

# Modelling urban networks: some results and their limitations

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# Introduction

## What this talk is about

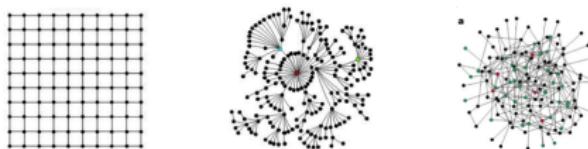
- How is human movement in an urban setting conditioned by the topology of the transportation networks?
- Are there any features common between large cities?
- How do quantitative results relate to the practice of urban planning?

## My aim

- Present recent results on urban transportation, joint work with Saray Shai, Emanuele Strano, and Marc Barthélemy
- Real-world data driving a study using network science
- The limitations we hit when theory met practice

# Complex networks

# Networks and processes



## Living between regularity and randomness<sup>1</sup>

- Heterogeneous degree distribution, fragile notion of “neighbourhood”
- Evaluate processes at each node, affecting behaviour of neighbours, often with a stochastic component
- Canonical example is the SIR model of disease propagation

<sup>1</sup> A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999.

URL [doi:/10.1126/science.286.5439.509](https://doi.org/10.1126/science.286.5439.509)

# Multiplexes

A *multiplex* (or more correctly a *multilayer network*) is a collection of two (or more) networks<sup>2</sup>

- Nodes in the different networks are *coupled*
- Study properties of the individual networks or of the ensemble
- One network may be “less wide” than the other, and so offer “shortcuts” for processes



<sup>2</sup>M. Kivelä, A. Arenas, M. Barthélémy, J. Gleeson, Y. Moreno, and M. Porter. Multilayer networks. *Journal of Complex Networks*, 2(3):203–271, 2014. URL [doi:10.1093/comnet/cnu016](https://doi.org/10.1093/comnet/cnu016)

# Spatial networks

## Planarity limits permissible topologies <sup>3</sup>

- Network embeds into Euclidean 2-space ( $\mathbb{R}^2$ )
- No crossings: all intersections form junctions
- (Doesn't work precisely for all cities, e.g., Edinburgh, which have significant 3D structure)
- Limits the possibility for long-distance connections
- Typically quite *modular*, with highly-connected locales

## Spatial multilayer networks

- Each *layer* is planar, but the *multiplex* typically isn't

<sup>3</sup> M. Barthélémy. Spatial networks. *Physics Reports*, 499:1–101, 2011

# Urban networks

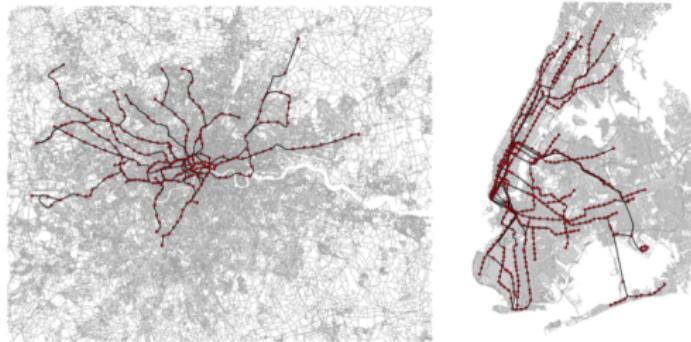
# Cities of different sizes and complexities



# The problem: Urban transportation

## Coupled transport networks <sup>4</sup>

- Street and tube/subway form a multilayer network
- How does the addition of the tube affect travel times?
- How does this change as the tube speeds up?

<sup>4</sup>

M. Batty. *The new science of cities*. MIT Press, 2013. ISBN 978-0-292-01952-1

# Framing the problem

## Our study<sup>5</sup>

- Simplify to treat as a purely topological problem
- Don't model traffic congestion *per se*

## Metrics

- Impact of tube speed on network usage, travel costs, and shortest paths
- Study the *betweenness centrality* of nodes as the relative speeds of the two networks changes
- How does *outreach* change?

