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Neutron star atmosphere models with McPHAC

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ABSTRACT

The *McGill Planar Hydrogen Atmosphere Code* (McPHAC) (Haakonsen et al. 2012) is an open-source radiative transfer code designed to model the spectra of neutron star atmospheres with a plane-parallel approximation. In this paper, we briefly describe the model and equations used in McPHAC, explain how to obtain and get the code running, and show results for a few different atmospheres. We analyze the importance of effective temperature, surface gravity and scattering geometry on the spectrum.

Keywords: Neutron stars — Radiative transfer

1. INTRODUCTION

Neutron stars have been prime targets for X-ray observations since their discovery more than fifty years ago because of all of the insight that they can provide on dense matter physics. But not only do the observations themselves require sophisticated space-bound telescopes, the interpretation of what is observed is entirely dependent on the modeling of radiative transfer in the atmospheres of these stars. This modeling is in many ways unlike the one done for typical, main-sequence stars. On one hand, spectral lines are more uncommon due to the high levels of ionization, rarity of heavy elements, and the spectral band itself, which simplifies the problem. On the other hand, the high frequencies and temperatures make the interplay between the frequency-dependent free-free opacity and the frequency-independent Thomson scattering challenging to analyze analytically. For instance, van Paradijs (1982) showed that the commonly used modified blackbody model failed to agree with simple numerical calculations, mainly because the model considers radiation from an isothermal layer, when in fact it is precisely the temperature gradient which in the end dictates the amount of flux carried by certain frequencies over others.

Nowadays, neutron star spectra are fit using tabulated atmosphere models, which are readily available in X-ray anaylsis tools such as XSPEC. A classic model is NSA (Zavlin et al. 1996), which calculates spectra for unmagnetized hydrogen atmospheres in radiative and thermodynamic equilibrium, under a plane parallel assumption. The *McGill Planar Hydrogen Atmosphere Code* (McPHAC) was developped by Haakonsen et al. (2012) with the goal of providing an open source code to the community for it to be tested and improved. McPHAC makes it easy to analyze the importance of some physical parameters and approximations over others, which was not necessarily the case with previous codes. The model and numerical methods used in McPHAC are largely similar to those in NSA, with one potentially important improvement being the ability to treat thomson scattering anisotropically.

This paper is meant to be an educationnal look at the model and equations on which McPHAC is built (sec. 2) and the numerical method it uses to solve these equations (sec. 3.1). McPHAC is very simple to obtain and run, and we show and this can be done in sec. 3.2. Finally, in sec. 4, we show results of different models, and analyze the importance that temperature, gravity and scattering geometry have on the computed spectra.

2. THE MODEL

All of the details of the model used in McPHAC can be found in Haakonsen et al. (2012), and we only go over the main equations here. The assumptions are zero magnetic field, coherent scattering (no Compton effects), hydrostatic and radiative equilibrium. There is no convection or electron heat conduction, meaning the only mechanism for energy transport is radiation. Also, since the scale height of the atmosphere $kT/mg \approx 1$ cm is much smaller than the radius R of the star, the plane-parallel assumption can be used.

The radiation propagates through an atmosphere with a temperature profile written as $T = T(y)$, where the column depth y is a coordinate giving the amount of mass above r:

$$y(r) = \int_r^\infty \rho(r')dr' \tag{1}$$

This is useful because it leads to a a very simple description of hydrostatic equilibrium:

$$P = gy \tag{2}$$

This expression states that at any point, the gas pressure P has to exactly balance out the gravitational pressure of all of the material above. This pressure can be calculated given ρ and T using equation of state tables. The surface gravity g has a curvature term from the Schwarzchild metric to take into account general relativistic effects:

$$g = \frac{GM}{R^2}\left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} \tag{3}$$

McPHAC solves the radiative transfer equation using the Feautrier method, combining the ingoing and outgoing intensities into a single expression:

$$P_\nu(y,\mu) = \frac{1}{2}\left[I_\nu(y,\mu) + I_\nu(y,-\mu)\right] \tag{4}$$

where $\mu = \cos\theta$ goes from 0 to 1. The boundary condition are zero ingoing intensity at the surface and blackbody at large depth:

$$I_\nu(y_{\rm min}, -\mu) = 0 \tag{5}$$

$$P(y_{\rm max}, \mu) = B_\nu(T(y_{\rm max})) \tag{6}$$

With the absorption opacity κ_ν and scattering opacity $\Sigma_T = \sigma_T/m_H$, σ_T being the standard Thomson cross-section, the probability of absorption is:

