HBS Case

The Harvard Management Company and Inflation-Indexed Bonds

1. READING: HMC's Approach

1. There are thousands of individual risky assets in which HMC can invest. Explain why MV optimization across 1,000 securities is infeasible.

Calculating mean-variance optimization requires us to have the expected value of each asset and a large covariance matrix. These assumptions mean that we need to assume some level of stability for every single asset, despite the fact that noise will obscure our singles. Additionally, for an asset manager, investing in 1,000 securities instead of the basket of securities means that we are at risk of losing 100% of an asset if it goes to 0 versus in an ETF, where the downfall of an asset is minimally felt by the overall return.

2. Rather than optimize across all securities directly, HMC runs a two-stage optimization.

1. They build asset class portfolios with each one optimized over the securities of the specific asset class.
2. HMC combines the asset-class portfolios into one total optimized portfolio.

In order for the two-stage optimization to be a good approximation of the full MV-optimization on all assets, what must be true of the partition of securities into asset classes?

The securities must be "separable," as in their covariances are roughly block-diagonal and similar within-class exposure to other classes.

3. Should TIPS form a new asset class or be grouped into one of the other 11 classes?

TIPS should form a new asset class, because they have a distinct return behavior that is inflation-hedged. They have a very different risk and return profile compared to other nominal bonds.

4. Why does HMC focus on real returns when analyzing its portfolio allocation? Is this just a matter of scaling, or does using real returns versus nominal returns potentially change the MV solution?

Because Harvard's main goal is to maximize purchasing power of endowment spending, they need to focus on real returns. Rather than scaling, this changes the MV solution because we are targeting a different goal; the inclusion of TIPS in our portfolio is an inflation hedge that removes our inflationary risks from our portfolio.

5. The case discusses the fact that Harvard places bounds on the portfolio allocation rather than implementing whatever numbers come out of the MV optimization problem.

How might we adjust the stated optimization problem in the lecture notes to reflect the extra constraints Harvard is using in their bounded solutions given in Exhibits 5 and 6?

Exhibit 5 shows bounds of 0% <= w_i <= 100% for all assets and cash as low as -50%. Exhibit 6 gives us lower and upper bounds with TIPS specifically allocated between [0,100] percentage points. We can implement these as linear equalities besides sum of w_i = 1.

6. Exhibits 5 shows zero allocation to domestic equities and domestic bonds across the entire computed range of targeted returns, (5.75% to 7.25%). Conceptually, why is the constraint binding in all these cases? What would the unconstrained portfolio want to do with those allocations and why?

The constraint is bound at 0% because otherwise the maximizer would short these asset classes to allow higher allocations towards other asset classes. In a world where we seek the highest Sharpe ratio or better diversifiers, the portfolio would probably tend towards asset classes like private equity, hedge funds, and emerging markets.

7. Exhibit 6 changes the constraints, (tightening them in most cases.) How much deterioration do we see in the mean-variance tradeoff that Harvard achieved?

The efficient frontier shifts downwards and towards the left with the higher constraints. Since our optimizer has added constraints, we have a lower achievable Sharpe. We see a closer allocation to the mix in Exhibit 1 (1999 policy mix).

2 Mean-Variance Optimization

COLAB: PM HW1 - Harvard Case Study.ipynb

Data

You will need the file in the github repo, `data/multi_asset_etf_data.xlsx`.

- The time-series data gives monthly returns for the 11 asset classes and a short-term Treasury-bill fund return, (`SHV`.)
- The case does not give time-series data, so this data has been compiled outside of the case, and it intends to represent the main asset classes under consideration via various ETFs. For details on the specific securities/indexes, check the “Info” tab of the data.

Excess Returns

- We consider `SHV` as the risk-free asset.
- We are going to analyze the problem in terms of **excess** returns, where `SHV` has been subtracted from the other columns.
- The risk-free rate changes over time, but the assumption is that investors know it’s value one-period ahead of time. Thus, at any given point in time, it is a risk-free rate for the next period. (This is often discussed as the “bank account” or “money market account” in other settings.)

Adjustment

For ease of analysis, drop `QAI` from the dataset. Analyze the remaining 10 assets.

Not Considered

- These are nominal returns-they are not adjusted for inflation, and in our calculations we are not making any adjustment for inflation.
- The exhibit data that comes via Harvard with the case is unnecessary for our analysis.

Format

In the questions below, **annualize the statistics** you report.

