Seismo 512 HW 4

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1/30/2023

1

Consider two types of monochromatic plane waves propagating in the x direction in a uniform medium. For each case, derive expressions for the nonzero components of the stress tensor. Refer to 2.17 to get the components of the strain tensor; then use 2.30 to obtain the stress components. Hint: Look at Example 3.4.1.

a.

P -wave in which $U_x = A\sin(\omega t - kx)$

$$\epsilon_{ij} = \frac{1}{2}\left[\partial_i U_j + \partial_j U_i\right] = \begin{bmatrix} -Ak\cos(\omega t - kx) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_{ij} = \lambda\delta_{ij}\epsilon_{kk} + 2\mu\epsilon_{ij} = \begin{bmatrix} -(\lambda+2\mu)Ak\cos(\omega t - kx) & 0 & 0 \\ 0 & -\lambda Ak\cos(\omega t - kx) & 0 \\ 0 & 0 & -\lambda Ak\cos(\omega t - kx) \end{bmatrix}$$

b.

S-wave with displacements in the y direction, i.e., $U_y = A\sin(\omega t - kx)$.

$$\epsilon_{ij} = \begin{bmatrix} 0 & -Ak\cos(\omega t - kx) & 0 \\ -Ak\cos(\omega t - kx) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_{ij} = \begin{bmatrix} 0 & -(\lambda+2\mu)Ak\cos(\omega t - kx) & 0 \\ -(\lambda+2\mu)Ak\cos(\omega t - kx) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2

(4 points) Assume harmonic P -waves are traveling through a solid with $\alpha = 10\frac{km}{s}$. If the maximum strain is 10^{-8}, what is the maximum particle displacement for waves with periods of:

a

1 s $\quad\rightarrow\quad$ Use relationship $k = \frac{2\pi f}{\alpha}$ and $T = \frac{1}{f}$ to write $k = \frac{2\pi}{\alpha T}$ and $\omega = \frac{2\pi}{T}$, so we take the real and write:

$$U(x,t) = Ae^{-i(\omega t - kx)} = Ae^{-i\left(\frac{2\pi}{T}t - \frac{2\pi}{\alpha T}x\right)} = Acos\left[\left(\frac{2\pi}{T}\right)\left(t - \frac{x}{\alpha}\right)\right]$$

Strain is the spacial derivative of displacement and the P wave will be polarized in x, so we really need ϵ_{xx}:

$$\epsilon_{xx} = \left[-\frac{2\pi}{\alpha T}A\sin\left[\left(\frac{2\pi}{T}\right)\left(t - \frac{x}{\alpha}\right)\right]\right] \quad\rightarrow\quad \epsilon_{max} = 10^{-8} = \frac{2\pi}{\alpha T}A \quad\rightarrow\quad A = \frac{\alpha T}{2\pi} * 10^{-8} = \frac{10^4(1)}{2\pi} * 10^{-8} = \frac{10^{-4}}{2\pi}m$$

b

10 s

$$A = \frac{10^4\,(10)}{2\pi} * 10^{-8} = \frac{10^{-3}}{2\pi} m$$

c

100 s

$$A = \frac{10^4\,(100)}{2\pi} * 10^{-8} = \frac{10^{-2}}{2\pi} m$$

3

(2 points) Is it possible to have spherical symmetry for S-waves propagating away from a point source? Under what conditions could an explosive source generate shear waves?

No. Define spherical symmetry as $\nabla \times A = 0$

$$U_s = \nabla \times \psi \quad \rightarrow \quad \nabla \times U_s = \nabla \times \nabla \times \psi = \nabla\left(\nabla \cdot \psi\right) - \nabla^2 \psi$$

The divergence term goes to zero, and the laplacian term gives us the wave equation, which is non-zero. Under conditions of anisotropy we should get shear waves, from energy not perfectly spreading in a sphere due to different velocity gradients in the material.