AMATH 481/581 - Autumn 2022

Homework #0 - Practice assignment

Submissions accepted (not required) until Wednesday, October 12, 2022

This assignment is for **practice** and will **not be** included in your grade calculation. Make sure to upload to Gradescope your main homework file (.m, .py, or .ipynb) as well as any auxiliary files (function files or data files). Additionally you may upload a .pdf you create to test the submission of writeup problems.

1. Creating matrices and assigning them to different variables will be necessary in this course. Define the following matrix,

$$A = \begin{pmatrix} 12 & 37 \\ -9 & 0 \end{pmatrix}.$$

   Deliverables: Save the matrix A to the variable `A1`.

2. Root-finding is the task of finding zeros of a function. The two most common ways to do this are with the Bisection Method and Newton's method. You can find information about both of these methods on Canvas. Later in this class we will use root finding to solve boundary-values problems.

   Consider the function $f(x) = -x - \cos(x)$. We will try to find the root near $x = -0.74$. In other words, we want to find $\bar{x}$ such that $f(\bar{x}) = 0$. First use Newton's Method with the initial guess $x_0 = -3$ to find the root near $x = -0.74$ such that $f(\bar{x})$ is within a tolerance of 10^{-6}. Keep track of the iterations required to converge, noting that the first guess of $x_0 = -3$ counts as an iteration. Next use the Bisection Method with the interval end points $x_a = -3$ and $x_b = 1$, *i.e.,* the initial interval is $[x_a, x_b]$. Keep track of the calculated mid point values and the number of midpoints calculated (iterations) until an accuracy of 10^{-6} (on the value of the function) is achieved.

   Deliverables: Save the iterations from Newton's Method to the variable `A2` as a column vector. Save the iterations (calculated midpoints) from the Midpoint Method to the variable `A3` as a row vector. Finally, save the variable `A4` which is a 1×2 vector with the number of iterations for the Newton and Bisection Methods respectively as the two components.

To practice demonstrating mastery of the “Discussing results from a mathematical perspective” presentation skill, discuss the results you see and what this implies about using either of the two methods for this problem.

3. Polynomial curve fitting is the act of finding a (usually) low-degree polynomial to fit a number of data points. The most common polynomial curve fit is a line of best fit (or ”least-squares line”). Consider the datasets

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x = [1, 3, 4, 8, 9]
y = [3, 4, 5, 7, 12]
```

   Use a built-in tool (`polyfit` in MATLAB or `numpy.polyfit` in python) to find the line of best fit assuming the data represents some function $y = f(x)$.

   Deliverables: Save the slope of the best-fit line to the variable `A5`.

   To practice demonstrating mastery of the “2D plotting” presentation skill, create a plot of the data and the line of best fit.

4. Consider the matrix-vector equation $Ax = b$, where

$$A = \begin{pmatrix} -0.1 & 3 \\ 3 & -0.1 \end{pmatrix}, \qquad b = \begin{pmatrix} -0.2 \\ 0.2 \end{pmatrix}.$$

   We will be required to solve large systems of this form once we get to solving PDEs later in the course.

   Deliverables: Solve for the unknown x (using \ in MATLAB or `numpy.linalg.solve` in python)and save the resultant column vector to the variable `A6`.