

A complex cocktail of networks and reality

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Introduction

What this talk is about

- This is the story of a paper
- ... and what it tells us about the interface between network science, data science, and scientific applications

My aim

- Using real-world data to drive abstract study
- The limitations we hit as a result
- How we might address these limitations

Complex networks

Networks and processes



Living between regularity and randomness¹

- Heterogeneous degree distribution, fragile notion of “neighbourhood”
- Evaluate processes at each node, affecting behaviour of neighbours, often with a stochastic component
- Canonical example is the SIR model of disease propagation

¹ A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999.

URL [doi:/10.1126/science.286.5439.509](https://doi.org/10.1126/science.286.5439.509)

Multiplexes

A *multiplex* (or more correctly a *multilayer network*) is a collection of two (or more) networks ²

- Nodes in the different networks are *coupled*
- Study properties of the individual networks or of the ensemble
- One network may be “less wide” than the other, and so offer “shortcuts” for processes



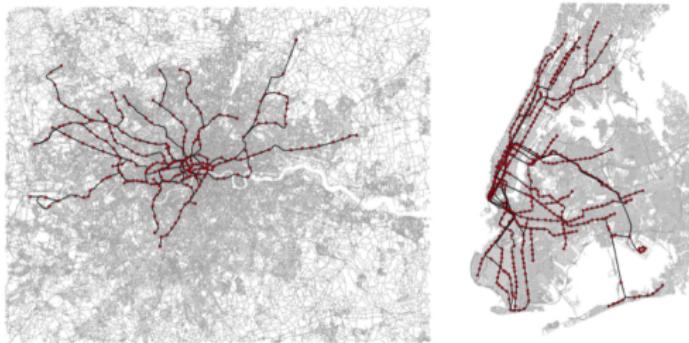
²M. Kivelä, A. Arenas, M. Barthélémy, J. Gleeson, Y. Moreno, and M. Porter. Multilayer networks. *Journal of Complex Networks*, 2(3):203–271, 2014. URL [doi:10.1093/comnet/cnu016](https://doi.org/10.1093/comnet/cnu016)

Studying the real world

The problem: Urban transportation

Coupled transport networks ³

- Street and tube/subway form a multilayer network
- How does the addition of the tube affect travel times?
- How does this change as the tube speeds up?

³

M. Batty. *The new science of cities*. MIT Press, 2013. ISBN 978-0-262-01952-1

Framing the problem

Our study ⁴

- Simplify to treat as a purely topological problem
- This study doesn't include explicit consideration of flows (although later work does)
- Study the *betweenness centrality* of nodes as the relative speeds of the two networks changes
- How does *outreach* change?

⁴ E. Strano, S. Shai, S. Dobson, and M. Barthélémy. Multiplex networks in metropolitan areas: generic features and local effects. *Journal of the Royal Society Interface*, 12(111), October 2015. URL
doi://10.1098/rsif.2015.0651

Setting up the study

Acquired street and tube data from Open Street Map

- London: $N_s = |V_s| = 325K$ street nodes, $N_t = |V_t| = 263$ tube nodes
- New York: $N_s = 68K$, $N_t = 454$

Data hygiene

- Needed substantial manual cleaning
- Streets don't meet, tubes don't come up where they should, ...
- Some slight simplifications, *i.e.*, tubes always emerge at street junctions

Methodology

Travel costs

- $\tau_s(i, j)$ the travel cost (in time units) between $i, j \in V_s$ using only street edges
- $\tau_m(i, j)$ the travel cost using the multiplex (street and tube)
- $0 \leq \beta \leq 1$ the ratio of speed between street and tube (tube is faster for smaller β)

Shortest paths

- $\sigma_{i,j}$ the number of shortest paths between $i, j \in V_s$ using only the street network
- Similarly define $\sigma_{i,j}^m$ the number of shortest paths using the multiplex

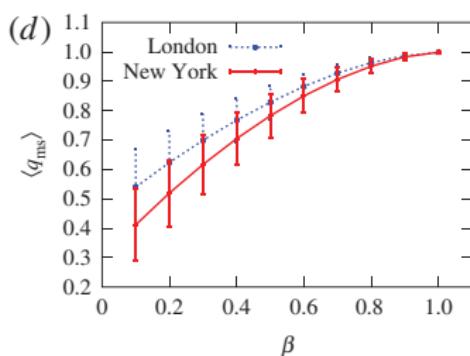
How much does the tube affect travel costs?

Metric

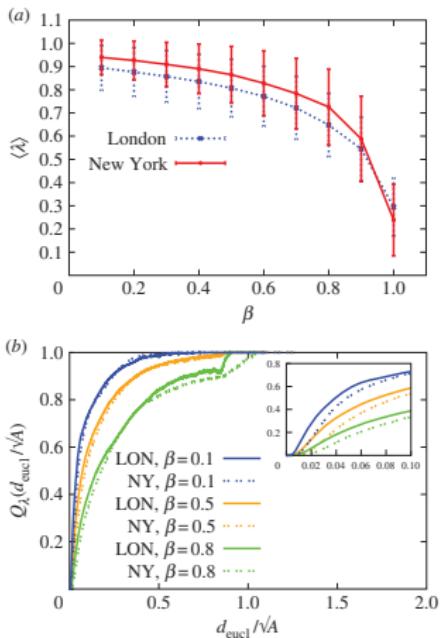
- Ratio of travel costs from a node $i \in V_s$ to all other nodes using the multiplex vs using the streets only

$$q_{ms}(i) = \frac{1}{N_s - 1} \sum_{j \in V_s} \frac{\tau_m(i, j)}{\tau_s(i, j)}$$

