

BACKGROUND



EXPLORING DISEASES ON NETWORKS



TRACK AND TRACE



A possible smeared phase transition in epidemic track-and-trace

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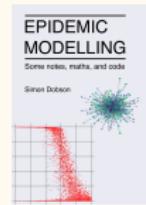


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INTRODUCTION

Exploring epidemics using network science¹

- ▶ Disease models and parameters
- ▶ Contact structures
- ▶ Countermeasures



In the course of writing about these ideas I stumbled across something I can't readily explain

- ▶ A change in behaviour that should be crisp, but doesn't seem to be
- ▶ Deserves more exploration

¹S. Dobson. **Epidemic modelling – Some notes, maths, and code**. Independent Publishing Network, 2020.
ISBN 978-183853-565-0. URL <https://simoninireland.github.io/introduction-to-epidemics/>

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STRUCTURE OF THIS TALK

Background

Measuring diseases

Compartmented models of disease

Epidemics on networks

Exploring diseases on networks

Changing the contact network

Immunity

Physical countermeasures

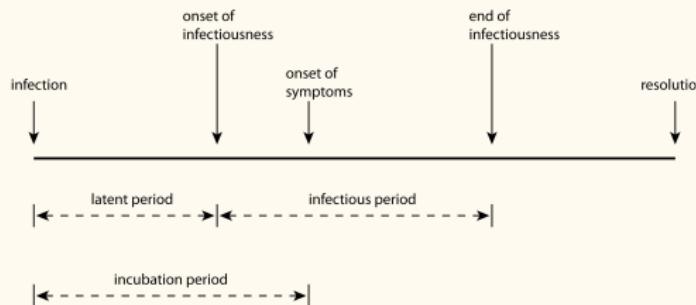
Track and trace

SEIR infections

Wild speculations



REAL DISEASES – 1



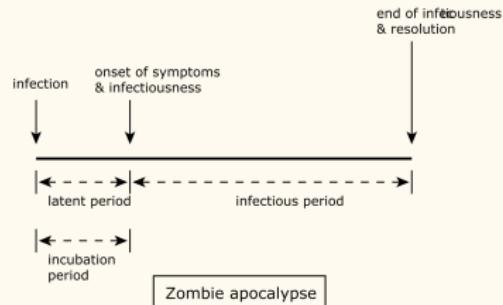
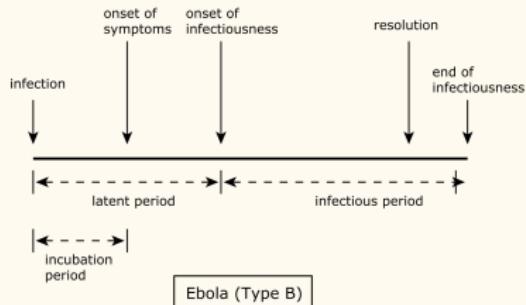
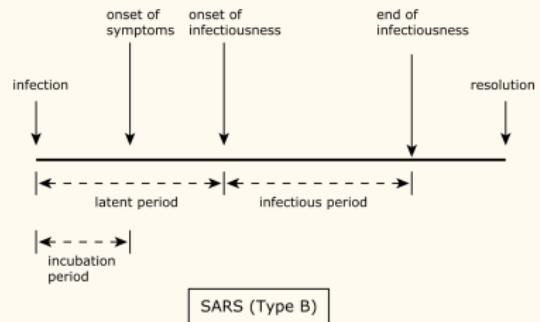
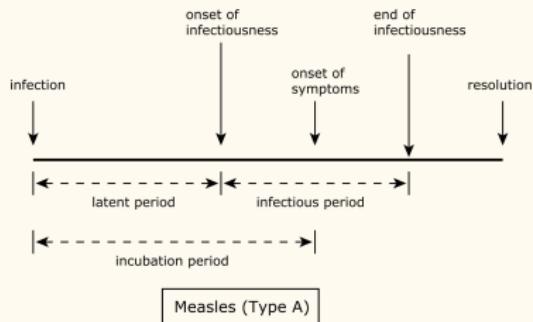
Different periods

