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**MAKING IT LAZY: NEVER EVALUATE  
ANYTHING MORE THAN ONCE**

fun

<https://github.com/simonjohnthompson/streams>

# FUNCTIONS AS DATA

**“Functions are first-class citizens”**

**A function actively represents  
behaviour of some sort, and we  
deal with it just like any other  
kind of data.**





Original image: <https://www.thishopeanchors.com/single-post/2017/04/06/Rock-Paper-Scissors>



# What is a strategy?

Random

Echo

No repeats

Statistical

...

# What is a strategy?

We choose what to play,  
depending on your last  
move, or the history of  
all your moves.



# What is a strategy?

```
-type play() :: rock | paper | scissors.  
-type strategy() :: fun([play()]) -> play()).
```

We choose what to play,  
depending on your last  
move, or the history of  
all your moves.

Random  
Echo  
No repeats  
Statistical  
...

```
echo( []) ->  
    random_play();  
echo( [X|_Xs] ) ->  
    X.  
  
beat( []) ->  
    random_play();  
beat( [X|_] ) ->  
    case X of  
        rock -> scissors;  
        paper -> rock;  
        scissors -> paper  
    end.
```

```
% The second argument here is the accumulated input from the player
% Note that this function doesn't cheat: the Response is chosen
% before the Play from the player.
```

```
-spec interact(strategy(), [play()]) -> ok.
```

```
interact(Strategy, Xs) ->
    Response = Strategy(Xs),
    {ok, [Play|_]} = io:fread('play rock, paper, scissors, stop: ', "~a"),
    case Play of
        stop -> ok;
        _ ->
            Result = result({Play, Response}),
            io:format("Machine plays ~p, result is ~p~n", [Response, Result]),
            interact(Strategy, [Play|Xs])
    end.
```

**What is a strategy combinator?**

**Choose randomly between these strategies.**

**Apply them all and choose most popular result.**

**Replay each of these strategies on the history so far and apply the one that's been best so far.**

**Take home**

**Toy example**

**Generality: not just a finite set . . .**

**Up a level: combining strategies**



# *WORLD RPS SOCIETY*

Serving the needs of decision makers since 1918

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# Worldrps.com has a new look

**Say goodbye to the old  
cluttered look of the World  
RPS Society site.**

The IT Brigade told us it would take them four weeks to re-do the worldrps.com web site. So after consuming four years, 4 palettes of Mellow Yellow, dozens of crates of Pringles, and surviving a few health scares, the team has done it.



<http://worldrps.com>



**EVALUATION  
ON DEMAND**

# function evaluation in Erlang

# function evaluation in Erlang

evaluate the arguments  
before the body

```
switch(N,Pos,Neg) ->  
  case N>0 of  
    true -> Pos;  
    _     -> Neg  
  end.
```

# function evaluation in Erlang

evaluate the arguments  
before the body

```
switch(N,Pos,Neg) ->  
  case N>0 of  
    true -> Pos;  
    _     -> Neg  
  end.
```

fully evaluate  
the argument

```
sum_first_two([A,B|_Rest])  
  -> A+B.
```

but if an argument is a  
**function** then it's  
passed unevaluated.

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```
fun () -> Stuff end
```



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**function** then it's  
passed unevaluated.

```
fun () -> Stuff end
```

```
fun () -> Stuff end ()
```

**DELAY!**

# a lazy switch

```
-spec lswitch(number(), fun(() ->T), fun(() ->T)) -> T.
```

```
lswitch(N,Pos,Neg) ->  
  case N>0 of  
    true -> Pos();  
    _     -> Neg()  
  end.
```

# a lazy switch

```
-spec lswitch(number(), fun(() ->T), fun(() ->T)) -> T.
```

```
lswitch(N,Pos,Neg) ->  
  case N>0 of  
    true -> Pos();  
    _     -> Neg()  
  end.
```

```
lex1() -> lswitch(1,fun() -> 3+4 end,fun () -> 1/0 end).
```

```
-spec lswitch(number(), fun(() ->T), fun(() ->T)) -> T.
```

```
lswitch(N,Pos,Neg) ->  
    case N>0 of  
        true -> Pos();  
        _    -> Neg()  
    end.
```

```
-define(switch(N,Pos,Neg),  
    lswitch(N,fun() -> Pos end,fun() -> Neg end)).
```

```
-spec lswitch(number(), fun(() ->T), fun(() ->T)) -> T.
```

```
lswitch(N,Pos,Neg) ->  
  case N>0 of  
    true -> Pos();  
    _    -> Neg()  
  end.
```

```
-define(switch(N,Pos,Neg),  
  lswitch(N,fun() -> Pos end,fun() -> Neg end)).
```

```
lex2() -> ?switch(1,3+4,1/0).
```



**STREAMS**



Original image: <http://www.metso.com/services/spare-wear-parts-conveyors/conveyor-belts/>

# streams

## build

```
cons(X, Xs) ->  
  fun() -> {X, Xs} end.
```

# streams

build

```
cons(X, Xs) ->  
  fun() -> {X, Xs} end.
```

deconstruct

```
head(L) ->  
  case (L()) of  
    {H, _} -> H  
  end.
```

```
tail(L) ->  
  case (L()) of  
    {_, T} -> T  
  end.
```

# streams

## build

