$$\begin{array}{c|c}
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|\alpha\beta\beta\beta\beta\beta\rangle |\alpha\rangle \\
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|\alpha\beta\beta\beta\rangle |\alpha\rangle \\$$

 $\hat{S}^{\beta\beta\beta}$ +

 $\hat{S}^{\alpha\alpha\alpha+}$

 $|\alpha\alpha\alpha\rangle|\alpha\rangle$

 $|\beta\beta\beta\rangle|\beta\rangle$

 $|\alpha\alpha\alpha\rangle|\beta\rangle$