- ▶ **Incubation**: from infection to onset of symptoms
- ▶ **Latent**: from exposure to infectiousness
- ▶ **Infectious**: overlapping with symptoms (usually)

Periods defined by biology, of both disease and host



REAL DISEASES – 2





\mathcal{R} AND ALL THAT²

\mathcal{R} , the case reproduction number

- ▶ Number of secondary cases per primary
- ▶ Especially \mathcal{R}_0 , reproduction absent immunity or countermeasures

r , the case reproduction rate

- ▶ Doubling time for an epidemic
- ▶ Also sometimes see T_g , the inter-generation time

²Royal Society SET-C group. Reproduction number (R) and growth rate (r) of the COVID-19 epidemic in the UK: methods of estimation, data sources, causes of heterogeneity, and use as a guide in policy formulation, August 2020. URL <https://royalsociety.org/-/media/policy/projects/set-c/set-covid-19-R-estimates.pdf>



THE “WICKEDNESS” OF COVID-19

$\mathcal{R}_0 \approx 3$ is not particularly infectious

- ▶ Straightforward to get $\mathcal{R} \approx 1.5$; harder to get $\mathcal{R} < 1$

Case fatality rate is about 1%

- ▶ Too large to comfortably ignore, but not so large as to admit no arguments

Infection seems to convey only temporary immunity

Substantial asymptomatic transmission

- ▶ Asymmetric costs (spreading *vs* dying, “long COVID”)
- ▶ Effective countermeasures are collective (and expensive)



COMPARTMENTED MODELS

Traditional epidemic modelling uses the framework of a **compartmented model** of a disease

- ▶ A number of compartments holding some fraction of the population
- ▶ (Can also think of this as the state of each individual)
- ▶ Rules on how these fractions change over time



CONTINUOUS SIR

The model

- ▶ Susceptible individuals can catch the infection from Infected individuals
- ▶ ...who then are Removed from the dynamics by recovery (or death)

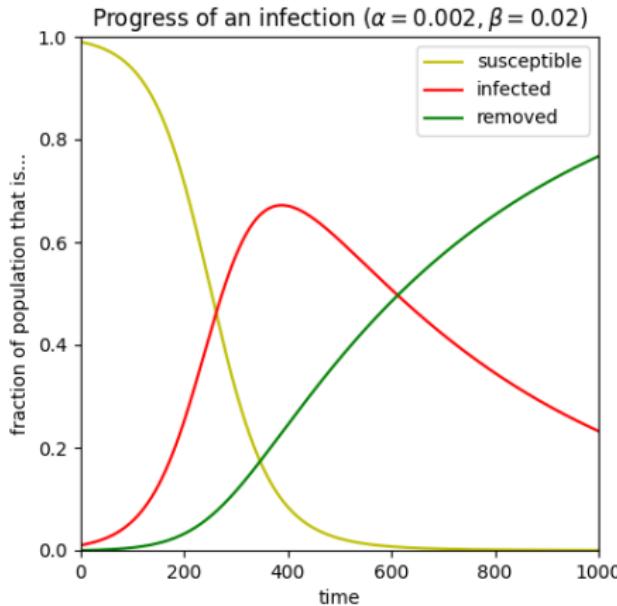
Epidemic dynamics

- ▶ Susceptibles infected per contact with probability β
- ▶ Infecteds removed with probability α

$$\frac{dS}{dt} = -\beta SI \quad \frac{dI}{dt} = \beta SI - \alpha I \quad \frac{dR}{dt} = \alpha I$$



SOLUTION





A RANGE OF MODELS WITH INCREASING COMPLEXITY

Can model different disease structures ³

- ▶ SIR – simple infection offering complete immunity post-infection
- ▶ SIS – infection confers no immunity
- ▶ SEIR – exposed individuals are infectious before symptoms
- ▶ MSEIR – initial immunity passed from mother to child
- ▶ SEIRS – immunity wears off with time
- ▶ ...