- Halving β reduces $\langle q_{ms} \rangle$ by about 20%



Interdependence

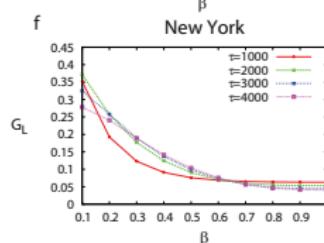
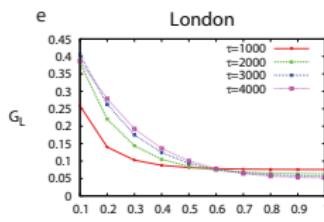
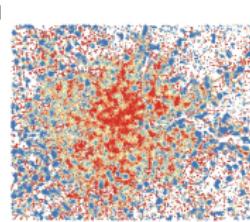
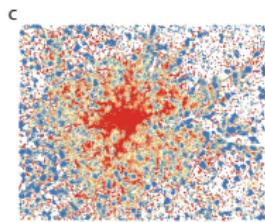
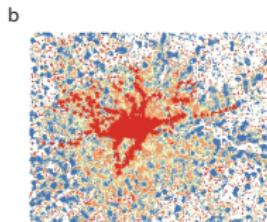
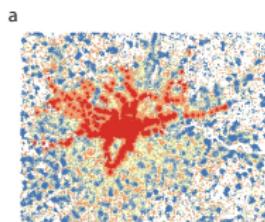


Ratios of shortest paths

- Already known in the planning literature as *inter-modal connectivity*
- $\lambda(i,j) = \frac{\sigma_{i,j}^m}{\sigma_{i,j}}$
- Large tube influence
- For $\beta = 0.8$, $\langle \lambda \rangle = 0.7$: 70% of journeys use the tube

Outreach

- *Spatial outreach* of a node i is the average Euclidean distance to all nodes reachable with a travel cost τ
- $L_\tau(i) = \frac{1}{N(\tau)} \sum_{j \in \{k | \tau_m(i,k) \leq \tau\}} d(i,j)$
- How “commutable” is a city



So far so good...

Analysis of real cities

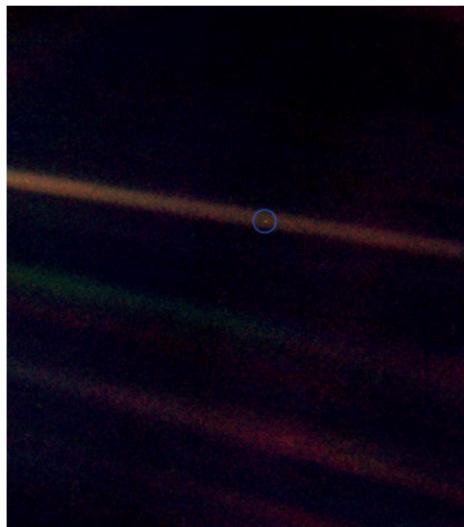
- Mathematically and computationally tractable
- Use publicly-available data (with a little work)

Implications for urban planning

- Impact of planning decisions
- Provides a quantitative underpinning for ideas that urban planners already use informally

Reality intrudes

But there's a problem with reality...



But there's a problem with reality...



- There's only one of it

(Carl Sagan's Pale Blue Dot, taken on Valentine's Day 1976.)

Universality

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- ... so we changed the title of the paper ☺

Repetition and reality

The usual network science approach

- Characterise a network topology (e.g., $\langle k \rangle$)
- Generate thousands of random instances
- Compute summary statistics over processes on each different instance

This doesn't work for the topologies of real cities (and other networks)

- A limited number of instances
- Topologies that *really* are complex
- Multiplexes with complex coupling constraints and probabilities

Faking it

A challenge

Can we synthesise realistic “really complex” networks?

- Complex dependencies between node degrees, spatial locations, and couplings between networks
- It's known that degree couplings have a major impact on epidemic processes, for example⁵



⁵ S. Shai and S. Dobson. Effect of resource constraints on intersimilar coupled networks. *Physical Review E*, 86 (6), December 2012. URL [doi:10.1103/PhysRevE.86.066120](https://doi.org/10.1103/PhysRevE.86.066120)

A possible approach

Study the topologies independently

- Of each component network in the multiplex
- Degree distribution, degree correlations between adjacent nodes, ...

Two ways to form the multiplex

1. Identify and couple corresponding nodes
2. Identify a growth process – the dynamics *of* the multiplex

Benefits

Aim for universality

- Even in problems that are sensitive to the detailed mathematical structure
- Project forward *and* back
- Easier “what if?” analysis

Canonical example is the Black Death

- Why was it so deadly?
- Interactions between topology, survival times, disease dynamics
- Which factors were the most critical?

Conclusion

- Realistic to study urban-scale networks using network science
- Data publicly-available, but needs care and cleaning
- Multiplex networks of interest are too complex to synthesise currently
- ... but it would be nice to learn how to, and would enormously increase the phenomena we could study
- Including the effects of past and projected changes to actual locations

Thank you

Topology! The stratosphere of human thought! In the twenty-fourth century it might possibly be of use to someone...

Alexander Solzhenitsyn, The First Circle

References

-  A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999. URL [doi://10.1126/science.286.5439.509](https://doi.org/10.1126/science.286.5439.509).
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