must indeed be interior on $(0, v_A) \times (0, v_B)$.

I note two more properties of optimal bids that hold across all three mechanisms: First, the flexible bidder bids less than what she would bid for a single object, that is $w_A^m < \frac{v_A}{2}$ and $w_B^m < \frac{v_B}{2}$ for all mechanisms $m \in \mathcal{M}$. Second, the comparative statics of optimal bids are such that $\frac{\partial w_A^m}{\partial v_A} > 0$, $\frac{\partial w_A^m}{\partial v_B} < 0$, $\frac{\partial w_B^m}{\partial v_A} < 0$, and $\frac{\partial w_B^m}{\partial v_B} > 0$ for all m. Hence, all auction formats considered allow the flexible bidder to express the substitutability between A and B to some, but varying, degree.

2.2 Profits, payments, revenue, welfare

The three auction formats are evaluated with respect to the flexible bidder's utility $\mathcal{U}^m$ and the flexible bidder's payment $\mathcal{P}^m$, the auctioneer's revenue $\mathcal{R}^m$, and total welfare $\mathcal{W}^m$. All outcomes are computed in expected values across the local bidders' bids. In the simulations in Section 4, I also average over the flexible bidder's values.

In the Product-Mix auction and the simultaneous auction, the auctioneer's expected revenue is given by

$$\begin{aligned}\mathcal{R}^m(w_A, w_B) = {} & P_A^m(w_A, w_B)\mathbb{E}_{s_B}\left[w_A + s_B \mid \text{flexible bidder wins } A\right] \\ & + P_B^m(w_A, w_B)\mathbb{E}_{s_A}\left[s_A + w_B \mid \text{flexible bidder wins } B\right] \\ & + P_{AB}^m(w_A, w_B)\left[w_A + w_B\right] \\ & + P_\emptyset^m(w_A, w_B)\mathbb{E}_{s_A, s_B}\left[s_A + s_B \mid \text{flexible bidder wins nothing}\right]\end{aligned}$$

[10]For general distributions of the local bidders' bids, a zero bid in the first auction may be optimal even for $v_A \neq 0$.

In the sequential auction, the auctioneer's expected revenue is given by

$$\begin{aligned}\mathcal{R}^{seq}(w_A, w_B, \overline{w}_B) = {} & P_A^{seq}(w_A, w_B)\mathbb{E}_{s_B}\left[w_A + s_B \mid \text{flexible bidder wins } A\right] \\ & + P_B^{seq}(w_A, \overline{w}_B)\mathbb{E}_{s_A}\left[s_A + \overline{w}_B \mid \text{flexible bidder wins } B\right] \\ & + P_{AB}^{seq}(w_A, w_B)[w_A + w_B] \\ & + P_{\emptyset}^{m}(w_A, w_B)\mathbb{E}_{s_A, s_B}\left[s_A + s_B \mid \text{flexible bidder wins nothing}\right]\end{aligned}$$

The flexible bidder's payment in the Product-Mix and the simultaneous auctions is given by

$$\mathcal{P}^m(w_A, w_B) = P_A^m(w_A, w_B)w_A + P_B^m(w_A, w_B)w_B + P_{AB}^m(w_A, w_B)\left[w_A + w_B\right]$$

In the sequential auction, the flexible bidder's payment is given by

$$\mathcal{P}^{seq}(w_A, w_B, \overline{w}_B) = P_A^{seq}(w_A, w_B)w_A + P_B^{seq}(w_A, \overline{w}_B)\overline{w}_B + P_{AB}^{seq}(w_A, w_B)[w_A + w_B]$$

Welfare is given by $\mathcal{W}^m = \mathcal{U}^m + \mathcal{R}^m$.

3 Experimental design

Each auction mechanism in a given environment corresponds to one treatment. I run a between-subject design, in which each subject is randomly assigned to one of six treatments. In the *computerised environment*, each subject competes against two computerised bidders for the objects A and B. The computerised bidders represent the two groups of local bidders interested in object A and object B, respectively. In the *human environment*, subjects compete in groups of three for the two objects. In each treatment, subject play a number of bidding rounds. The outcome of each round is shown to the subject at the end of each round and payments are disclosed at the end of the experiment. The study is implemented in oTree (Chen et al. 2016).

3.1 Notation

Each subject $i \in [i] = \{1, ..., I\}$ is assigned to one auction mechanism, denoted by $m \in \mathcal{M}$, and to one environments $e \in [e] = \{1, 2\}$, where $e = 1$ represents the *computerised environment* and $e = 2$ represents *human environment*. Treatments are denoted by $\tau \in \mathcal{T} := \mathcal{M} \times [e]$. Each subject plays R auction rounds, denoted by $r \in [r] = \{1, ..., R\}$. In round r, subject i draws values v_{ir}^A, v_{ir}^B and submits bids w_{ir}^A and w_{ir}^B. The subject's *expected* utility (profit or loss) is denoted by $\mathcal{U}_{ir}$. In each corresponding auction, the expected flexible bidder's payments, the auctioneer's revenue, and welfare are denoted by $\mathcal{P}_{ir}$, $\mathcal{R}_{ir}$, and $\mathcal{W}_{ir}$. In environment $e = 1$, computerised bidders draw values s_{ir}^A, s_{ir}^B. For each subject i, I also observe a vector of covariates c_i. The covariates are established in a survey after the auction rounds. Note that v_{ir}^A and v_{ir}^B are not ordered in the experiment, i.e. a subject's value for object B may be higher or lower than its value for object A, unlike in the theoretical model.[11]

[11] For the analysis, objects may be relabelled such that $v_{ir}^A \leq v_{ir}^B$.

3.2 Treatments and environments

The experiment is run in a 3-by-2 between-subject design. Each of the six treatment groups corresponds to a combination of auction format and environment. The group size is balanced in each session between treatments. The two environments are run in separate sessions for grouping purposes.

Product-Mix Auction. In the Product-Mix auction, subject i submits a bid w_{ir}^A for object A and a bid w_{ir}^B for object B, and is guaranteed to win at most one of the two objects. The winning bid on A and the winning bid on B are those that maximise the sum of the valid combinations of submitted bids. In environment $e = 1$, the flexible bidder's bids can be combined with either of the computerised bidders' bids, i.e. the valid combinations of potential winning bids are $C_1 := \{(w_i^A, s^B), (s^A, w_i^B)\}$. In environment $e = 2$, let $(w_i^A, w_i^B)_{i=1,2,3}$ denote the bids of three subjects competing against each other. All subjects are flexible bidders and the nature of their bids allows winning at most one object. Thus, the valid combinations of potential winning bids are $C_2 := \{(w_i^A, w_j^B)\}_{i,j \in 1,2,3, i \neq j}$.

Sequential Auction. Subject i participates in two sequential first-price auctions. Bids are submitted using the strategy method, whereby the subject makes three simultaneous choices: a bid w_{ir}^J for object J in the first auction, a bid w_{ir}^K for object K in the second auction conditional on having won in the first auction, and a bid $\overline{w}_{ir}^K$ for K conditional on *not* having won in the first auction.[12] In environment $e = 1$, the less valuable object is always sold first, i.e. the two sequential auctions are ordered such that $J = j, K = k$ if values are drawn such that $v_{ir}^j \leq v_{ir}^k$, $j, k \in \{A, B\}$. In environment $e = 2$, the object to be sold first is chosen at random.[13]

Simultaneous Auction. Subject i participates in two simultaneous fist-price auctions, submitting a bid w_{ir}^A for object A and a bid w_{ir}^B for object B. The subject can win in both auctions and would then have to pay for both objects. In each of the two simultaneous auctions, the highest submitted bid wins.

'Subject vs. computerised bidders' ($e = 1$). Each subject plays against two computerised agents. One of the computerised agents is only interested in object A, the other agent is only interested in object B. The computerised agents' exact bids are unknown to the subject, but the subject learns the distribution from which these bids are drawn.

'Groups of 3 subjects' ($e = 2$). In each round, subjects within the same treatments are randomly assigned to groups of three. Each subject can be matched with any other two subjects within the same treatment, including previously matched players, and subjects are made aware of this. The three subjects compete for the two object A and B in the auction mechanism corresponding to their treatment. Each subject knows that it is playing against two opponents with preferences just like themselves, i.e. that each opponent is interested in either of the two objects but not both. To each subject, their opponents' exact values are unknown, but they know the distribution of their opponents' values. In the sequential auction mechanism, one

[12] The strategy method (due to Selten (1967)) has been shown to induce no different behaviour than exposing a subject to the actual move in the game (Brandts & Charness (2011)). Using this method for bid elicitation allows me to collect all relevant bids to compute expected outcomes. Moreover, eliciting bids this way mitigates confounding effects from the two-stage nature of the sequential auction (e.g. wealth effects, emotions).

[13] In environment $e = 2$, I am only interested data points from subjects whose values are such that $v_{i2r}^A \leq v_{i2r}^B$. To account for discarded data points in $e = 2$, I may recruit additional subjects for the sequential treatment (but the overall sample size will be guided by power calculations for environment $e = 1$).

object will be randomly chosen to be sold first, i.e. some bidders will bid for their more valuable object first.

3.3 Rounds

In every bidding round, subjects are assigned values for object A and B. In environment $e = 1$, computerised bidders are also assigned values for object A and B. Subjects' and computerised bidders' values are drawn uniformly (with equal probability) from integer values between 0 and 100 (inclusive), approximating a continuous uniform distribution on $[0, 100]$. The draws are independent between objects, subjects, and rounds. After drawing their values, each subject submits bids corresponding to their treatment, and winners are determined using the respective treatment's allocation rule. Three practice rounds are played first, which are not payoff-relevant. The practice rounds are followed by 20 'actual' auction rounds. Any of those rounds can be payoff-relevant (more detail below).

3.4 Survey

After the auction rounds, each subject is asked to complete a survey and several additional tasks, including a Holt & Laury (2002) risk aversion (probability weighting) test, a risk aversion test following Drichoutis & Lusk (2016), an ambiguity aversion test Gneezy et al. (2015), the adaptive Berlin numeracy test by Cokely et al. (2012)), and a questionnaire on bidding experience, demographics, and self-reported risk attitude (Dohmen et al. 2011). Subjects are also asked to give a brief explanation of their rationale in choosing their bids.

3.5 Payoffs

The auction tasks and three multiple price lists are incentivised with monetary payments. Among the 20 actual rounds played, 3 rounds will be randomly selected to determine payoffs. Note that the average payoffs in each auction round (across all possible induced values) are 15.1 points in the Product-Mix format, 15.0 points in the sequential format, and 14.4 points in the simultaneous format (cf. Table 1).[14] The payoffs from the 3 randomly selected rounds are added together. If this sum is non-negative, it will be added to the subject's payoff tally. If the sum is negative, the payoff tally remains at zero.

One list among the three multiple price lists is randomly selected (with equal probability). From the decision sheet of this list, one decision is selected to be payoff-relevant. The payoff from this decision is added to the subject's payoff tally. The average payment from the three multiple price lists is 15.28 points.

The subject's final tally will be converted to GBP, at a rate of 10 points = 1 GBP for the earnings from the auction task in all three treatments, and at a rate of 10 points = 1 GBP for the earnings from the multiple price lists. Thus, subjects have identical perceived incentives across treatments. Average payoffs are slightly higher in the Product-Mix and the sequential auction format, but subjects have an equal chance of being in any treatment, hence are treated ex-ante the same.

[14] The computerised environment is taken as a reference point for both environments, as theoretical predictions and simulations are only feasible in this environment.

Based on a small, technical pilot the study is expected to last approximately 30 minutes for treatments in the computerised environment, and 45 minutes for treatments in the human environment. The increased duration in the human environment is due to additional waiting time for the arrival of all participants (within treatment) to form subgroups of 3 players.

The variable average payments for the study are therefore {£12.16/hour, £12.14/hour, £11.72/hour} for the {Product-Mix, sequential, simultaneous} treatments in the computerised environment, and {£8.11/hour, £8.09/hour, £7.81/hour} for the {Product-Mix, sequential, simultaneous} treatments in the human environments. In addition, all participants who complete the study receive a completion fee of £3. Total hourly average payoffs are higher in the computerised environments, but subjects have an ex-ante equal chance of being in either environment.