³ H. Hethcote. The mathematics of infectious diseases. *SIAM Review*, 42(4):599–653, December 2000. URL [doi://10.1137/S0036144500371907](https://doi.org/10.1137/S0036144500371907)



EPIDEMICS ON NETWORKS – 1

Complete homogeneous mixing implies populations have no structure, which clearly isn't true

- ▶ Not everyone meets everyone else
- ▶ Some are massively more (or less) connected than average

One man linked to 56 COVID cases in Ireland after failing to restrict his movements

The man failed to restrict his movements after returning from abroad, according to a new report from Ireland's Department of Public Health, HSE Mid-West.

Kerry O'Sullivan, who is Due 21/2020

A new report from Ireland's Health Service Executive Mid-West region details how one man was responsible for 56 confirmed cases in persons in Ireland when he failed to restrict his movements after returning from abroad.

The report, which was published on October 19 and can be downloaded here, was authored by Declan Kavanagh, Katie Evans, Margaret Morris, Deirdre Anne O'Byrne, Rose Fitzgerald, and Maia Murray on behalf of the



EPIDEMICS ON NETWORKS – 2

Use a network as the substrate for the epidemic ⁴

- ▶ Only nodes that are adjacent can interact
- ▶ Compartment = label on node

Leads to some more modelling decisions

- ▶ Degree distribution: probability p_k of node having degree k
- ▶ Often treat the mean degree $\langle k \rangle$ as “representative”
- ▶ Topological fine structure: loops, modules, layers, ...
- ▶ Adaptive behaviour to change these features in response to the progression of the disease

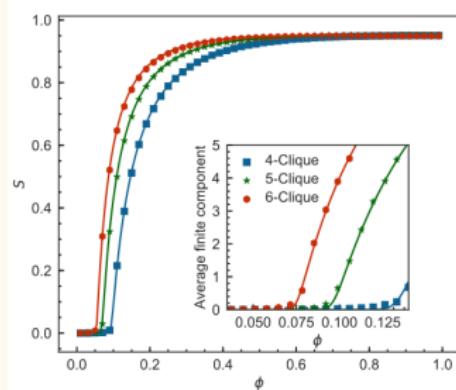
⁴ M. Newman. Spread of epidemic disease on networks. *Physical Review E*, 66, July 2002. URL
[doi://10.1103/PhysRevE.66.016128](https://doi.org/10.1103/PhysRevE.66.016128)



HOW TO DO ANALYSIS

The “gold standard” is to have an analytic model with numerical validation

- ▶ Find an analytic description for what happens under different infection parameters
- ▶ Run process on random networks with the given topology
- ▶ Lots of repetitions to squeeze out variance
- ▶ (Hopefully) sample points land on solutions of the equations⁵



⁵P. Mann, V. A. Smith, J. Mitchell, and S. Dobson. Percolation in random graphs with higher-order clustering.

URL <https://arxiv.org/abs/2006.06744>. Under review by Physical Review E. Preprint available on arXiv



HOW TO DO NUMERICAL VALIDATION

Discrete-event simulation⁶

- ▶ After a time τ we see an event e (infection, removal)
- ▶ Draw the next time and event from joint distribution $P(\tau, e)$
- ▶ Shape of the distribution changes as number of SI edges change

The code is quite involved, so we encapsulated the core behaviours as libraries

- ▶ `epydemic`: models, standard topologies, ...
- ▶ `epyc`: repeated experiments, parallelism, dataset management

<https://github.com/simoninireland>

⁶D. Gillespie. Exact stochastic simulation of coupled chemical reactions. *Journal of Physical Chemistry*, 81(25):2340–2361, 1977



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WHAT DO HUMAN CONTACT NETWORKS LOOK LIKE?

Networks with particular contact structure

- ▶ Tend to have cycles (friends of friends)
- ▶ Highly variable numbers of contacts
- ▶ Often modular, having different local and global structures

Construction

- ▶ Theoretically, using a process to generate the structures
- ▶ Empirically, structured from survey data ⁷

