Posture Optimization of 3-DOF Humanoid Robot Leg

Searching for the optimal angle to decrease the torque in each joint of the humanoid robot leg

Humanoid Team
Simon Kang, Juntae Kim, Hyunteak Lim, Jiyun Lee, Jungu Heo, Seokwon Hong

Approach and Methodology

Introduction

Stable Robot
Position

Approach and Methodology

Mathematical Modeling

Forward Kinematics
Jacobian
Dynamic Equation

3D Modeling

SOLIDWORKS -> URDF

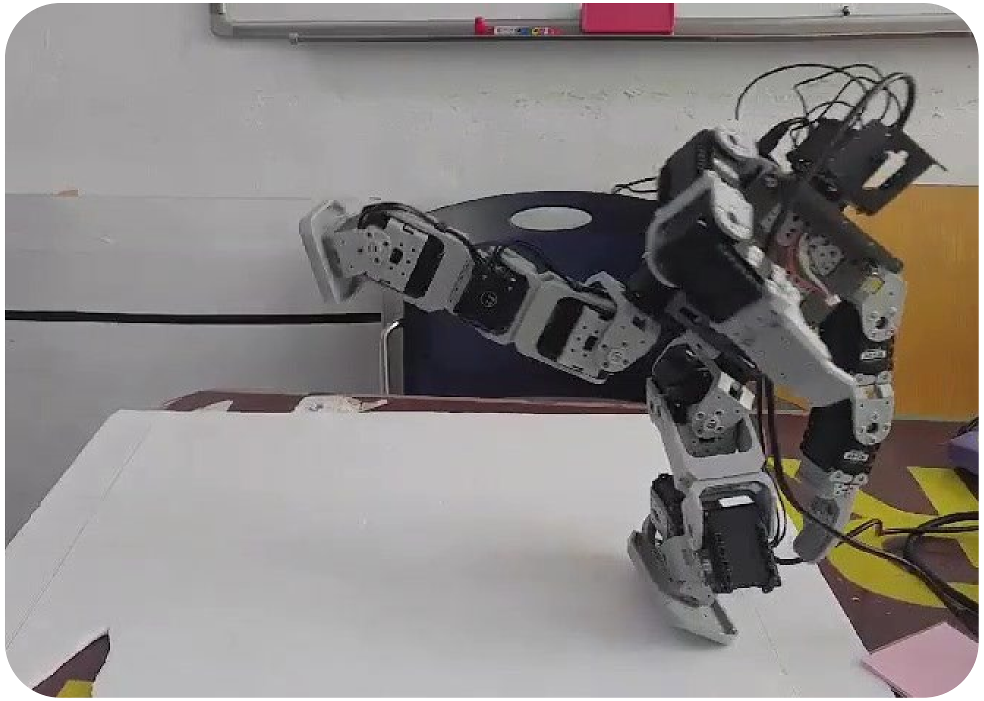
MATLAB(Simulink)

- PD controller
- Optimization

Conclusion

Optimal Θ and τ

Introduction : Soc Robot War / Stable position

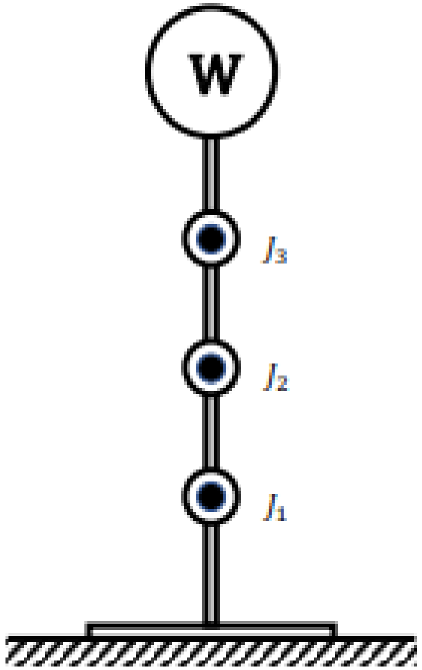


<Humanoid Robot>

Main Goal : Find the optimal knee angle to decrease torque in the humanoid robot leg.

Positioning the robot in this stance during standby mode minimizes the torque generated by the motors. This reduction in torque leads to less energy consumption, making it the most efficient posture for humanoid robots in standby

Main Methodology : Forward Kinematics, Jacobian, Dynamic Analysis



#	θ	d	a	α
0-1	θ_1	0	$76(a_1)$	0
1-2	θ_2	0	$76(a_2)$	0
2-H	θ_3	0	$99(a_3)$	0
U-0	0	0	$35(a_0)$	0

$$ {}^{U}_{0}T = \begin{bmatrix} 1 & 0 & 0 & a_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{0}_{1}T = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}_{H}T = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