In the Product-Mix format, subjects cannot make any losses. In the sequential and simultaneous format, however, subjects can make a loss if they do not bid wisely. Although it is optimal to bid such that one would never make a loss, i.e. $w^A + w^B \leq v^B$, it is possible that subjects overbid. The probabilities to win objects A and B are very low, on average 1.5 (3.6)%, if one bids optimally (Table 30).[15] Even if one were to overbid 'in the worst possible way', i.e. to bids one's induced values[16] the average probabilities of winning both objects are only at 25.7% (Table 31). Overbidding in this way results loss of -13.80 points in the sequential and the simultaneous auction (Table 32).[17] Of course, payments are bounded from below at zero. However, because I select multiple, payoff-relevant rounds to calculate payments, I believe the incentives in the sequential and simultaneous format are still intact.[18]

3.6 Further details

Comprehension quiz. After reading the instructions, subjects have to pass a comprehension quiz to be admitted for participation in the experiment. To pass, they must answer at least four questions of different types correctly, and two questions of different subtypes. Each type corresponds to a specific allocation to the 'participant' in the question.[19] In one subtype, subjects are asked to determine the winning bid for one of the two objects, and in the other subtype, subjects are asked to calculate earnings based on given values, allocations, and prices. Subjects have an unlimited number of attempts. After a wrong attempt, the correct answer is shown and an additional question of the same type is added to the end of the quiz. The questions alternate in subtypes while at most one question of each subtype is answered correctly. The initial order of the questions is randomised.

There is no time limit to answer any one question, but there is a (generous) overall time limit for passing the quiz, after which subjects would time out and be excluded from the study.

[15] The probabilities are averaged across all possible (uniformly distributed) induced values.

[16] Subjects are constrained to bid at most their value on each object.

[17] Overbidding this way always leads to payoffs of at most zero, but in those cases where the subject wins both objects, it will result in a loss amounting to the price paid for the lower value object. The losses are averaged across all possible (uniformly distributed) induced values.

[18] Endowments would provide more guarantee that the lower bound is not hit. However, endowments come with several drawbacks as well: they would diminish incentives to 'try hard' in each auction round, and it is difficult to determine an appropriate amount. In the Product-Mix auction, endowments are not necessary, and introducing treatment-specific endowments also distorts incentives between treatments. Therefore, I choose not to provide subjects with endowments.

[19] There are only three different types in the PMA treatments.

Attention checks. Throughout the study, subjects will be asked to complete attention checks, confirming their continued participation in the study. To do so, they must submit the page within a time limit of 1 minute. Failure to complete the attention check results in exclusion from the study without payment.

Timeouts. Subjects will have a fixed time of 45 seconds for each auction round in the experiment. A timer appears when 20 seconds are left to alert subjects to the remaining time. Subjects also have a fixed time of 4 minutes for each of the incentivised, additional tasks (timer appears when 90 seconds are left) and a fixed time of 8 minutes for the survey (timer appears when 2 minutes are left). For reading the instructions, subjects have 8 minutes (timer appears when 1 minute are left). For passing the comprehension quiz, subjects have 10 minutes (timer appears when 2 minutes are left).

On most pages, timing out does not result in exclusion from the study, but subjects are automatically advanced to the next page. In every auction round, in the comprehension quiz, in the survey, and in the additional, incentivised tasks (multiple price lists), subjects must advance within the time limit; otherwise, they are excluded from the study (and they are specifically made aware of it). Participants who fail to complete the comprehension quiz receive £2 for their participation. All other participants only receive payment if they complete the study.

Dropouts. A subject may qualify as a dropout if they manually exit the session, their browser or internet connection is interrupted for any reason, or if they time out during the quiz, the auction rounds, the survey, or the additional, incentivised tasks. If possible, the subject will be redirected to the end of the experiment. If the subject qualifies as a dropout, they do not receive any payments (except if they fail to complete the comprehension quiz) and this is emphasised in the instructions.

4 Simulations

I simulate bidding in the computerised environment building on the theoretical model set out in Section 2, assuming that flexible bidders are risk-neutral quasi-linear utility maximisers. From $10,000$ independent, uniform integer draws of $v_A, v_B \in [0, 100]$, I calculate optimal bids and corresponding expected utility of the flexible bidder, expected revenue, and expected welfare. Average outcomes, standard deviations, medians and quartiles Q1 and Q3 are presented in Table 1. The results are striking: the Product-Mix and the sequential format perform nearly identically on average. For the flexible bidder, these two formats could be thought of as more advantageous than the simultaneous auction, but it is surprising that these two mechanisms perform nearly *identically* with respect to all outcome variables. The simultaneous auction performs only slightly worse than the other two auction formats: approximately 5% in utility, 7% in flexible bidder payments, 0.6% in revenue, and 1% in welfare.

5 Model and evaluation

Let y_{ir} denote an outcome variable in $\{\mathcal{U}_{ir}, \mathcal{P}_{ir}, \mathcal{R}_{ir}, \mathcal{W}_{ir}\}$. Recall that each subject draws values v_{ir}^A, v_{ir}^B and submits bids w_{ir}^A, w_{ir}^B. I denote by t_i^τ, $\tau \in \mathcal{T}$ a binary treatment variable which equals one if subject i is in treatment τ and zero otherwise. The corresponding coefficients are

		Mean	SD	Median	Q1	Q3
$\mathcal{U}$	pma	15.18	8.73	15.17	7.86	22.08
	seq	15.14	8.73	15.05	7.85	21.99
	sim	14.44	8.06	14.56	7.63	21.17
$\mathcal{R}$	pma	106.97	3.79	107.11	103.79	110.17
	seq	106.78	3.67	106.91	103.71	109.89
	sim	106.29	3.33	106.46	103.54	109.17
$\mathcal{W}$	pma	122.16	12.5	122.28	111.7	132.29
	seq	121.92	12.37	122.06	111.54	131.93
	sim	120.73	11.32	121.09	111.26	130.33
$\mathcal{P}$	pma	13.51	7.23	13.85	7.47	19.71
	seq	13.56	7.34	13.81	7.42	19.77
	sim	12.57	6.67	12.91	7.08	18.34

Table 1: Simulated average outcomes and standard deviations across auction mechanisms

given by θ^τ, $\tau \in \mathcal{T}$. I evaluate each environment separately and compare mechanisms pairwise. The main hypotheses are tested using equivalence tests (e.g. with R-package 'TOSTER' by Lakens (2017)). For robustness checks, I use regressions. For all treatments $(j,e) \neq (k,e) \in \mathcal{T}$, $e = 1, 2$, the model is given by

$$y_{ir} = \theta^{(j,e)} + \theta^{(k,e)} t_i^{(k,e)} + \epsilon_{ir}$$

where ϵ_{ir} is an error term orthogonal to the treatment variables and $\epsilon_{ir} \sim \mathcal{N}(0, \sigma_\epsilon)$. Due to the randomisation of the treatments and values in each round, population orthogonality and the rank condition are satisfied, so $\theta^\tau, \tau \in \mathcal{T}$ can be consistently estimated using least squares.

Additional robustness checks include covariates c_i and drawn values v_{ir}^A and v_{ir}^B (and potentially their interaction). To account for any additional subject-specific variation I also estimate a model with random subject-specific intercepts, given by

$$y_{ir} = \theta^{(j,e)} + \theta^{(k,e)} t_i^{(k,e)} + z_i + u_{ir}$$

where the error term is decomposed in a residual between-subject error $u_{ir} \sim \mathcal{N}(0, \sigma_u)$ and the random effect $z_i \sim \mathcal{N}(0, \sigma_z)$. The standard assumptions $E(u_{ir} \mid t_i^{(k,e)}, z_i) = 0$ and $E(z_i \mid t_i^{(k,e)}) = E(z_i) = 0$ clearly hold because of the randomisation into treatments and the random assignment of values in each round; hence z_i is independent of $t_i^{(k,e)}, k \in \mathcal{M}$. I may also include additional round-specific random intercepts. The random effect model can be estimated, e.g., with the R-package 'lme4' (Bates et al. 2015).

Hypotheses. My hypotheses are formulated for absolute differences between outcomes.[20] I test for pairwise equivalence of outcomes between all auction formats. My hypotheses are formulated below.

[20] A previous version set out to compare relative differences. After a few sessions, however, it was evident that a non-trivial number of subjects obtained negative or zero payoffs even in expectation. Therefore, a log-transformation is not feasible.

(1) $\theta^{(pma,e)}$ and $\theta^{(seq,e)}$ are equivalent up to ±0.2SD equivalence bounds (Null hypothesis $\theta^{(pma,e)} \neq \theta^{(seq,e)}$)

(2) $\theta^{(pma,e)}$ and $\theta^{(sim,e)}$ are equivalent up to ±0.2SD equivalence bounds (Null hypothesis $\theta^{(pma,e)} \neq \theta^{(sim,e)}$)

(3) $\theta^{(seq,e)}$ and $\theta^{(sim,e)}$ are equivalent up to ±0.2SD equivalence bounds (Null hypothesis $\theta^{(seq,e)} \neq \theta^{(sim,e)}$)

The hypotheses for $e = 1$ follow immediately from the simulations building on the theoretical model, and the hypotheses for $e = 2$ use this as a reference point. The standard deviation for computing the equivalence bounds will be taken from the simulated data also for the equivalence tests of the real data.[21] Hypotheses (1), (2), and (3) are powered for $e = 1$, and the analysis for $e = 2$ should be seen as exploratory. Further analysis will consider within-treatment effects across environments, aiming to understand, e.g., which bidding environment is more competitive for the flexible bidder, with its implications for revenue and welfare. From a theoretical angle, it is not feasible to predict if a competitive fringe or a multi-product oligopoly market are easier to compete against.

5.1 Power analysis

I power the analysis with respect to hypotheses (1), (2), and (3). Effects between the computerised and the human environment will be studied from an exploratory perspective. I conduct the power analysis with the R package 'TOSTER' (Lakens 2017). I also correct for serial correlation within observations from a single subject between rounds (cf. Diggle et al. (2002), Potvin & Schutz (2000), Liu & Wu (2005)), by setting $\tilde{n} = (1 + (R-1)\rho) \cdot n/R$ as the serial-correlation-adjusted sample size, where R is the number of observations per subject and ρ is the correlation between observations. Setting ρ a priori is not trivial. I follow the example of Drichoutis et al. (2015) who demonstrate that their study is powered for a variety of parameter constellations, including the values $\rho \in \{0.1, 0.3, 0.5\}$.[22] The pooled standard deviation of the expected payoff for the comparison of the PMA and the SEQ treatment, for example, is given by $SD = sqrt((8.73^2 + 8.73^2)/2) = 8.73$. The hypothesised effect size is $\delta = 0$ and I choose a significance level of $\alpha = 0.05$ and equivalence bounds $\pm\Delta = \pm 0.2SD$, where the pooled standard deviation SD depends on the outcome and the two treatments that are tested for equivalence. For testing equivalence of expected payoffs between PMA and SEQ, e.g., the bounds are given by $\pm 0.2 * 8.73$. Note that the required sample size will be identical for all outcomes, as the bounds vary with the corresponding standard deviation. I present power $1 - \beta$ and sample sizes for various scenarios of within-subject correlation ρ in Table 7. I choose a sample size of 185 subjects per treatment arm of the computerised treatments. This powers my hypotheses conservatively for a serial correlation between rounds of up to $\rho = 0.4$. I expect $\rho = 0.4$ to be a

[21] The subject data may have an even greater variance, such that basing equivalence bounds the corresponding standard deviation may not be meaningful.

[22] Drichoutis et al. (2015) base their results on computations for previous auction experiments. In their experiment, they have 24 rounds with 3 blocks of 8 randomly chosen, induced values, from a range of only 8 values to choose from in total. Because in my setup bidders play 20 rounds with completely independent values for *two* goods instead of one, their chosen correlation may serve as an upper bound for my study.

strict upper bound (cf. Footnote 22). The sample size for the human treatments may be chosen somewhat lower as they serve an exploratory analysis.

6 Results

6.1 Data collection

Subjects participated in this study in a virtual lab environment on their own device. CESS at Nuffield College, Oxford, provided the virtual lab environment and collected the data. At the time of writing, the data collection is not yet complete and two thirds of the desired sample size remain to be collected. However, the existing data already reveals interesting, albeit preliminary results. The data collection took place in September and October 2022 with subjects between 18 and 65 years of age from all over the UK. Subjects interacted only anonymously and their non-anonymous data is protected by the experimental laboratory CESS, from whom I obtained completely anonymised and unidentifiable datasets. Data was collected on 10 different days, where on most days 2 sessions were run, one with the computerised environment, and one with the human environment. The order of environments was randomised for each days. On few days, 2 computerised and 2 human environment sessions were run. For each session, between 50 and 75 subjects were recruited with approximately 15% no-shows. The sessions on the first two days served as technical pilot sessions and were helpful in making improvements to the design, e.g. inclusion of attention checks, spreading the presentation of examples, and simplifying the comprehension test.

