COMP 350 Numerical Computing
Assignment #4. Solving a nonlinear equation.

Date given: Monday, Oct 17. Date due: 5pm, Monday, Oct 31, 2016.

1. (a) (6 points) We say r is a multiple root of $f(x) = 0$ with multiplicity m if $f^{(k)}(r) = 0$ for $0 \leq k < m$, but $f^{(m)}(r) \neq 0$. Show that in the case of a root of multiplicity m, the **modified Newton's method**

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}$$

is quadratic convergent.

Hint: Use the Taylor series expansions of $f(x_n)$ and $f'(x_n)$ about r. You have to think about how many terms should be used.

(b) (6 points) Write a MATLAB program for the modified Newton's method with $m = 2$. Then use this program and `newton.m` on the course web site to find the root of $f(x) = (x-1)^2 \sin(x)$, respectively. In you test, you may take $x_0 = 2$, `xtol=1.e-14`, `ftol=1.e-14` and `n_max=40`. From your results comment on the convergence rates of the two methods.

2. (8 points) (**Regular Falsi Method**) This method combines features of the bisection method and the secant method. In each iteration, we have an interval $[a, b]$ such that $f(a)f(b) < 0$. If $f(x)$ were a linear function, an easy calculation shows that the root would be

$$c = b - \frac{f(b)(a-b)}{f(a) - f(b)}.$$

We then compute $f(c)$, and proceed to the next step with the interval $[a, c]$ if $f(a)f(c) < 0$ or the interval $[c, b]$ if $f(c)f(b) < 0$. Program this algorithm by MATLAB and test it on the function given in Question 1(b) with the initial intervals $[0, 4]$. You choose some stopping criteria.