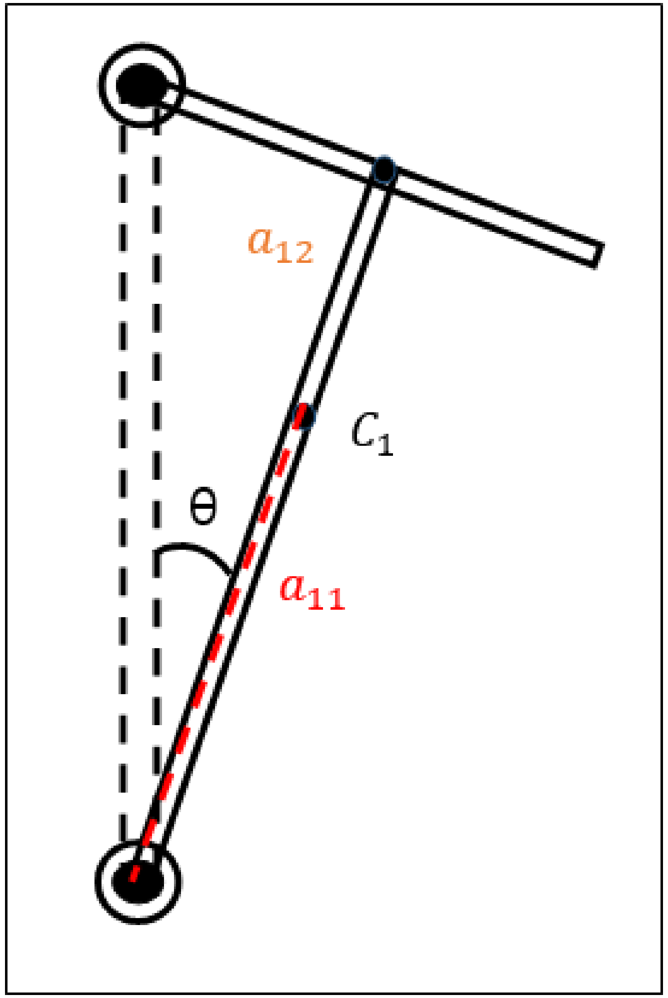
$$ {}^{U}_{1}T = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 + a_0 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{U}_{2}T = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_2C_{12} + a_1C_1 + a_0 \\ S_{12} & C_{12} & 0 & a_2S_{12} + a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{U}_{H}T = \begin{bmatrix} C_{123} & -S_{123} & 0 & a_3C_{123} + a_2C_{12} + a_1C_1 + a_0 \\ S_{123} & C_{123} & 0 & a_3S_{123} + a_2S_{12} + a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ J = \begin{bmatrix} -a_3S_{123} - a_2S_{12} - a_1S_1 & -a_3S_{123} - a_2S_{12} & -a_3S_{123} \\ a_3C_{123} + a_2C_{12} + a_1C_1 & a_3C_{123} + a_2C_{12} & a_3C_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} $$

Forward Kinematics, Jacobian, Dynamic Analysis



Dynamics equation(자중에 의한 torque(G(q))만 계산)

Link-1

$$a_{11} = 58.41(mm)$$

$$\theta = 6.146°$$

$${}^{0}_{C_1}P = \begin{bmatrix} a_{11}cos(\theta_1 - \theta) \\ a_{11}sin(\theta_1 - \theta) \\ 0 \end{bmatrix}$$

$${}^{0}_{C_2}P = \begin{bmatrix} a_1C_1 + \frac{a_2C_2}{2} \\ a_1S_1 + \frac{a_2S_2}{2} \\ 0 \end{bmatrix}$$

$${}^{0}_{C_3}P = \begin{bmatrix} a_3C_3 + a_2C_2 + a_1C_1 \\ a_3S_3 + a_2S_2 + a_1S_1 \\ 0 \end{bmatrix}$$

$${}^{0}_{v_1}J = \begin{bmatrix} -a_{11}sin(\theta_1 - \theta) & 0 & 0 \\ a_{11}cos(\theta_1 - \theta) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad {}^{0}_{v_2}J = \begin{bmatrix} -a_1S_1 & -\frac{a_2S_2}{2} & 0 \\ a_1C_1 & \frac{a_2C_2}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$${}^{0}_{v_3}J = \begin{bmatrix} -a_1S_1 & -a_2S_2 & -a_3S_3 \\ a_1C_1 & a_2C_2 & a_3C_3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore G(q) = -[J_{v_1}^T(m_1g) + J_{v_2}^T(m_2g) + J_{v_3}^T(m_3g)]$$

$$at\ g = (-g \quad 0 \quad 0)^T$$

$$G(q) = \begin{bmatrix} f(\theta_1) \\ f(\theta_2) \\ f(\theta_3) \end{bmatrix}$$

Where

$$f(\theta_1) = -a_{11}sin(\theta_1 - \theta)m_1g - a_1S_1(m_2g + m_3g)$$

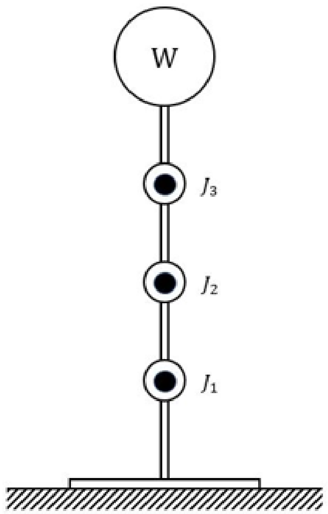
$$f(\theta_2) = m_2g(-\frac{a_2S_2}{2} + a_2S_2)$$

$$f(\theta_3) = -a_3S_3$$

$$(m_1 = 89g, m_2 = 30g, m_3 = 618g)$$

Forward Kinematics, Jacobian, Dynamic Analysis

Model



Parameter

$a_1 = 76(mm)$ $m_1 = 89(g)$
$a_2 = 76(mm)$ $m_2 = 30(g)$
$a_3 = 99(mm)$ $m_3 = 618(g)$

$\alpha = 12.153(degree)$
$a_{c1} = 58.41(mm)$

Equation oof Motion

$$\boldsymbol{T} = \boldsymbol{M}(q)[\ddot{\boldsymbol{q}}] + \boldsymbol{C}(q)[\dot{\boldsymbol{q}}^2] + \boldsymbol{B}(q)[\dot{\boldsymbol{q}}\dot{\boldsymbol{q}}] + \boldsymbol{G}(q)$$

$$M(q) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

$$m_{11} = a_{c1}^2 m_1 + a_1^2 m_2 + a_1^2 m_3 + I_{c1} + I_{c2} + I_{c3}$$

$$m_{12} = m_{21} = \left(\frac{m_2 a_1 a_2}{2} + m_3 a_1 a_2\right)\cos(\theta_1 - \theta_2) + I_{c2} + I_{c3}$$

$$m_{13} = m_{31} = m_3 a_1 a_3 \cos(\theta_1 - \theta_3) + I_{c3}$$

$$m_{22} = \frac{1}{4} m_2 a_2^2 + m_3 a_2^2 + I_{c2} + I_{c3}$$

$$m_{23} = m_{32} = m_3 a_2 a_3 \cos(\theta_2 - \theta_3) + I_{c3}$$

$$m_{33} = m_3 a_3^2 + I_{c3}$$

$$C(q) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$c_{11} = c_{22} = c_{33} = 0$$

$$c_{12} = \left(\frac{m_2 a_1 a_2}{2} + m_3 a_1 a_2\right)\sin(\theta_1 - \theta_2)$$

$$c_{13} = m_3 a_1 a_3 \sin(\theta_1 - \theta_3)$$

$$c_{21} = -\left(\frac{m_2 a_1 a_2}{2} + m_3 a_1 a_2\right)\sin(\theta_1 - \theta_2)$$

$$c_{23} = m_3 a_2 a_3 \sin(\theta_2 - \theta_3)$$

$$c_{31} = -m_3 a_1 a_3 \sin(\theta_1 - \theta_3)$$

$$c_{32} = -m_3 a_2 a_3 \sin(\theta_2 - \theta_3)$$

$$\boldsymbol{B}(q) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{G}(q) = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}$$