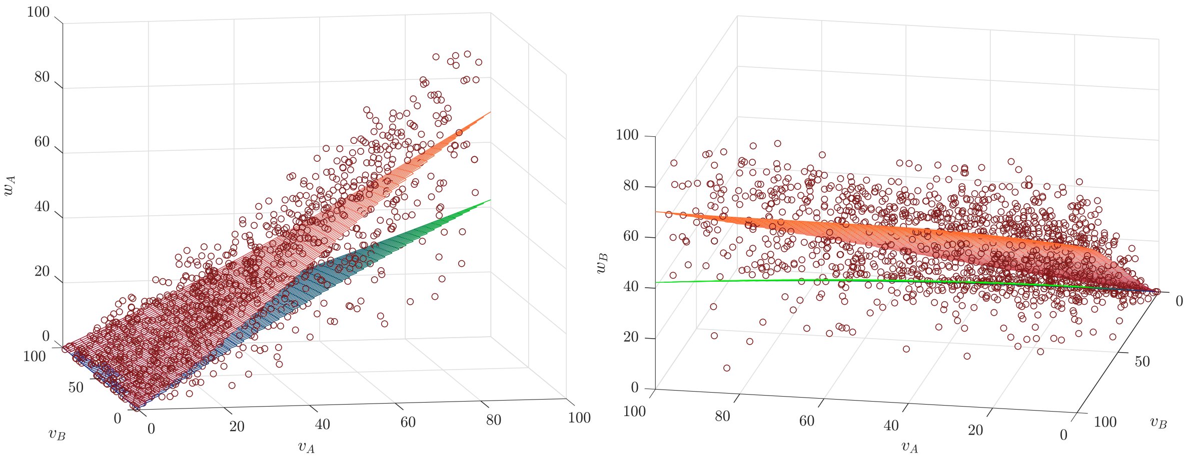
Across the 24 sessions, 370 subjects completed the study. About 11% did not pass the comprehension test and were therefore excluded from the study. Further details are given in Table 6 in Appendix C. Among those who completed the study, the average and median time spent on the study were between 40 and 41 minutes. The quickest participant finished in 15 minutes, and the slowest participant took 64 minutes. Of the 370 subjects, each played 20 (non-practice) auction rounds. In the following, I provide a graphical analysis of bidding behaviour as well as descriptive summaries of outcomes.

6.2 Bidding behaviour

Subject draw independent value for object A and object B in each round. In the experiment, either object could be the less valuable one, but for the analysis I relabelled objects such that object A is always worth less object B.[23] For the computerised environments I calculate, given the subjects' drawn values, their optimal bids. In Figs. 2, 4 and 5 I show the observed bids in the computerised environments as scatter plots, together with fitted polynomial surfaces and the corresponding optimal bid surfaces. In the computerised treatments, we observe significant overbidding compared to the risk-neutral Bayes-Nash equilibrium. Relative overbidding appears to be strongest in the simultaneous auction, and in bids for object B conditional on winning A in the sequential auction. In Figs. 3, 6 and 7, I plot the observed bids in the human environments,

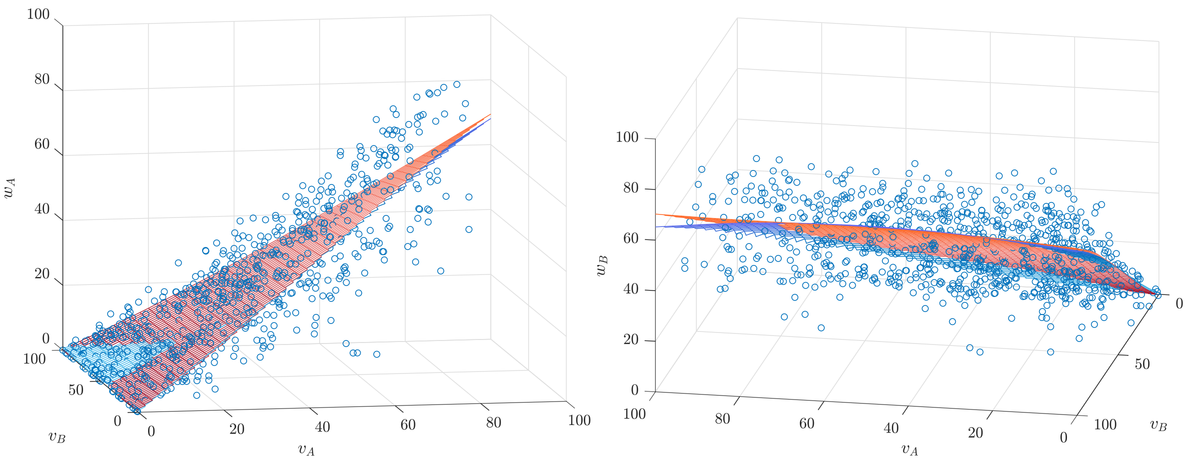
[23] Note that in the sequential auction in the human environment, this is not possible. To keep priors about the distribution of values independent, it could not be the case that the cheaper object is sold first for every bidder. Therefore, I have to discard about half of the data in the SEQ/h treatment.

together with fitted polynomial surfaces and the fitted polynomials from the corresponding computerised treatment. In most auction formats and bids, the fitted surfaces are very close, with an exception of the sequential auction, where bids for the more valuable object B are higher in the human than the computerised environment.



(a) Bids for object A (b) Bids of object B

Figure 2: Product-Mix format computerised: observed bids, fitted polynomial surface (red), and optimal bid function (blue/green)



(a) Bids for object A (b) Bids for object B

Figure 3: Product-Mix format: observed bids and fitted polynomial surface (blue) for human environment, fitted polynomial surface computerised (red)

6.3 Outcomes

In all treatments, payoffs are observed as direct outcomes of the experiment and compute revenue and welfare. For the computerised treatments, I also consider the payments of the flexible bidder. In computerised treatments, I compute expected outcomes based on the observed bids and values.[24] The main goal of this section is to compare expected outcomes in

[24]For a very large number of observations, the expected outcomes are, on average, equal to average realised outcomes, as the opponents' bids replicate the density which they are drawn from. However, the distribution of

the computerised treatments with expected outcomes based on optimal bids (cf. simulations in Section 4). In human environments, i.e. auction sessions in which subjects played against other human participants in groups of three, I cannot compute expected or optimal outcomes. However, I compare realised outcomes of the human treatments to the realised outcomes of the corresponding computerised treatments. As the two environments are very alike in the market setting, except for the type and strength of opponents a subject is facing, I obtain some results on the robustness of the theoretical model with competitive fringes.

In Figs. 9 and 10 I plot the densities of all considered outcomes, and the outcomes based on optimal bidding in the computerised version as a benchmark. We observe that realised payoffs have a spike at zero, which is to be expected as the realised payoff is zero whenever the subject does not win anything. Expected payoffs, however, still have a spike close to zero, indicating that a substantial number of subjects may have overbid by a significant amount, even close to their value, and this is confirmed by the data. These results hold across treatments, as well as the fact that realised and expected payoff distributions are skewed to the left and are tailing more than optimal payoffs. Revenues are consequently also much more tailed, and, at least in expectation, some probability mass is shifted to the right: the bidders' loss in payoff is often the auctioneer's gain in revenue. Surprisingly, the distributions of expected and optimal welfare very close. This immediately suggest that my theoretical model may be doing well in predicting efficiency of the considered auction formats. However, the distribution of welfare between the auctioneer and bidders is skewed towards the auctioneer in the experimental data. In the PMA treatment, realised welfare is shifted to the right compared to optimal and expected outcomes. Payments of the flexible bidder (the subjects) have, as expected from the preceding discussion, a spike at zero and a long tail to the right compared to payments under optimal bidding.

In Fig. 11, I plot the densities of expected outcomes, comparing the three computerised treatments in each outcome. It is striking how similar the distributions are, encouraging the detailed equivalence analysis I set out to do in the pre-analysis (see Section 6.5). The density plots of outcomes in the three human treatments (Fig. 12), however, exhibit a different pattern: while revenues across all treatments and other outcomes between SEQ and SIM are very similarly distributed, the density of payoffs, efficiency, and bidder surplus in the Product-Mix format is shifted significantly to the right, compared to SEQ and SIM. Note that the flexible bidder payments equal revenue in the human environment, whereas bidder surplus equals payoff in the computerised environment (computerised agents make zero profit).

I briefly present some summary statistic of outcomes. Expected outcomes are shown in Table 2. Comparing to the theoretical predictions (Table 1), average expected payoffs $\mathcal{U}$ are between 35% and 47% lower than predicted. Correspondingly, payments of the flexible bidder $\mathcal{P}$ are between double or triple of the predicted amount, with a large standard deviation, and revenue is also somewhat, between 6% and 9% higher. Surprisingly, the disadvantage of the flexible bidder does not come at the cost of efficiency.

When looking at the realised outcomes (Table 3 and Table 4), I first note that the outcomes in the computerised treatments are, on average, identical to the expected outcomes, meaning the number of observations is large enough to account for the whole distribution of the opponents'

expected outcomes and realised outcomes will still differ. For this reason and to avoid any small sample bias, I chose to work with expected outcomes.

values. Naturally, the variability in realised outcomes is higher.

The comparison of outcomes between the computerised and human environments needs some qualification. Note that the environments were chosen to be as similar as possible, while transforming the competitive fringe into a less competitive kind of oligopoly. Each flexible bidder makes a bid in structure identical to the computerised environment, and they may win either object A, object B, neither object, or, in treatments SEQ and SIM, both objects. However, their guess about the opponents' highest competing bid on each good, is much more difficult, as it depends on their values and strategies. In the sequential and simultaneous auctions, this resulted in lower average payoffs for each bidder. However, in the Product-Mix format, bidders did remarkably well, comparing to the payoffs against competitive fringes and, even more, against the other auction formats in the oligopoly environment: the average payoff is 90% higher than in the sequential and 156% higher than in the simultaneous auction.

Although the auctioneer's revenue is lower in the PMA than other auctions, the higher surplus translate also into stronger performance in terms of efficiency: the PMA is 12% more efficient than the other auction formats. I note that for the computation of average payoffs in the sequential treatment, I only use subjects for whom the cheaper object was sold first, in order to not distort the comparison further.[25] However, revenue, welfare, and bidder surplus can only sensibly reported for 'complete markets', i.e. groups of three bidders, and therefore all subjects in SEQ/h are included.

Table 2: Expected outcomes summary statistics

		Mean	SD	Median	Q1	Q3
$\mathcal{U}$	pma	9.84	7.33	8.58	3.74	14.79
	seq	8.60	9.82	7.94	2.87	15.01
	sim	6.80	10.68	6.59	1.79	13.69
$\mathcal{R}$	pma	114.73	11.58	112.39	105.44	121.71
	seq	113.65	11.97	111.05	104.55	119.73
	sim	116.22	13.51	113.00	105.63	123.13
$\mathcal{W}$	pma	124.56	15.25	123.65	111.86	136.67
	seq	122.25	13.85	122.40	109.95	133.63
	sim	123.02	13.12	123.76	111.59	133.83
$\mathcal{P}$	pma	27.65	20.80	23.93	10.57	42.25
	seq	27.30	23.93	22.10	9.11	39.47
	sim	32.44	27.02	26.00	11.25	46.25

6.4 Covariates

For robustness checks, I also collected a number of covariates after the auction task in the study. I show contingency tables of the means of expected outcomes for all computerised environments in Appendix C.3. The most obvious trend is suggested by Table 15, where higher scores show higher average expected payoffs in all three treatments. Otherwise, there are no obvious trends in covariates. The tables suggest, however, that some variation across covariate

[25]The difference to including all subjects in SEQ/h is small.

Table 3: Outcomes summary statistics computerised treatments

		Mean	SD	Median	Q1	Q3
$\mathcal{U}$	pma	9.34	12.66	2.00	0.00	15.00
	seq	8.78	17.62	0.00	0.00	18.00
	sim	6.63	17.99	0.00	0.00	15.00
$\mathcal{R}$	pma	115.27	34.39	117.00	92.00	140.00
	seq	113.90	36.06	115.00	90.00	140.00
	sim	117.09	35.35	119.00	91.00	143.25
$\mathcal{W}$	pma	124.61	33.56	125.00	102.00	149.00
	seq	122.68	36.81	124.00	96.00	152.25
	sim	123.72	36.37	127.00	97.00	151.00
$\mathcal{P}$	pma	27.31	29.27	20.00	0.00	52.25
	seq	27.77	34.07	10.00	0.00	50.00
	sim	32.77	37.55	22.50	0.00	60.00

Table 4: Outcomes summary statistics human treatments

		Mean	SD	Median	Q1	Q3
$\mathcal{U}$	pma	13.20	13.92	10.00	0.00	22.00
	seq	6.96	16.64	1.50	0.00	15.00
	sim	5.16	16.98	0.00	0.00	13.00
$\mathcal{R}$	pma	105.98	24.98	106.00	87.00	124.00
	seq	109.42	28.51	110.00	89.00	129.00
	sim	113.73	27.86	115.00	95.00	134.00
$\mathcal{W}$	pma	145.57	28.61	148.00	124.00	169.00
	seq	129.71	36.13	131.00	97.00	160.00
	sim	129.22	36.78	135.00	95.00	160.00
$\mathcal{S}$	pma	39.59	18.63	38.00	25.00	51.00
	seq	20.29	31.81	23.00	10.00	40.00
	sim	15.49	32.87	22.00	4.00	36.00

outcomes does exist. For example, there may be a difference in payoffs between female and male participants in SEQ and SIM, but not PMA.[26] Payoffs also vary somewhat with age, ethnicity, and socio-economic status.[27] We see payoff variation in income, e.g. subjects with doctoral degrees have the highest payoffs in SIM but lowest payoffs in SEQ. Subjects with intermediate self-reported risk aversion appear to have higher payoffs than those with extreme risk attitudes. The multiple-price-list tests after Drichoutis et al. (2015) and Holt & Laury (2002) confirm only that more risk-seeking subjects may have lower payoffs. Strongly ambiguity-seeking subjects also seem to have lower payoffs, but a trend is, as with risk-aversion, not apparent. In the PMA and SIM format, the perceived difficulty of bidding in the auction rounds seems to be correlated with expected payoffs, while in the sequential such trend is not apparent.