$$\epsilon_\nu = \frac{\kappa_\nu}{\kappa_\nu + \Sigma_T} = \frac{\kappa_\nu}{k_\nu} \tag{7}$$

where k_ν is the total opacity. With it, the radial optical depth can be written as a function of column depth:

$$d\tau_\nu = k_\nu dy \tag{8}$$

allowing us to write the transfer equation with y:

$$\frac{\mu^2}{k_\nu}\frac{d}{dy}\left(\frac{1}{k_\nu}\frac{dP_\nu(y,\mu)}{dy}\right) = -S_\nu(y,\mu) + P_\nu(y,\mu) \tag{9}$$

$$S_\nu(y,\mu) = \epsilon_\nu B_\nu(T(y)) + \frac{2}{k_\nu}\int_0^1 P_\nu(y,\mu')\frac{d\Sigma_T}{d\mu'}(\mu,\mu')d\mu' \tag{10}$$

Since the derivation of equations 9 and 10 was not included in Haakonsen et al. (2012), it is detailed in appendix A of this paper for reference. The source function S_ν includes an integral term for scattering of photons from beams μ' into μ, which depends on scattering geometry. If scattering is isotropic, then:

$$\frac{d\Sigma_T}{d\mu'}(\mu,\mu') = \frac{1}{2}\Sigma_T \tag{11}$$

For anisotropic scattering, McPHAC uses an expression from Hummer (1962):

$$\frac{d\Sigma_T}{d\mu'}(\mu,\mu') = \frac{3}{16}\Sigma_T(3 + 3\mu^2\mu'^2 - \mu^2 - \mu'^2) \tag{12}$$

3. THE CODE

3.1. *Numerical Method*

McPHAC finds atmosphere solutions by iteratively solving equation 9 via matrix inversion and applying corrections to the temperature profile. Since scattering is assumed to be coherent, each frequency can be solved independently. The initial temperature profile is calculated with the Eddington approximation with the values of $T_{\rm eff}$ and g, the latter of which connects pressure (and therefore the EOS) to column depth y, which is the spatial coordinate. The variables are discretized into N_ν frequencies, logarithmically spaced around $kT_{\rm eff}$, N_y column depths, logarithmically spaced between user-specified minimum and maximum values, and N_μ angles. There are some requirements to user-set values. For example, $y_{\rm max}$ should be high enough such that $\tau_{\nu,\rm max} > 80$ for every ν. Full details can be found in Haakonsen et al. (2012) and/or the McPHAC source code (sec. 3.2).

To set up the matrix equation, the scattering integral is written as a Gauss-Legendre quadrature with N_μ points μ_i and weights a_i:

$$\int_0^1 P_\nu(y,\mu')\frac{d\Sigma_T}{d\mu'}(\mu,\mu')d\mu' \approx \sum_{i=1}^{N_\mu} P_\nu(y,\mu_{i'})\frac{d\Sigma_T}{d\mu'}(\mu,\mu_{i'})a_{i'} \tag{13}$$

At this stage, equation 9 has been reduced to N_ν second order differential equations. However, each of these equations is to be solved on a (N_y,N_μ) grid, so to solve them is not as simple as inverting a $N_y \times N_y$ tridiagonal matrix, as in the classic Feautrier problem with no angle dependence. Instead, a block-tridiagonal system has to be solved. The cumbersome expressions for this system can be found in Auer (1976) and are omitted here for the sake of brevity. As suggested by Haakonsen et al. (2012), this approach by Auer (1976) seems to be robust and generally accepted.

The temperature correction procedure is well described in previous work (Auer & Mihalas 1968; Zavlin et al. 1996; Haakonsen et al. 2012). The general idea is to integrate equation 9 over angle and frequency and manipulate it into:

$$\frac{1}{4\pi}\frac{d}{dy}\int_0^\infty F_\nu(y)d\nu = \int_0^\infty (J_\nu(y) - B_\nu(T(y))\kappa_\nu d\nu \tag{14}$$