```
-define(cons(X,Xs),  
        fun() -> {X,Xs} end).
```

## deconstruct

```
head(L) ->  
  case (L()) of  
    {H,_} -> H  
  end.
```

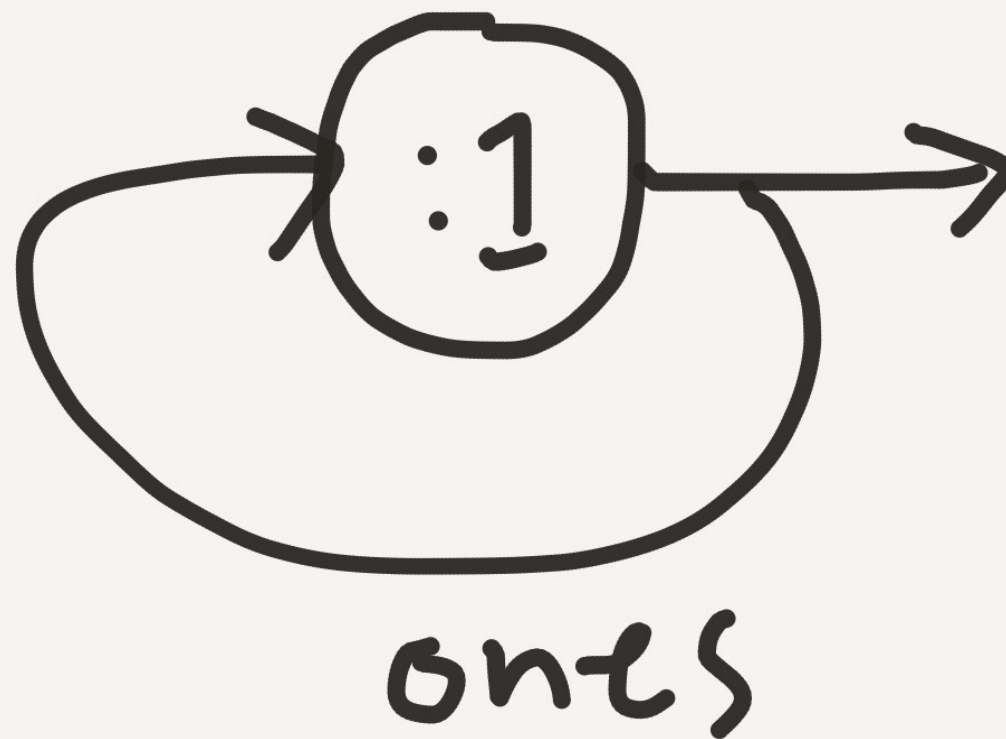
```
tail(L) ->  
  case (L()) of  
    {_,T} -> T  
  end.
```

```
ones() ->  
  ?cons(1, ones()).
```

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, ...



```
ones() ->  
  ?cons(1, ones()).
```



```
ns(N) ->  
  ?cons(N, ns(N+1)) .
```

42, 43, 44, 45, 46, 47, 48, 49, 50, ...

2, 3, 5, 7, 11,  
13, 17, 19,  
23, 29, 31,  
37, 41, 43,  
47, ...

```
primes() -> sieve(ns(2)).
```

```
sieve(Ns) ->  
  H = head(Ns),  
  ?cons(H, sieve(cut(H, tail(Ns)))).
```

```
cut(N, Ns) ->  
  H = head(Ns),  
  case H rem N of  
    0 -> cut(N, tail(Ns));  
    _ -> ?cons(H, cut(N, tail(Ns)))  
  end.
```

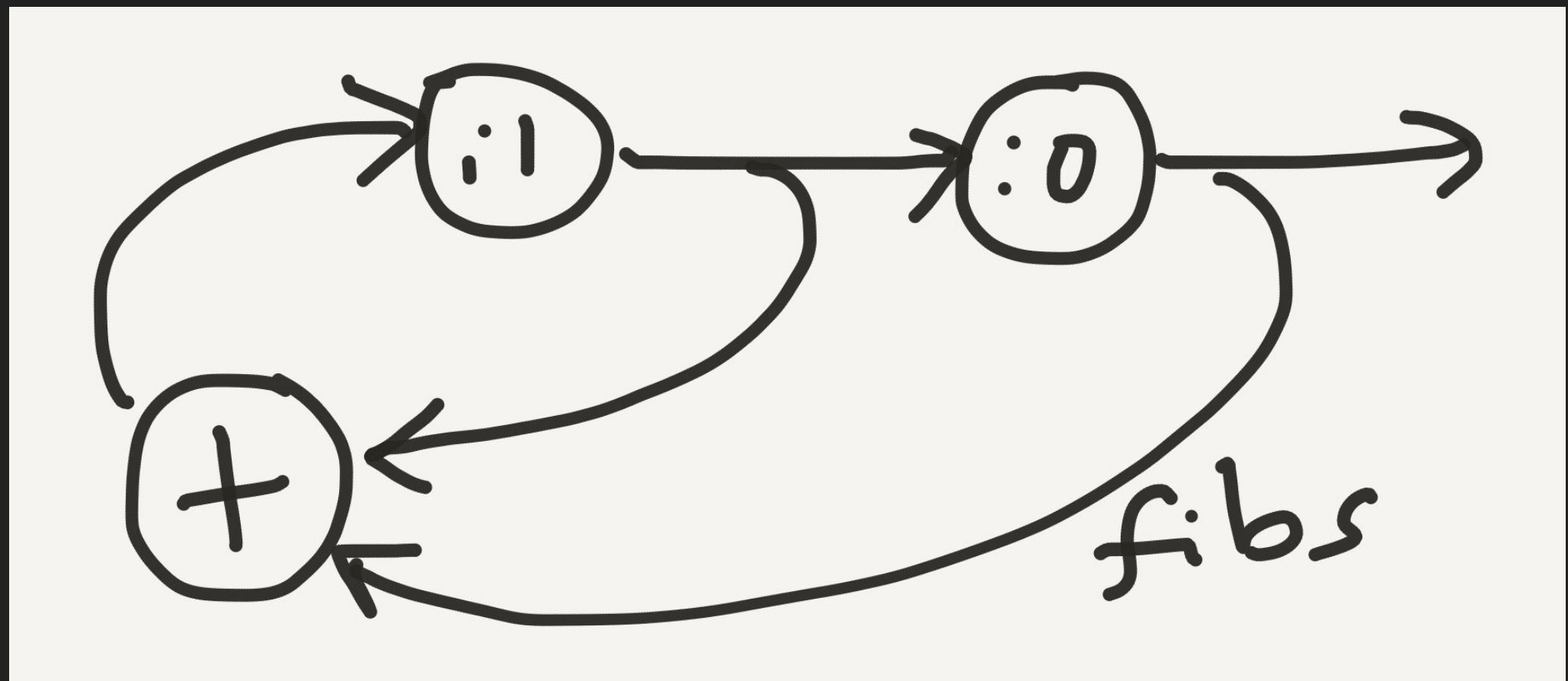
```
fibs() ->
  ?cons(0,
    ?cons(1,
      addZip(fibs(), tail(fibs())))).

addZip(Xs, Ys) ->
  ?cons(head(Xs)+head(Ys), addZip(tail(Xs), tail(Ys))).
```

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

```
fibs() ->  
  ?cons(0,  
    ?cons(1,  
      addZip(fibs(), tail(fibs())))).
```