A more in-depth analysis is beyond the scope of this thesis; nonetheless, details (also on revenue, efficiency, and flexible bidder payments) are shown in Tables 9 to 15 and 17 to 20.

6.5 Hypothesis testing

In this section, I formally test the hypotheses of pairwise equivalence between the auction formats with respect to the outcomes of expected payoffs, expected revenue, expected efficiency, and expected payments of the flexible bidder. These equivalence tests are only done for the computerised environments. I report the results from two-one-sided-tests procedures (TOST) for payoffs in Table 8. Details are given in Tables 25 to 27. The equivalence bounds for each outcome are set at $\pm 0.2 * SD$, where SD is the pooled standard deviation across the considered treatments. The significance level is $\alpha = 0.05$ and 0.9-confidence intervals are used. All equivalence tests are run with the R-package 'TOSTER' (cf. Lakens (2017)).

Equivalence is rejected for almost all pairwise comparisons at the pre-selected bounds (the tests fail to reject the null hypothesis of equivalence). The exception is equivalence in welfare between the sequential and the simultaneous auction, but other confidence intervals are also

[26] Other gender categories have a too low number of subjects to report averages.

[27] Some variation like very low payoffs for subjects who prefer not to reveal their socio-economic status is puzzling.

just beyond the set equivalence bounds: payoffs and flexible bidder payments in SEQ vs. SIM and welfare in PMA vs SIM. I plot the distribution of mean differences in Fig. 8. I also conduct robustness checks with the Wilcoxon TOST equivalence test (Lakens 2017) and TOST equivalence test with bootstrapped standard errors (Lakens 2017). The robustness checks confirm the findings of the standard equivalence tests, with the exception of the Wilcoxon test for the equivalence of payoffs between PMA and SEQ.[28]

6.6 Regression analysis

To check the difference in outcomes for robustness, I run the linear regressions (Table 5), also including covariates (Table 29) and subject-specific random effects (Table 28). This section only analyses the outcomes from computerised environments.[29]

Table 5: Linear models, testing for difference in expected outcomes, computerised environment

	Dependent variable:			
	Exp. payoff	Exp. revenue	Exp. welfare	Exp. payments
	(1)	(2)	(3)	(4)
PMA/c vs. SEQ/c	1.238***	1.078**	2.315***	0.346
	(0.326)	(0.448)	(0.557)	(0.849)
SEQ/c	8.597***	113.651***	122.248***	27.301***
	(0.241)	(0.331)	(0.412)	(0.628)
PMA/c vs. SIM/c	3.032***	−1.491***	1.541***	−4.791***
	(0.335)	(0.462)	(0.524)	(0.884)
SIM/c	6.804***	116.219***	123.023***	32.438***
	(0.240)	(0.331)	(0.376)	(0.633)
SEQ/c vs. SIM/c	1.794***	−2.568***	−0.775	−5.137***
	(0.397)	(0.494)	(0.519)	(0.988)
SIM/c	6.804***	116.219***	123.023***	32.438***
	(0.271)	(0.338)	(0.355)	(0.675)
Note:				*p<0.1; **p<0.05; ***p<0.01

For the inclusion of covariates, I run a step-wise search (backward direction) for the best combination of covariates, which outputs nearly all covariates for all comparisons. For simplicity, I therefore include all covariates in the analysis. In the standard linear model, I find, in line with Table 8, a significant difference in payoffs and revenue in all pairwise comparisons. The difference in welfare is not significant in the comparison of SEQ/c and SIM/c, and the difference in payment is not significant in the comparison of PMA and SEQ.

[28] The standard test for equivalence, the bootstrap version, and the test for difference provide, together, overwhelming evidence that equivalence does not hold.

[29] A regression analysis for the human environment is planned in future work, but beyond the scope of this thesis.

The random effect model confirms the difference between payoffs, but standard errors increase substantially for the revenue comparison; as a result, the difference PMA vs. SEQ and PMA vs. SIM is not significant anymore, and similarly for the comparison of payments between PMA and SEQ. The difference in welfare between SEQ and SIM is non-significant throughout; indeed the TOST procedure confirmed equivalence in welfare given the set bounds. Overall, the results of the random effect model are encouraging: the subject-specific variance is small relative to the residual variance for most estimates. This indicates that observations are largely independent between rounds. The variation in my estimated differences is predominantly (but of course not exclusively) by subject-independent.

Finally, I note that the inclusion of covariates naturally changes the estimates in size, but not in direction, which provides some robustness. Because, at the time of writing, the sample is not complete yet, the results from the regression with covariates are to be interpreted with caution. Some categories like the category 'Prefer not so say' in gender do not have enough observations to allow for a meaningful interpretation of the effect. Similarly, the number of auction experiments participated in appears to have some significant, but in size rather erratic effects, that may be only spurious. The subjects' numeracy score has a significant effect on welfare but not on payoffs. This may be due to some endogeneity, as Table 15 shows an unambiguous, increasing trend in payoffs in all treatments. The number of errors subjects made in the comprehension quiz seems to be a significant predictor consistent across payoffs, payments, and revenue. However, a closer look at Table 16 suggests that this may be driven mainly by the simultaneous auction treatment.

7 Conclusion

My experiment delivers the first evidence for the comparison of strategic bidding in the Product-Mix auction, a sequential auction, and a simultaneous auction. While the results should still be considered preliminary, there is some clear indication that the Product-Mix format performs best in terms of bidder surplus and efficiency, whereas from the auctioneer's perspective, a simultaneous auction may be preferred as it achieves the highest revenue. These effects are even more pronounced in the treatments with symmetric, flexible bidders. These results contrast the theory developed alongside the experimental study: all pairwise equivalence tests fail to reject a hypothesised difference between outcomes, with the exception of efficiency in the sequential and simultaneous auction.

Bidders with unit demand face a difficult strategic choice when several independent auctions offer the opportunity to win an object. Regardless if a sophisticated, flexible bidders and competitive fringes, or several sophisticated bidders participate in the auction, optimal bidding behaviour is non-trivial. Indeed, bidding in the experiment was far from optimal; a brief analysis of bidding behaviour showed strong evidence of overbidding. Future work may attempt to shed light on why the different auction formats generate such different outcomes. For example, systematic differences in bid-shading across the auction formats may be a possible channel. Nonetheless, the presented results already indicate that an auction format with combinatorial bids like the Product-Mix auctions is the vastly more efficient solution in markets for heterogeneous substitutes.

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A Proofs

Proof of Proposition ??.

The flexible bidder's payoff function simplifies to

$$\begin{aligned}&\mathcal{U}^{pma}(w_A, w_B) = \\ &-\frac{1}{2}{w_A}^3 + {w_A}^2\left(\frac{1}{2}v_A - \frac{1}{2}v_B + \frac{3}{2}w_B - 1\right) + v_B w_B - {w_B}^2 + v_A w_A(1 - w_B)\end{aligned}$$

First-order conditions are

$$-\frac{3}{2}{w_A}^2 + w_A\left(-2 + 3w_B + v_A - v_B\right) + v_A(1 - w_B) = 0 \tag{5}$$

$$\frac{3}{2}{w_A}^2 + v_B - 2w_B - v_A w_A = 0 \tag{6}$$

Substituting (6) into (5) yields

$$9{w_A}^3 - (9v_A + 6)\, {w_A}^2 + \left(2{v_A}^2 + 4v_A + 2v_B - 8\right) w_A - 2v_A v_B + 4v_A = 0 \tag{7}$$

If an interior solution exists, it must be a stationary point of $\mathcal{U}^{pma}(w_A, w_B)$, i.e. the optimal w_A must be a root of equation (7). Let $p(w_A) := \alpha {w_A}^3 + \beta {w_A}^2 + \gamma w_A + \delta$ denote the polynomial on the left hand side of (7), where

$$\begin{aligned}\alpha :=& 9 \\ \beta :=& -(9v_A + 6) \\ \gamma :=& 2{v_A}^2 + 4v_A + 2v_B - 8 \\ \delta :=& -2v_A v_B + 4v_A\end{aligned}$$

Using *Mathematica*, one can easily verify that the discriminant of $p(w_A)$

$$\Delta_p = 18\alpha\beta\gamma\delta - 4\beta^3\delta + \beta^2\gamma^2 - 4\alpha\gamma^3 - 27\alpha^2\delta^2$$

is strictly positive for $v_A, v_B < 2$, hence $p(w_A)$ possesses three distinct real roots. Existence of at least one real root follows from the argument given below for the existence of an interior solution. Furthermore, we know that $p(w_A) \to -\infty$ as $w_A \to -\infty$ and $p(w_A) \to \infty$ as $w_A \to \infty$. It is also easy to check that

$$\begin{aligned} p(0) &= \delta && > 0 && \text{if } v_B < 2 \text{ and} \\ p(1) &= -5 - v_A + 2{v_A}^2 + 2v_B - 2v_Av_B && < -5 + 2v_B < 0 && \text{if } v_B < 2 \end{aligned}$$

Thus, by the intermediate value theorem, $p(w_A)$ has exactly one real root on $(0,1)$. Moreover, one can easily check that $p(v_A) < 0$ for $v_A < 2$, hence the solution is such that $w_A < v_A$. From equation (6) one can derive w_B as a function of w_A. Therefore, if w_A is uniquely defined, and so is w_B. Global optimality is immediate from Proposition **??**. □

Comparative statics for the product-mix auction. The implicit function theorem applied to the first-order conditions yields

$$\begin{aligned} \frac{\partial w_A}{\partial v_A} &= \frac{\frac{1}{2}w_A(3w_A - v_A) + w_B - w_A - 1}{\frac{1}{2}(3w_A - v_A)^2 + 3(w_B - w_A) + v_A - v_B - 2} \\ \frac{\partial w_A}{\partial v_B} &= \frac{\frac{1}{2}(v_A - w_A)}{\frac{1}{2}(3w_A - v_A)^2 + 3(w_B - w_A) + v_A - v_B - 2} \\ \frac{\partial w_B}{\partial v_A} &= \frac{\partial w_A}{\partial v_A}\frac{3w_A - v_A}{2} - \frac{w_A}{2} \\ \frac{\partial w_B}{\partial v_B} &= \frac{\partial w_A}{\partial v_B}\frac{3w_A - v_A}{2} + \frac{1}{2} \end{aligned}$$

First, we note that $w_A \leq \frac{v_A}{2}$. This follows because $p(\frac{v_A}{2})$ is strictly negative. From Eq. (4), it follows then that $w_B \leq \frac{v_B}{2}$. Using these upper bounds on the optimal bids w_A and w_B, one can establish that the following holds: $\frac{\partial w_A}{\partial v_A} > 0$, $\frac{\partial w_A}{\partial v_B} < 0$, $\frac{\partial w_B}{\partial v_A} < 0$, and $\frac{\partial w_B}{\partial v_B} > 0$.

Proof of Proposition ??. Assuming $v_B \leq 1$, we can restrict the analysis to $0 \leq w_B < 1$.
Case 1: Bidder won the auction for object A. The payoff function is given by

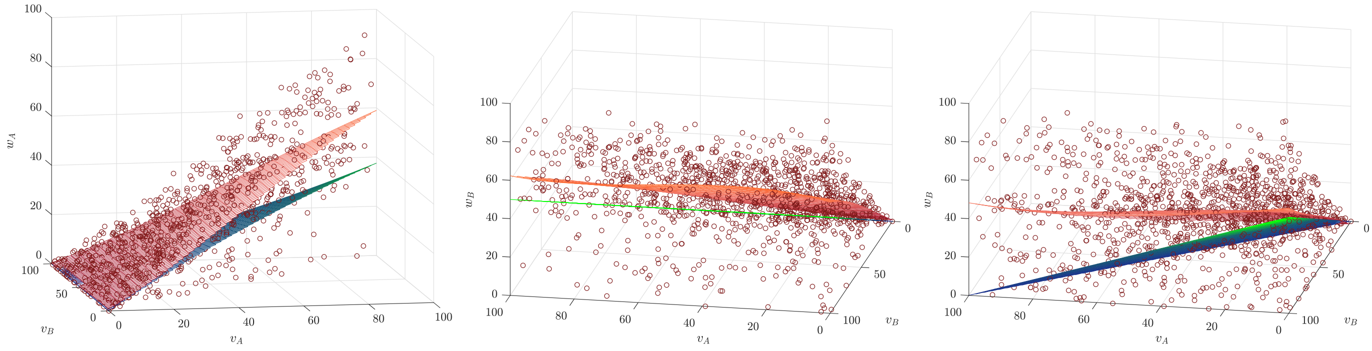
$$\begin{aligned} &\mathit{Prob}\,(\text{bidder wins } B)\,(v_B - w_B - w_A) + \mathit{Prob}(\text{bidder loses } B)(v_A - w_A) \\ =&w_B(v_B - w_B - w_A) + (1 - w_B)(v_A - w_A) \end{aligned}$$

The first-order condition yields ${w_B}^* = \frac{v_B - v_A}{2}$ and this is the global maximum if $v_B < 2 + v_A$. The bidder's conditional expected second stage payoff is $\frac{(v_B - v_A)^2}{4} + v_A - w_A$.

