The Art of Strategy:

The Game Theorist's Guide to Success in Life

by

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It's Your Move, Charlie Brown

In a recurring theme in the cartoon strip "Peanuts," Lucy holds a football on the ground and invites Charlie Brown to run up and kick it. At the last moment, Lucy pulls the ball away. Charlie Brown, kicking air, lands on his back, and this gives Lucy great perverse pleasure.

Anyone could have told Charlie that he should refuse to play Lucy's game. Even if Lucy had not played this particular trick on him last year (and the year before and the year before that), he knows her character from other contexts and should be able to predict her action.

At the time when Charlie is deciding whether or not to accept Lucy's invitation, her action lies in the future. However, just because it lies in the future does not mean Charlie should regard it as uncertain. He should know that of the two possible outcomes --- letting him kick and seeing him fall --- Lucy's preference is for the latter. Therefore he should forecast that when the time comes, she is going to pull the ball away. The logical possibility that Lucy will let him kick the ball is realistically irrelevant. Reliance on it would be, to borrow Dr. Johnson's characterization of remarriage, a triumph of hope over experience. Charlie should disregard it, and forecast that acceptance will inevitably land him on his back. He should decline Lucy's invitation.

Two Kinds of Strategic Interactions

The essence of a game of strategy is the interdependence of the players' decisions. These interactions arise in two ways. The first is *sequential,* as in the Charlie Brown story. The players make alternating moves. Charlie, when it is his turn, must look ahead to how his current actions will affect the future actions of Lucy, and his own future actions in turn.

The second kind of interaction is simultaneous, as in the prisoners' dilemma tale of Chapter 1. The players act at the same time, in ignorance of the others' current actions. However, each must be aware that there are other active players, who in turn are similarly aware, and so on. Therefore each must figuratively put himself in the shoes of all, and try to calculate the outcome.
His own best action is an integral part of this overall calculation.

When you find yourself playing a strategic game, you must determine whether the interaction is simultaneous or sequential. Some games such as football have elements of both. Then you must fit your strategy to the context. In this chapter, we develop in a preliminary way the ideas and rules that will help you play sequential games; simultaneous-move games are the subject of Chapter 3. We begin with really simple, sometimes contrived, examples, such as the Charlie Brown story. This is deliberate; the stories are not of great importance in themselves, and the right strategies are usually easy to see by simple intuition, so the underlying ideas stand out that much more clearly. The examples get increasingly realistic and more complex in the case studies and in the later chapters.


PEANUTS
featuring
"Good ol' Charlie Brown"
by SCHULZ
CHARLIE BROWNNN...
I'LL HOLD THE BALL, CHARLIE BROWN, AND YOU COME RUNNING UP AND KICK IT..
NOPE, I REFUSE! YOU'LL PULL THE BALL AWAY, AND I'LL COME CRASHING DOWN AND KILL MYSELF!
BUT YOU CAN'T BACK OUT NOW... THE PROGRAMS HAVE ALREADY BEEN PRINTED...
PROGRAMS?
"AT ONE O'CLOCK LUCILLE VAN PELT WILL HOLD THE FOOTBALL AND CHARLES BROWN WILL RUN UP AND KICK IT"
SHE'S RIGHT..IF THE PROGRAMS HAVE ALREADY BEEN PRINTED, IT'S TOO LATE TO BACK OUT...
THIS YEAR I'M GONNA KICK THAT BALL CLEAR OUT OF THE UNIVERSE!
AAUGH!
WHAM!
IN EVERY PROGRAM, CHARLIE BROWN, THERE ARE ALWAYS A FEW LAST MINUTE CHANGES!

The First Rule of Strategy

The general principle for sequential-move games is that each player should figure out the other players' future responses, and use them in calculating his own best current move. So important is this idea that it is worth codifying into a basic rule of strategic behavior:

> *Rule 1:* **Look ahead and reason back.**

Anticipate where your initial decisions will ultimately lead, and use this information to calculate your best choice.

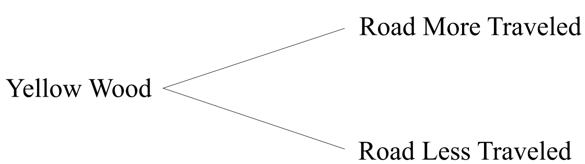
In the Charlie Brown story, this was easy to do for anyone (except Charlie Brown). He had just two alternatives, and one of them led to Lucy's decision between two possible actions. Most strategic situations involve a longer sequence of decisions with several alternatives at each. A "tree diagram" of the choices in the game sometimes serves as a visual aid for correct reasoning in such games. Let us show you how to use these trees.

Decision Trees and Game Trees

A sequence of decisions, with the need to look ahead and reason back, can arise even for a solitary decision-maker not involved in a game of strategy with others. For Robert Frost in the yellow wood:

> Two roads diverged in a wood, and I
> I took the road less traveled by,
> And that has made all the difference.[i]

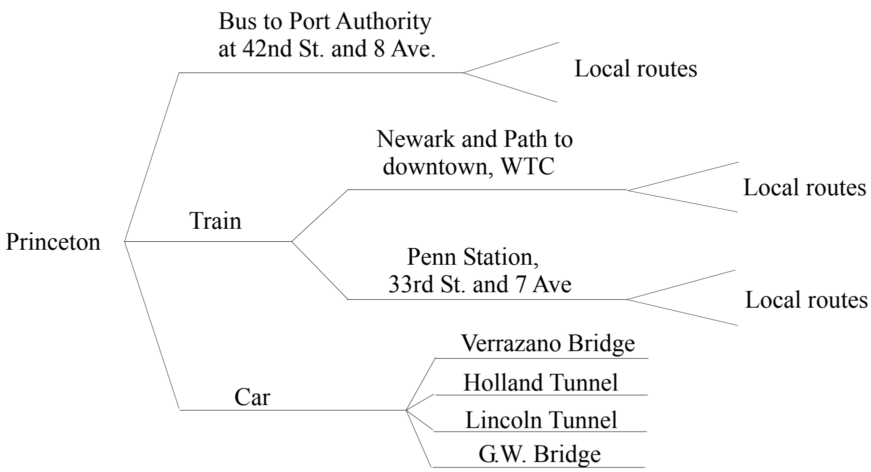
We can show this schematically.



This need not be the end of the choice. Each road might in turn have further branches. The road map becomes correspondingly complex. Here is an example from our own experience.

Travelers from Princeton to New York have several choices. The first decision point involves selecting the mode of travel: bus, train, or car. Those who drive then have to choose among the Verrazano Narrows Bridge, the Holland Tunnel, the Lincoln Tunnel, and the George Washington Bridge. Rail commuters must decide whether to switch to the PATH train at Newark or continue to Penn Station. Once in New York, rail and bus commuters must choose among going by foot, subway (local or express), bus, or taxi to get to their final destination. The best choices depend on many factors, including price, speed, expected congestion, the final destination in New York, and one's aversion to breathing the air on the Jersey Turnpike.

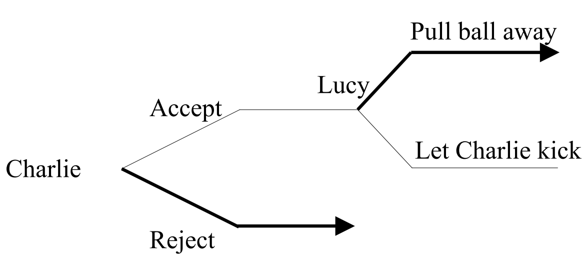
This road map, which describes one's options at each junction, looks like a tree with its successively emerging branches --- hence the term "decision tree." The right way to use such a map or tree is not to take the route whose first branch looks best --- for example, because you would prefer driving to taking the train when all other things are equal --- and then “cross the Verrazano Bridge when you get to it.” Instead, you anticipate the future decisions and use them to make your earlier choices. For example, if you want to go downtown, the PATH train would be superior to driving because it offers a direct connection from Newark.



We can use just such a tree to depict the choices in a game of strategy, but one new element enters the picture. A game has two or more players. At various branching points along the tree, it may be the turn of different players to make the decision. A person making a choice at an earlier point must look ahead, not just to his own future choices, but to those of others. He must forecast what the others will do, by putting himself figuratively in their shoes, and thinking as they would think. To remind you of the difference, we will call a tree showing the decision sequence in a game of strategy a *game tree,* reserving the term *decision tree* for situations in which just one person is involved.