```
addZip(Xs, Ys) ->  
  ?cons(head(Xs)+head(Ys), addZip(tail(Xs), tail(Ys))).
```



demo

**Take home**

**“infinite” streams**

**apparently circular**

**repeated re-computation**

# LAZY EVALUATION

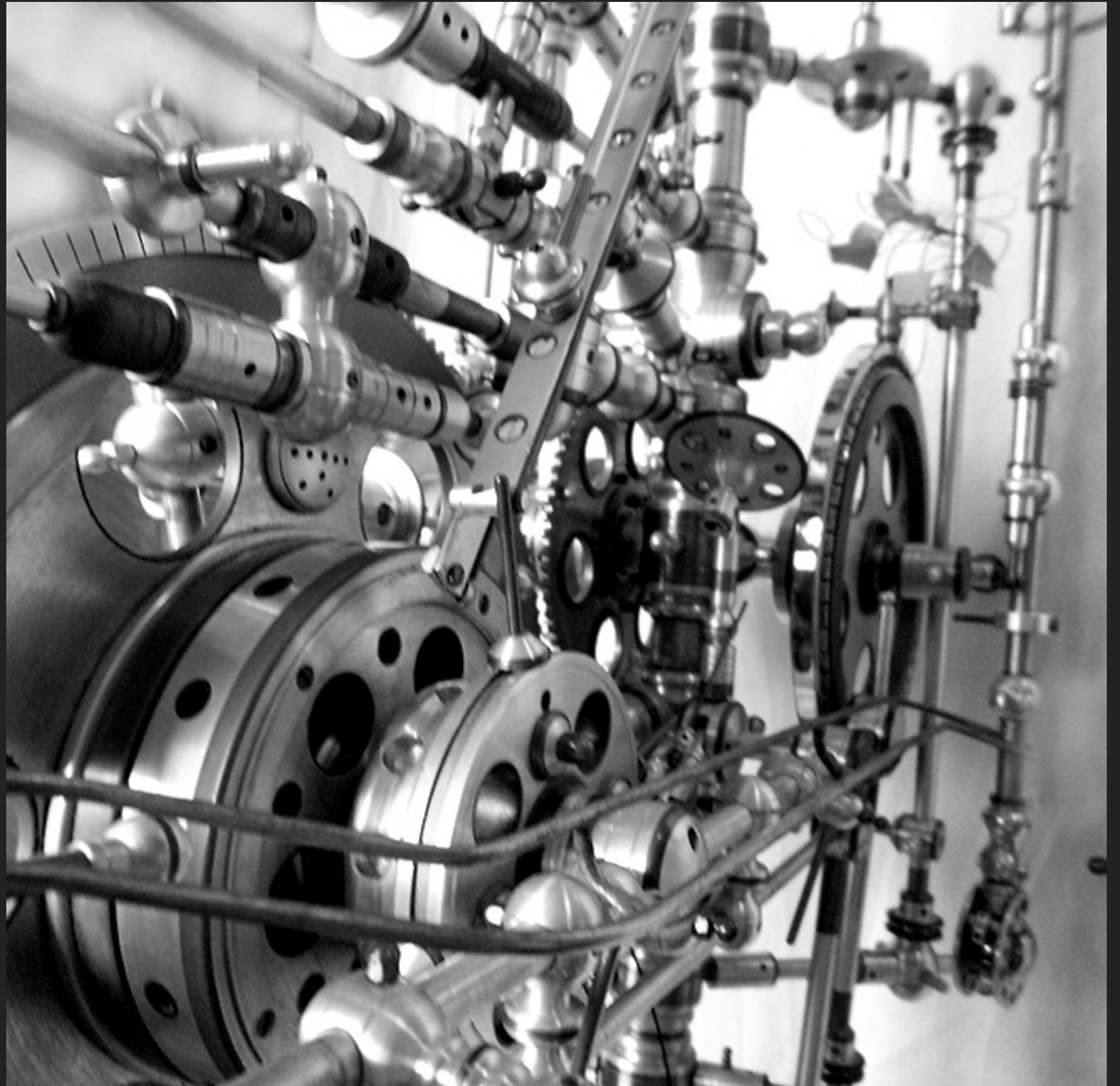


**ensure that each argument is  
evaluated at most once**

**ensure that each argument is  
evaluated at most once**

**we must ensure that results  
are memoised in some way**

but isn't  
that a job  
for the  
compiler?



**key idea**

**we explicitly manage how  
results are stored once evaluated**

use an ETS table to keep track  
of evaluated results, or ...

... model the store functionally,  
thread it through the calculations

**MEMOISATION**

**use ETS for general memoisation**

# use ETS for general memoisation

```
fib(0) -> 0;  
fib(1) -> 1;  
fib(N) -> fib(N-1) + fib(N-2).
```

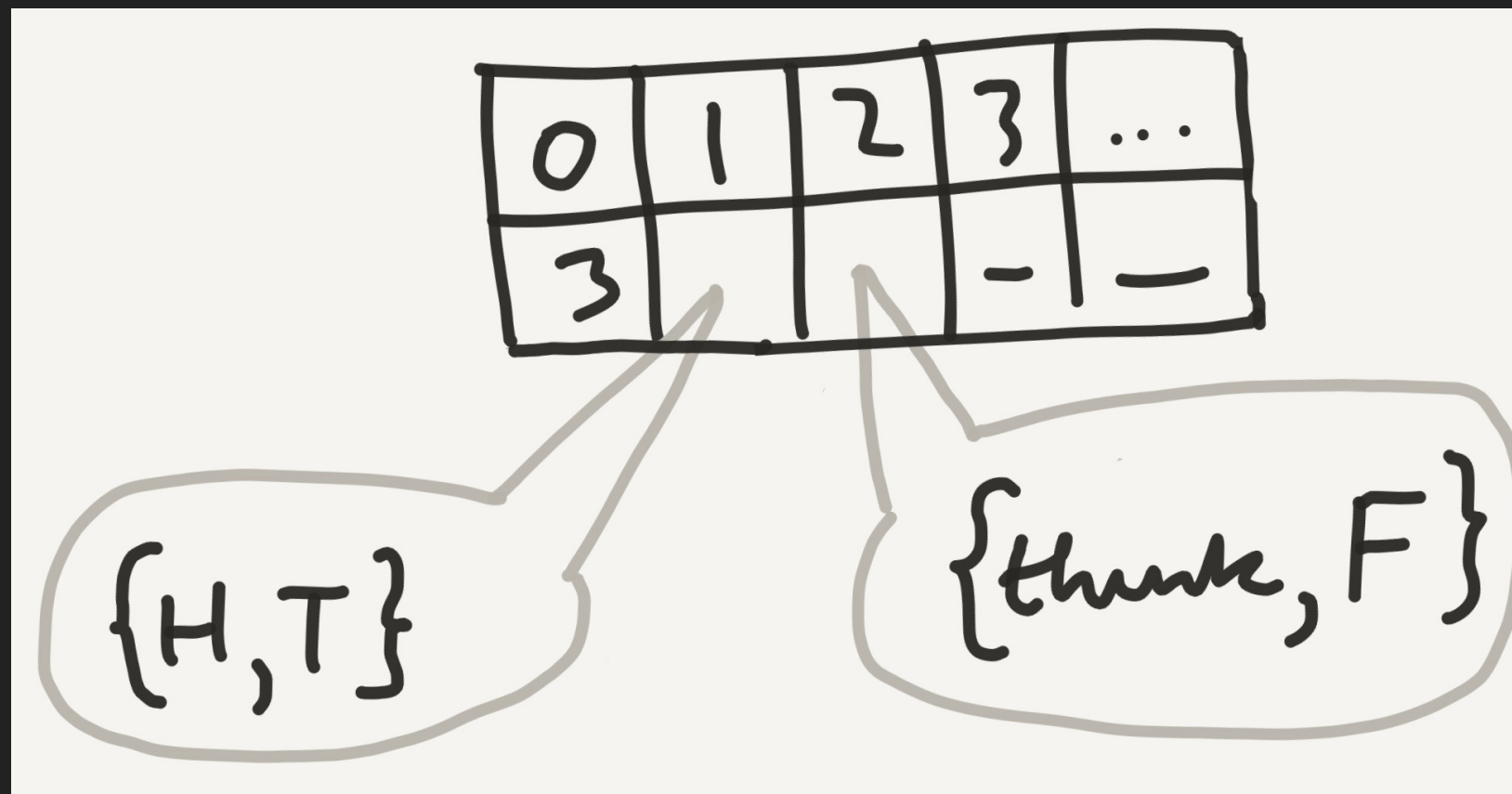