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<sup>5</sup>E. Strano, S. Shai, S. Dobson, and M. Barthélémy. Multiplex networks in metropolitan areas: generic features and local effects. *Journal of the Royal Society Interface*, 12(111), October 2015. URL  
doi://10.1098/rsif.2015.0651

# Methodology

## Topological properties

- Compute metrics between all pairs  $(i, j)$  of nodes in the street network  $V_s$
- Compute ratio of metrics between travel using the streets only *versus* using the whole multiplex

## Geographical properties

- Network is spatial, so nodes have location in space, and a distance  $d(i, j)$  between pairs of street nodes
- Often scale distances according to network diameter,  $\frac{d(i,j)}{\sqrt{A}}$
- Compare network metrics to geographical distances

# Setting up the study – 1

Acquired street and tube data from Open Street Map

- Street network consisting of  $v \in V_s$  nodes
- Tube network  $V_t$
- Coupled at access points to form a multiplex

	$N_s$	$N_t$	Street diameter	Tube diameter
London	325K	263	89km	60km
New York	68K	454	55km	57km

# Data hygiene

Needed substantial manual cleaning

- Streets don't meet, tubes don't come up where they should, ...

Choices to be made

- Tubes sometimes emerge mid-street, not at a junction
- Add a pseudo-junction for the tube to be coupled to
- Couple to junction at one end of the street or the other
- Do these choices make a difference?

## Setting up the study – 2

### Travel costs

- $\tau_s(i, j)$  the travel cost (in time units) between  $i, j \in V_s$  using only street edges
- $\tau_m(i, j)$  the travel cost using the multiplex (street and tube)
- $0 \leq \beta \leq 1$  the ratio of speed between street and tube (tube is faster for smaller  $\beta$ )

### Shortest paths

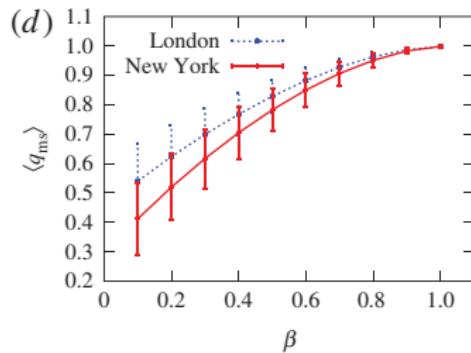
- $\sigma_{i,j}$  the number of shortest paths between  $i, j \in V_s$  using only the street network
- Similarly define  $\sigma_{i,j}^m$  the number of shortest paths using the multiplex

# How much does the tube affect travel costs?

## Metric

- Ratio of travel costs from a node  $i \in V_s$  to all other nodes using the multiplex *vs* using the streets only

$$q_{ms}(i) = \frac{1}{N_s - 1} \sum_{j \in V_s} \frac{\tau_m(i, j)}{\tau_s(i, j)}$$

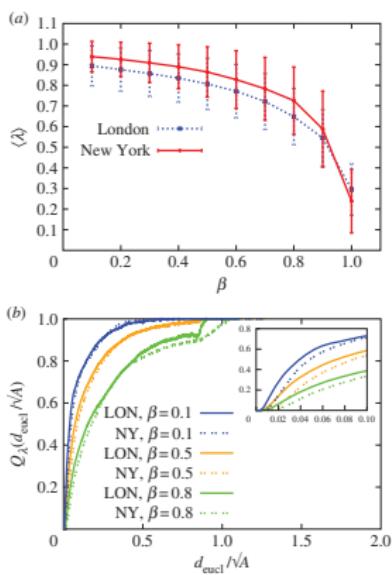


## Impact

- Halving  $\beta$  reduces  $\langle q_{ms} \rangle$  by about 20%
- Most journeys have a large street component that can't be removed

# Interdependence

## Ratios of shortest paths



- *Inter-modal connectivity*

$$\bullet \quad \lambda(i,j) = \frac{\sigma_{i,j}^m}{\sigma_{i,j}}$$

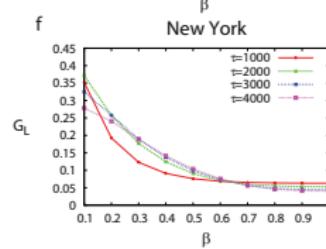
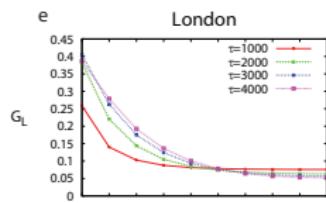
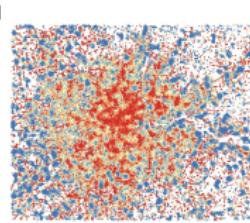
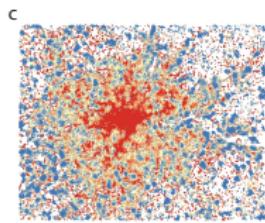
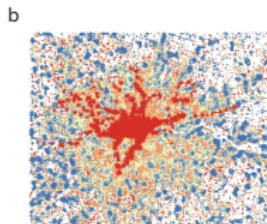
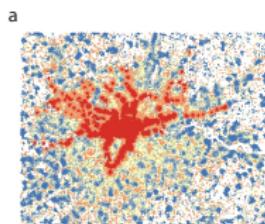
- For  $\beta = 0.8$ ,  $\langle \lambda \rangle = 0.7$ : 70% of journeys use the tube