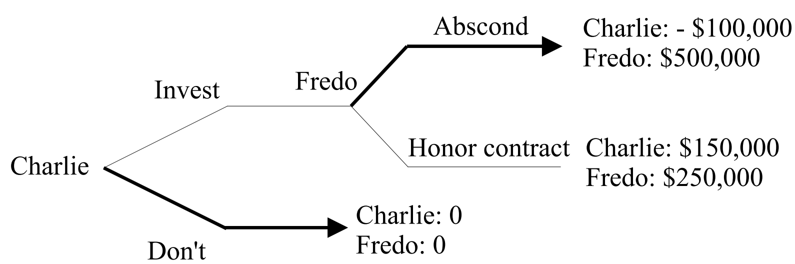
Charlie Brown in Football and in Business

The story of Charlie Brown that opened this chapter is absurdly simple, but you can become familiar with game trees by casting that story in such a picture. Start the game when Lucy has issued her invitation, and Charlie faces the decision of whether to accept. If Charlie refuses, that is the end of the game. If he accepts, Lucy has the choice between letting Charlie kick and pulling the ball away. We can show this by adding another fork along this road.



As we said earlier, Charlie should forecast that Lucy will choose the upper branch. Therefore he should figuratively prune the lower branch of her choice from the tree. Now if he chooses his own upper branch, it leads straight to a nasty fall. Therefore his better choice is to follow his own lower branch. We show these selections by making the branches thicker and marking them with arrowheads.

Are you thinking that this game is too frivolous? Here is a business version of it. Imagine the following scenario. Charlie, now an adult, is vacationing in the newly reformed formerly Marxist country of Freedonia. He gets into a conversation with a local businessman named Fredo, who talks about the wonderful profitable opportunities that he could develop given enough capital, and then makes a pitch: “Invest $100,000 with me, and in a year I will turn it into $500,000, which I will share equally with you. So you will more than double your money in a year.” The opportunity Fredo describes is indeed attractive, and he is willing to write up a proper contract under Freedonian law. But how secure is that law? If at the end of the year Fredo absconds with all the money, can Charlie, back in the U.S., enforce the contract in Freedonian courts? They may be biased in favor of their national, or too slow, or bribed by Fredo. So Charlie is playing a game with Fredo, and the tree is as shown here. (Note that if Fredo honors the contract, he pays Charlie $250,000; therefore Charlie’s profit is that minus the initial investment of $100,000, that is, $150,000).



What do you think Fredo is going to do? In the absence of a clear and strong reason to believe his promise, Charlie should predict that Freedo will abscond, just as young Charlie should have been sure that Lucy would pull the ball away. In fact the trees of the two games are identical in all essential respects. But how many Charlies have failed to do the correct reasoning in such games?

What reasons can there be for believing Fredo’s promise? Perhaps he is engaged in many other enterprises that require financing from the U.S., or export goods to the U.S. Then Charlie may be able to retaliate by ruining his reputation in the U.S., or seizing his goods. So this game may be a part of a larger game, perhaps an ongoing interaction, that ensures Fredo’s honesty. But in the one-time version we showed above, the logic of backward reasoning is clear.

We would like to use this game to make three remarks. First, different games may have identical or very similar mathematical forms (trees, or the tables used for depictions in later chapters). Thinking about them using such formalisms highlights the parallels, and makes it easy to transfer your knowledge about a game in one situation to that in another. This is an important function of the “theory” of any subject: it distils the essential similarities in apparently dissimilar contexts, and enables one to think about them in a unified and therefore simplified manner. Many people have an instinctive aversion to theory of any kind. But we think this is a mistaken reaction. Of course, theories have their limitations. Specific contexts and experiences can often add to or modify the prescriptions of theory in substantial ways. But to abandon theory altogether would be to abandon a valuable starting point for thought, or a beachhead for conquering the problem. You should make game theory your friend, and not a bugbear, in your strategic thinking.

The second remark is that Fredo should recognize that a strategic Charlie would be suspicious of his pitch and not invest at all, losing Fredo the opportunity of making $250,000. Therefore Fredo has a strong incentive to make his promise credible. As an individual businessman, he has little influence over Freedonia's weak legal system, and cannot allay the investor's suspicion that way. What other methods may be at his disposal? We will examine the general issue of credibility, and devices for achieving it, in Chapters 6 and 7.

The third, and perhaps most important, remark concerns comparisons of the different outcomes that could result corresponding to different choices the players could make. It is not always the case that more for one player means less for the other. The situation where Charlie invests and Fredo honors the contract is better for both than the one where Charlie does not invest at all. Unlike sports or contests, games don't have to have winners and losers; in the jargon of game theory, they don't have to be *zero-sum*. Games can have win-win or lose-lose outcomes. In fact, some combination of commonality of interest (as when Charlie and Fredo can both gain if there is a way for Fredo to commit credibly to honoring the contract) and some conflict (as when Fredo can gain at Charlie's expense by absconding after Charlie has invested) coexist in most games in business, politics, and social interactions. And that is precisely what makes the analysis of these games so interesting and challenging.

More Complex Trees

We turn to politics for an example of a slightly more complex game tree. A caricature of American politics says that Congress likes pork-barrel expenditures and presidents try to cut down the bloated budgets that Congress passes. Of course presidents have their own likes and dislikes among such expenditures, and would like to cut only the ones they dislike. To do so, they would like to have the power to cut out specific items from the budget, or a "line-item veto." Ronald Reagan in his State of the Union Address in January 1987 said this eloquently: "Give us the same tool that 43 governors have, a line-item veto, so we can carve out the boondoggles and pork—those items that would never survive on their own."

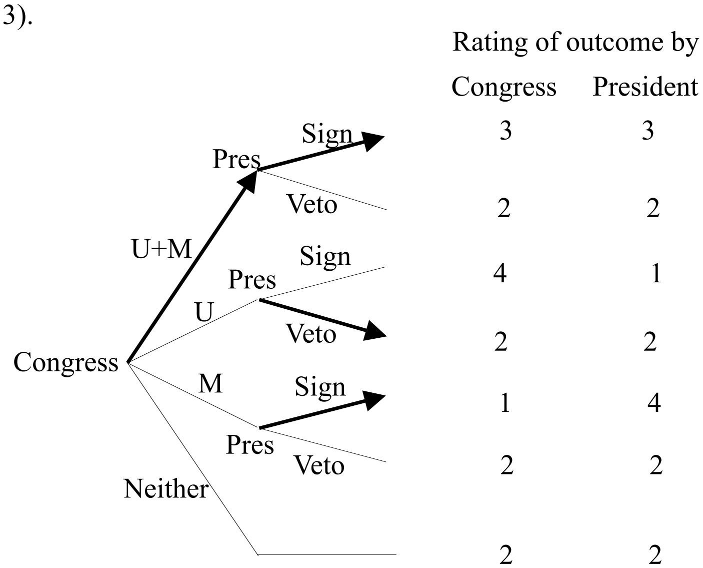
At first sight, it would seem that the added freedom to veto parts of a bill can only increase the president's power and never yield him any worse outcomes. Yet it is possible that the president may be better off without this tool. The point is that the existence of a line-item veto will influence the Congress's strategies in passing bills. A simple game shows how.

For this purpose, the essence of the situation in 1987 was as follows. Suppose there are two items of expenditure under consideration: urban renewal (U), and an anti-ballistic missile system (M). Congress liked the former and the President liked the latter. But both preferred a package of the two to the status quo. The following table shows the ratings of the possible scenarios by the two players, in each case 4 being best and 1 worst.