- Annualize the mean of monthly returns with a scaling of 12.
- Annualize the volatility of monthly returns with a scaling of
- Note that we are not scaling the raw timeseries data, just the statistics computed from it (mean, vol, Sharpe).

1. Summary Statistics

- Calculate and display the mean and volatility of each asset's excess return. (Recall we use volatility to refer to standard deviation.)
- Which assets have the best and worst Sharpe ratios?

Recall that the Sharpe Ratio is simply the ratio of the mean-to-volatility of excess returns:

Be sure to annualize all three stats (mean, vol, Sharpe).

	Annual Excess Mean	Annual Vol	Annual Sharpe
SPY	0.128141	0.142839	0.897103
HYG	0.041371	0.075928	0.544873
IYR	0.074916	0.168675	0.444143
PSP	0.092561	0.213370	0.433804
EFA	0.061775	0.150903	0.409372
TIP	0.020502	0.051115	0.401091
IEF	0.016404	0.063442	0.258569
EEM	0.029339	0.176164	0.166542
DBC	-0.005292	0.166553	-0.031774
BWX	-0.007716	0.082789	-0.093202

```
Best Sharpe:  SPY  (0.897)
Worst Sharpe: BWX (-0.093)
```

The best Sharpe is SPY (0.897) and the worst Sharpe is BWX (-0.093).

2. Descriptive Analysis

- Calculate the correlation matrix of the returns. Which pair has the highest correlation? And the lowest?
- How well have TIPS done in our sample? Have they outperformed domestic bonds? Foreign bonds?

	BWX	DBC	EEM	EFA	HYG	IEF	IYR	PSP	SPY	TIP
BWX	1.000000	0.191116	0.621673	0.602820	0.602555	0.580891	0.552557	0.526692	0.439994	0.675151
DBC	0.191116	1.000000	0.511667	0.500922	0.461887	-0.300207	0.280518	0.453303	0.432162	0.109006
EEM	0.621673	0.511667	1.000000	0.819925	0.691167	0.026704	0.584063	0.750109	0.687751	0.378792
EFA	0.602820	0.500922	0.819925	1.000000	0.787191	0.042639	0.699292	0.895320	0.845863	0.394821
HYG	0.602555	0.461887	0.691167	0.787191	1.000000	0.187258	0.739356	0.812157	0.793518	0.538648
IEF	0.580891	-0.300207	0.026704	0.042639	0.187258	1.000000	0.316532	0.022436	0.000815	0.754102
IYR	0.552557	0.280518	0.584063	0.699292	0.739356	0.316532	1.000000	0.749836	0.754711	0.598742
PSP	0.526692	0.453303	0.750109	0.895320	0.812157	0.022436	0.749836	1.000000	0.891687	0.408005
SPY	0.439994	0.432162	0.687751	0.845863	0.793518	0.000815	0.754711	0.891687	1.000000	0.381625
TIP	0.675151	0.109006	0.378792	0.394821	0.538648	0.754102	0.598742	0.408005	0.381625	1.000000

```
Highest cor: ('EFA', 'PSP') = 0.895
Lowest cor: ('DBC', 'IEF') = -0.300
```

	annual mean	annual vol	annual sharpe
TIP	0.020502	0.051115	0.401091
IEF	0.016404	0.063442	0.258569
BWX	-0.007716	0.082789	-0.093202

```
TIPS vs Domestic Bonds (annual mean): 0.021 vs 0.016
TIPS vs Foreign Bonds  (annual mean): 0.021 vs -0.008
```

EFA and PSP have the highest correlation (0.895). DBC and IEF have the lowest correlation (-0.3).

TIPS has done moderately well in our sample. It has outperformed both domestic and foreign bonds on an annual basis.

3. The MV frontier.

- Compute and display the weights of the tangency portfolios: w^{tan}.
- Does the ranking of weights align with the ranking of Sharpe ratios?
- Compute the mean, volatility, and Sharpe ratio for the tangency portfolio corresponding to w^{tan}.

	Tangency Weights (sum=1)
SPY	1.059632
IEF	0.881186
HYG	0.290614
TIP	0.175293
EFA	0.068682
EEM	0.026437
DBC	-0.071623
IYR	-0.246582
PSP	-0.332995
BWX	-0.850643

	Weight Rank	Sharpe Rank
SPY	1.000000	1.000000
IEF	2.000000	7.000000
HYG	3.000000	2.000000
TIP	4.000000	6.000000
EFA	5.000000	5.000000
EEM	6.000000	8.000000
DBC	7.000000	9.000000
IYR	8.000000	3.000000
PSP	9.000000	4.000000
BWX	10.000000	10.000000

```
Tangency portfolio performance (annualized):
Mean:   0.1285
Vol:    0.1779
Sharpe: 0.723
```

The ranking of the weights do not align with the ranking of the Sharpe ratios. The mean, vol, and Sharpe ratios are 0.1285, 0.1779, and 0.723 respectively.