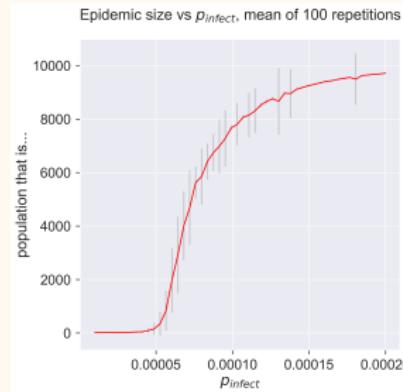
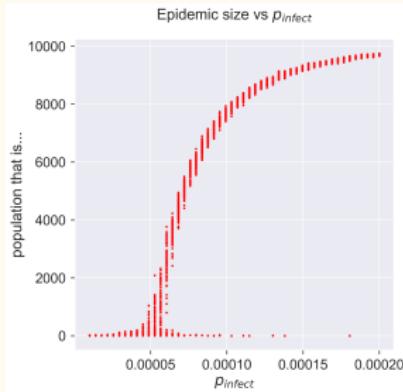
⁷ K. Prem, A. Cook, and M. Jit. Projecting social contact matrices in 152 countries using contact surveys and demographic data. *PLOS Computational Biology*, 13(9), 2017. URL
<https://doi.org/10.1371/journal.pcbi.1005697Ed>



ER NETWORKS AND THE EPIDEMIC THRESHOLD

Erdős-Rényi (ER) networks

- ▶ For N nodes build the complete network K_N
- ▶ For each edge, retain (“occupy”) it with probability p_{infect}
- ▶ Leads to p_k normally distributed around a mean $\langle k \rangle = \frac{p_{infect}}{N}$

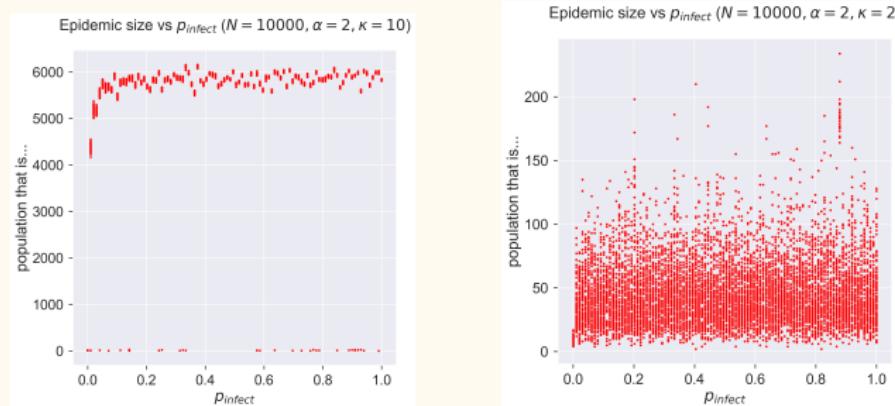




NOT ALL NETWORKS BEHAVE LIKE THIS

Too “even” to be a good model of human contacts

- ▶ Powerlaw with cutoff, $p_k \propto k^{-\alpha} e^{K/\kappa}$



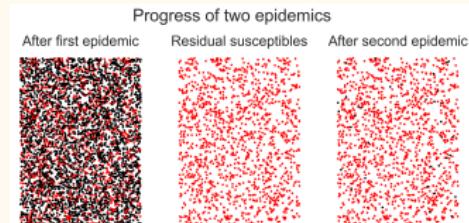
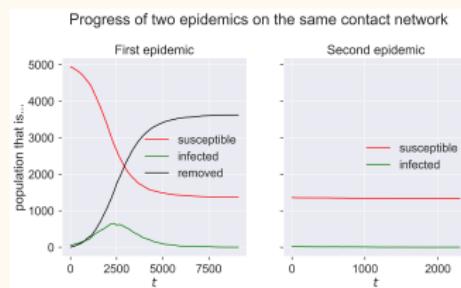
- ▶ Relatively insensitive to p_{infect} , but sensitive to α and κ



HERD IMMUNITY

Sufficient immune/recovered individuals to stop an epidemic propagating

- ▶ Infecteds never adjacent to enough susceptibles
- ▶ First epidemic changes the effective topology
- ▶ “Effective” $\langle k \rangle$ drops from 20 to 5.5
- ▶ (Interested to explore other topological changes)