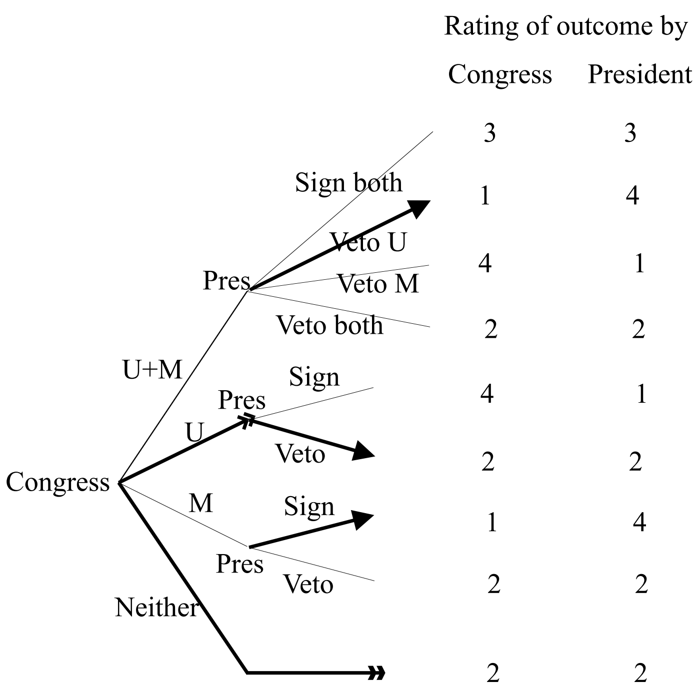
Outcomes	Congress	President
Both U and M	3	3
U only	4	1
M only	1	4
Neither	2	2

When the president does not have a line-item veto, the tree for the game is shown below. The president will sign a bill containing the package of U and M, or one with M alone, but will veto one with U alone. Knowing this, the Congress chooses the package. Once again we show the selections at each point by thickening the chosen branches and giving them arrowheads. Note that we have to do this for all the points where the President might conceivably be called upon to choose, even though some of these are rendered moot by Congress's previous choice. The reason is that Congress's actual choice is crucially affected by its calculation of what the President would have done if Congress had counterfactually made a different choice; to show this logic we must show the President's actions in all logically conceivable situations.

Our analysis of the game yields an outcome in which both sides get their second-best preference (rating 3).



Next suppose the president has a line-item veto. The game changes to the following:



Now Congress foresees that if it passes the package, the President will selectively veto U, leaving only M. Therefore Congress's best action is now either to pass U only to see it vetoed, or pass nothing. Perhaps it may have a preference for the former, if it can score political points from a Presidential veto, but perhaps the President may equally score political points by this show of budgetary discipline. Let us suppose the two offset each other, and Congress is indifferent between the two choices. But either gives each party only their third-best outcome (rating 2). Even the President is left worse-off by his extra freedom of choice.[1]

This game illustrates an important general conceptual point. In single-person decisions, greater freedom of action can never hurt. But in games, it can hurt because its existence can influence other players' actions. Conversely, tying your own hands can help. We will explore this "advantage of commitment" in Chapters 6 and 7.

We have applied the method of backward reasoning in a game tree to a very trivial game (Charlie Brown), and extended it to a slightly more complicated game (the line-item veto). The general principle remains applicable, no matter how complicated the game may be. But trees for games where each player has several choices available at any point, and where each player gets several turns to move, can quickly get too complicated to draw or use. In chess, for example, 20 branches emerge from the root – the player with the white pieces can move any of his/her eight pawns forward one square or two, or move one of his two knights in one of two ways. For each of these, the player with the black pieces has 20 moves, so we are up to 400 distinct paths already. The number of branches emerging from later nodes in chess can be even larger. Solving chess fully using the tree method is beyond the ability of the most powerful computer that exists or might be developed during the next several decades, and other methods of partial analysis must be sought. We will discuss later in the chapter how chess experts have tackled this problem.

Between the two extremes lie many moderately complex games that are played in business, politics, and everyday life. Two approaches can be used for these. Computer programs are available to construct trees and compute solutions.[ii] Alternatively, many games of moderate complexity can be solved by the logic of tree analysis, without drawing the tree explicitly. We illustrate this using a game that was played in a TV show that is all about games, where each player tries to "outplay, outwit, and outlast" the others.

Strategies for "Survivors"

The CBS reality TV series *Survivor* features many interesting games of strategy. In the sixth episode of *Survivor Thailand*, the two teams or tribes played a game that provides an excellent example of thinking ahead and reasoning backward in theory and in practice.[iii] Twenty-one flags were planted in the field of play between the tribes, who alternated turns in removing the flags. Each tribe at its turn could choose to remove one or two or three flags. (Thus zero— passing up one's turn—was not permitted; neither was it within the rules to remove four or more at one turn.) The team to take the last flag, whether standing alone or as a part of a group of two or three flags, won the game. The losing tribe had to vote out one of its own members, thus weakening it in future contests. In fact the loss proved crucial in this instance, and a member of the other tribe went on to win the ultimate prize of a million dollars. Thus the ability to figure out the correct strategy for this game would prove to be of great value.

[1] In many states, governors do have the power of line-item veto. Do they have significantly lower budget expenditures and deficits than states without line-item vetoes? A statistical analysis by Professor Douglas Holtz-Eakin of Syracuse University (now director of the Congressional Budget Office) showed that they do not. ("The Line Item Veto and Public Sector Budgets," Journal of Public Economics 1988, 269-292.)

The two tribes were named Sook Jai and Chuay Gahn, and Sook Jai had the first move. They started by taking two flags. Before reading on, pause a minute and think. If you were in their place, how many would you have chosen?

Write down your choice somewhere, and read on. To understand how the game should be played, and compare the correct strategy with how the two tribes actually played, it helps to focus on two very revealing incidents.

First, each tribe had a few minutes to discuss the game among its own members before the play started. During this discussion within Chuay Gahn, one of their members, Ted Rogers, their African-American software developer, pointed out "At the end, we must leave them with four flags." This is correct: if Sook Jai faces four flags, it must take 1 or 2 or 3, leaving Chuay Gahn to take the remaining 3 or 2 or 1 respectively at its next turn and win the game. Chuay Gahn did in fact get and exploit this opportunity correctly; facing six flags, they took two.

But here is the other revealing incident. At the previous turn, just as Sook Jai returned having taken three flags out of the nine facing them, the realization hit one of their members Shii Ann, a feisty and articulate competitor who took considerable pride in her analytical skills: "If Chuay Gahn now takes two, we are sunk." So Sook Jai's just-completed move was wrong. What should it have done?

Shii Ann or one of her Sook Jai colleagues should have reasoned as Ted Rogers did, but carried the logic of leaving the other tribe with four flags to its next step. How do you ensure leaving the other tribe with four flags at its next turn? By leaving it with eight flags at its previous turn. When it takes 1 or 2 or 3 out of eight, you take 3 or 2 or 1 at your next turn, leaving them with four as planned. Therefore Sook Jai should have turned the tables on Chuay Gahn, and taken just one flag out of the nine. Shii Ann's analytical skill kicked into high gear one move too late! Ted Rogers, the software developer, perhaps had the better analytical insights. But did he?

How did Sook Jai come to face nine flags at its previous move? Because Chuay Gahn had taken two from 11 at *its* previous turn. Ted Rogers should have carried his own reasoning one step further. Chuay Gahn should have taken three, leaving Sook Jai with eight, which would be a losing position.

The same reasoning can be carried even farther back. To leave the other tribe with eight flags, you must leave them with 12 at their previous turn; for that you must leave them with 16 at the turn before that, and 20 at the turn before that. So Sook Jai should have started the game by taking just one flag, not two as it actually did. Then it could have had a sure win by leaving Chuay Gahn with 20, 16, … 4 at their successive turns.[2]

Now think of Chuay Gahn's very first turn. It faced 19 flags. If it had carried its own logic back far enough, it would have taken three, leaving Sook Jai with 16 and already on the way to certain defeat. Starting from any point in the middle of the game where the opponent has played

[2] Does the first mover always have a sure win in all games? No. If the flags game started with 20 flags instead of 21, the second mover would have a sure win. And in some games, for example the simple 3-by-3 tic-tac-toe, either player can ensure a tie with correct play.

incorrectly, the team with the turn to move can seize the initiative and win. But Chuay Gahn did not play the game perfectly either.[3]

The table below shows the comparison between the actual and the correct moves at each point in the game. (The entry "None" means that all moves are losing moves if the opponent plays correctly.) You can see that almost all the choices were wrong, except Chuay Gahn's move when facing 13 flags, and that must have been accidental, because at their next turn they faced 11 and took two when they should have taken three.

