#### **HBS** Case

The Harvard Management Company and Inflation-Indexed Bonds

# 1. READING: HMC's Approach

**1.** There are thousands of individual risky assets in which HMC can invest. Explain why MV optimization across 1,000 securities is infeasible.

Calculating mean-variance optimization requires us to have the expected value of each asset and a large covariance matrix. These assumptions mean that we need to assume some level of stability for every single asset, despite the fact that noise will obscure our singles. Additionally, for an asset manager, investing in 1,000 securities instead of the basket of securities means that we are at risk of losing 100% of an asset if it goes to 0 versus in an ETF, where the downfall of an asset is minimally felt by the overall return.

- **2.** Rather than optimize across all securities directly, HMC runs a two-stage optimization.
  - 1. They build asset class portfolios with each one optimized over the securities of the specific asset class.
  - 2. HMC combines the asset-class portfolios into one total optimized portfolio.

In order for the two-stage optimization to be a good approximation of the full MV-optimization on all assets, what must be true of the partition of securities into asset classes?

The securities must be "separable," as in their covariances are roughly block-diagonal and similar within-class exposure to other classes.

**3.** Should TIPS form a new asset class or be grouped into one of the other 11 classes?

TIPS should form a new asset class, because they have a distinct return behavior that is inflation-hedged. They have a very different risk and return profile compared to other nominal bonds.

**4.** Why does HMC focus on real returns when analyzing its portfolio allocation? Is this just a matter of scaling, or does using real returns versus nominal returns potentially change the MV solution?

Because Harvard's main goal is to maximize purchasing power of endowment spending, they need to focus on real returns. Rather than scaling, this changes the MV solution because we are targeting a different goal; the inclusion of TIPS in our portfolio is an inflation hedge that removes our inflationary risks from our portfolio.

**5.** The case discusses the fact that Harvard places bounds on the portfolio allocation rather than implementing whatever numbers come out of the MV optimization problem.

How might we adjust the stated optimization problem in the lecture notes to reflect the extra constraints Harvard is using in their bounded solutions given in Exhibits 5 and 6?

Exhibit 5 shows bounds of  $0\% \le w_i \le 100\%$  for all assets and cash as low as -50%. Exhibit 6 gives us lower and upper bounds with TIPS specifically allocated between [0,100] percentage points. We can implement these as linear equalities besides sum of  $w_i = 1$ .

**6.** Exhibits 5 shows zero allocation to domestic equities and domestic bonds across the entire computed range of targeted returns, (5.75% to 7.25%). Conceptually, why is the constraint binding in all these cases? What would the unconstrained portfolio want to do with those allocations and why?

The constraint is bound at 0% because otherwise the maximizer would short these asset classes to allow higher allocations towards other asset classes. In a world where we seek the highest Sharpe ratio or better diversifiers, the portfolio would probably tend towards asset classes like private equity, hedge funds, and emerging markets.

**7.** Exhibit 6 changes the constraints, (tightening them in most cases.) How much deterioration do we see in the mean-variance tradeoff that Harvard achieved?

The efficient frontier shifts downwards and towards the left with the higher constraints. Since our optimizer has added constraints, we have a lower achievable Sharpe. We see a closer allocation to the mix in Exhibit 1 (1999 policy mix).

# 2 Mean-Variance Optimization

COLAB: • PM HW1 - Harvard Case Study.ipynb

#### Data

You will need the file in the github repo, data/multi\_asset\_etf\_data.xlsx.

- The time-series data gives monthly returns for the 11 asset classes and a short-term Treasury-bill fund return, (SHV.)
- The case does not give time-series data, so this data has been compiled outside of the case, and it intends to represent the main asset classes under consideration via various ETFs. For details on the specific securities/indexes, check the "Info" tab of the data.

#### **Excess Returns**

- We consider SHV as the risk-free asset.
- We are going to analyze the problem in terms of **excess** returns, where SHV has been subtracted from the other columns.
- The risk-free rate changes over time, but the assumption is that investors know it's value one-period ahead of time. Thus, at any given point in time, it is a risk-free rate for the next period. (This is often discussed as the "bank account" or "money market account" in other settings.)