## Compare to scaled distances

- Scale based on  $\frac{d(i,j)}{\sqrt{A}}$
- $Q_\lambda(d) = \frac{1}{N(d)} \sum_{i,j \in V_s | d(i,j)=d} \lambda(i,j)$
- Spatially short journeys benefit from hopping on the tube

# Outreach

- *Spatial outreach* of a node  $i$  is the average Euclidean distance to all nodes reachable with a travel cost  $\tau$
- $L_\tau(i) = \frac{1}{N(\tau)} \sum_{j \in \{k | \tau_m(i,k) \leq \tau\}} d(i,j)$
- How “commutable” is a city



# Betweenness centrality

## Metric

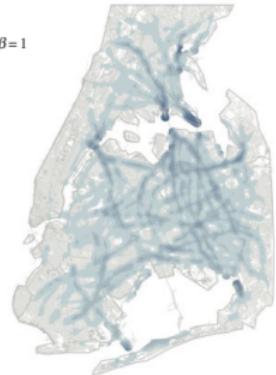
- Compute  $\sigma_{i,j}^m(v)$  the fraction of shortest paths that go through  $v \in V_s$
- $bc_m(v) = \frac{1}{(N_s-1)(N_s-2)} \sum_{i,j \in V_s} \frac{\sigma_{i,j}^m(v)}{\sigma_{i,j}^m}$

## Impact

- Shift congestion from roads to nodal points of tubes as  $\beta$  decreases
- Tubes “decentralise” congestion to the ends of lines
- Betweenness centrality doesn’t move to the centre, as happens with mesh networks

# Shifting spatial patterns

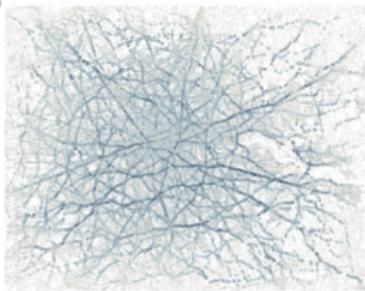
(a)



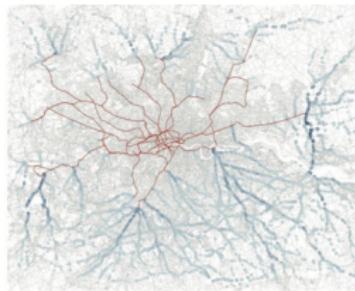
(b)



(c)



(d)



# Limitations

# Topology only

## No congestion “agents”

- Not modelling the traffic *per se*
- When we suck traffic into the tube, we assume that we *can*
- Either roads are “sufficiently big” or traffic “sufficiently light”
  - neither of which is actually the case

## A more detailed model

- Make cost dependent on centrality?
- Limit capacity of edges?

# Wrong metrics

Betweenness centrality is all-to-all

- Shortest paths between all pairs  $i$  and  $j$

A commuter model would capture which routes were more important

- Probabilistically weight the routes that people actually use
- Drive from real data, *i.e.*, TfL turnstile measurements
- Recent results show we can estimate route weights from census data on living and working population densities

# Universality

Want to know that ideas work everywhere

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Getting more towards universality

- Can we synthesise cities with realistic (coupled) topologies?
- Generate plausible alternative topologies to explore

# Faking it

## How do cities form and evolve?

- Villages coalesce over time, interconnections grow, . . .
- Certain topological structures seem to be very persistent over time

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## Study historical events, for example the Black Death

- A combination of disease, healthcare, and social structure
- Is breaking one of these features sufficient to stop an epidemic?
- Do network features make some modern epidemics worse?

# Conclusion

- Realistic to study urban-scale networks computationally using network science
- Data is publicly available, but needs care and cleaning
- Topology-driven analysis still shows useful results
- A commuter model would be useful, and seems to be possible from observed patterns (in London at least)
- Universal results are elusive and would require significant advances in synthetic urban network generation

# Thank you

*Topology! The stratosphere of human thought! In the twenty-fourth century it might possibly be of use to someone...*

*Alexander Solzhenitsyn, The First Circle*

# References

-  A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999. URL [doi://10.1126/science.286.5439.509](https://doi.org/10.1126/science.286.5439.509).
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