Tribe	No. of flags before move	No. of flags taken	Move to put team on path to sure victory
Sook Jai	21	2	1
Chuay Gahn	19	2	3
Sook Jai	17	2	1
Chuay Gahn	15	1	3
Sook Jai	14	1	2
Chuay Gahn	13	1	1
Sook Jai	12	1	No move
Chuay Gahn	11	2	3
Sook Jai	9	3	1
Chuay Gahn	6	2	2
Sook Jai	4	3	No move
Chuay Gahn	1	1	1

Before you judge the tribes harshly, you should recognize that it takes time and some experience to learn how to play even very simple games. We have played this game between pairs or teams of students in our classes, and found that it takes even Ivy League freshmen three or even four plays before they figure out the complete reasoning and play correctly all the way through from the first move. (By the way, what number had you chosen initially when we asked you to, and what was your reasoning?) Incidentally, people seem to learn faster by watching others play than by playing themselves; perhaps the perspective of an observer is more conducive to seeing the game as a whole and reasoning about it coolly than that of a participant.

To fix your understanding of the logic of the reasoning, we offer you the first of our Trips to the Gym—questions on which you can exercise and hone your developing skills in strategic thinking. The answers are in the Workouts section in the end of the book.

Now that you are invigorated by this exercise, let us proceed to think about some general issues of strategy in this whole class of games.

> Trip to the Gym No. 1
>
> Let us turn the flag game into a hot potato: now you win by forcing the other team to take the last flag. It's your move and there are 21 flags. How many do you take?

What Makes a Game Fully Solvable by Backward Reasoning?

[3] The fates of the two key people were also interesting. Shii Ann made another key miscalculation in the next episode and was voted out, at number 10 among the 16 contestants who started the game. Ted, more quiet but perhaps somewhat more skillful, made it to the last five.

The 21-flags game had a special property that made it fully solvable, namely absence of uncertainty of any kind: whether about some natural chance elements, or about the other players' motives and capabilities, or about their actual actions.[iv] This seems a simple point to make, but it needs some elaboration and clarification.

First, at any point in the game where one tribe had the move, it knew exactly what the situation was, namely how many flags remained. In many actual games there are elements of pure chance, thrown up by nature or by the gods of probability. For example, in many card games, when a player makes a choice, he/she does not know for sure what cards the other players hold, although their previous actions may give some basis for drawing some inferences about that. In many subsequent chapters, our examples and analysis will involve games that have this natural element of chance.

Secondly, the tribe making its choice in 21-flags also knew the other tribe's objective, namely to win. And Charlie Brown should have known that Lucy enjoyed seeing him fall flat on his back. Players have such perfect knowledge of the other player's or players' objectives in many simple games and sports, but that is not necessarily the case in games people play in business, politics, and social interactions. Motives in such games are complex combinations of selfishness and altruism, concern for justice or fairness, short-run and long-run considerations, and so on. To figure out what the other players will choose at future points in the game, you need to know what their objectives are, and in the case of multiple objectives, how will they trade one off against the other. You can almost never know this for sure, and must make educated guesses. You must not assume that other people will have broadly the same preferences as you do, or as a hypothetical "rational person" does, but must genuinely think about their situation. This "putting yourself in the other person's shoes" is a very difficult task, and it is often made even more complicated by your emotional involvement in your own aims and pursuits. We will have more to say about this kind of uncertainty later in this chapter and at various points throughout the book. Here we merely point out that the uncertainty about other players' motives is an issue for which it may be useful to seek advice from an objective third party, a "strategic consultant."

Finally, players in many games must face uncertainty about other players' choices, sometimes called strategic uncertainty to distinguish it from the natural aspects of chance such as a distribution of cards or the bounce of a ball from an uneven surface. In 21-flags there was no strategic uncertainty, because each tribe saw and knew exactly what the other had done previously. But in many games, players take their actions simultaneously or in such rapid sequence that one cannot see what the other has done and react to it. A soccer goalie facing a penalty kick must decide whether to move to his/her own right or left without knowing which direction the shooter will aim for; a good shooter will conceal his/her own intentions up to the last microsecond, by which time it is too late for the goalie to react. The same is true for serves and passing shots in top-level tennis, and many other sports. Each participant in a sealed-bid auction must make his/her own choice without knowing what the other bidders are choosing at the same time. In other words, in many games the players make their moves simultaneously, and not in a preassigned sequence. The kind of thinking that is needed for these choosing one's action in such games is different and in some respects harder than the pure backward reasoning of sequential-move games like 21-flags; each player must be aware of the fact that others are making conscious choices, and are in turn thinking about what he/she is thinking, and so on. The games we consider in the next several chapters will elucidate the reasoning and solution concepts for simultaneous-move games. In this chapter we focus on purely sequential-move games, as exemplified by the 21-flags game, or at a much higher level of complexity, by chess.

Do People Actually Solve Games by Backward Reasoning?

Backward reasoning along a tree is the correct way to analyze and solve games where the players move sequentially. Those who fail to do so either explicitly or intuitively are harming their own objectives; they should read our book or hire a strategic consultant. But that is an advisory or normative use of the theory of backward reasoning. Does the theory have the more usual explanatory or positive value that most scientific theories do? In other words, do we observe the correct outcomes from the play of actual games? Researchers in the new and exciting fields of "behavioral economics" and "behavioral game theory" have conducted experiments that yield mixed evidence.

Seemingly the most damaging criticism comes from the ultimatum game. This is the simplest possible negotiation game: there is just one take-it-or-leave-it offer. The ultimatum game has two players, a "proposer," say A, a "respondent", say B, and a sum of money, say 100 dollars. Player A begins the game by proposing a division of the 100 dollars between the two. Then B decides whether to agree to A's proposal. If B agrees, the proposal is implemented; each player gets what A proposed and the game ends. If B refuses, then neither player gets anything, and the game ends.

Pause a minute and think. If you were playing this game in the A role, what division would you propose?

Now think how this game would be played by two people who are "rational" as in conventional economic theory, that is, each of them is concerned only with his or her self-interest, and can calculate perfectly the optimal strategies to pursue that interest. The proposer (A) would think as follows. "No matter what split I propose, B is left with the choice between that and nothing. (The game is played only once, so B has no reason to develop a reputation for toughness, or in future plays where B may be the proposer, to engage in any tit-for-tat response to A's actions, or anything of the kind.) So B will accept whatever I offer. I can do best for myself by offering B as little as possible, for example just one cent if that is the minimum permissible under the rules of the game." Therefore A would offer this minimum and B would accept.[4]

Pause and think again. If you were playing this game in the B role, would you accept one cent?

Numerous experiments have been conducted on this game.[v] Typically, two dozen or so subjects are brought together, and are matched randomly in pairs. In each pair, the roles of proposer and responder are assigned, and the game is played once. New pairs are formed at random, and the game played again. Usually the players do not know with whom they are matched in any one play of the game. Thus the experimenter gets several observations from the same pool in the same session, but there is no possibility of forming ongoing relationships that can affect behavior. Within this general framework, many variations of conditions are attempted, to study their effects on the outcomes.

Your own introspection of how you would act as proposer and as responder has probably led you to believe that the results of actual play of this game should differ from the "theoretical" prediction above. And indeed they differ, often dramatically so. The amounts offered to the responder differ across proposers, but one cent or one dollar, or in fact anything below 10 percent of the total sum at stake, is very rare. The median offer (half of the proposers offer less than that and half offer more) is in the 40-50 percent range; in many experiments a 50:50 split is the single

[4] This argument is another example of "tree logic without drawing a tree".

most frequent proposal. Proposals that would give the responder less than 20 percent are rejected about half the time.

Irrationality Versus Other-Regarding Rationality

Why do proposers offer substantial shares to the responders? Three reasons suggest themselves. First, the proposers may be unable to do the correct backward reasoning. Second, the proposers may have motives other than the pure selfish desire to get as much as they can; they act altruistically, or care about fairness. Third, they may fear that respondents would reject low offers.

The first is unlikely because the logic of backward reasoning is so simple in this game. In more complex situations, players may fail to do the necessary calculations fully or correctly, especially if they are novices to the game being played, as we saw in the 21-flags game. But the ultimatum game is surely simple enough, even for novices. The explanation must be the second, the third, or a combination.

> Reverse Ultimatum Game
>
> In this variant of the ultimatum game, A makes an offer to B about how to divide up the 100 dollars. If B says yes, the money is divided up and the game is over. But if B says no, then A must decide whether to make another offer or not. Each subsequent offer from A must be more generous to B. The game ends when either B says yes or A stops making offers. How do you predict this game will end up?
>
> Now we can suppose that A will keep on making offers until he has proposed 99 to B and 1 for himself. Thus, according to tree logic, B should get almost all of the pie. If you were B, would you hold out for 99:1? We'd advise against it.