Case 2: Bidder lost the auction for object A. She bids her half of her valuation, i.e. $\frac{v_B}{2}$ with a conditional expected second stage payoff of $\frac{{v_B}^2}{4}$. This is a valid solution for $v_B < 2$.

We can now analyse the first stage auction. The bidder's payoff function is

$$\mathcal{U}^{seq}(w_A, w_B) = w_A\left(\frac{(v_B - v_A)^2}{4} + v_A - w_A\right) + (1 - w_A)\,\frac{{v_B}^2}{4}$$



(a) Bids for object A (b) Bids for object B if lost A (c) Bids for object B if won A

Figure 4: Sequential format computerised: observed bids, fitted polynomial surface (red), and optimal bid function (blue/green)

The first-order condition yields ${w_A}^* = \frac{1}{4}\left(\frac{{v_A}^2}{2} + v_A(2 - v_B)\right)$, which is a global and interior solution. □

Proof of Proposition ??.

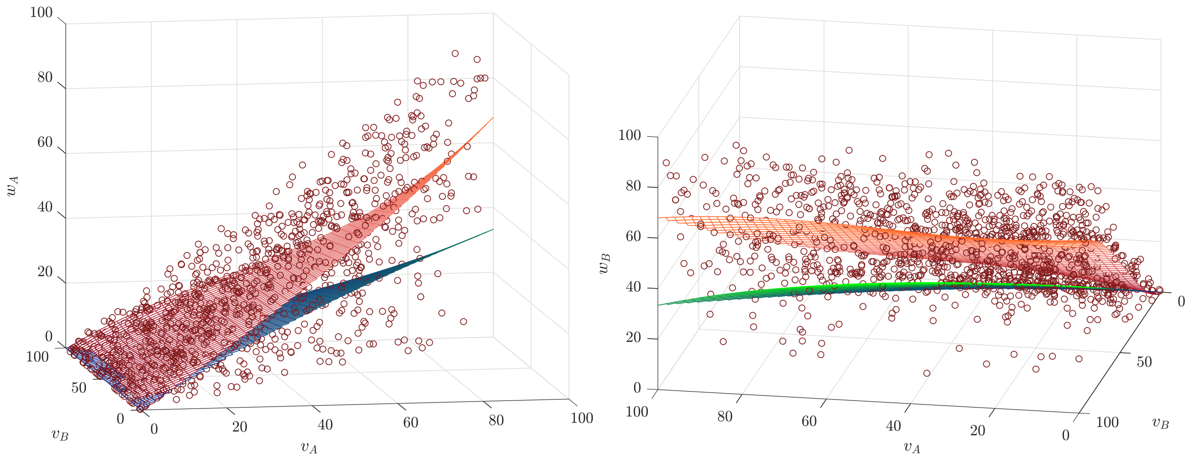
The bidder's payoff function is given by

$$\mathcal{U}^{sim}(w_A, w_B) = w_A(1 - w_B)v_A - {w_A}^2 + w_B(v_B - w_B)$$

First-order conditions yield ${w_A}^* = v_A \frac{2-v_B}{4-{v_A}^2}$ and ${w_B}^* = \frac{2v_B - {v_A}^2}{4-{v_A}2}$. Second-order conditions hold if $v_A < 2$. For an interior solution, we need $v_B < 2$ as an additional necessary and sufficient condition. □

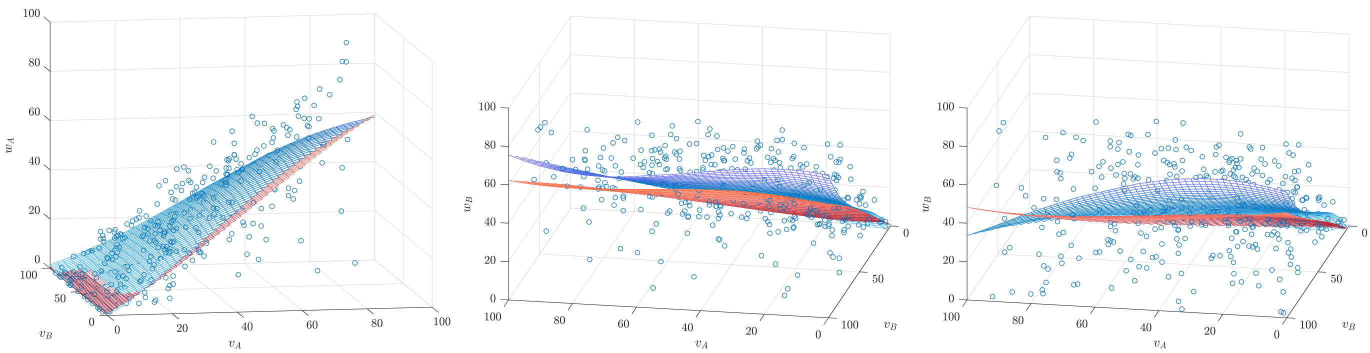
Comparative statics for the simultaneous auction. We have $\frac{\partial w_A}{\partial v_A} = \frac{(2-v_B)(4+{v_A}^2)}{(4-{v_A}^2)} > 0$, $\frac{\partial w_A}{\partial v_B} = \frac{-v_A(4-{v_A}^2)}{(4-{v_A}^2)} < 0$, $\frac{\partial w_B}{\partial v_A} = \frac{4v_A(v_B-2)}{(4-{v_A}^2)} < 0$, $\frac{\partial w_B}{\partial v_B} = 2 > 0$, for all $v_A, v_B \in (0, 1)$.

B Figures



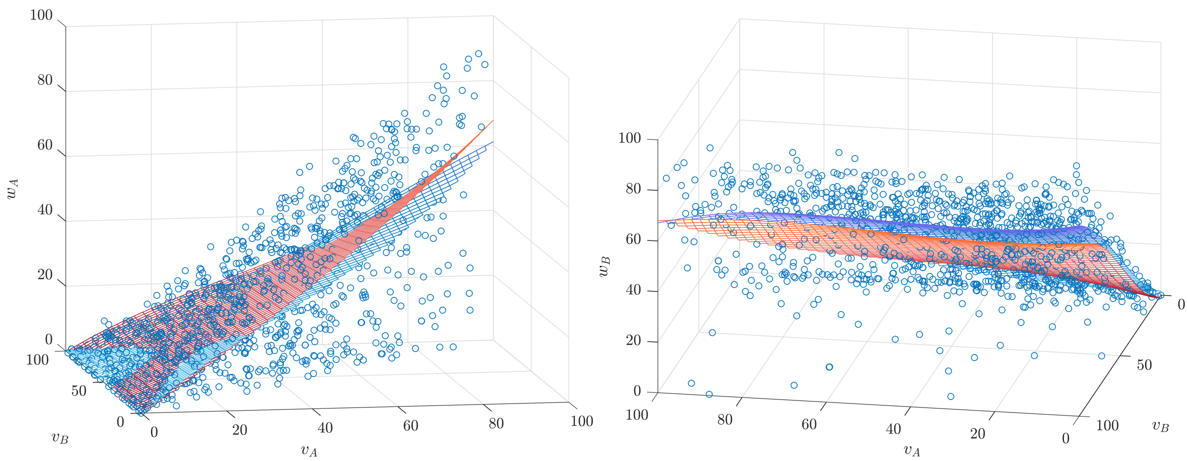
(a) Bids for object A (b) Bids for object B

Figure 5: Simultaneous format computerised: observed bids, fitted polynomial surface (red), and optimal bid function (blue/green)



(a) Bids for object A (b) Bids for object B if lost A (c) Bids for object B if won A

Figure 6: Sequential format: observed bids and fitted polynomial surface (blue) for human environment, fitted polynomial surface computerised (red)



(a) Bids for object A (b) Bids for object B

Figure 7: Simultaneous format: observed bids and fitted polynomial surface (blue) for human environment, fitted polynomial surface computerised (red)

C Tables

C.1 Descriptive statistics and power analysis

Table 6: Overview participants

	PMA/c	SEQ/c	SIM/c	PMA/h	SEQ/h	SIM/h	All
failed quiz	5	11	7	14	12	9	58
passed quiz	78	75	78	76	75	88	470
dropout after quiz	2	12	6	9	9	5	43
with dropout	0	0	0	9	12	4	25
unmatched	0	0	0	11	11	10	32
completed study	76	63	72	47	43	69	370

Table 7: Sample sizes (per treatment arm) needed to power study for different serial correlation. Numbers in bold indicate that the required sample size is lower than the planned sample of 185 subjects per treatment arm.

		$b = \pm 0.2SD$	$b = \pm 0.15SD$
$p = 0.8$	$\rho = 0.1$	**63**	**111**
	$\rho = 0.2$	**103**	**183**
	$\rho = 0.3$	**144**	256
	$\rho = 0.4$	**185**	328
	$\rho = 0.5$	226	401
$p = 0.9$	$\rho = 0.1$	**79**	**140**
	$\rho = 0.2$	**131**	232
	$\rho = 0.3$	**182**	323
	$\rho = 0.4$	233	414
	$\rho = 0.5$	285	506

C.2 Hypothesis testing

C.3 Expected outcomes by covariates for computerised treatments

C.4 Equivalence tests for computerised treatments

Table 8: Overview equivalence tests for expected outcomes in computerised environment

		equivalence test significant	t-test for difference significant
PMA/c vs. SEQ/c	payoff	-	yes
	revenue	-	yes
	welfare	-	yes
	payment	-	-
PMA/c vs. SIM/c	payoff	-	yes
	revenue	-	yes
	welfare	-	yes
	payment	-	yes
SEQ/c vs. SIM/c	payoff	-	yes
	revenue	-	yes
	welfare	yes	-
	payment	-	yes

Table 9: Expected mean outcomes by gender

		Female	Male	Other	Prefer not to say
#subjects		123	85	1	2
$\mathcal{U}$	pma	9.71	10.15	-	-
	seq	7.62	9.47	-	17.38
	sim	5.31	7.96	13.04	9.76
$\mathcal{R}$	pma	114.55	115.19	-	-
	seq	114.55	112.56	-	113.40
	sim	117.35	115.19	114.39	116.60
$\mathcal{W}$	pma	124.27	125.34	-	-
	seq	122.18	122.03	-	130.78
	sim	122.66	123.15	127.43	126.37
$\mathcal{P}$	pma	27.32	28.51	-	-
	seq	29.11	25.13	-	26.80
	sim	34.70	30.39	28.78	33.20

Table 10: Expected mean outcomes by age

		18-24	25-34	35-44	45-54	55-64
#subjects		40	53	42	44	32
$\mathcal{U}$	pma	10.66	9.64	9.38	10.58	8.61
	seq	10.95	7.06	7.58	8.58	8.61
	sim	7.89	6.97	7.40	4.50	8.00
$\mathcal{R}$	pma	113.04	115.56	114.74	114.19	116.68
	seq	112.78	113.52	115.13	112.59	115.17
	sim	113.23	117.40	114.89	117.69	116.09
$\mathcal{W}$	pma	123.71	125.20	124.12	124.77	125.29
	seq	123.72	120.58	122.72	121.16	123.78
	sim	121.12	124.37	122.29	122.18	124.09
$\mathcal{P}$	pma	24.52	29.26	27.77	26.47	31.22
	seq	25.55	27.04	30.26	25.18	30.34
	sim	26.46	34.80	29.79	35.38	32.18