WHY THIS IS A BAD IDEA

Herd immunity has been seriously advocated as a strategy for COVID-19⁸

Ignores some rather inconvenient facts

- ▶ A 1% death rate = 700K UK deaths, about one year's excess
- ▶ At a rate that would collapse health services
- ▶ Immunity may not be permanent – which makes herd immunity behave differently (or not appear at all)
- ▶ Long COVID not accounted for in the costs

⁸See the “Great Barrington Declaration”, <https://gbdeclaration.org>



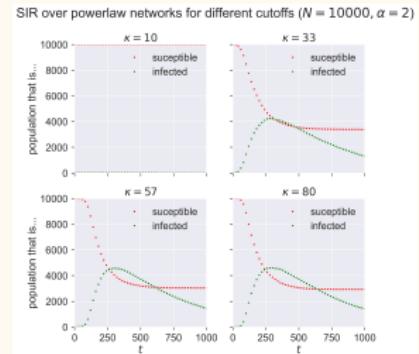
VACCINATION

“Herd immunity without the bad bits”

- ▶ Aim for the herd immunity threshold, generally about 60% of the population
- ▶ ... without anyone actually being infected

Epidemic proceeds at different rates depending on topology

- ▶ “Enough” contacts stabilise the size of outbreak





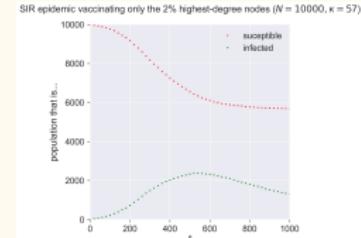
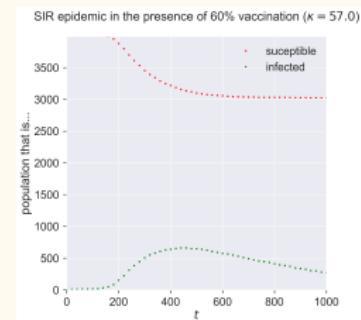
VACCINATION STRATEGIES

Randomly vaccinate 60% of the population

- ▶ Massive reduction in epidemic size
- ▶ Only catching high-degree nodes at random
- ▶ Sensitive to missing people

If we target vaccination we can reduce the threshold needed to get the same effect

- ▶ Target highest-degree 2% of nodes
- ▶ Take out the super-spreaders





PHYSICAL DISTANCING

What does a physically-distanced network look like?

- ▶ Good question
- ▶ (I have two SH projects looking at this at University level)

One possible model

- ▶ Normally-distributed, fully connected family “bubbles” of mean size 4
- ▶ A couple of members with outside contacts
- ▶ Exponentially-distributed connections between the contacts in different bubbles



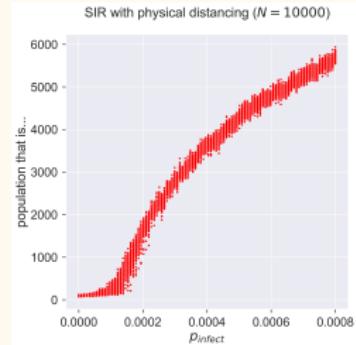
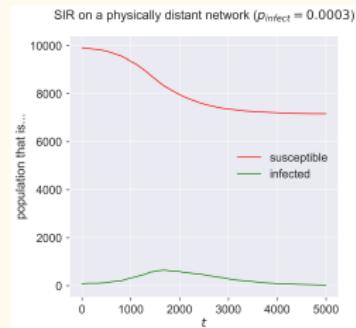
LOCKDOWN CHANGES PROPAGATION

Changes the epidemic threshold compared to an ER network

- ▶ Needs a higher infectivity to take off

Slower take-off

- ▶ Not like a powerlaw network
- ▶ Get bursts of infection if the infection gets into a bubble





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ASYMPTOMATIC TRANSMISSION

Because covid-19 is essentially SEIR (or maybe SEIRS) it invites other countermeasures

- ▶ Self-isolating on showing symptoms is ineffective
- ▶ Try to find the asymptomatic carriers

This is the basis for track-and-trace

- ▶ Identify contacts of that person
- ▶ Quarantine them if they're infected – means we catch infecting individuals before they knew to self-isolate
- ▶ Quarantine the symptomatic individual too



TRACK AND TRACE IN PRACTICE

A large-scale procedure, unlike the local procedure of self-isolation when symptomatic

- ▶ Requires organisation by some authority
- ▶ What can possibly go wrong?...