Early results from ultimatum experiments favored the third. In fact, Harvard's Al Roth and his coauthors found that, given the pattern of rejection thresholds that prevailed in their subject pool, the proposers were choosing their offers to achieve an optimal balance between the prospect of obtaining a greater share for oneself against the risk of rejection. This suggests a remarkable conventional rationality on part of the proposers.

However, later work to distinguish the second and the third possibilities led to a different idea. To distinguish between altruism and strategy, experiments were done using a variant called the dictator game. Here the proposer dictates how the available total is to be split; the other player has no say in the matter at all. Proposers in the dictator game give away significantly smaller sums on the average than they offer in the ultimatum game, but they give away substantially more than zero. Thus there is something to both of those explanations; proposers' behavior in the ultimatum game has both generous and strategic aspects.

Is the generosity driven by altruism or by a concern for fairness? Both explanations are different aspects of what might be called a "regard for others" in people's preferences. Another variation of the experiment helps tell these two possibilities apart.

In the basic setup, the pairs are formed, and then the roles of proposer and responder are assigned by a random mechanism like a coin toss. This may build in a notion of equality or fairness in the players' minds. To remove this, a variant assigns the roles by holding a preliminary contest, such as a test of general knowledge, and making its winner the proposer. This creates some sense of entitlement to the proposer, and indeed leads to offers that are on the average about 10 percent smaller. However, the offers remain substantially above zero, indicating that proposers have an element of altruism in their thinking. Remember that they do not know the identity of the responders, so this must be a generalized sense of altruism, not concern for the well-being of a particular person.

Observe two things about these experiments. First, they follow the standard methodology of science: one tests hypotheses by designing appropriate variations of controls in the experiment. We will mention a few prominent variations of this kind here. (Many more are discussed in the Camerer's book cited in Endnote vi.(tk)) Second, in the social sciences, multiple causes often coexist, each contributing a part of the explanation for the same phenomenon. Hypotheses don't have to be either fully correct or totally wrong, and accepting one need not mean rejecting all others.

Now consider the behavior of the responders. Why do they reject an offer when they know that the alternative is to get even less? The reason cannot be to establish a reputation for being a tough negotiator that may bear fruit in future plays of this game or of other games of division: the same pair does not play repeatedly, and no track record of one player's past behavior is made available to future partners. Even if a reputational motive is implicitly present, it must take a deeper form: a general rule for action or a routine that the responder follows without doing any explicit thinking or calculation in each instance, or instinctive action, or an emotion-driven response. And that is indeed the case. In a new emerging line of experimental research called neuroeconomics, the subjects' brain activity is scanned using functional magnetic resonance imaging (fMRI) or positron emission tomography (PET) while they take various economic decisions. When ultimatum games are played under such conditions, it is found that the responders' anterior insula shows more activity as the proposers' offers become more unequal. Since the anterior insula is active for some emotions such as anger and disgust, this result helps explain why second movers reject unequal offers. Conversely, the left-side prefrontal cortex is more active when an unequal offer is accepted, indicating that conscious control is being exercised to balance between acting on one's disgust and getting more money.[vi]

Many people (especially economists) argue that responders may reject small shares of the small sums that are typically on offer in laboratory experiments; in the real world, where stakes are often much larger, rejection must be very unlikely. To test this, ultimatum game experiments have been conducted in poorer countries where the amounts were worth several months' income for the participants. Rejection does become somewhat less likely, but offers do not become significantly less generous. The consequences of rejection become more serious for the proposers just as they do for the respondents, so proposers fearing rejection are likely to behave more cautiously.

Although behavior can be explained in part by instincts, hormones, or emotions hardwired into the brain, part of it varies from one culture to another. In experiments conducted across many countries, it was found that the perception of what constitutes a reasonable offer varied by up to 10 percent across cultures, but properties like aggressiveness or toughness varied less. Only one group differed substantially from the rest: among the Machiguenga of the Peruvian Amazon, the offers were much smaller (average 26 percent) and only one offer was rejected. Anthropologists explain that the Machiguenga live in small family units, are socially disconnected, and have no norms of sharing. Conversely, in two cultures the offers exceeded 50 percent; these have the custom of lavish giving when one has as stroke of good luck; this places an obligation on the recipients to return the favor even more generously in the future. This norm or habit seems to carry over to the experiment even though the players do not know whom they are giving to or receiving from.

Evolution of Altruism and Fairness

What should we learn from the findings of these experiments on the ultimatum game, and others like them? Many of the outcomes do differ significantly from what we would expect based on the theory of backward reasoning based on the assumption that each player cares only about his or her own reward. Which of the two, correct backward calculation, or selfishness, is the wrong assumption, or is it a combination? And what are the implications?

Consider backward reasoning first. We saw the players in the 21-flags game in *Survivor* fail to do this correctly or fully. But they were playing the game for the first time, and even then, their discussion revealed glimpses of the correct reasoning. Our classroom experience shows that students learn the full strategy after playing the game, or watching it played, just three or four times. Many experiments inevitably and almost deliberately work with novice subjects, whose actions in the game are often steps in the process of learning the game. In the real world of business, politics, and professional sports, where people are experienced at playing the games they are involved in, we should expect that the players have accumulated much more learning, and that they play generally good strategies either by calculation or by trained instinct. For somewhat more complex games, strategically aware players can use computers or consultants to do the calculations; this practice is still somewhat rare but is sure to spread. Therefore we believe that backward reasoning should remain our starting point for analysis of such games, and for predicting their outcomes. This first pass at the analysis can then be modified as necessary in a particular context, to recognize that beginners may make mistakes, and that some games may become too complex to solve unaided.

We believe that the more important lesson from the experimental research is that people bring many considerations and preferences into their choices besides their own rewards that have been the focus of conventional economic theory. Game theorists should include in their analysis of games the players' concerns for fairness or altruism. "Behavioral game theory *extends* rationality rather than abandoning it."[vii]

This is all to the good; a better understanding of people's motives enriches our understanding of economic decision-making and strategic interactions alike. And that is already happening; frontier research in game theory increasingly includes in the players' objectives their concerns for equity, altruism, and similar concerns (and even a "second-round" concern to reward or punish others whose behavior reflects or violates these precepts).[viii]

But we should not stop there; we should go one step further and think why concerns for altruism and fairness, and anger or disgust when someone else violates these precepts, have such a strong hold on people. This takes us into the realm of speculation, but one plausible explanation can be found in evolutionary psychology. Groups that instill norms of fairness and altruism into their members will have less internal conflict than groups consisting of purely selfish individuals. Therefore they will be more successful in taking collective action, such as provision of goods that benefit the whole group, and conservation of common resources, and they will spend less effort and resources in internal conflict. As a result, they will do better, both in absolute terms and in competition with groups that do not have similar norms. In other words, some measure of fairness and altruism may have evolutionary survival value.

Some biological evidence for rejecting unfair offers comes from an experiment run by Terry Burham.[5] In his version of the ultimatum game, the amount at stake was $40 and the subjects were all male Harvard graduate students. The divider was given only two choices: offer $25 and keep $15 or offer $5 and keep $35. Among those offered only $5, twenty students accepted and six rejected, giving themselves and the divider both zero. Now for the punch line. It turns out that the six who rejected the offer had testosterone levels 50% higher than those who accepted the offer. To the extent that testosterone is connected with status and aggression, this could provide a genetic link that might explain an evolutionary advantage of what evolutionary biologist Robert Trivers has called "moralistic aggression."

In addition to a potential genetic link, societies have non-genetic ways of passing on norms, namely the processes of education and socialization of infants and children in families and schools. We see parents and teachers telling impressionable children the importance of caring for others, sharing, and being nice; some of this undoubtedly remains imprinted in their minds and influences their behavior throughout their lives.

Finally, we should point out that fairness and altruism have their limit. Long-run progress and success of a society need innovation and change. These in turn require individualism, and a willingness to defy social norms and conventional wisdom; selfishness often accompanies these characteristics. Therefore we need the right balance between self-regarding and other-regarding behaviors.

