Seismo 512 HW 5

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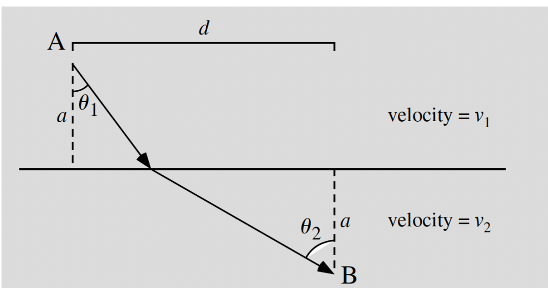

(8 points) Show that the minimum time path between points A and B for the ray geometry in Figure 4.26 gives the same result as Snell's law.
(Hint: what is the total travel time of the ray in this configuration as a function of distances and velocities?)

Snell's Law: $\frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2}$. Let the base of the triangle of θ_1 be x and the base of the triangle of θ_2 be $(d-x)$.

Travel time $t = \frac{distance}{velocity}$

$$t = \frac{\sqrt{a^2+x^2}}{V_1} + \frac{\sqrt{a^2+(d-x)^2}}{V_2}$$

A minimum time path between points means $\frac{dt}{dx} = 0$, so:

$$\frac{dt}{dx} = V_1^{-1}\frac{1}{\cancel{2}}\cancel{2}x\left(a^2+x^2\right)^{-\frac{1}{2}} + V_2^{-1}\frac{1}{\cancel{2}}\cancel{2}(d-x)\left(a^2+(d-x)^2\right)^{-\frac{1}{2}} = 0$$

$$\frac{x}{V_1\sqrt{a^2+x^2}} - \frac{d-x}{V_2\sqrt{a^2+(d-x)^2}} = 0 \quad \rightarrow \quad \frac{x}{V_1\sqrt{a^2+x^2}} = \frac{d-x}{V_2\sqrt{a^2+(d-x)^2}}$$

$\frac{x}{\sqrt{a^2+x^2}} = \sin\theta_1$ and $\frac{d-x}{\sqrt{a^2+(d-x)^2}} = \sin\theta_2$

QED