```
fib(0) -> 0;  
fib(1) -> 1;  
fib(N) -> fib(N-1) + fib(N-2).
```

```
fibM(0) -> 0;  
fibM(1) -> 1;  
fibM(N) ->  
    case ets:lookup(tab,N) of  
    [] -> V = fibM(N-1) + fibM(N-2),  
        ets:insert(tab,{N,V}),  
        V;  
    [{N,V}] -> V  
    end.
```

**USING  
ETS TABLES**

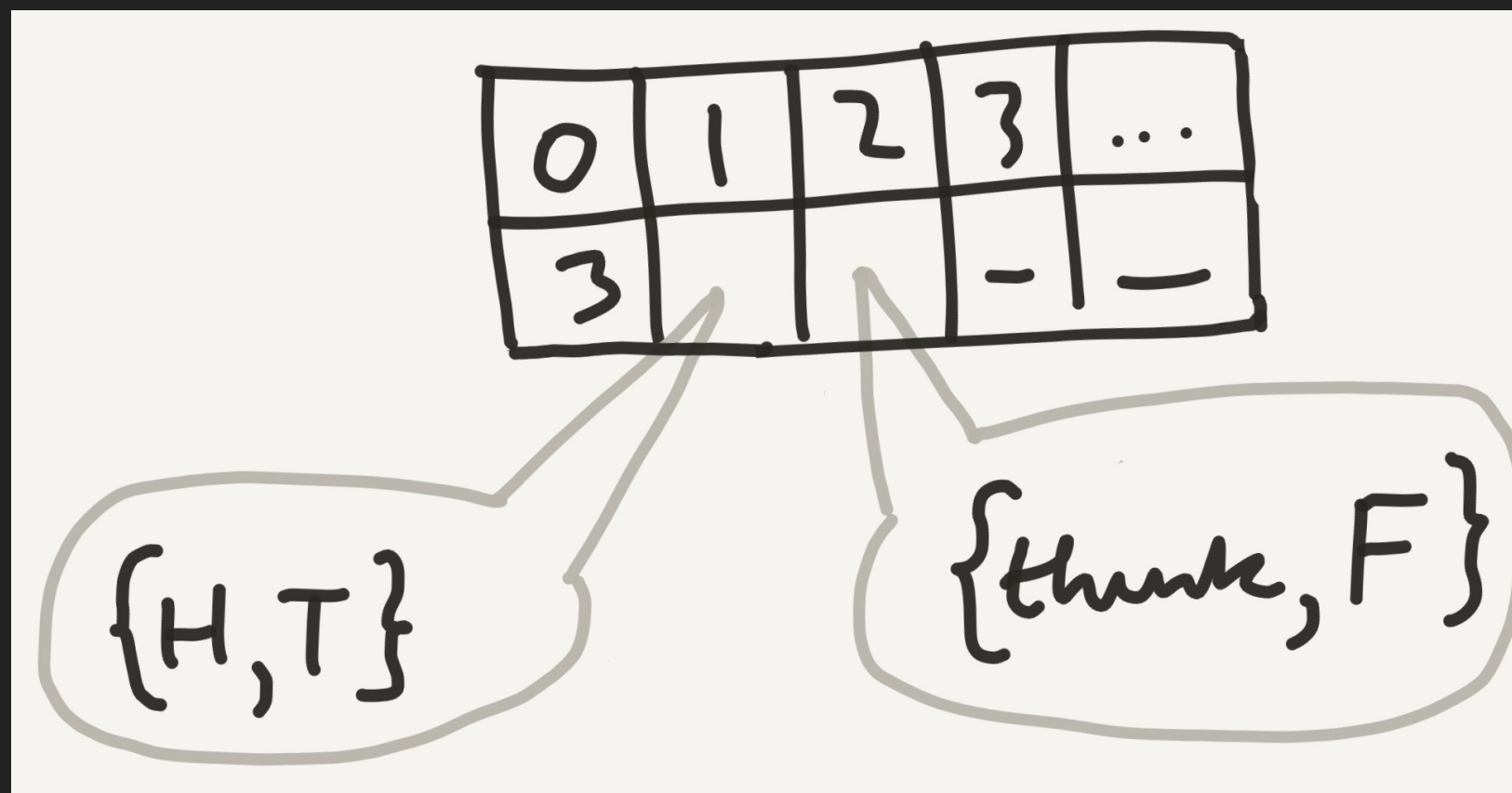
store either the head and tail,  
or a “thunk” to be evaluated



```

-define(cons(X,Xs), begin ets:insert(tab,{0,next_ref()+1}) ,
                        ets:insert(tab, {next_ref(),
                                         {thunk, fun () -> {X,Xs} end}}),
                        % io:format("done cons insert~n"),
                        {ref,next_ref()} end).