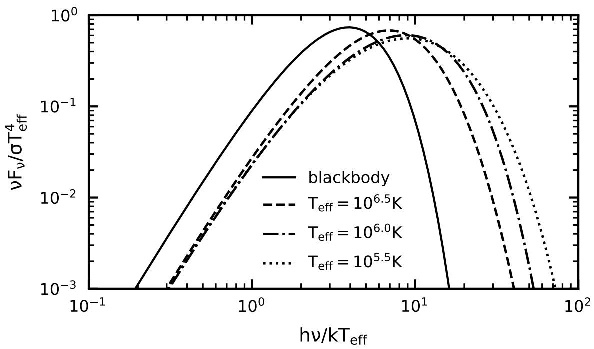

Figure 1. Normalized spectra calculated with McPHAC at three effective temperatures and blackbody spectrum for comparison. Reproduced from Haakonsen et al. (2012)

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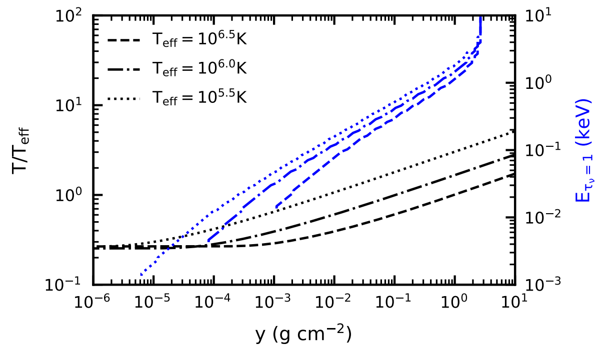

Figure 2. Column depth profiles of models with three different effective temperatures. Left y-axis and black curves: temperature profiles. Right y-axis and blue curves: photon energy for which $\tau_\nu = 1$.

where J_ν and F_ν are the mean intensity and flux, which can be calculated from P_ν with:

$$J_\nu(y) = \int_0^1 P_\nu(y,\mu)d\mu \tag{15}$$

$$F_\nu(y) = \frac{4\pi}{k_\nu}\frac{d}{dy}\int_0^1 \mu^2 P_\nu(y,\mu)d\mu \tag{16}$$

In radiative equilibrium, the total flux must be conserved across the atmosphere, so the left-hand side of equation 14 must vanish. The right-hand side won't be zero unless the temperature profile is exact, and it is this error which sets the correction procedure. Expanding the Planck function to first order around the initial temperature profile T_0:

$$B_\nu(T(y)) \approx B_\nu(T_0(y)) + \frac{dB_\nu}{dT}\Delta T(y) \tag{17}$$

Then, right-hand side of 14 rearranges to:

$$\Delta T(y) = \frac{\int_0^\infty (J_\nu - B_\nu(T_0(y))\kappa_\nu d\nu}{\int_0^\infty \frac{dB_\nu}{dT}\kappa_\nu d\nu} \tag{18}$$

This correction is applied at every depth, and the radiation field re-calculated. This process repeats until convergence is reached, i.e the correction ΔT reaches a minimum threshold at every depth.

3.2. *Getting it to run*

McPHAC can be obtained via `git` directly from the source:

```
git clone https://github.com/McPHAC/McPHAC
```

C and Fortran compilers should be set in `Makefile`, then the code can be compiled with:

```
make McPHAC
```

To run the code, parameters such as $T_{\mathbf{eff}}$, g and grid limits and resolution can be set in `McPHAC.bash`. Then:

```
bash ./McPHAC.bash
```

The code should finish running in no more than ~1 min, and store the results in `OUT/`.

4. RESULTS

4.1. *Impact of temperature*

We first analyze the effect that the atmosphere's effective temperature has on the spectrum, leaving surface gravity and all other parameters constant. The results for three different temperatures are shown with normalized units in figure 1 (which is a reproduction of figure 1 in Haakonsen et al. (2012)). The first striking result is that the spectra are significantly harder than a blackbody. This is due to the ν^{-3} dependence of free-free opacity, which makes higher energies probe deeper and hotter regions of the atmosphere. But this opacity is also proportional to $T^{-1/2}$, such that at higher temperatures, free-free is less important relative to electron scattering, than at lower temperatures. This explains why the the spectrum becomes closer to a blackbody at higher temperature, perhaps counter-intuitively.