4. TIPS

Assess how much the tangency portfolio (and performance) change if…

- TIPS are dropped completely from the investment set.
- The expected excess return to TIPS is adjusted to be 0.0012 higher than what the historic sample shows.

Based on the analysis, do TIPS seem to expand the investment opportunity set, implying that Harvard should consider them as a separate asset?

Tangency Portfolio: Baseline (with TIPS)
Mean: 0.1285 Vol: 0.0875 Sharpe: 1.469

	Weights
SPY	1.059632
IEF	0.881186
HYG	0.290614
TIP	0.175293
EFA	0.068682
EEM	0.026437
DBC	-0.071623
IYR	-0.246582
PSP	-0.332995
BWX	-0.850643

Tangency Portfolio: Drop TIPS
Mean: 0.1327 Vol: 0.0905 Sharpe: 1.467

	Weights
SPY	1.100973
IEF	1.019957
HYG	0.316300
EFA	0.060153
EEM	0.032493
DBC	-0.063742
IYR	-0.247445
PSP	-0.339398
BWX	-0.879292

Tangency Portfolio: Boost TIPS mean by +0.0012 monthly
Mean: 0.1204 Vol: 0.0746 Sharpe: 1.612

	Weights
TIP	1.282487
SPY	0.798513
HYG	0.128372
EFA	0.122554
IEF	0.004667
EEM	-0.011815
DBC	-0.121397

Harvard should include them as their own assets. We see a trend where the Sharpe ratio is highest in the boosted TIPS > normal TIPS > dropped TIPS portfolios.

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3. Allocations

COLAB: PM HW3 Part3.ipynb

Build the following portfolios:

Equally-weighted (EW)

“Risk-parity” (RP)

Risk-parity is a term used in a variety of ways, but here we have in mind setting the weight of the portfolio to be proportional to the inverse of its full-sample variance estimate.

Mean-Variance (MV)

As described in `Section 2`.

Comparing

- Calculate the performance of each of these portfolios over the sample.
- Report their mean, volatility, and Sharpe ratio.
- How does performance compare across allocation methods?

```
Report Annualized Mean,Volatility, and Sharpe ratio:
```

Portfolio	Annualized Mean	Annualized Vol	Annualized Sharpe	SumWeights
Equal Weights	0.12	0.291527	0.411625	2.987328
Risk Parity	0.12	0.309515	0.387703	5.507806
Regularized Portfolio	0.12	0.102604	1.169541	0.789321
Tangency Portfolio	0.12	0.074765	1.605037	−0.227585

```
Report scaled weight of four allocation methods:
```

	Equal Weights	Risk Parity	Regularized Portfolio	Tangency Portfolio
BWX	0.271575	0.522879	−0.675374	−0.621364
DBC	0.271575	0.122905	−0.136195	0.025317
EEM	0.271575	0.107717	−0.066165	0.140124
EFA	0.271575	0.150554	0.037302	−0.120777
HYG	0.271575	0.583857	0.316781	0.189482
IEF	0.271575	0.873276	0.393506	1.065661
IYR	0.271575	0.121273	0.080277	−0.199617
PSP	0.271575	0.075284	0.049547	−0.063041
QAI	0.271575	1.452202	−0.156500	−1.643297

Date	Equal Weights	Risk Parity	Regularized Portfolio	Tangency Portfolio
2011-02-28	0.060980	0.047690	0.024063	0.015249
2011-03-31	0.025603	0.032566	-0.011978	-0.003583
2011-04-30	0.104131	0.139159	0.002404	-0.024090
2011-05-31	-0.035098	0.002996	0.020293	0.020357
2011-06-30	-0.046328	-0.032283	-0.008892	-0.000398
...	...	...	...	...
2024-05-31	0.063925	0.077448	0.041103	0.031043
2024-06-30	0.004151	0.007010	0.031826	0.057063
2024-07-31	0.063089	0.091001	0.014307	-0.016777
2024-08-31	0.032720	0.045060	0.009409	-0.001928
2024-09-30	0.063672	0.077376	0.001839	-0.001426

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