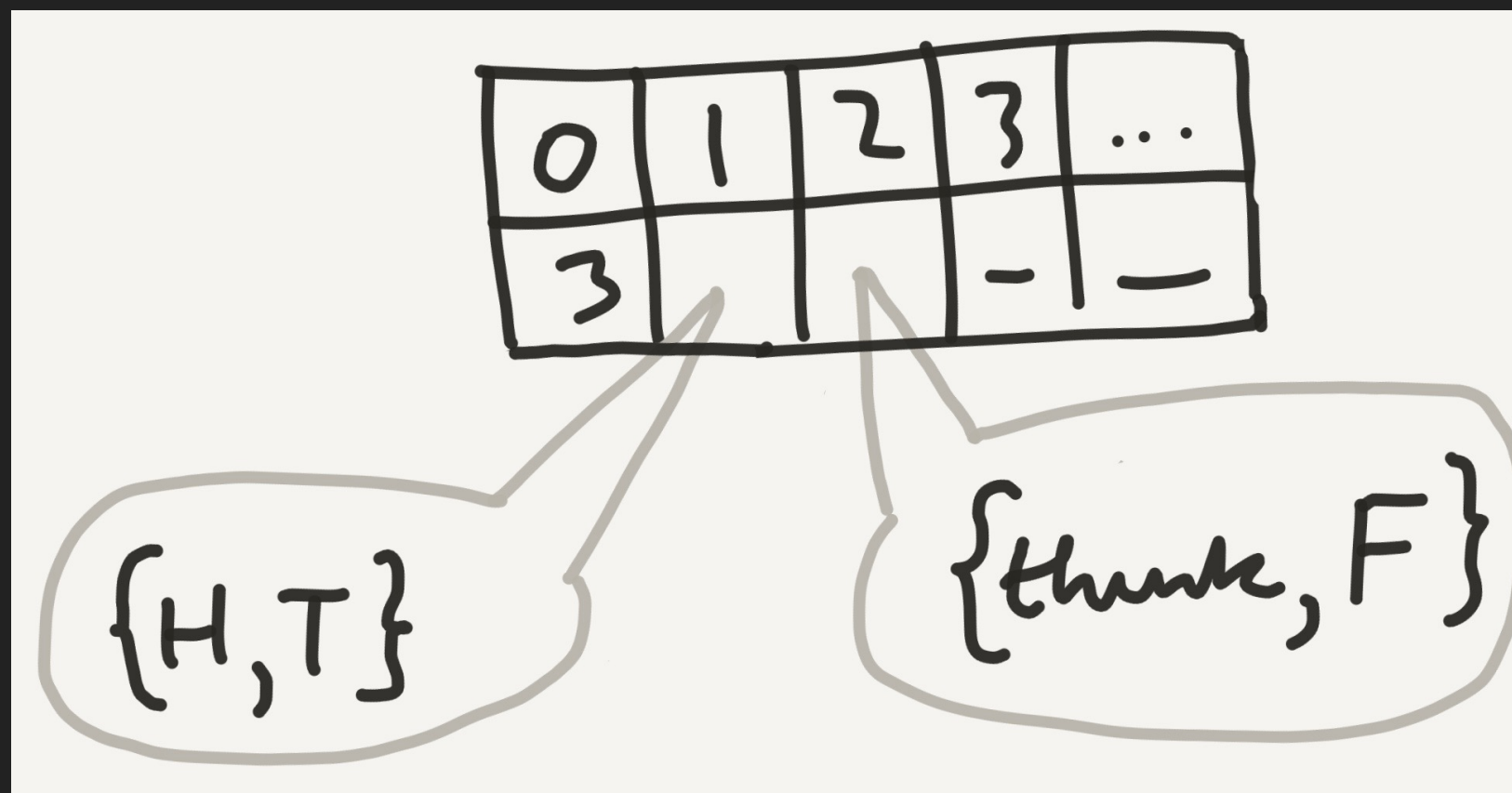
```



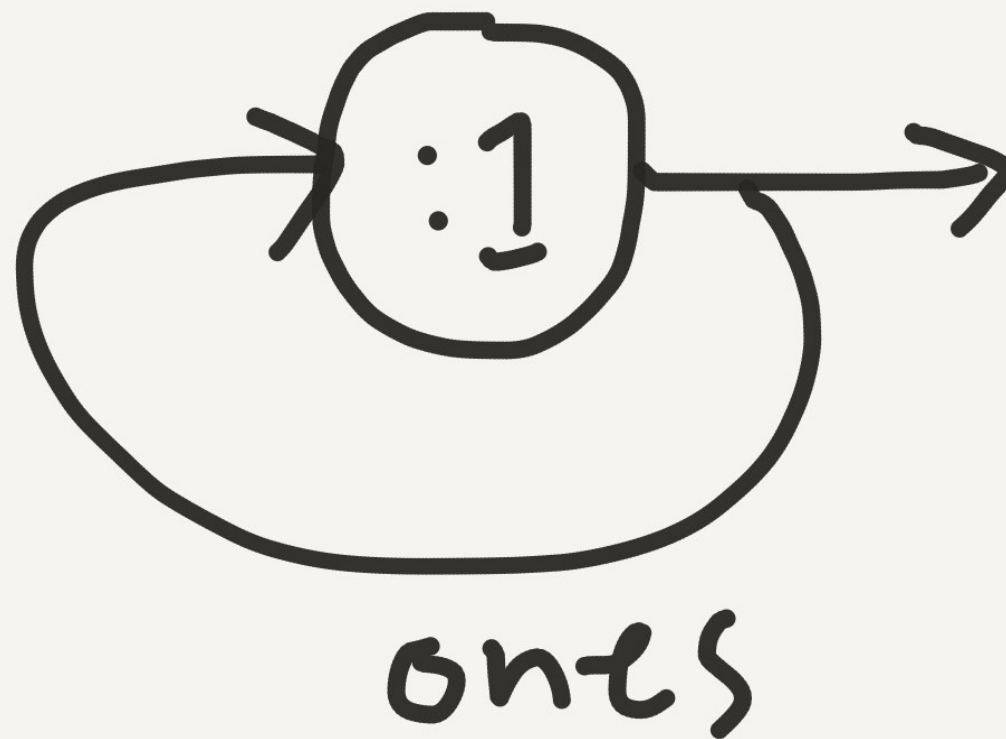
```

head({ref, Ref}) ->
  case ets:lookup(tab, Ref) of
    [{Ref, {thunk, F}}] -> Val = F(),
                          ets:insert(tab, {Ref, Val}),
                          {H, _} = Val,
                          H;
    [{Ref, {H, _}}]      -> H
  end.