### Adjustment

For ease of analysis, drop QAI from the dataset. Analyze the remaining 10 assets.

### **Not Considered**

- These are nominal returns-they are not adjusted for inflation, and in our calculations we are not making any adjustment for inflation.
- The exhibit data that comes via Harvard with the case is unnecessary for our analysis.

#### **Format**

In the questions below, annualize the statistics you report.

- Annualize the mean of monthly returns with a scaling of 12.
- Annualize the volatility of monthly returns with a scaling of
- Note that we are not scaling the raw timeseries data, just the statistics computed from it (mean, vol, Sharpe).

### 1. Summary Statistics

- Calculate and display the mean and volatility of each asset's excess return. (Recall we use volatility to refer to standard deviation.)
- Which assets have the best and worst Sharpe ratios?

Recall that the Sharpe Ratio is simply the ratio of the mean-to-volatility of excess returns:

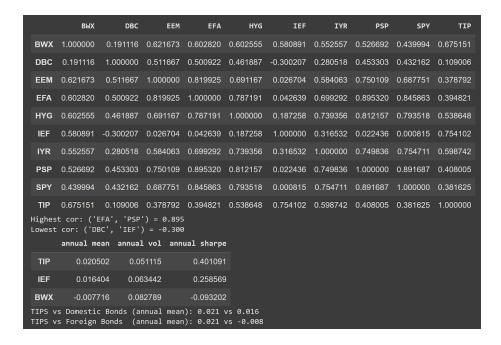
Be sure to annualize all three stats (mean, vol, Sharpe).

|  | Annual | Excess Mean | Annual Vol | Annual Sharpe |
|--|--------|-------------|------------|---------------|
| SPY  |        | 0.128141    | 0.142839   | 0.897103      |
| HYG  |        | 0.041371    | 0.075928   | 0.544873      |
| IYR  |        | 0.074916    | 0.168675   | 0.444143      |
| PSP  |        | 0.092561    | 0.213370   | 0.433804      |
| EFA  |        | 0.061775    | 0.150903   | 0.409372      |
| TIP  |        | 0.020502    | 0.051115   | 0.401091      |
| IEF  |        | 0.016404    | 0.063442   | 0.258569      |
| EEM  |        | 0.029339    | 0.176164   | 0.166542      |
| DBC  |        | -0.005292   | 0.166553   | -0.031774     |
| BWX  |        | -0.007716   | 0.082789   | -0.093202     |
| Best Sharpe: SPY (0.897)<br>Worst Sharpe: BWX (-0.093) |        |             |            |               |

The best Sharpe is SPY (0.897) and the worst Sharpe is BWX (-0.093).

### 2. Descriptive Analysis

- Calculate the correlation matrix of the returns. Which pair has the highest correlation? And the lowest?
- How well have TIPS done in our sample? Have they outperformed domestic bonds? Foreign bonds?



EFA and PSP have the highest correlation (0.895). DBC and IEF have the lowest correlation (-0.3).

TIPS has done moderately well in our sample. It has outperformed both domestic and foreign bonds on an annual basis.

#### 3. The MV frontier.

- Compute and display the weights of the tangency portfolios: w<sup>tan</sup>.
- Does the ranking of weights align with the ranking of Sharpe ratios?
- Compute the mean, volatility, and Sharpe ratio for the tangency portfolio corresponding to w<sup>tan</sup>.

|     | Tangency Weig | hts (sum=1) |
|-----|---------------|-------------|
| SPY |               | 1.059632    |
| IEF |               | 0.881186    |
| HYG |               | 0.290614    |
| TIP |               | 0.175293    |
| EFA |               | 0.068682    |
| EEM |               | 0.026437    |
| DBC |               | -0.071623   |
| IYR |               | -0.246582   |
| PSP |               | -0.332995   |
| BWX |               | -0.850643   |

|               | Weight Rank | Sharpe Rank |               |
|---------------|-------------|-------------|---------------|
| SPY           | 1.000000    | 1.000000    |               |
| IEF           | 2.000000    | 7.000000    |               |
| HYG           | 3.000000    | 2.000000    |               |
| TIP           | 4.000000    | 6.000000    |               |
| EFA           | 5.000000    | 5.000000    |               |
| EEM           | 6.000000    | 8.000000    |               |
| DBC           | 7.000000    | 9.000000    |               |
| IYR           | 8.000000    | 3.000000    |               |
| PSP           | 9.000000    | 4.000000    |               |
| BWX           | 10.000000   | 10.000000   |               |
| Mean:<br>Vol: |             | performance | (annualized): |