Very Complex Trees

When you have acquired a little experience of backward reasoning, you will find that many strategic situations in everyday life or work are amenable to "tree logic" without the need to draw and analyze trees explicitly. Many other games at an intermediate level of complexity can be solved using computer software packages that are increasingly available for this purpose. But for some very complex games such as chess, a complete solution by backward reasoning is infeasible.

In principle, chess is the ideal game of sequential moves amenable to solution by backward reasoning.[ix] The players have alternate moves; all previous moves are observable and irrevocable; there is no uncertainty about the position or the players' motives. The rule that the game is a draw if the same position is repeated ensures that the game ends within a finite total number of moves. Therefore we can start with the terminal nodes and work backward. However, practice and principle are two very different things. It has been estimated that the total number of nodes in chess is about 10 raised to the power of 120, that is, 1 with 120 zeroes after it. A supercomputer 1,000 times as fast as the typical PC will need 10 raised to the power 103 years to examine them all. Waiting for that is futile; foreseeable progress in computers is not likely to improve matters significantly. In the meantime, what have chess players and programmers of chess-playing computers done?

Chess experts have been very successful at characterizing optimal strategies near the end of the game. Once the chess-board has only a small number of pieces on it, experts are able to look ahead to the end of the game and determine by backward reasoning whether one side has a

[5] Terry was on the faculty at Harvard and is the author of *Mean Genes* and *Mean Markets and Lizard Brains: How to Profit from the New Science of Irrationality*. His paper on this experiment is "High-testosterone men reject low ultimatum game offers," *Proceedings of the Royal Society*, 2007.

guaranteed win or whether the other side can obtain a draw. But the middle of the game, when several pieces remain on the board, is far harder. Looking ahead five pairs of moves, which is about as much as can be done by experts in a reasonable amount of time, is not going to simplify the situation to a point where the end-game can be solved completely from there on.

The pragmatic solution is a combination of forward-looking analysis and value judgment. The former is the science of game theory—looking ahead and reasoning backward. The latter is the art of the practitioner—being able to judge the value of a position from the number and interconnections of the pieces without doing explicit solution of the game from that point onward. Chess players often speak of this as "knowledge," but you can call it experience or instinct or art. The best chess players are usually distinguished by the depth and subtlety of their knowledge.

Knowledge can be distilled from the observation of many games and many players, and codified into rules. This has been done most extensively with regard to openings, that is, the first ten or even 15 moves of a game. There are huge books that analyze many different openings and discuss their relative merits and drawbacks.

How do computers fit into this picture? At one time, the project of programming computers to play chess was seen as an integral part of the emerging science of artificial intelligence; the aim was to design computers that would think as humans do. This did not succeed for many years. Then the attention shifted to using computers to do what they do best, namely crunch numbers. Computers can look ahead to more moves and do this more quickly than humans could.[6] Using pure number crunching, by the late 1990s dedicated chess computers like Fritz and Deep Blue had achieved levels where they could compete on equal terms with the top human players. More recently, computers have been improved further by programming into them some knowledge of mid-game positions acquired by questioning some of the best human players.

Human players have ratings determined by their performances; the best-ranked computers are already achieving ratings comparable to the 2800 enjoyed by the world's strongest human player, Garry Kasparov. In November 2003, Kasparov played a four-game match against the latest version of the Fritz computer, X3D. The result was one victory each and two draws. In July 2005, the Hydra chess computer demolished Michael Adams, ranked number 13 in the world, winning five games and drawing one in a six game match. It may not be long before the rival computers rank at the top and play each other for world championships.

What should you take away from this account of chess? It should be the method of thinking about any highly complex games you may face. You should combine the rule of looking ahead and reasoning back, and your experience that guides you in evaluating the intermediate positions that are reached at the end of your span of forward calculation. Success will come from such synthesis of the science of game theory and the art of playing a specific game, not from either alone.

Being of Two Minds

Chess illustrates another important practical feature of looking forward and reasoning backward. To do so, you have to play the game from the perspective of both players. While it is hard to calculate your best move in a complicated tree, it is even harder to predict what the other side will do.

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[6] But good chess players can use their knowledge to disregard immediately those moves that are likely to be bad without pursuing their consequences four or five moves ahead, thereby saving their calculation time and effort for the moves that are more likely to be good ones.

If you really could analyze all possible moves and countermoves, and the other player could as well, then the two of you would agree upfront as to how the entire game would play out. But once the analysis is limited to looking down only some branches of the tree, the other player may see something you didn't or miss something you've seen. Either way, the other side or sides may then make a move you didn't anticipate.

To really look forward and reason backward, you have to predict what the other players will actually do, not what you would have done in their shoes. The problem is that when you try to put yourself in the other players' shoes, it is hard if not impossible to leave your own shoes behind. You know too much about what you are planning to do next move and it is hard to erase that knowledge when you are looking at the game from the other player's perspective. Indeed, that explains why people don't play chess (or poker) against themselves. You certainly can't bluff against yourself or make a surprise attack.

There is no perfect solution to this problem. When you try to put yourself in the other players' shoes, you have to know what they know and not know what they don't know. And your objectives have to be their objectives, not what you wish they had as an objective. In practice, firms trying to simulate the move and countermoves of a potential business scenario will hire outsiders to play the role of the other players. That way, they can ensure that their game partners don't know too much. Often the biggest learning comes from seeing the moves that were not anticipated and then understanding what led to that outcome, so that it can be either avoided or promoted.

To end this chapter, we return to Charlie Brown's problem of whether or not to kick the football. This question became a real issue for football coach Tom Osborne in the final minutes of his championship game. We think he too got it wrong. Backward reasoning will reveal the mistake.

Case Study: The Tale of Tom Osborne and the '84 Orange Bowl

In the 1984 Orange Bowl the undefeated Nebraska Cornhuskers and the once-beaten Miami Hurricanes faced off. Because Nebraska came into the Bowl with the better record, it needed only a tie in order to finish the season with the number-one ranking.

But Nebraska fell behind by 31--17 in the fourth quarter. Then the Cornhuskers began a comeback. They scored a touchdown to make the score 31--23. Nebraska coach Tom Osborne had an important strategic decision to make.

In college football, a team that scores a touchdown then runs one play from a hash mark 2½ yards from the goal line. The team has a choice between trying to run or pass the ball into the end zone, which scores two additional points, or trying the less risky strategy of kicking the ball through the goalposts, which scores one extra point.

Coach Osborne chose to play it safe, and Nebraska successfully kicked for the one extra point. Now the score was 31--24. The Cornhuskers continued their comeback. In the waning minutes of the game they scored a final touchdown, bringing the score to 31--30. A one-point conversion would have tied the game and landed them the title. But that would have been an unsatisfying victory. To win the championship with style, Osborne recognized that he had to go for the win.

The Cornhuskers went for the win with a two-point conversion attempt. Irving Fryer got the ball, but failed to score. Miami and Nebraska ended the year with equal records. Since Miami beat Nebraska, it was Miami that was awarded the top place in the standings.

Put yourself in the cleats of Coach Osborne. Could you have done better?

Case Discussion

Many Monday morning quarterbacks fault Osborne for going for the win rather than the tie. But that is not the bone of our contention. Given that Osborne was willing to take the additional risk for the win, he did it the wrong way. Tom Osborne would have done better to first try the two-point attempt, and then if it succeeded go for the one-point, while if it failed attempt a second two-pointer.

Let us look at this more carefully. When down by 14 points, he knew that he needed two touchdowns plus three extra points. He chose to go for the one and then the two. If both attempts were made, the order in which they were made becomes irrelevant. If the one-point conversion was missed but the two-point was successful, here too the order is irrelevant and the game ends up tied, with Nebraska getting the championship. The only difference occurs if Nebraska misses the two-point attempt. Under Osborne's plan, that results in the loss of the game and the championship. If, instead, they had tried the two-point conversion first, then if it failed they would not necessarily have lost the game. They would have been behind 31--23. When they scored their next touchdown this would have brought them to 31--29. A successful two-point attempt would tie the game and win the number-one ranking![7]

We have heard the counterargument that if Osborne first went for the two-pointer and missed, his team would have been playing for the tie. This would have provided less inspiration and perhaps they might not have scored the second touchdown. Moreover, by waiting until the end and going for the desperation win-lose two-pointer his team would rise to the occasion knowing everything was on the line. This argument is wrong for several reasons. Remember that if Nebraska waits until the second touchdown and then misses the two-point attempt, they lose. If they miss the two-point attempt on their first try, there is still a chance for a tie. Even though the chance may be diminished, something is better than nothing. The momentum argument is also flawed. While Nebraska's offense may rise to the occasion in a single play for the championship, we expect the Hurricanes' defense to rise as well. The play is equally important for both sides. To the extent that there is a momentum effect, if Osborne makes the two-point attempt on the first touchdown, this should increase the chance of scoring another touchdown. It also allows him to tie the game with two field goals.