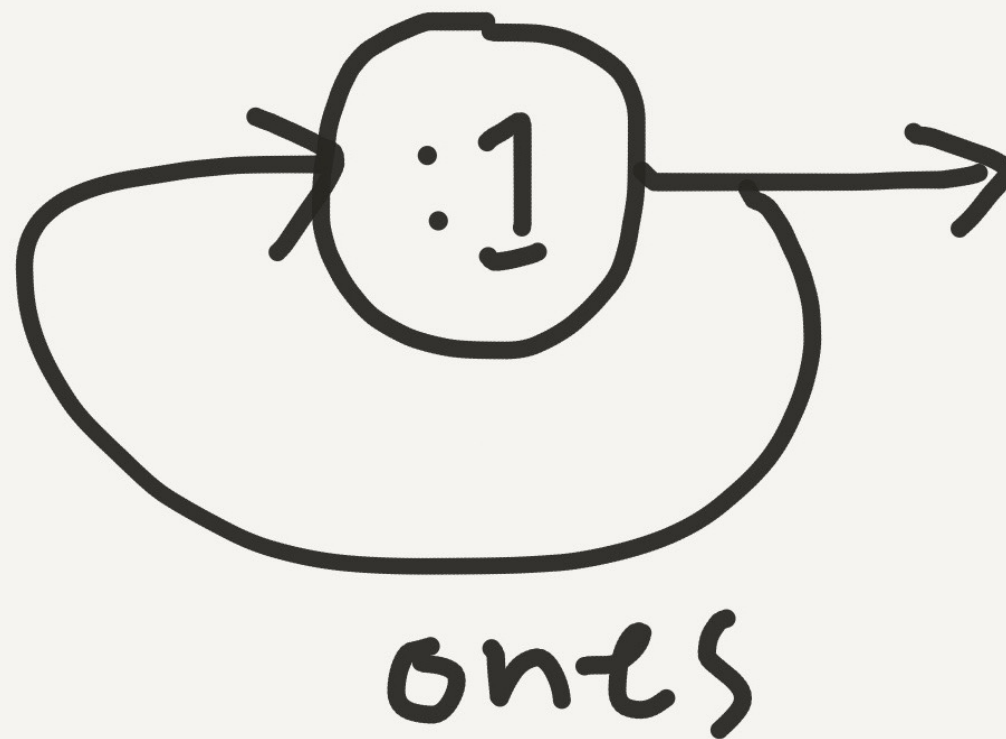
```



```
ones() ->  
  ?cons(1, ones()).
```

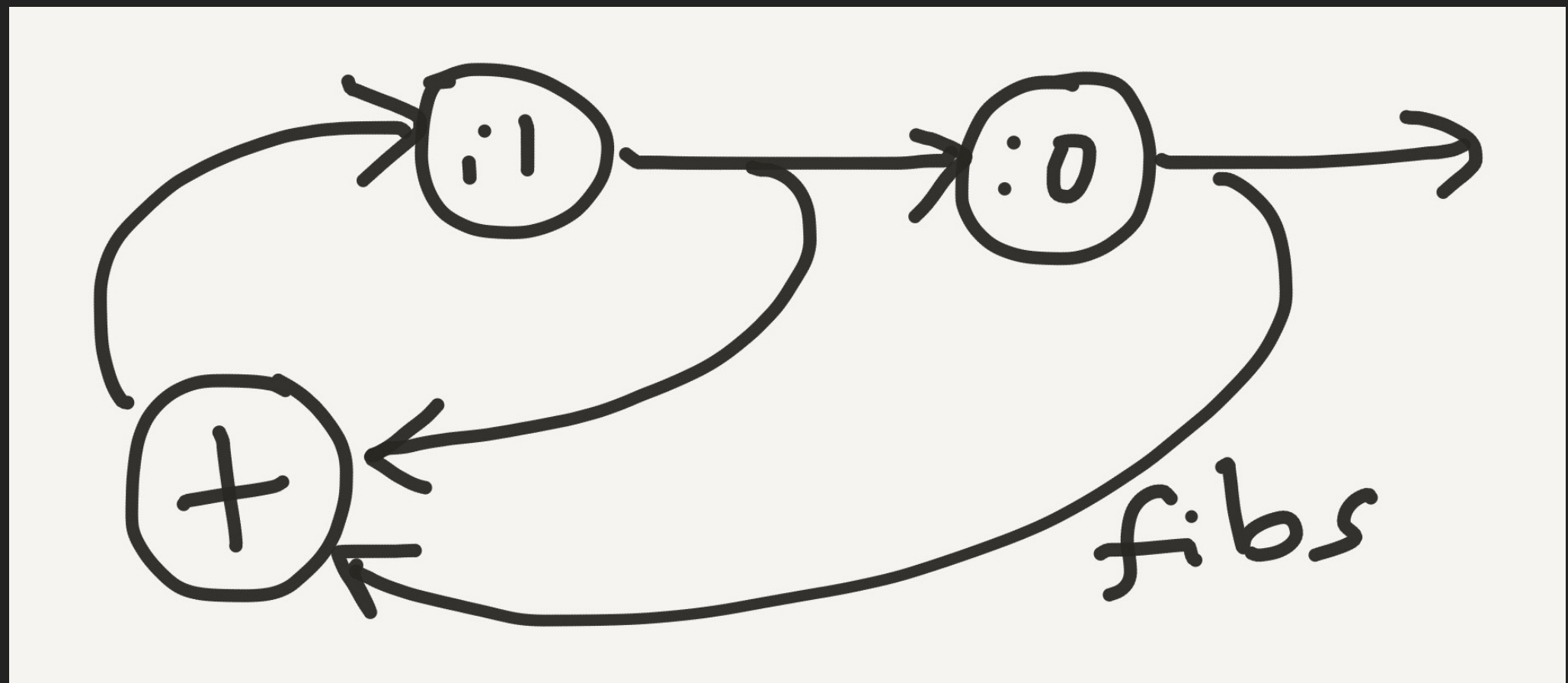