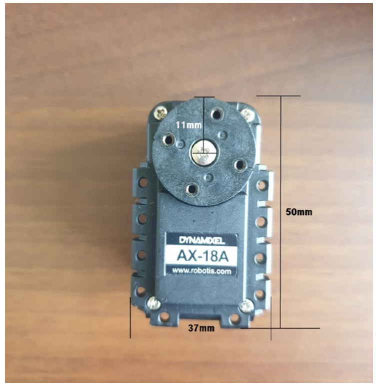
$$g_1 = \frac{a_{c1}}{2} m_1 g \sin(\theta_1 - \alpha) + a_1 m_3 g \sin(\theta_1) + a_1 m_4 g \sin(\theta_1)$$

$$g_2 = \frac{a_2}{2} m_3 g \sin(\theta_2) + a_2 m_4 g \sin(\theta_2)$$

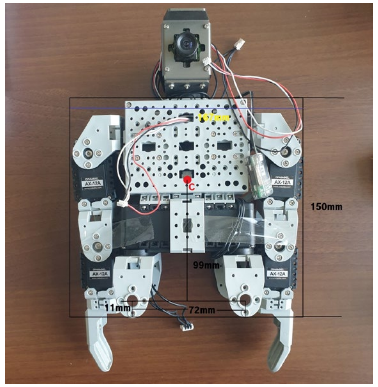
$$g_3 = a_3 m_4 g \sin(\theta_3)$$

3D Modeling for URDF

Leg
- 3 –LINK
- Mass = 425g