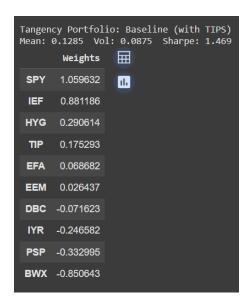
The ranking of the weights do not align with the ranking of the Sharpe ratios. The mean, vol, and Sharpe ratios are 0.1285, 0.1779, and 0.723 respectively.

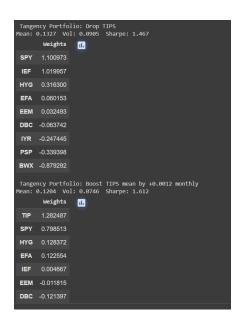
#### 4. TIPS

Assess how much the tangency portfolio (and performance) change if...

- TIPS are dropped completely from the investment set.
- The expected excess return to TIPS is adjusted to be 0.0012 higher than what the historic sample shows.

Based on the analysis, do TIPS seem to expand the investment opportunity set, implying that Harvard should consider them as a separate asset?





Harvard should include them as their own assets. We see a trend where the Sharpe ratio is highest in the boosted TIPS > normal TIPS > dropped TIPS portfolios.

## 3. Allocations

COLAB: • PM HW3 Part3.ipynb

Build the following portfolios:

### **Equally-weighted (EW)**

### "Risk-parity" (RP)

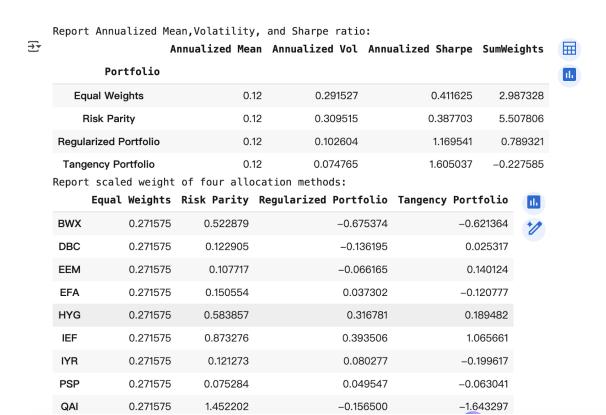
Risk-parity is a term used in a variety of ways, but here we have in mind setting the weight of the portfolio to be proportional to the inverse of its full-sample variance estimate.

### **Mean-Variance (MV)**

As described in Section 2.

### **Comparing**

- Calculate the performance of each of these portfolios over the sample.
- Report their mean, volatility, and Sharpe ratio.
- How does performance compare across allocation methods?



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#### Equal Weights Risk Parity Regularized Portfolio Tangency Portfolio 🚃





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| Date       |           |           |           |           |
|------------|-----------|-----------|-----------|-----------|
| 2011-02-28 | 0.060980  | 0.047690  | 0.024063  | 0.015249  |
| 2011-03-31 | 0.025603  | 0.032566  | -0.011978 | -0.003583 |
| 2011-04-30 | 0.104131  | 0.139159  | 0.002404  | -0.024090 |
| 2011-05-31 | -0.035098 | 0.002996  | 0.020293  | 0.020357  |
| 2011-06-30 | -0.046328 | -0.032283 | -0.008892 | -0.000398 |
|            |           |           |           |           |
| 2024-05-31 | 0.063925  | 0.077448  | 0.041103  | 0.031043  |
| 2024-06-30 | 0.004151  | 0.007010  | 0.031826  | 0.057063  |
| 2024-07-31 | 0.063089  | 0.091001  | 0.014307  | -0.016777 |
| 2024-08-31 | 0.032720  | 0.045060  | 0.009409  | -0.001928 |
| 2024-09-30 | 0.063672  | 0.077376  | 0.001839  | -0.001426 |
|            |           |           |           |           |

164 rows × 4 columns