```
onesC() ->  
  This = next_ref()+1,  
  ?cons(1,{ref,This}).
```



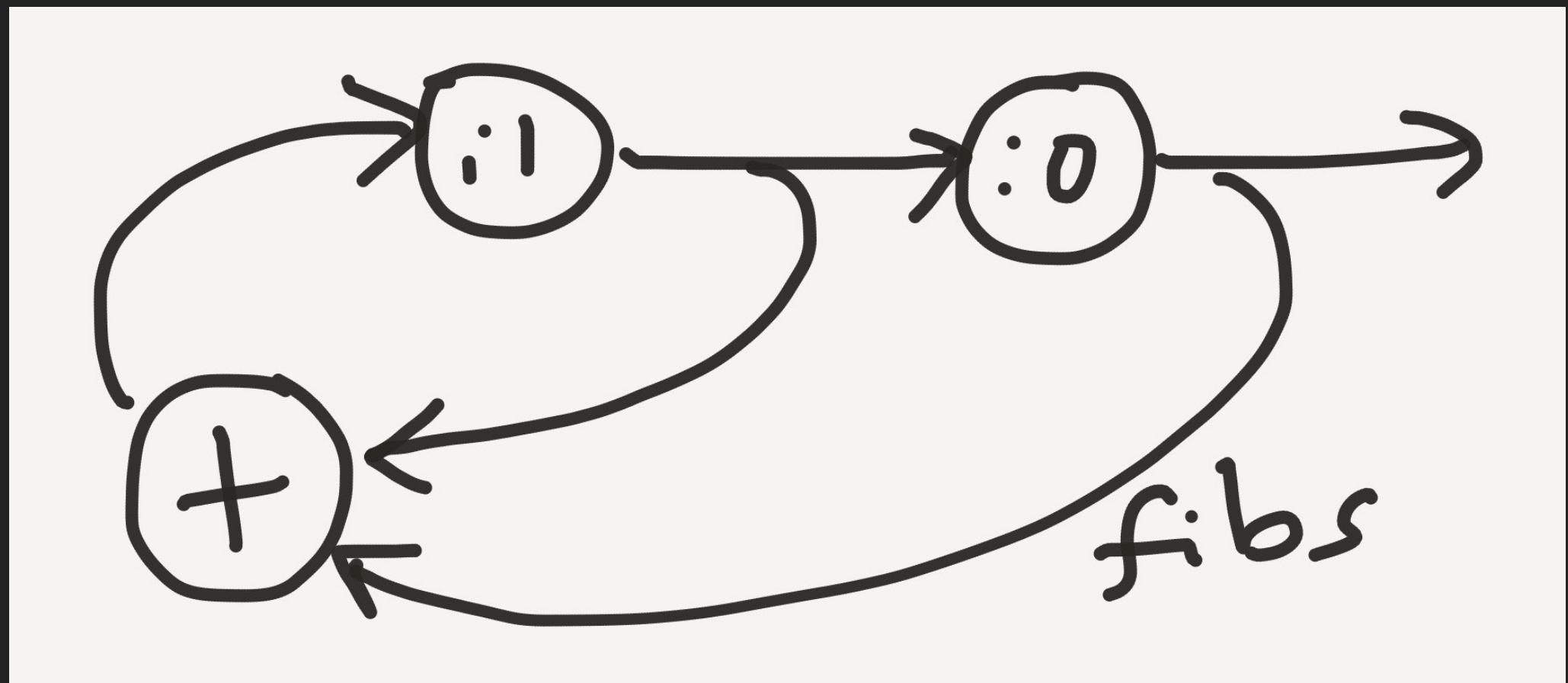
```
fibs() ->  
  ?cons(0,  
    ?cons(1,  
      addZip(fibs(), tail(fibs())))).
```

```
addZip(Xs, Ys) ->  
  ?cons(head(Xs)+head(Ys), addZip(tail(Xs), tail(Ys))).
```

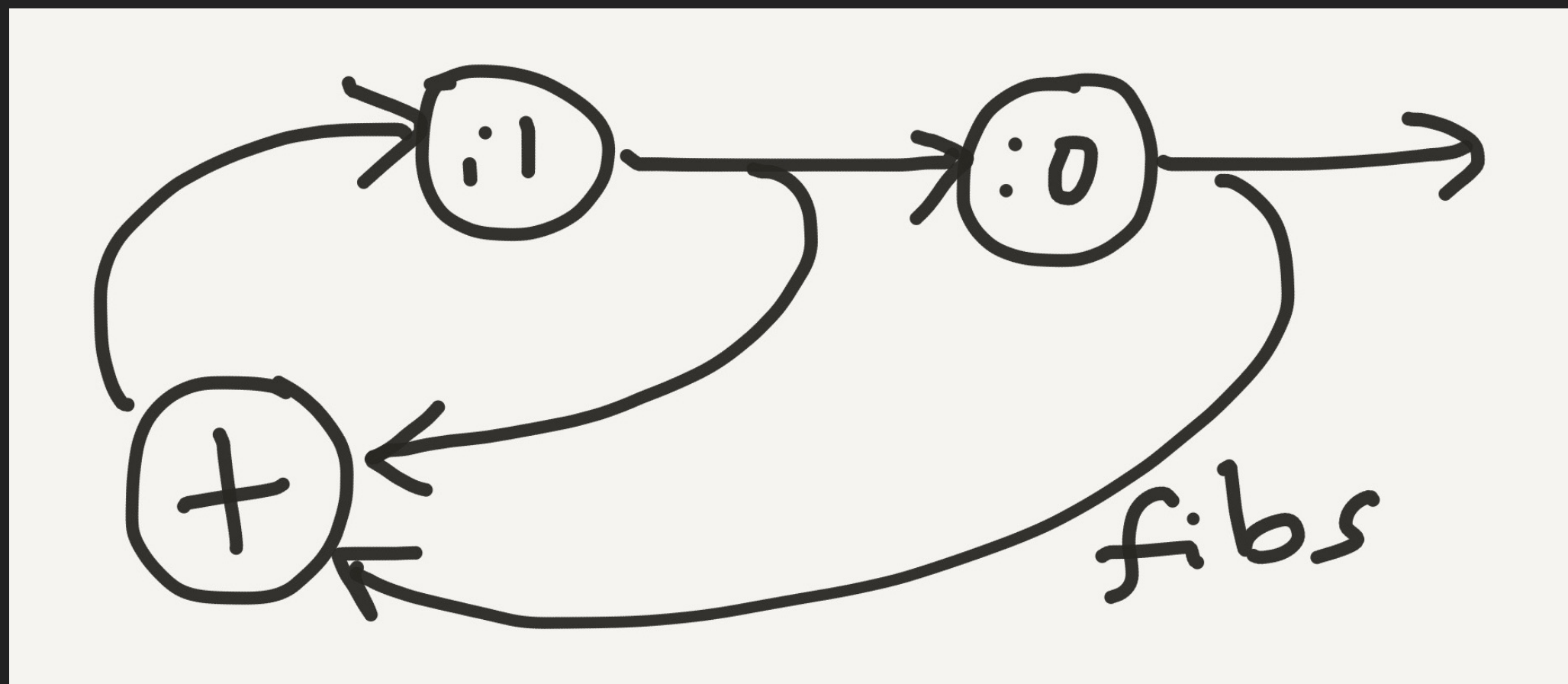