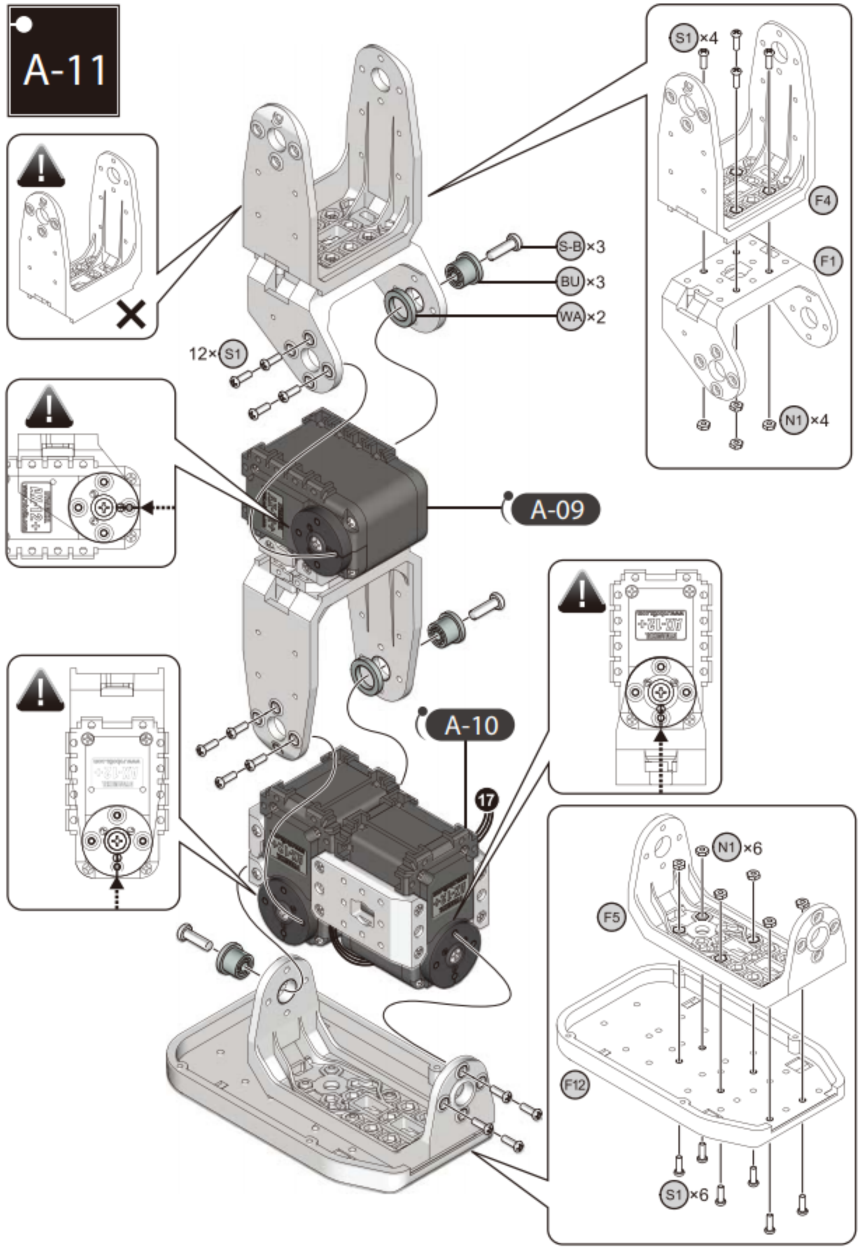


Motor
- Mass = 59g

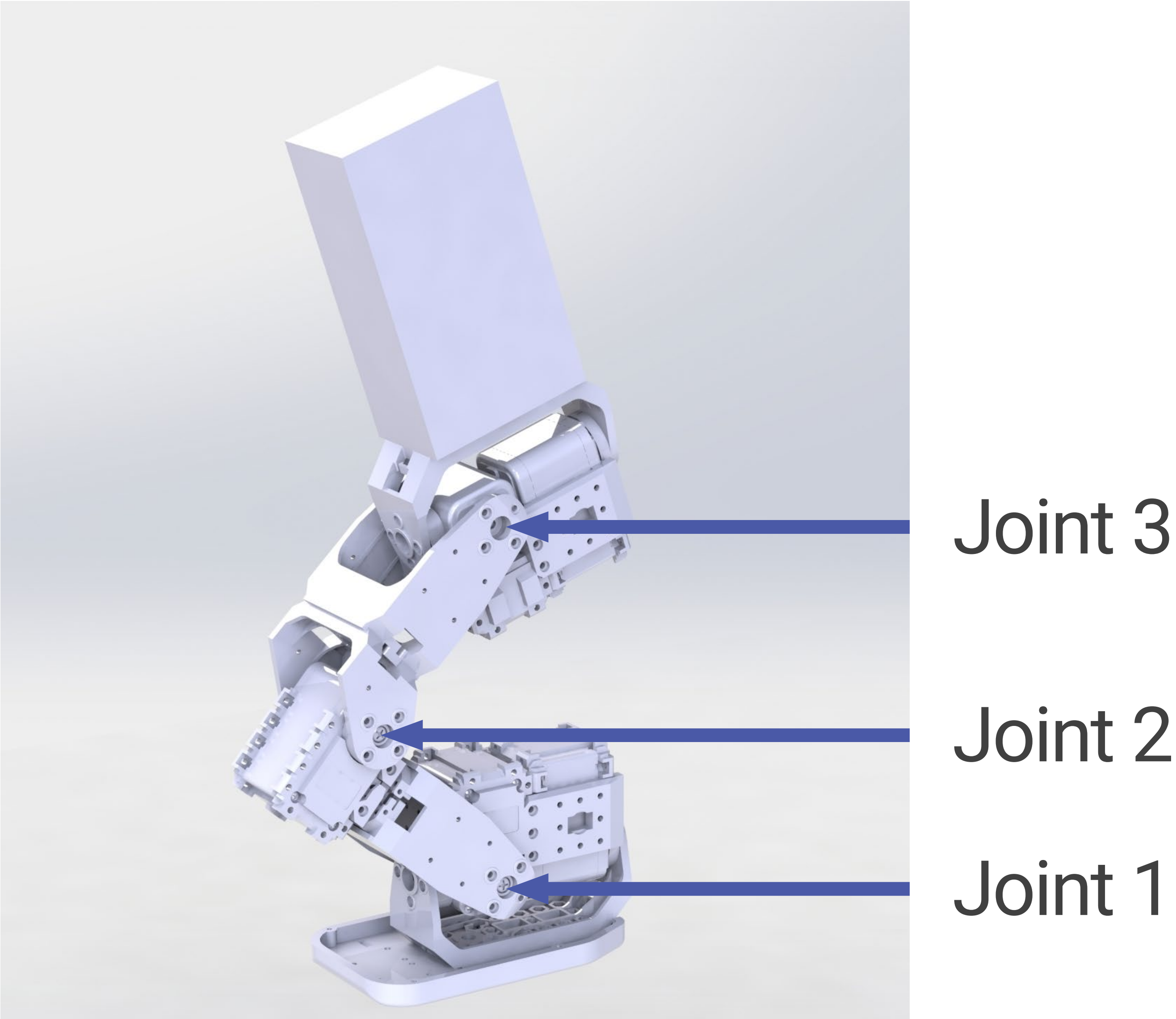


Upper body
- Mass = 1000g

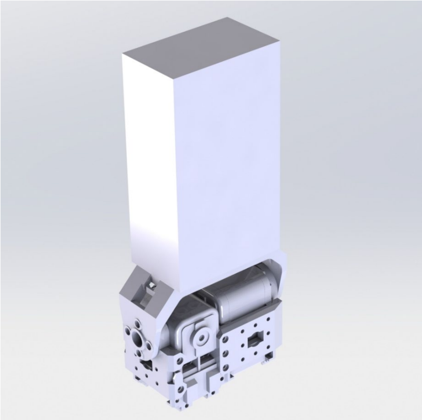
Leg Link
- Mass = 30g



3D Modeling for URDF

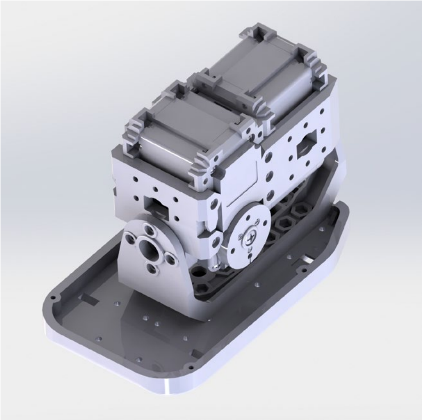


Solidworks Rendering

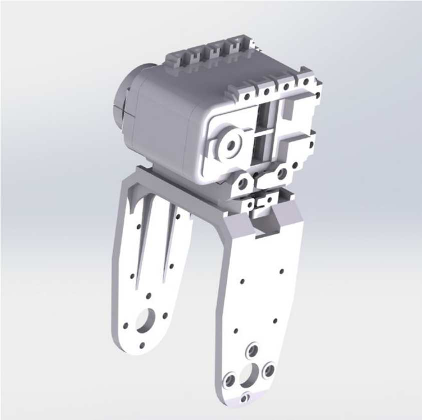


<Link 3>