To investigate this further, we look at the temperature profiles of these same models in figure 2. Similar to a grey atmosphere, the temperature decreases exponentially with column depth and eventually flattens out near the surface at $\sim T_{\rm eff}/4$. If we look at the photospheres, which we call the location where $\tau_\nu = 1$ (blue curves in fig. 2), we get a more detailed look at the temperatures that different frequencies probe as they escape the atmosphere. In particular, we see that at high effective temperature, while a given frequency has a hotter photosphere than at low effective temperature, the increase in photospheric temperature from lower frequen-

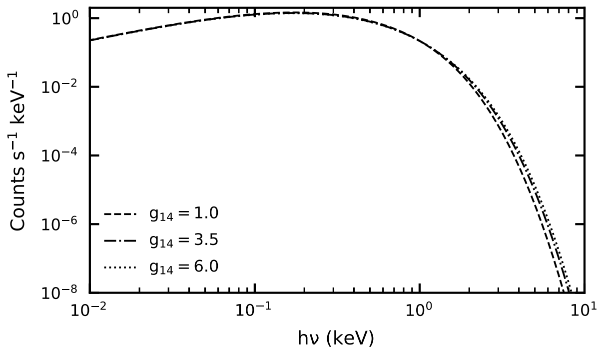

Figure 3. Spectra seen at 10 kpc by a detector with a 1 m^2 area, for three values of surface gravity $g_{14} = g/10^{14}$.

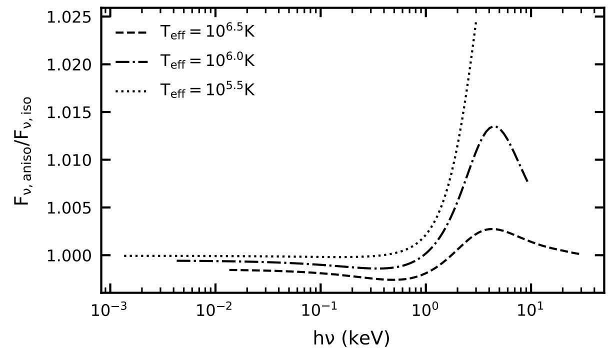

Figure 4. Flux ratios for models with anisotropic and isotropic scattering. Reproduced from Haakonsen et al. (2012).

cies is not as large. Essentially, the change in slope of the blue curves as T_{eff} increases reflects the the temperature dependence of free-free opacity and the movement of the spectrum towards a blackbody.

4.2. *Impact of surface gravity*

One of the most important aspects of neutron star research is trying to determine values for masses and radii to constrain the dense matter equation of state. In figure 3, we calculate the spectrum for envelopes with $T_{\mathrm{eff}} = 10^{6.5}$ K and three different surface gravities. The spectra are those observed at earth with a distance of 10 kpc, by a telescope with a 1 m^2 area detector. The units are photon counts per second per keV, a common choice for these observations. The values chosen for surface gravities span the parameter space of mass and radius of different equation of state models. Surface gravities below 10^{14} or above 6×10^{14} cm^2 s^{-1} are most likely not possible (Bejger & Haensel 2004). Therefore, according to figure 3, it is very difficult to constrain the surface gravity from the X-ray spectrum of a neutron star atmosphere, since the only significant differences appear at high energies, where the count rate is small.

4.3. *Impact of scattering geometry*

McPHAC improved over other codes by allowing for anisotropic thomson scattering in the form of equation 12. In figure 4, we show the flux ratios for anisotropic versus isotropic scattering. This figure is again a reproduction of figure 2 from Haakonsen et al. (2012). The difference in flux that anisotropic scattering cause are overall small, especially at low energies where the final flux is smaller than with isotropic scattering. At high energies, the flux is in excess, peaking at around 5 keV. The high energy excess appears to grow as effective temperature becomes smaller, which is convenient because, simultaneously, these high energies become less relevant in terms of total flux. Overall, it does not seem essential to consider anisotropic scattering in these models, as the flux corrections are of only a few percent or less.

5. CONCLUSION

McPHAC is not only good for educationnal purposes, it is also a great tool for members of the neutron star community to study the importance of different variables and approximations in the spectra that we observe. In the process of making this paper, we were easily able to observe the behavior of the atmosphere spectra the effective temperature increases, and gain insight on this behavior by analyzing the temperature and photosphere profiles. We also found that neither surface gravity nor anisotropic scattering are particularly important when it comes to the final spectrum. As scientists working in an ever so connected community, we should hope that all future codes, radiative transfer and otherwise, follow the philosophy with which McPHAC was written by being made open-source and transparent.