```
fibs() ->  
  ?cons(0,  
    ?cons(1,  
      addZip(fibs(),tail(fibs()))))
```



```
fibsCVar() ->  
  This = next_ref()+1,  
  ?cons(0,  
    ?cons(1,  
      addZip({ref,This},tail({ref,This}))))).
```



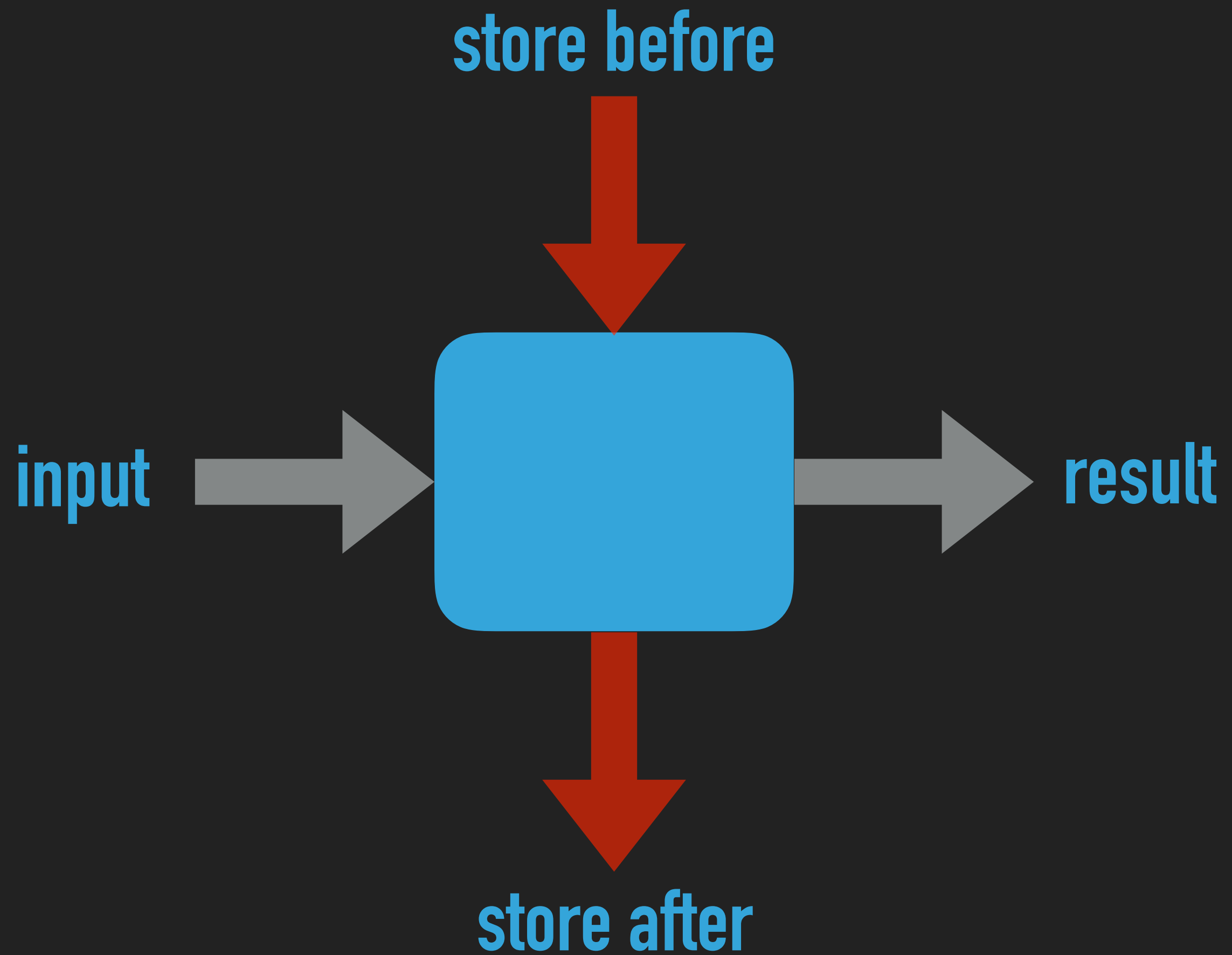
**Explicitly managed refs**

**Simulates full lazy implementation**

**Uses impure features . . .**

**. . . but a smooth transition**

**AN EXPLICIT  
STORE**



# Printing out the first N values

```
ss(_Xs, 0, _T) ->  
  io:format("~n");  
  
ss(Xs, N, Sto) ->  
  io:format("~w, ", [head(Xs, Sto)]),  
  {T, Sto1} = tail(Xs, Sto),  
  ss(T, N-1, Sto1).
```

Node to {Head, {think, Tail}}

Think takes **state** as argument . . .

. . . so that the suspended  
computation can be evaluated in  
the context of the current state.

# Construct a list

```
-define(cons(X,F,Sto),  
  begin {{ref,next_ref(Sto)},  
        Sto#{next_ref(Sto) => {X, {thunk, F}}},  
        0 => next_ref(Sto)+1}}  
end).
```



```

tail({ref,Ref},Sto) ->
  case maps:get(Ref,Sto) of
    {ref,R} ->
      Hd = head({ref,R}, Sto),
      {Tl,Sto1} = tail({ref,R},Sto),
      Sto2 = Sto1#{Ref => {Hd,Tl}},
      {Tl,Sto2};
    {Hd,{thunk,F}} ->
      {Tl,StoC} = F(Sto),
      Sto1 = StoC#{Ref => {Hd,Tl}},
      {Tl,Sto1};
    {_,T} ->
      {T,Sto}
  end.

```

# Fibonacci numbers

```
fibsC(Sto) ->  
  This = next_ref(Sto),  
  ?cons(0, fun(T) ->  
    ?cons(1, fun(S) ->  
      begin  
        {Tl,S1} = tail({ref,This},S),  
        addZip({ref,This},Tl,S1)  
      end  
    end,  
    T)  
  end,  
  Sto).
```

**TO CONCLUDE**

**functions are flexible and  
powerful modelling tool**

**strategies  
simulations  
suspensions**

**pure modelling of effects  
is not straightforward**

**monads, monad transformers,  
effects, . . . provide some  
useful patterns**

reify?

can model DSLs of strategies,  
parsers, and write interpreters  
for these DSLs into the  
functions we've seen here

**data and types**

**all the data we used here was  
well understood 30 years ago**

**it is just that the types have changed**

<https://github.com/simonjohnthompson/streams>



fun