Table 11: Expected mean outcomes by ethnicity

		Arabic	Black	Central Asian	East Asian	Latin American	South Asian	White	Other	Prefer not to say
#subjects		1	4	2	9	1	18	158	11	7
$\mathcal{U}$	pma	-	9.55	-	9.83	-	10.64	9.71	9.36	11.43
	seq	-	-	-	5.86	-	9.94	8.67	-	6.40
	sim	14.13	5.61	2.54	3.07	2.82	0.16	7.44	7.32	10.90
$\mathcal{R}$	pma	-	118.03	-	112.27	-	113.38	114.76	117.62	113.33
	seq	-	-	-	118.04	-	113.79	113.58	-	111.64
	sim	111.98	116.44	113.39	118.07	121.03	123.10	115.72	116.89	112.47
$\mathcal{W}$	pma	-	127.58	-	122.10	-	124.01	124.48	126.98	124.76
	seq	-	-	-	123.90	-	123.73	122.26	-	118.04
	sim	126.11	122.05	115.94	121.15	123.85	123.26	123.16	124.21	123.37
$\mathcal{P}$	pma	-	33.42	-	23.33	-	24.96	27.76	32.70	25.26
	seq	-	-	-	36.08	-	27.57	27.16	-	23.28
	sim	23.96	32.88	26.79	36.14	42.07	46.20	31.44	33.78	24.94

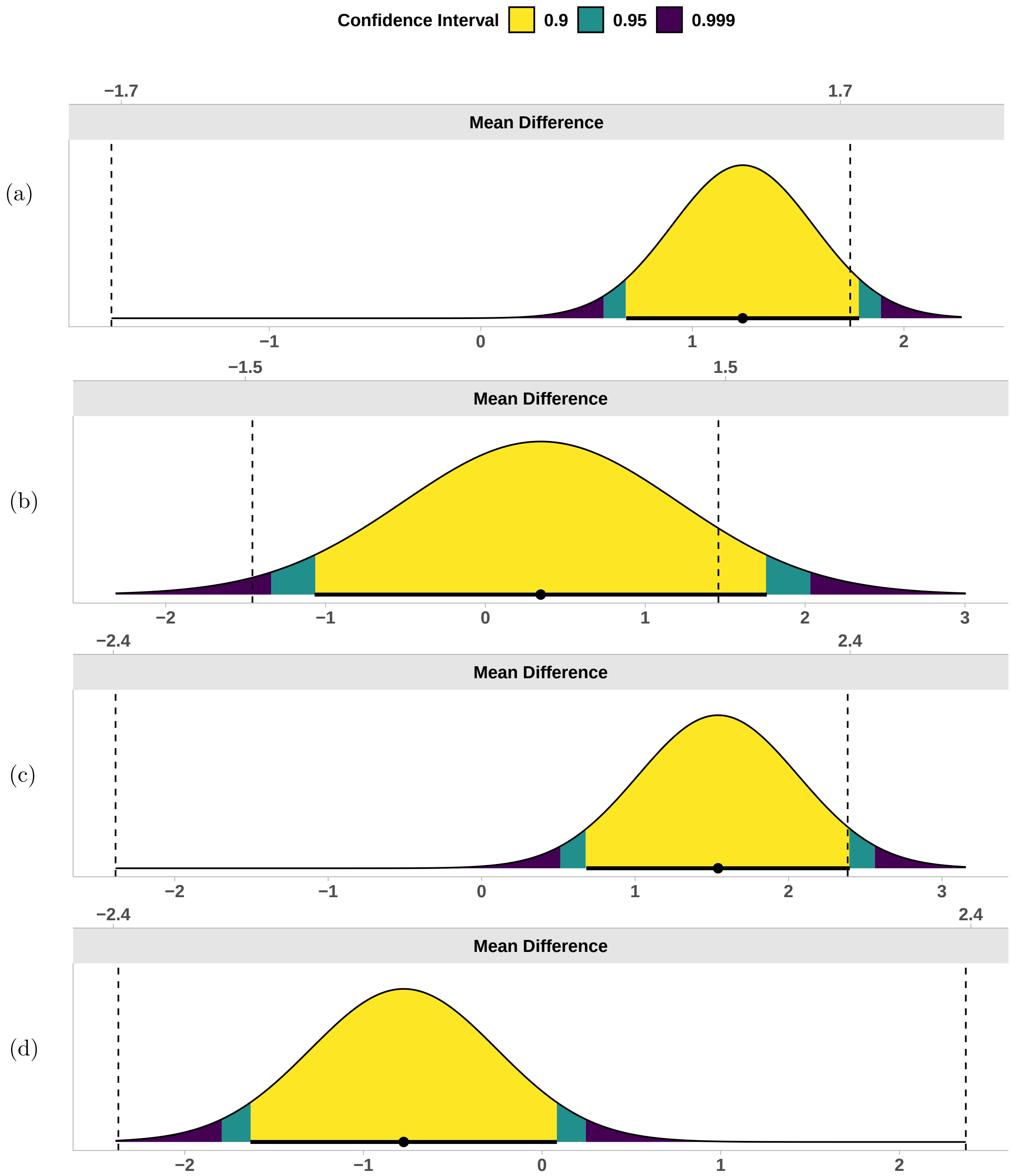
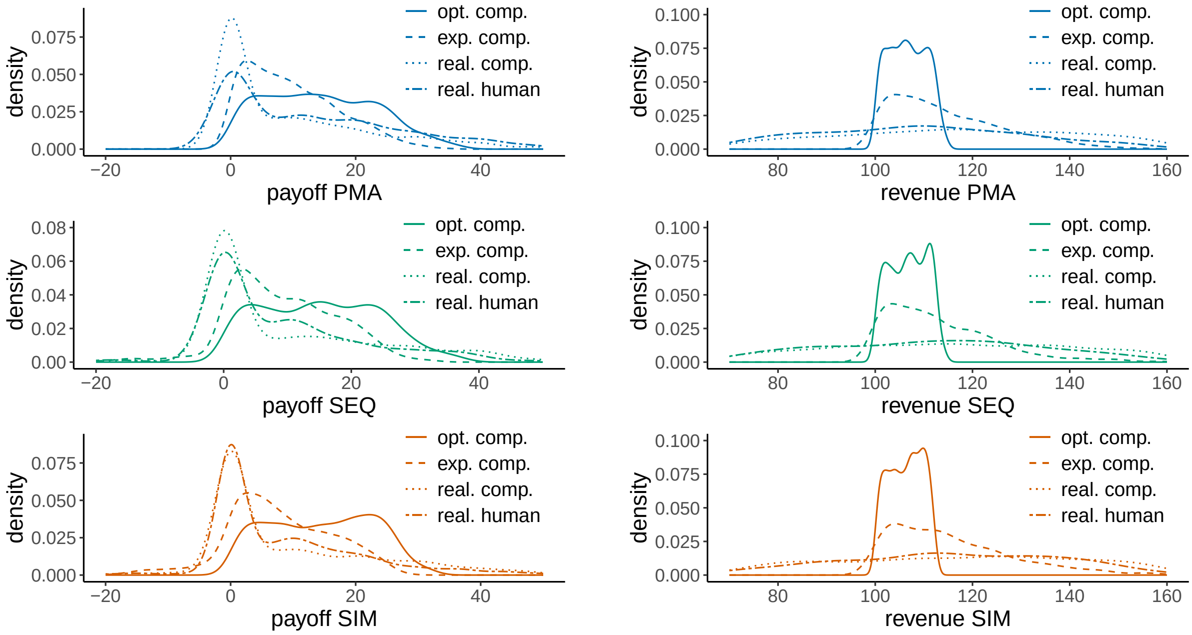


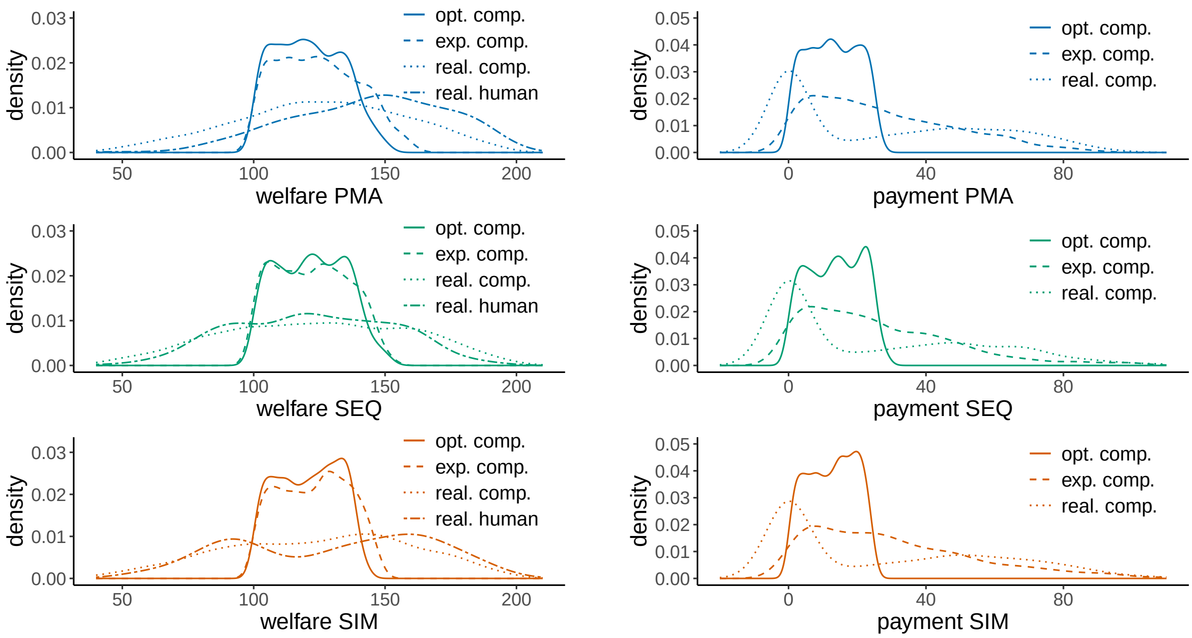
Figure 8: Distribution of mean differences in equivalence tests of expected outcomes, computerised env.: a) payoffs SEQ vs. SIM, b) payments SEQ vs. SIM, c) welfare PMA vs. SIM, d) welfare SEQ vs. SIM



(a) payoff

(b) revenue

Figure 9: Density plots of outcomes payoff and revenue



(a) welfare

(b) payments flexible bidder

Figure 10: Density plots of outcomes welfare and flexible bidder payments

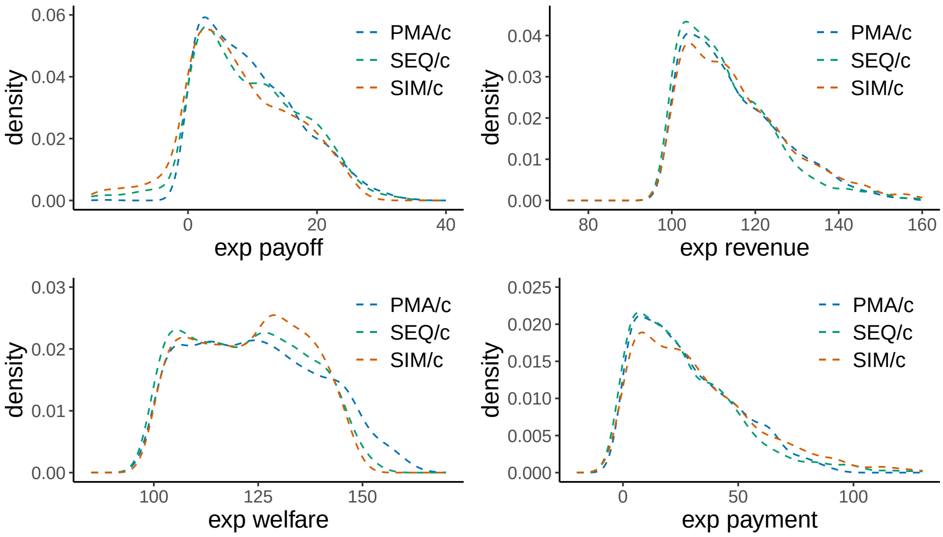


Figure 11: Density plots of expected outcomes comparing computerised treatments PMA, SEQ, SIM