<Link 2>

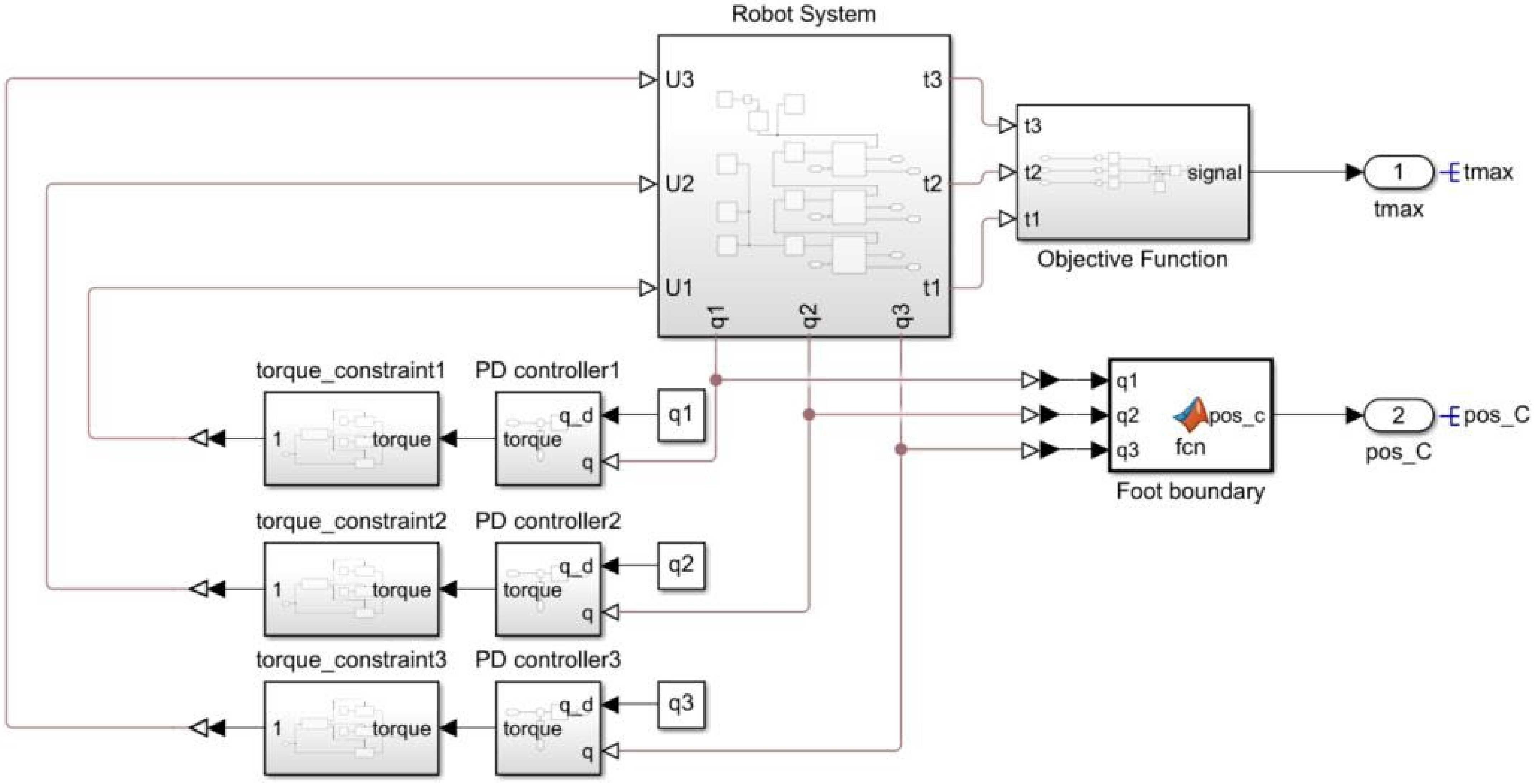


<Base>



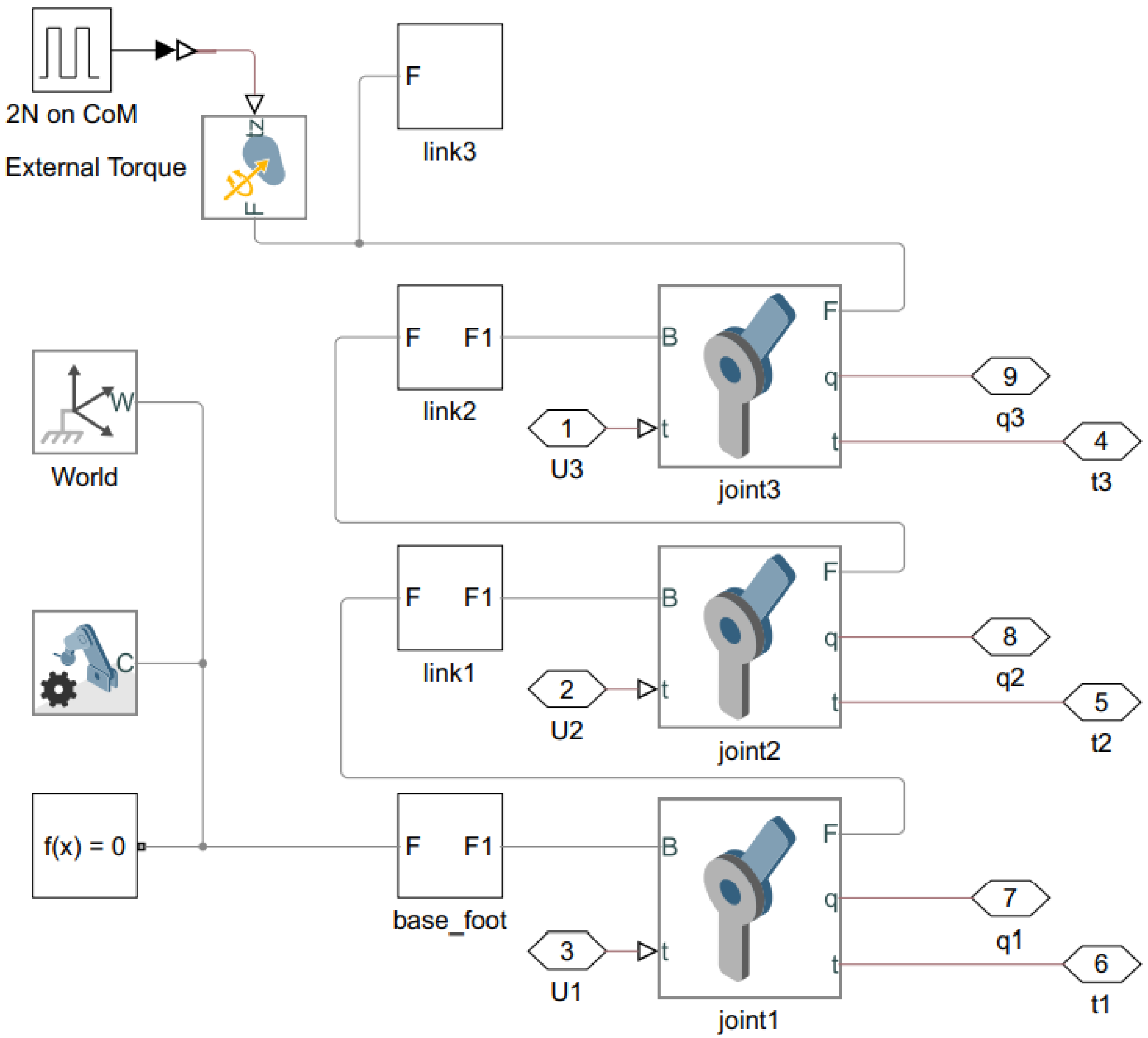
<Link 1>

Simulation : MATLAB Simulink



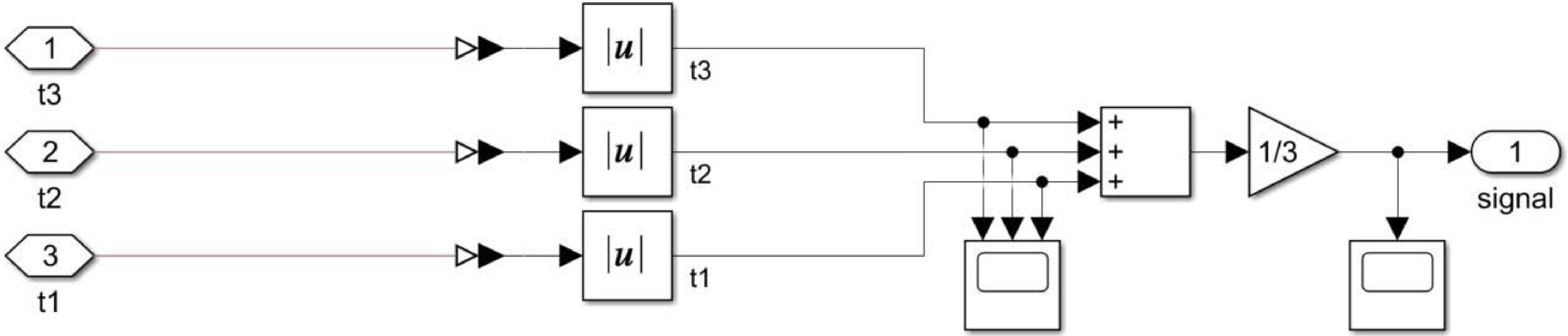
<Block Diagram for the Robot System on Simulink>

Simulation : MATLAB Robot System



<Block Diagram for the Robot(Plant) using Simulink>

Simulation : MATLAB Objective Function