Unlikely to be fully accurate even if done competently

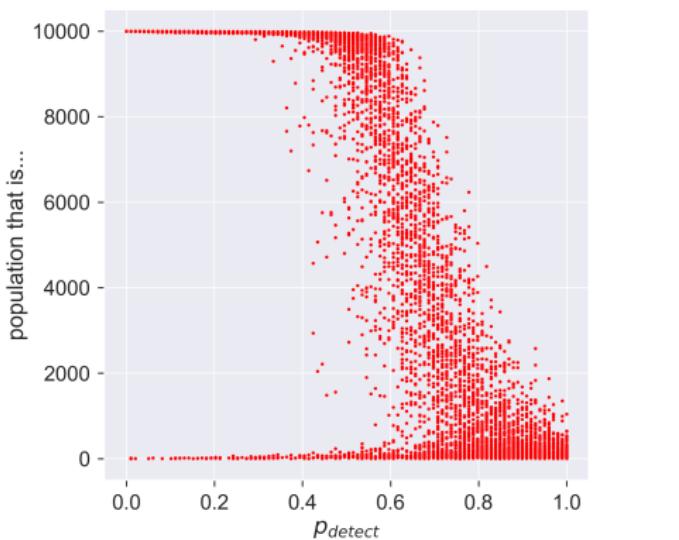
- ▶ Some proportion of people don't quarantine? (p_{rewire})
- ▶ Only test some proportion of contacts? (p_{detect})



THE IMPACT OF DETECTION RATES

Hold p_{rewire} constant and vary p_{detect}

SEIR epidemic size vs p_{detect} ($N = 10000$, $\langle k \rangle = 40$, $p_{\text{rewire}} = 0.8$)





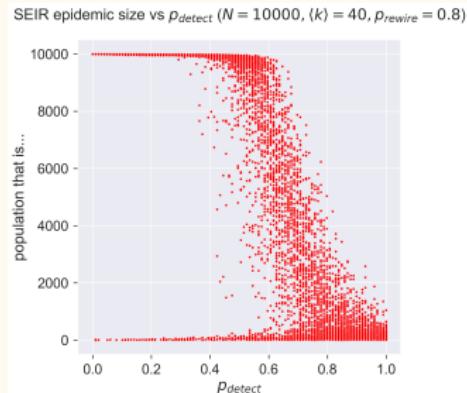
INTERPRETING THE PLOT

Very dependent on detection rate

- Quarantine can still sometimes damp epidemic anyway
- High detection is very effective

Seems to be unstable in the mid range

- All sizes of epidemic are possible
- No clean break
- Phase transition is “smeared out” rather than occurring cleanly





WHAT'S HAPPENING?

This phenomenon only entered the literature last year ⁹

Possible explanations

- ▶ It's an artefact of ER networks
- ▶ It's an artefact of not simulating long enough to crush the variance
- ▶ It's a function of network fine structure like clustering
- ▶ SEIR propagation is really driven by asymptomatic infection

⁹ L. Hébert-Dufresne and A. Allard. Smeared phase transitions in percolation on real complex networks. **Physical Review Research**, 1, August 2019. URL <https://doi.org/10.1103/PhysRevResearch.1.013009>



POSSIBLE IMPLICATIONS

Designing track-and-trace

- ▶ Very sensitive to the proportion of contacts tested
- ▶ Need to check at least 40% to have any effect at all

Effectiveness

- ▶ Even very effective tracing doesn't guarantee elimination
- ▶ Although it does reduce the peak significantly once above about 80%



FOUR THINGS TO TAKE AWAY

1. Epidemic spreading still isn't fully understood, even now – there's lots of exciting work still to do, mathematically and computationally
2. Explore of possible public policy decisions
3. Need (in my copious free time) to understand what's going on with this possible smeared phase transition
4. Especially interested in how small-scale topological structures affect network-based processes



REFERENCES

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