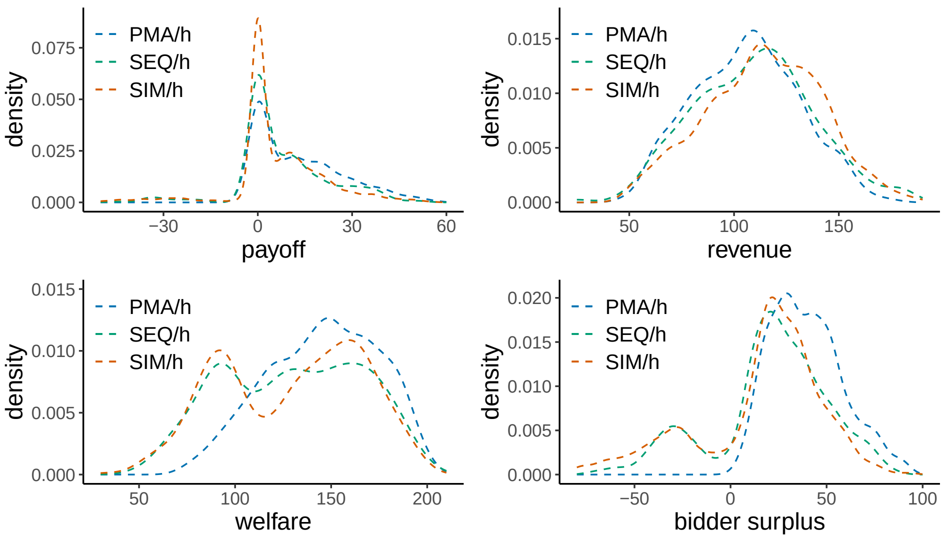


Figure 12: Density plots of outcomes comparing human treatments PMA, SEQ, SIM

Table 12: Expected mean outcomes by socio-economic status

		Upper class	Upper middle class	Lower middle class	Working class	Lower class	Prefer not to say
#subjects		0	59	78	46	5	23
$\mathcal{U}$	pma	-	9.82	9.36	10.30	14.18	10.57
	seq	-	10.57	7.67	8.08	8.72	7.44
	sim	-	8.64	6.98	5.63	9.09	1.32
$\mathcal{R}$	pma	-	114.54	115.17	113.92	108.38	115.27
	seq	-	113.80	113.75	114.63	108.97	113.34
	sim	-	113.79	116.78	117.95	105.61	121.64
$\mathcal{W}$	pma	-	124.36	124.53	124.22	122.57	125.84
	seq	-	124.37	121.41	122.72	117.69	120.78
	sim	-	122.43	123.76	123.58	114.70	122.96
$\mathcal{P}$	pma	-	27.49	28.40	26.12	16.06	28.61
	seq	-	27.60	27.49	29.27	17.93	26.68
	sim	-	27.59	33.56	35.90	11.22	43.28

Table 13: Expected mean outcomes by income

		£0	£1 to £9,999	£10,000 to £24,999	£25,000 to £49,999	£50,000 to £99,999	£100,000 to £149,999	£150,000 and more	Prefer not to say
#subjects		7	44	46	62	22	3	0	27
$\mathcal{U}$	pma	10.45	10.50	9.49	9.26	10.28	14.89	-	10.16
	seq	9.48	9.20	7.14	8.82	8.47	16.03	-	8.27
	sim	-5.18	6.82	8.02	6.24	9.12	2.64	-	5.39
$\mathcal{R}$	pma	114.22	113.26	115.08	115.15	116.45	113.41	-	113.51
	seq	110.53	113.00	114.55	114.38	114.38	110.01	-	113.23
	sim	124.50	114.75	115.73	117.86	112.91	116.50	-	118.37
$\mathcal{W}$	pma	124.67	123.76	124.57	124.41	126.73	128.31	-	123.67
	seq	120.01	122.20	121.69	123.20	122.85	126.04	-	121.50
	sim	119.33	121.57	123.75	124.10	122.03	119.14	-	123.76
$\mathcal{P}$	pma	26.09	24.90	28.53	28.37	30.76	25.51	-	25.37
	seq	21.06	26.01	29.09	28.76	28.75	20.02	-	26.46
	sim	49.00	29.50	31.45	35.73	25.82	33.00	-	36.73

Table 14: Expected mean outcomes by education

		Early childhood education / no education	Primary education	Lower secondary education	Upper secondary education	Post-secondary non-tertiary education	Short-cycle tertiary education	Bachelor or equivalent	Master or equivalent	Doctoral or equivalent
#subjects		0	0	2	21	12	6	92	68	10
$\mathcal{U}$	pma	-	-	-	7.61	8.99	7.73	10.74	9.37	7.71
	seq	-	-	-	9.59	6.71	9.65	9.16	8.45	6.32
	sim	-	-	5.02	5.72	5.17	9.67	6.43	7.18	11.48
$\mathcal{R}$	pma	-	-	-	117.25	115.44	116.96	113.36	115.79	116.17
	seq	-	-	-	111.02	113.53	113.81	114.34	114.08	112.43
	sim	-	-	116.69	117.84	113.69	115.81	116.10	116.24	114.16
$\mathcal{W}$	pma	-	-	-	124.87	124.43	124.69	124.10	125.15	123.88
	seq	-	-	-	120.61	120.24	123.46	123.50	122.53	118.75
	sim	-	-	121.72	123.56	118.86	125.48	122.53	123.42	125.64
$\mathcal{P}$	pma	-	-	-	31.20	28.88	32.03	25.25	29.60	30.23
	seq	-	-	-	22.04	27.06	27.62	28.68	28.16	24.86
	sim	-	-	33.39	35.67	27.38	31.63	32.20	32.48	28.31

Table 15: Expected mean outcomes by numeracy score, 4 being the best score

		1	2	3	4
#subjects		46	75	28	62
$\mathcal{U}$	pma	9.01	9.06	9.88	11.60
	seq	6.38	9.34	9.95	8.58
	sim	5.40	6.45	7.20	8.20
$\mathcal{R}$	pma	115.72	114.88	115.39	113.43
	seq	113.54	112.70	114.20	114.49
	sim	117.90	116.09	114.15	115.59
$\mathcal{W}$	pma	124.73	123.94	125.27	125.02
	seq	119.92	122.04	124.15	123.07
	sim	123.29	122.54	121.35	123.79
$\mathcal{P}$	pma	29.62	27.94	28.54	25.33
	seq	27.08	25.40	28.40	28.97
	sim	35.79	32.17	28.30	31.19

Table 16: Expected mean outcomes by number of comprehension errors

		0	1-2	3-5	6-10	≥ 11
#subjects		84	68	34	19	6
$\mathcal{U}$	pma	11.07	9.10	8.11	10.15	9.38
	seq	10.59	7.96	4.44	10.01	9.00
	sim	9.41	8.69	2.43	1.94	0.84
$\mathcal{R}$	pma	113.06	116.08	114.46	115.56	119.62
	seq	113.39	114.18	115.14	112.82	105.50
	sim	114.28	114.51	120.02	118.62	124.42
$\mathcal{W}$	pma	124.13	125.18	122.57	125.71	129.00
	seq	123.98	122.14	119.59	122.84	114.50
	sim	123.69	123.20	122.45	120.56	125.26
$\mathcal{P}$	pma	24.65	30.08	27.30	28.98	36.28
	seq	26.78	28.37	30.29	25.65	10.99
	sim	28.56	29.03	40.05	37.23	48.84

Table 17: Expected mean outcomes by self-reported risk preferences, 10 being the most risk-averse

		0-1	2-3	4-5	6-7	8-9	10
#subjects		17	53	86	38	17	0
$\mathcal{U}$	pma	8.03	10.28	9.87	10.21	8.63	-
	seq	6.51	9.32	9.11	7.48	6.36	-
	sim	5.56	6.76	9.30	3.91	4.81	-
$\mathcal{R}$	pma	116.64	114.67	114.36	113.81	118.05	-
	seq	110.29	114.16	114.02	112.89	113.24	-
	sim	115.59	115.90	115.05	118.36	119.59	-
$\mathcal{W}$	pma	124.67	124.95	124.23	124.02	126.69	-
	seq	116.80	123.49	123.13	120.36	119.60	-
	sim	121.15	122.66	124.36	122.27	124.40	-
$\mathcal{P}$	pma	31.15	27.54	26.95	26.11	33.36	-
	seq	20.57	28.33	28.05	25.78	26.48	-
	sim	31.18	31.80	30.11	36.73	39.18	-

Table 18: Expected mean outcomes by risk aversion measure with test after Drichoutis et al. (2015), 10 being the most risk-averse

		0-1	2-3	4-5	6-7	8-9	10
#subjects		41	21	65	52	21	11
$\mathcal{U}$	pma	7.59	10.79	10.93	9.86	9.73	9.34
	seq	4.41	6.80	8.05	10.64	14.04	7.54
	sim	5.24	3.18	7.05	9.86	4.35	7.53
$\mathcal{R}$	pma	117.84	117.43	113.66	113.07	114.55	113.82
	seq	115.75	114.56	113.46	113.19	110.97	111.46
	sim	117.55	118.86	116.27	113.22	118.44	116.27
$\mathcal{W}$	pma	125.44	128.22	124.59	122.93	124.28	123.16
	seq	120.15	121.36	121.51	123.83	125.01	119.00
	sim	122.79	122.04	123.32	123.08	122.80	123.80
$\mathcal{P}$	pma	33.29	32.30	25.78	24.59	27.50	25.88
	seq	31.49	29.13	26.93	26.39	21.94	22.92
	sim	35.10	37.72	32.54	26.44	36.88	32.55

Table 19: Expected mean outcomes by risk aversion (probability weights) measure after test by Holt & Laury (2002), 10 being the most risk-averse

		0-1	2-3	4-5	6-7	8-9	10
#subjects		20	12	76	66	27	10
$\mathcal{U}$	pma	7.14	8.05	9.70	11.38	10.86	8.26
	seq	4.63	9.66	6.94	10.52	10.30	10.10
	sim	5.29	1.28	7.47	7.98	5.27	4.91
$\mathcal{R}$	pma	119.35	118.16	115.18	112.67	112.11	115.03
	seq	112.19	105.58	115.36	113.21	113.20	112.92
	sim	115.35	119.03	116.59	114.19	120.30	118.06
$\mathcal{W}$	pma	126.49	126.21	124.89	124.05	122.98	123.29
	seq	116.82	115.24	122.30	123.74	123.51	123.02
	sim	120.64	120.31	124.06	122.17	125.57	122.96
$\mathcal{P}$	pma	36.29	33.75	28.34	23.95	22.95	28.45
	seq	24.38	11.16	30.72	26.43	26.40	25.83
	sim	30.70	38.07	33.17	28.38	40.60	36.11

Table 20: Expected mean outcomes by ambiguity aversion measure after Gneezy et al. (2015), 20 being the most ambiguity-averse

		0-2	3-5	6-8	9-11	12-14	15-17	18-20
#subjects		26	53	25	23	16	38	30
$\mathcal{U}$	pma	8.11	10.18	9.08	10.89	9.60	10.25	10.02
	seq	5.13	9.09	5.53	11.24	9.88	10.53	8.26
	sim	5.46	7.58	4.22	7.34	9.17	6.94	6.88
$\mathcal{R}$	pma	118.64	112.56	116.26	115.67	117.25	114.04	113.87
	seq	113.40	114.45	116.62	112.92	112.50	112.54	112.44
	sim	118.56	116.14	118.88	114.06	114.69	115.43	114.70
$\mathcal{W}$	pma	126.75	122.74	125.34	126.56	126.86	124.29	123.89
	seq	118.53	123.54	122.14	124.16	122.37	123.07	120.70
	sim	124.02	123.72	123.10	121.40	123.85	122.37	121.58
$\mathcal{P}$	pma	34.91	23.73	30.45	29.41	32.03	26.40	25.99
	seq	26.79	28.91	33.23	25.84	24.99	25.08	24.88
	sim	37.13	32.27	37.76	28.12	29.37	30.86	29.40

Table 21: Expected mean outcomes by self-reported, perceived competition in auction

		0-1	2-3	4-5	6-7	8-9	10
#subjects		10	20	31	68	63	19
$\mathcal{U}$	pma	6.06	10.03	9.99	9.94	9.51	12.96
	seq	9.62	7.75	7.93	8.34	9.18	9.34
	sim	10.38	5.61	9.20	7.85	4.93	5.16
$\mathcal{R}$	pma	117.73	115.47	114.88	114.49	114.81	111.34
	seq	112.05	112.78	114.07	113.68	113.98	113.64
	sim	113.08	119.09	112.74	116.44	117.64	117.22
$\mathcal{W}$	pma	123.79	125.50	124.87	124.43	124.32	124.30
	seq	121.67	120.54	122.00	122.02	123.17	122.98
	sim	123.46	124.70	121.94	124.29	122.57	122.38
$\mathcal{P}$	pma	33.36	28.99	28.09	27.17	27.73	21.54
	seq	24.10	25.57	28.14	27.36	27.97	27.28
	sim	26.16	38.18	25.48	32.88	35.29	34.44

Table 22: Expected mean outcomes by self-reported, perceived difficulty of bidding in auction

		0-1	2-3	4-5	6-7	8-9	10
#subjects		7	28	30	68	66	12
$\mathcal{U}$	pma	11.18	10.59	10.92	10.19	8.81	5.38
	seq	7.38	9.31	8.70	8.30	9.17	3.08
	sim	10.44	8.13	10.59	6.71	5.11	4.81
$\mathcal{R}$	pma	113.29	112.22	115.23	113.89	116.06	122.41
	seq	115.12	110.33	111.87	114.99	113.69	115.65
	sim	111.94	116.40	112.55	116.58	117.88	114.93
$\mathcal{W}$	pma	124.47	122.81	126.15	124.08	124.87	127.79
	seq	122.50	119.64	120.57	123.29	122.86	118.73
	sim	122.37	124.53	123.14	123.29	122.99	119.74
$\mathcal{P}$	pma	25.30	23.00	28.80	26.12	30.10	40.83
	seq	30.24	20.65	23.73	29.98	27.37	31.31
	sim	23.87	32.80	25.11	33.17	35.76	29.86