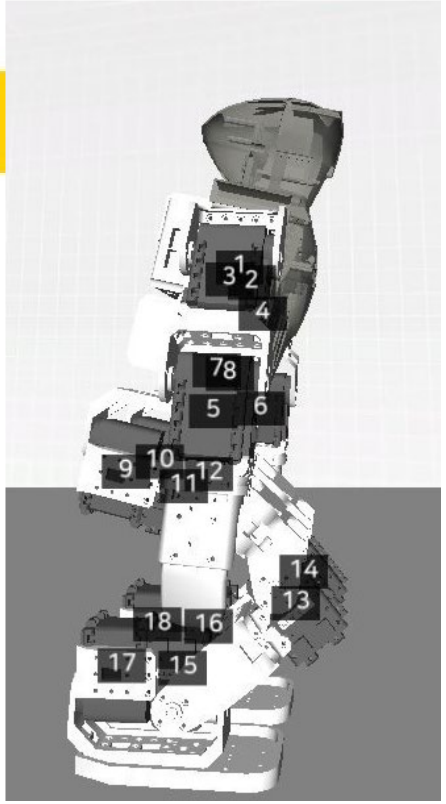


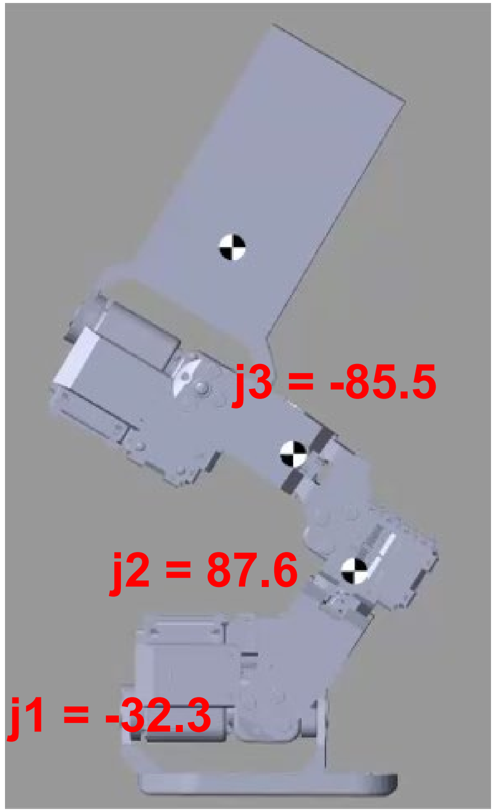
<Objective Function inside the Block Diagram>

Simulation : MATLAB Result

로봇 포즈

ID	3D 로봇	실제 로봇	
6	14.36	?	?
7	-45.12	?	?
8	45.12	?	?
9	-1.46	?	?
10	1.17	?	?
11	-61.82	j3 ?	?
12	61.52	?	?
13	-100.2	j2 ?	?
14	99.9	?	?
15	51.27	j1 ?	?
16	-51.56	?	?