One of the general morals from this story is that if you have to take some risks, it is often better to do this as quickly as possible. This is obvious to those who play tennis: everyone knows to take more risk on the first serve and hit the second serve more cautiously. That way, if you fail on your first attempt, the game won't be over. You may still have time to take some other options that can bring you back to or even ahead of where you were. Taking risks early is true for most aspects of life, whether it be career choices, investments, or dating.

[7] Furthermore, this would be a tie that resulted from the failed attempt to win, so no one would criticize Osborne for playing to tie.

For more practice using the principle of Look Forward, Reason Backward, have a look at the following case studies in Chapter 14: Here’s Mud in Your Eye; Red I Win, Black You Lose; The Shark Repellent that Backfired; Tough Guy, Tender Offer; The Three-way Duel; and Winning without Knowing How.

Many Contexts, One Concept

What do the following situations have in common?

• Two gas stations at the same corner, or two supermarkets in the same neighborhood, sometimes get into fierce price wars with each other.

• In general election campaigns, both the Democratic and the Republican parties in the U.S. often adopt similar centrist policies to attract the swing voters in the middle of the political spectrum, ignoring their core supporters who hold more extreme views to the left and the right respectively.

• "The diversity and productivity of New England fisheries was once unequalled. A continuing trend over the past century has been the overexploitation and eventual collapse of species after species. Atlantic halibut, ocean perch, Haddock and Yellowtail Flounder, …even the venerable Atlantic Cod, … [have joined] the ranks of species written-off as commercially extinct."[x]

• Near the end of Joseph Heller's celebrated novel Catch-22, the Second World War is almost won. Yossarian does not want to be among the last to die; it won't make any difference to the outcome. He explains this to Major Danby, his superior officer. When Danby asks, "But, Yossarian, suppose everyone felt that way?" Yossarian replies, "Then I'd certainly be a damned fool to feel any other way, wouldn't I?" [xi]

Answer: They are all instances of the prisoners' dilemma.[8] Just like the interrogation of Richard Handcock and Perry Edward from *In Cold Blood* recounted in Chapter 1, each has a personal incentive to do something that ultimately leads to a result that is bad for everyone when everyone similarly does what his or her personal interest dictates.[9] If one confesses, the other had better confess to avoid the really harsh sentence reserved for recalcitrants; if one holds out, the other can cut himself a much better deal by confessing. Indeed, the force is so strong that each prisoner's temptation to confess exists regardless of whether the two are guilty (as was the case in *In Cold Blood*) or innocent and being framed by the police (as was the case in *L.A. Confidential).*

Price wars are no different. If the Nexon gas station charges a low price, the Lunaco station had better set its own price low to avoid losing too many customers; if Nexon prices its gas high, Lunaco can divert many customers its way by pricing low. But, when both stations price low, neither make money (though customers are better off).

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[8] No prizes for correct answers—after all, the prisoners' dilemma is the subject of this chapter. But we take this opportunity to point out, as we did in Chapter 2, that the common conceptual framework of game theory can help us understand a great variety of diverse and seemingly unrelated phenomena. We should also point out that neighboring stores do not *constantly* engage in price wars, and political parties do not *always* gravitate to the center. In fact, analyses and illustrations of how the participants in such games can avoid or resolve the dilemma is an important part of this chapter.

[9] While Prisoner's dilemma is the more common usage, we prefer the plural, because unless there are two or more prisoners involved, there is no dilemma.

If the Democrats adopt a platform that appeals to the middle, the Republicans may stand to lose all these voters and therefore the election if they cater only to their core supporters in the economic and social right wings; if the Democrats cater to their core supporters in the minorities and the unions, then the Republicans can capture the middle and therefore a win a large majority by being more centrist. If all others fish conservatively, one fisherman's going for a bigger catch is not going to deplete the fishery to any significant extent; if all others are fishing aggressively, then any single fisherman would be a fool to try single-handed conservation.[xii] The result is overfishing and extinction. Yossarian's logic is what makes it so difficult to continue to support a failed war.

A Little History

How did theorists devise and name this game, which captures so many economic, political, and social interactions? It happened very early in the history of the subject. Harold Kuhn, himself one of the pioneers of game theory, recounted the story in a symposium held in conjunction with John Nash's Nobel Prize award ceremonies:[xiii]

> Al Tucker was on leave at Stanford in the Spring of 1950 and, because of the shortage of offices, he was housed in the Psychology Department. One day a psychologist knocked on his door and asked what he was doing. Tucker replied: 'I'm working on game theory,' and the psychologist asked if he would give a seminar on his work. For that seminar, Al Tucker invented prisoner's dilemma as an example of game theory, Nash equilibria, and the attendant paradoxes of non-socially-desirable equilibria. A truly seminal example, it inspired dozens of research papers and several entire books.

Others tell a slightly different story. According to them, the mathematical structure of the game predates Tucker, and can be attributed to two mathematicians, Merrill Flood and Melvin Dresher at the cold-war think-tank, the RAND Corporation.[xiv] Tucker's genius was to invent the story illustrating the mathematics. And genius it was, because presentation can make or break an idea; a memorable presentation spreads and is assimilated in the community of thinkers far better and faster, whereas a dull and dry presentation may be overlooked or forgotten.

A Visual Representation

We will develop the method for displaying and solving the game using a business example. Rainbow's End and B. B. Lean are rival mail-order firms that sell clothes. Every Fall they print and mail their Winter catalogs. Each firm must honor the prices printed in its catalog for the whole Winter season. The preparation time for the catalogs is much longer than the mailing window, so the two firms must make their pricing decisions simultaneously and without knowing the other firm's choices. They know that the catalogs go to a common pool of potential customers, who are smart shoppers and are looking for low prices.

Both catalogs usually feature an almost identical item, say a chambray deluxe shirt. The cost of each shirt to each firm is $20.[10] The firms have estimated that if they each charge $80 for this

[10] This includes not only the cost of buying the shirt from the supplier in China, but also the cost of transporting it to the U.S., any import duties, the cost of stocking it, and of order fulfillment. In other

item, each will sell 1,200 shirts, so each will make a profit of (80 – 20) * 1200 = 72,000 dollars. Moreover, it turns out that this price serves their joint interests best: if the firms can collude and charge a common price, $80 is the price that will maximize their combined profits.

The firms have estimated that if one of them cuts its price by $1 while the other holds its price unchanged, then the price-cutter gains 100 customers, 80 of whom shift to it from the other firm, and 20 are new, for example they might decide to buy the shirt when they would not have done so at the higher price, or might switch from the store in their local mall. Therefore each firm has the temptation for each firm to undercut the other to gain more customers; the whole purpose of this story is to figure out how these temptations play out.

We begin by supposing that each firm chooses between just two prices, $80 and $70. [11] If one firm cuts its price to $70 while the other is still charging $80, the price-cutter gains 1000 customers and the other loses 800. So the price-cutter sells 2200 shirts while the other's sales drop to 400; the profits are (70 – 20) * 2200 = $110,000 for the price-cutter, and (80 – 20) * 400 = $24,000 for the other firm.

What happens if both firms cut their price to $70 at the same time? If both firms reduce their price by $1, existing customers stay put but each gains the 20 new customers. So when both reduce their price by $10, each gains 10 * 20 = 200 net sales above the previous 1200. Each sells 1400, and each makes profit (70 – 20) * 1400 = $70,000.