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APPENDIX

A. DERIVATION OF THE FEAUTRIER TRANSFER EQUATION

We start with a general radiative transfer equation with absorption and scattering coefficients α_ν and β:

$$\frac{dI_\nu(\mu)}{ds} = j_\nu - \alpha_\nu I_\nu(\mu) - \beta_\nu I_\nu(\mu) + \int_{-1}^{1} I_\nu(\mu')\frac{d\beta_\nu}{d\mu'}(\mu,\mu')d\mu' \tag{A1}$$

Each quantity depending on the angle μ also depends on the column depth y, it is only omitted for brevity. The terms on the right hand side represent emission, absorption, scattering out of the beam and scattering into the beam. The $\frac{d\beta_\nu}{d\mu'}(\mu,\mu')$ term represents the differential amount of scattering from beams μ' into beams μ. If the scattering is coherent, the scattering coefficient does not depend on frequency, and we have the normalization:

$$\int_{-1}^{1} \frac{d\beta_\nu}{d\mu'}(\mu,\mu')d\mu' = \beta_T \equiv n\sigma_T \tag{A2}$$

The path length coordinate relates to radial displacement with $ds = \mu^{-1}dr$. We define the radial optical depth as $d\tau_\nu = -(\alpha_\nu + \beta_\nu)dr$, and make use of the absorption probability $\epsilon_\nu = \alpha_\nu/(\alpha_\nu + \beta_\nu)$, to re-write A1 as:

$$\mu\frac{dI_\nu(\mu)}{d\tau_\nu} = I_\nu(\mu) - \epsilon_\nu B_\nu - \frac{\epsilon_\nu}{\alpha_\nu}\int_{-1}^{1} I_\nu(\mu')\frac{d\beta_\nu}{d\mu'}(\mu,\mu')d\mu' \tag{A3}$$

where we assumed LTE to write $j_\nu = \alpha_\nu B_\nu$. So far, μ has covered all values from -1 to 1. Now we'll separate outgoing and ingoing intensities and write separate equations for each of them. With $0 \leq \mu \leq 1$:

$$\mu\frac{dI_\nu(\mu)}{d\tau_\nu} = I_\nu(\mu) - \epsilon_\nu B_\nu - \frac{\epsilon_\nu}{\alpha_\nu}\int_{-1}^{1} I_\nu(\mu')\frac{d\beta_\nu}{d\mu'}(\mu,\mu')d\mu' \tag{A4}$$

$$-\mu\frac{dI_\nu(-\mu)}{d\tau_\nu} = I_\nu(-\mu) - \epsilon_\nu B_\nu - \frac{\epsilon_\nu}{\alpha_\nu}\int_{-1}^{1} I_\nu(\mu')\frac{d\beta_\nu}{d\mu'}(-\mu,\mu')d\mu' \tag{A5}$$

We now define the quantities:

$$P_\nu(\mu) = \frac{1}{2}\left[I_\nu(\mu) + I_\nu(-\mu)\right] \tag{A6}$$

$$Q_\nu(\mu) = \frac{1}{2}\left[I_\nu(\mu) - I_\nu(-\mu)\right] \tag{A7}$$

Scattering symmetry should be expected, such that $\frac{d\beta_\nu}{d\mu'}(-\mu,\mu') = \frac{d\beta_\nu}{d\mu'}(\mu,-\mu') = \frac{d\beta_\nu}{d\mu'}(\mu,\mu')$. We can therefore combine A4 and A5 into:

$$\mu\frac{dQ_\nu(\mu)}{d\tau_\nu} = P_\nu(\mu) - \epsilon_\nu B_\nu - 2\frac{\epsilon_\nu}{\alpha_\nu}\int_{0}^{1} P_\nu(\mu')\frac{d\beta_\nu}{d\mu'}(\mu,\mu')d\mu' \tag{A8}$$

$$\mu\frac{dP_\nu(\mu)}{d\tau_\nu} = Q_\nu(\mu) \tag{A9}$$

Combining once more to eliminate Q_ν, we finally obtain:

$$\mu^2\frac{d^2P_\nu(\mu)}{d\tau_\nu^2} = P_\nu(\mu) - \epsilon_\nu B_\nu - 2\frac{\epsilon_\nu}{\alpha_\nu}\int_{0}^{1} P_\nu(\mu')\frac{d\beta_\nu}{d\mu'}(\mu,\mu')d\mu' \tag{A10}$$

In this paper (and in the McPHAC paper), the opacity coefficients are used instead, changing α_ν to κ_ν and β_ν to Σ_ν in A10. The equation can also be written in terms of y with $d\tau_\nu = (\kappa_\nu + \Sigma_\nu)dy$.