<Initial Degrees of the robot legs>

<Degrees found after optimization>

<Centroid(mm)>

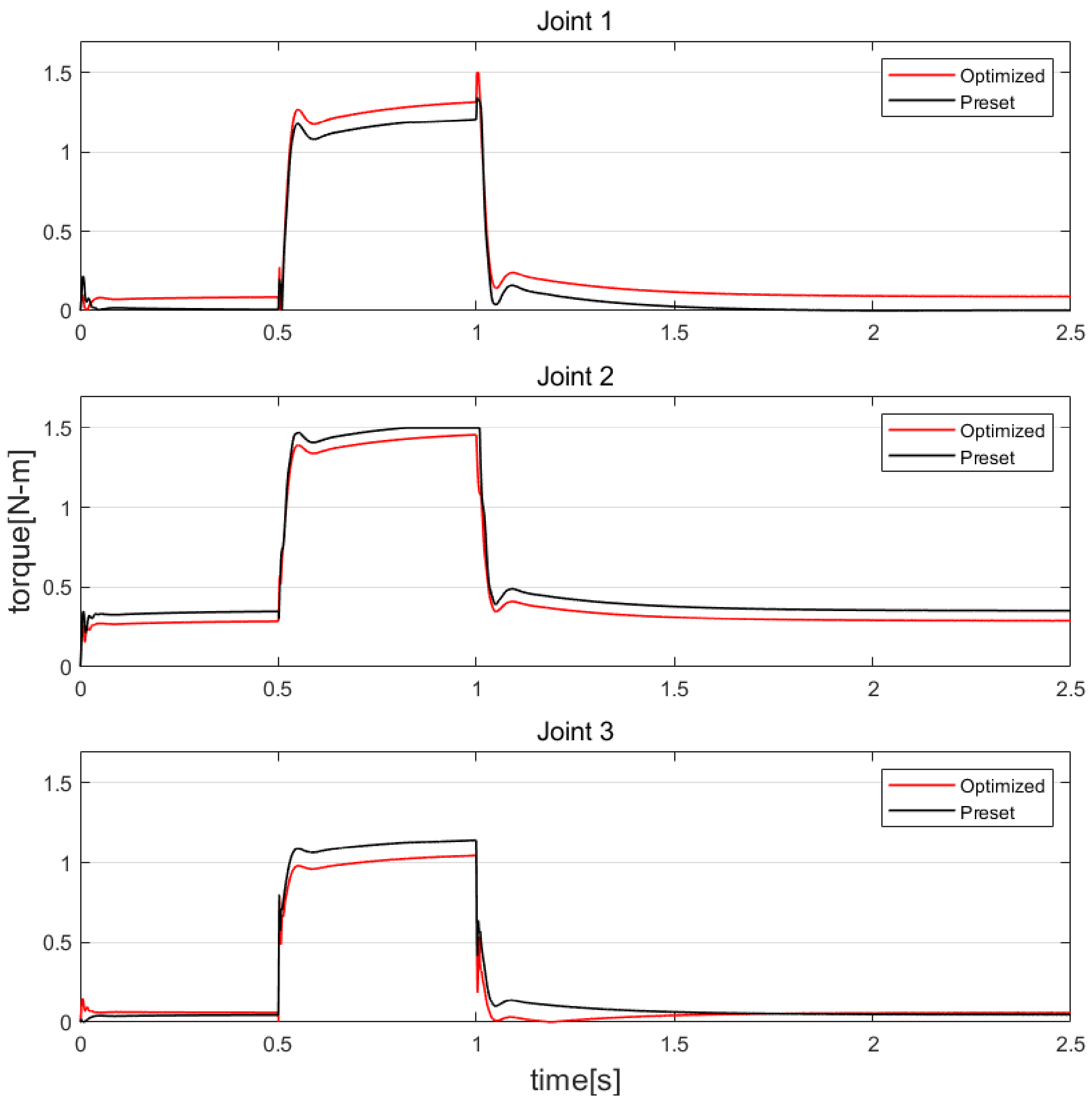
Coordinate	Preset	Optimized
x	-26.3128	-5.8665
y	100.4182	107.1931