Table 23: Expected mean outcomes by experience in auction experiments (number of auction experiments participated in)

		0	1	2	3	4	5 or more
#subjects		144	37	14	8	6	2
$\mathcal{U}$	pma	9.81	9.77	10.39	9.13	11.09	-
	seq	9.07	7.01	7.87	-	3.05	11.86
	sim	7.03	7.43	7.96	2.96	4.75	-
$\mathcal{R}$	pma	115.22	113.95	112.61	115.17	107.38	-
	seq	113.53	117.53	110.87	-	106.82	111.44
	sim	116.26	115.32	115.67	117.14	119.54	-
$\mathcal{W}$	pma	125.04	123.72	123.00	124.30	118.47	-
	seq	122.60	124.54	118.74	-	109.87	123.30
	sim	123.29	122.75	123.63	120.10	124.30	-
$\mathcal{P}$	pma	28.54	26.09	24.23	28.57	14.22	-
	seq	27.05	35.07	21.73	-	13.64	22.88
	sim	32.51	30.64	31.34	34.29	39.09	-

Table 24: Expected mean outcomes by experience in real life auctions (number of real life auctions participated in)

		0	1	2	3	4	5 or more
#subjects		111	35	19	5	6	35
$\mathcal{U}$	pma	9.68	9.78	11.86	11.33	8.00	10.15
	seq	8.50	8.83	9.83	13.06	3.05	8.28
	sim	8.03	6.27	4.50	-0.73	3.98	8.24
$\mathcal{R}$	pma	114.73	114.58	112.90	110.05	120.63	114.67
	seq	113.97	114.76	109.23	111.27	106.82	114.82
	sim	115.59	117.31	118.97	118.72	119.51	113.14
$\mathcal{W}$	pma	124.42	124.36	124.76	121.38	128.63	124.82
	seq	122.46	123.59	119.06	124.33	109.87	123.10
	sim	123.62	123.58	123.48	117.99	123.49	121.38
$\mathcal{P}$	pma	27.66	27.29	24.63	19.13	37.96	27.58
	seq	27.93	29.51	18.47	22.55	13.64	29.64
	sim	31.19	34.62	37.95	37.44	39.03	26.27

Table 25: Equivalence tests of expected outcomes in PMA vs. SEQ, computerised

TOST Results PMA/c vs. SEQ/c								
							Equivalence Bounds	
		estimate	SE	t	df	p-value	CI lower	CI upper
payoff							*-1.746*	*1.746*
	t-test	1.24	0.33	3.70	2285.85	2.22E-04	0.687	1.788
	TOST Lower		0.33	8.92	2285.85	4.77E-19		
	TOST Upper		0.33	-1.52	2285.85	6.44E-02		
revenue							*-0.746*	*0.746*
	t-test	1.08	0.45	2.40	2649.20	1.65E-02	0.338	1.817
	TOST Lower		0.45	4.06	2649.20	2.53E-05		
	TOST Upper		0.45	0.74	2649.20	7.69E-01		
welfare							*-2.487*	*2.487*
	t-test	2.32	0.55	4.19	2754.83	2.86E-05	1.406	3.224
	TOST Lower		0.55	8.69	2754.83	2.95E-18		
	TOST Upper		0.55	-0.31	2754.83	3.78E-01		
payment							*-1.458*	*1.458*
	t-test	0.35	0.86	0.40	2512.79	6.88E-01	-1.069	1.760
	TOST Lower		0.86	2.10	2512.79	1.80E-02		
	TOST Upper		0.86	-1.29	2512.79	9.80E-02		

Note: Welch Two Sample t-test, $\alpha = 0.05$, *90% confidence interval*

Table 26: Equivalence tests of expected outcomes in PMA vs. SIM, computerised

TOST Results PMA/c vs. SIM/c								
							Equivalence Bounds	
		estimate	SE	t	df	p-value	CI lower	CI upper
payoff							*-1.680*	*1.680*
	t-test	3.03	0.34	8.96	2532.32	6.37E-19	2.475	3.589
	TOST Lower		0.34	13.92	2532.32	8.43E-43		
	TOST Upper		0.34	3.99	2532.32	1.00E+00		
revenue							*-0.714*	*0.714*
	t-test	-1.49	0.46	-3.22	2837.31	1.32E-03	-2.254	-0.728
	TOST Lower		0.46	-1.68	2837.31	9.53E-01		
	TOST Upper		0.46	-4.76	2837.31	1.04E-06		
welfare							*-2.385*	*2.385*
	t-test	1.54	0.52	2.95	2931.26	3.18E-03	0.682	2.400
	TOST Lower		0.52	7.52	2931.26	3.56E-14		
	TOST Upper		0.52	-1.62	2931.26	5.29E-02		
payment							*-1.391*	*1.391*
	t-test	-4.79	0.89	-5.39	2701.82	7.86E-08	-6.255	-3.327
	TOST Lower		0.89	-3.82	2701.82	1.00E+00		
	TOST Upper		0.89	-6.95	2701.82	2.30E-12		

Note: Welch Two Sample t-test, $\alpha = 0.05$, *90% confidence interval*

Table 27: Equivalence tests of expected outcomes in SEQ vs. SIM, computerised

TOST Results SEQ/c vs. SIM/c								
							Equivalence Bounds	
		estimate	SE	t	df	p-value	CI lower	CI upper
payoff							*-1.680*	*1.680*
	t-test	1.79	0.39	4.54	2691.26	5.75E-06	1.144	2.443
	TOST Lower		0.39	8.80	2691.26	1.19E-18		
	TOST Upper		0.39	0.29	2691.26	6.13E-01		
revenue							*-0.701*	*0.701*
	t-test	-2.57	0.49	-5.24	2697.59	1.74E-07	-3.375	-1.762
	TOST Lower		0.49	-3.81	2697.59	1.00E+00		
	TOST Upper		0.49	-6.67	2697.59	1.55E-11		
welfare							*-2.371*	*2.371*
	t-test	-0.77	0.52	-1.49	2606.46	1.37E-01	-1.632	0.083
	TOST Lower		0.52	3.06	2606.46	1.11E-03		
	TOST Upper		0.52	-6.04	2606.46	9.08E-10		
payment							*-1.403*	*1.403*
	t-test	-5.14	0.98	-5.24	2697.59	1.74E-07	-6.750	-3.523
	TOST Lower		0.98	-3.81	2697.59	1.00E+00		
	TOST Upper		0.98	-6.67	2697.59	1.55E-11		

Note: Welch Two Sample t-test, $\alpha = 0.05$*, 90% confidence interval*

C.5 Random effects model

Table 28: Models with subject-specific random effects, testing for difference in expected outcomes, computerised environment

	Dependent variable:			
	Exp. payoff	Exp. revenue	Exp. welfare	Exp. payments
	(1)	(2)	(3)	(4)
PMA/c vs. SEQ/c	1.238*	1.078	2.315***	0.346
	(0.736)	(0.933)	(0.728)	(1.771)
SEQ/c	8.597***	113.651***	122.248***	27.301***
	(0.544)	(0.690)	(0.539)	(1.310)
Random effects:				
sd(Subject)	3.97	4.924	2.822	9.357
sd(Residual)	7.589	10.691	14.358	20.243
# subjects	139	139	139	139
PMA/c vs. SIM/c	3.032***	−1.491	1.541**	−4.791***
	(0.790)	(0.962)	(0.626)	(1.844)
SIM/c	6.804***	116.219***	123.023***	32.438***
	(0.566)	(0.689)	(0.449)	(1.321)
Random effects:				
sd(Subject)	4.463	5.262	2.134	10.094
sd(Residual)	7.965	11.415	14.092	21.833
# subjects	148	148	148	148
SEQ/c vs. SIM/c	1.794*	−2.568**	−0.775	−5.137**
	(0.978)	(1.017)	(0.701)	(2.034)
SIM/c	6.804***	116.219***	123.023***	32.438***
	(0.668)	(0.695)	(0.479)	(1.389)
Random effects:				
sd(Subject)	5.315	5.283	2.801	10.567
sd(Residual)	8.834	11.688	13.174	23.377
# subjects	135	135	135	135
Note:				*p<0.1; **p<0.05; ***p<0.01

C.6 Linear model with covariates

Table 29: Linear model for expected outcomes. Reference category for 'Auction experiments participated in' and 'Real auctions participated in': 0, for all other categorical variables: 'Prefer not to say'

		Dependent variable:			
		Payoff	Revenue	Welfare	Payments
	PMA/c vs. SIM/c	3.589***	−1.330***	2.259***	−4.418***
	SEQ/c vs. SIM/c	1.751***	−2.689***	−0.938***	−5.293***
	valueA	−0.048***	0.135***	0.087***	0.239***
	valueB	0.202***	0.291***	0.493***	0.577***
Gender	Female	−5.547***	4.344***	−1.203	7.954***
	Male	−4.094***	2.679*	−1.414	4.687*
	Other	−1.200	−0.332	−1.532	−1.475
	Age	−0.054***	0.071***	0.017**	0.139***
Ethnicity	Arabic	4.117**	−2.867	1.251	−5.299
	Black	0.994	1.366	2.360***	2.132
	Central Asian	−1.840	0.740	−1.100	1.496
	East Asian	−3.241***	5.273***	2.032***	10.245***
	Latin American	−2.860	4.538**	1.678	9.189**
	South Asian	−2.610***	4.232***	1.622***	8.424***
	White	−0.908	1.803**	0.894*	3.518**
	Other	−0.093	1.533	1.439**	2.819
	Lower class	3.685***	−6.665***	−2.980***	−12.942***
	Working class	2.296***	−1.823***	0.473	−3.376***
	Lower middle class	1.898***	−1.697***	0.201	−3.325***
	Upper middle class	3.078***	−2.484***	0.593*	−4.770***
Income	£0	−1.967**	0.577	−1.389**	0.726
	£1 to £9,999	−0.140	1.272**	1.132***	2.395**
	£10,000 to £24,999	0.276	1.120**	1.396***	2.209**
	£25,000 to £49,999	−1.463***	3.208***	1.745***	6.189***
	£50,000 to £99,999	0.498	0.194	0.692*	0.155
	£100,000 to £149,999	−0.074	1.428	1.354*	2.693
Education	Upper secondary education	3.071**	0.503	3.574***	0.813
	Post-secondary non-tertiary education	3.938***	0.387	4.325***	0.703
	Short-cycle tertiary education	3.397**	2.129	5.527***	4.226
	Bachelor or equivalent	3.593***	−0.152	3.441***	−0.084
	Master or equivalent	3.136**	0.905	4.041***	1.818
	Doctoral or equivalent	2.785*	0.337	3.122***	0.677
	Risk av. self report (0-10)	−0.318***	0.297***	−0.021	0.580***
	Risk aversion (0-10)	0.297***	−0.313***	−0.017	−0.627***
	Risk av./Prob. weight (0-10)	0.167***	−0.228***	−0.060	−0.394***
	Ambiguity av. (0-20)	0.056**	−0.015	0.041***	−0.039
	Perceived competition (0-10)	0.120*	−0.111*	0.008	−0.169
	Perceived difficulty (0-10)	−0.420***	0.645***	0.225***	1.188***
Auction experiments participated in	1	−0.884**	1.342***	0.458*	2.768***
	2	0.970*	−2.484***	−1.514***	−4.589***
	3	−4.631***	3.176***	−1.455**	6.273***
	4	−1.232	−2.027**	−3.259***	−3.472*
	5 or more	2.650**	−1.547	1.103	−3.153
Real auctions participated in	1	−0.294	0.907**	0.613**	1.803**
	2	−0.151	−0.406	−0.557	−0.714
	3	1.274	−1.931**	−0.657	−3.532*
	4	−1.652	−1.053	−2.705***	−2.686
	5 or more	−0.077	0.029	−0.047	0.090
	Numeracy score (1-4)	0.068	0.258*	0.326***	0.516*
	#comprehension errors	−0.207***	0.215***	0.009	0.428***
	SIM/c	−1.173	80.768***	79.595***	−36.215***
Note:					*p<0.1; **p<0.05; ***p<0.01

C.7 Additional tables

Table 30: Average probabilities of winning both objects with optimal bidding

		Mean	SD	Median	Q1	Q3
P(win AB)	pma	0	0	0	0	0
	seq	0.015	0.010	0.013	0.006	0.023
	sim	0.036	0.028	0.031	0.013	0.056

Table 31: Average probabilities of winning both objects with worst possible overbidding

		Mean	SD	Median	Q1	Q3
P(win AB)	pma	0	0	0	0	0
	seq	0.257	0.223	0.194	0.072	0.395
	sim	0.257	0.223	0.194	0.072	0.395

Table 32: Average loss with worst possible overbidding

		Mean	SD	Median	Q1	Q3
$\mathcal{U}$	pma	0	0	0	0	0
	seq	-13.80	18.25	-5.70	-20.03	-1.00
	sim	-13.80	18.25	-5.70	-20.03	-1.00