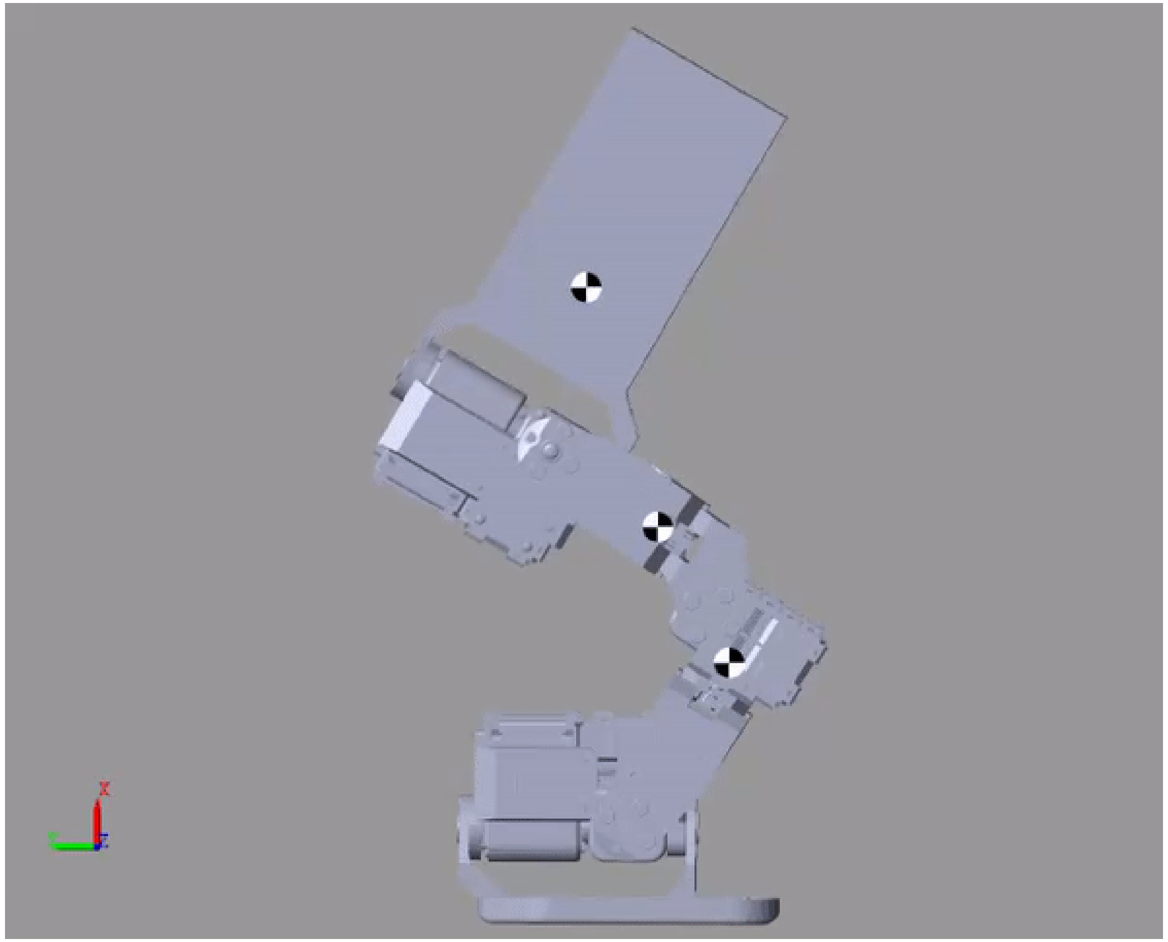
<Torque(N-m)>

Preset	Optimized
0.3730	0.3673

- Total torque decreased by 1.5%.
- Knee torque decreased by 11.34%

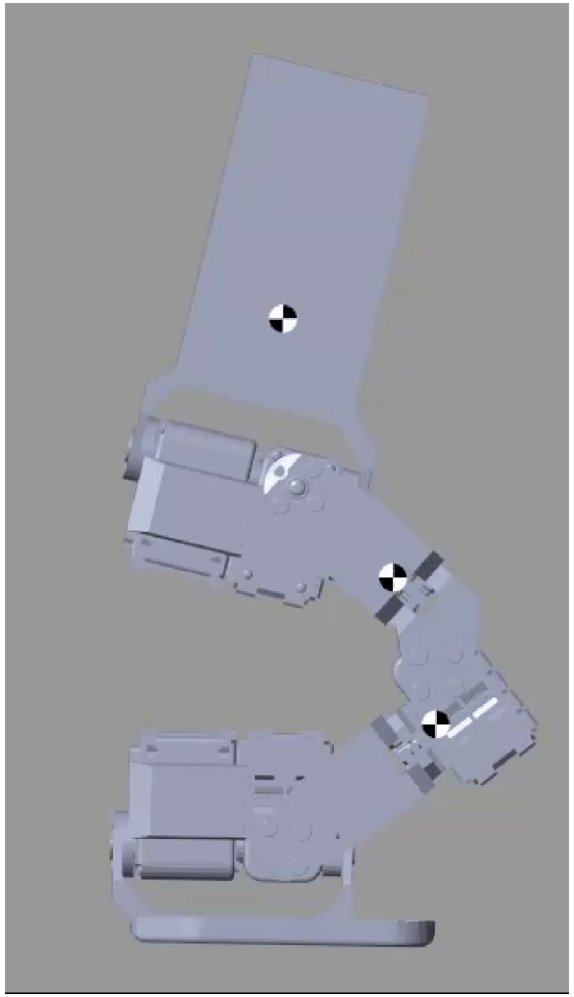
Simulation : MATLAB Result



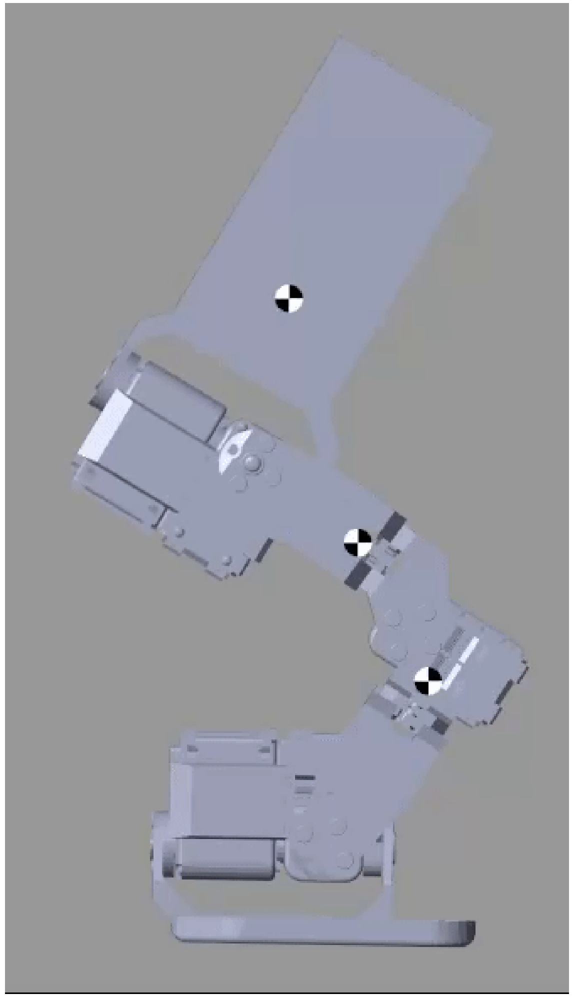


<MATLAB Simulation>

Conclusion



<Preset Posture>



<Optimized Posture>

Comparison between preset and optimized posture

1. Total torque decreased by 1.5%

2. Knee torque decreased by 11.34%

3. Positioning the robot in this stance during standby mode minimizes the torque generated by the motors. This reduction in torque leads to less energy consumption, making it the most efficient posture for humanoid robots in standby