We want to display the profit consequences (the firms' payoffs in their game) visually. However, we cannot do this using a game tree like the ones in Chapter 2. Here the two players act simultaneously. Neither can make his move knowing what the other has done, or anticipating how the other will respond. Instead, each must think what the other is thinking at the same time. A starting point for this thinking about thinking is to lay out all the consequences of all the combinations of the simultaneous choices the two could make. Since each has two alternatives, $80 or $75, there are four such combinations. We can display them most easily in a spreadsheet-like format of rows and columns, which we will generally refer to as a game table. The choices of Rainbow's End (RE for short) are arrayed along the rows, and those of B. B. Lean (BB) along the columns. In each of the four cells corresponding to each choice of a row by RE and of a column by the BB, we show two numbers, namely the profit, in thousands of dollars, from selling this shirt. In each cell, the number in the south-west corner belongs to the row player, and the number in the north-east corner belongs to the column player.[12] In the jargon of game theory, these numbers are called payoffs.[13]

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words, it includes all costs specifically attributable to this item. The intention is to have a comprehensive measure of what economists would call marginal cost.

11 This specification, and in particular the assumption that there are just two possible choices for the price, is just to build the analytical method for such games in the simplest possible way. In the following chapter we will allow the firms much greater freedom in choosing their prices.

12 Thomas Schelling invented this way of representing both players' payoffs in the same table, while making clear which payoff belongs to which player. With excessive modesty he writes: "If I am ever asked whether I ever made a contribution to game theory, I shall answer yes … the invention of staggered payoffs in a matrix." Actually Schelling developed many of the most important concepts of game theory—focal points, credibility, commitment, threats and promises, tipping, and much more – for which he shared the 2005 Nobel prize in economics. We will cite him and his work frequently in the chapters to come.

13 Generally, higher payoff numbers are better for each player. Sometimes, as with prisoners under interrogation, the payoff numbers are years in jail, so each player prefers a smaller number for himself. The

		B. B. Lean (BB)	
		$80	$70
Rainbow’s End (RE)	$80	$72,000 $72,000	$110,000 $24,000
	$70	$24,000 $110,000	$70,000 $70,000

Before we “solve” the game, let us observe and emphasize one feature of it. Compare the payoff pairs across the four cells. A better outcome for RE does not always imply a worse outcome for BB, or vice versa. Specifically, both of them are better off in the top left cell than in the bottom right cell. This game need not end with a winner and a loser; it is not zero-sum. We similarly pointed out in Chapter 2 that the Charlie Brown investment game was not zero-sum, and neither are most games we meet in reality. In many games, as in the prisoner’s dilemma, the issue will be how to avoid a lose-lose outcome, or to achieve a win-win outcome.

The Dilemma

Now consider the reasoning of RE’s manager. “If BB chooses $80, I can get $110,000 instead of $72,000 by cutting my price to $70. If BB chooses $70, then my payoff is $70,000 if I also charge $70 but only $24,000 if I charge $80. Therefore in both cases, choosing $70 is better than choosing $80. My better choice (in fact my best choice, since I have only two alternatives) is the same no matter what BB chooses. Therefore I don’t need to think through their thinking at all; I should just go ahead and set my price at $70.”

When a simultaneous-move game has this special feature, namely that for a player the best choice is the same regardless of what the other player or players choose, it greatly simplifies the players’ thinking and the game theorists’ analysis. Therefore it is worth making a big deal of it, and looking for it to simplify the solution of the game. The name given by game theorists for this property is dominant strategy. A player is said to have a *dominant strategy* if that same strategy is better for him than all of his other available strategies, for each strategy or strategy combination that the other player or players could choose. And we have a simple rule for behavior in simultaneous-move games: [14]

Rule 2: **If you have a dominant strategy, use it.**

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same can happen if the payoff numbers are rankings where 1 is best. When looking at a game table, you should check the interpretation of the payoff numbers for that game.

[14] In Chapter 2, we could offer a single, unifying principle to devise the best strategies for games with sequential moves. This was our Rule 1: look ahead and reason back. It won’t be so simple for simultaneous-move games. But the thinking about thinking required for simultaneous moves can be summarized in three simple rules for action. These rules in turn rest on two simple ideas --- dominant strategies and equilibrium. Rule 2 is given here; Rules 3 and 4 will follow in the next chapter. We develop all of these through simple examples.

The prisoners' dilemma is an even more special game—not just one player, but both (or all) players have dominant strategies. The reasoning of BB's manager is exactly analogous to that of RE's manager, and you should fix the idea by going through it on your own. You will see that $70 is also the dominant strategy for BB.

The result is the outcome shown in the bottom right cell of the game table; both charge $70 and make a profit of $70,000 each. And here is the feature that makes the prisoners' dilemma such an important game. When both players use their dominant strategies, both do worse than they would have if somehow they could have jointly and credibly agreed that each would choose the other, dominated, strategy. In this game, that would have meant charging $80 each to obtain the outcome in the top left cell of the game table, namely $72,000 each.[15]

It would not be enough for just one of them to price at $80; then that firm would do very badly. Somehow they must both be induced to price high, and this is hard to achieve given the temptation each of them has to try to undercut the other. Each firm pursuing its own self-interest does not lead to an outcome that is best for them all, in stark contrast to what conventional theories of economics from Adam Smith onward have taught us.[16]

This opens up a whole host of questions. Some of these pertain to more general aspects of game theory. What happens if only one player has a dominant strategy? What if none of the players has a dominant strategy? When the best choice for each varies depending on what the other is choosing simultaneously, can they see through each other's choices and arrive at a solution to the game? We will take up these questions in the next chapter, where we develop a more general concept of solution for simultaneous-move games, namely John Nash's beautiful equilibrium. In this chapter we focus on questions about the prisoners' dilemma game per se.

In the general context, the two strategies available to each player are labeled "Cooperate" and "Defect" (or sometimes "Cheat") and we will follow this usage. Defect is the dominant strategy for each, and the combination where both choose Defect yields both a worse outcome than the outcome if both choose Cooperate.

Some Preliminary Ideas for Resolving the Dilemma

The players caught on the horns of this dilemma have strong incentives to make joint agreements to avoid it. For example, the fishermen in New England might agree to limit their catch to preserve the fish stocks for the future. The difficulty is to make such agreements stick, when each faces the temptation to cheat; for example to take more than one's allotted quota of fish. What does game theory have to say on this issue? And what happens in the actual play of such games?

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[15] Actually $80 is the common price that yields the two the highest possible joint profit; that is the price they would choose if they could get together and cartelize the industry. Rigorous proof of this statement requires some math, so just take our word for it. For readers who want to follow the calculation, it is on the book's web site.

[16] The beneficiaries from this price-cutting by the firms are of course the consumers, who are not active players in this game. Therefore it is often in the larger society's interest to prevent the two firms from resolving their pricing dilemma. That is the role of antitrust policies in the U.S. and some other countries.

In the fifty years since the prisoners' dilemma game was invented, its theory has advanced a great deal, and much evidence has accumulated, both from observations about the real world and from controlled experiments in laboratory settings. Let us look at all this material, and see what we can learn from it.

The flip side of achieving cooperation is avoiding defection. A player can be given the incentive to choose cooperation rather than the originally dominant strategy of defection by giving him a suitable reward, or deterred from defecting by creating the prospect of a suitable punishment.

The reward approach is problematic for several reasons. Rewards can be internal—one player pays the other for taking the cooperative action. Sometimes they can be external; some third party that also benefits from the two players' cooperation pays them for cooperating. In either case, the reward cannot be given before the choice is made; otherwise the player will simply pocket the reward and then defect. If the reward is merely promised, the promise may not be credible: after the promisee has chosen cooperation, the promisor may renege.

These difficulties notwithstanding, rewards are sometimes feasible and useful. At an extreme of creativity and imagination, the players could make simultaneous and mutual promises, and make these credible by depositing the promised rewards in an escrow account controlled by a third party.[xv] More realistically, sometimes the players interact in several dimensions, and cooperation in one can be rewarded with reciprocation in another. For example, among groups of female chimpanzees, help with grooming is reciprocated by sharing food or help with child-minding. Sometimes, third parties may have sufficiently strong interests in bringing about cooperation in a game. For example, in the interest of bringing to an end various conflicts around the world, the United States and the European Union have from time to time promised economic assistance to combatants as a reward for peaceful resolutions of their disputes. The U.S. rewarded Israel and Egypt in this way for cooperating to strike the Camp David accord in 1978.

Punishment is the more usual method of resolving prisoners' dilemmas. This could be immediate. In a scene from the movie *L.A. Confidential*, Sergeant Ed Exley promises Leroy Fontain, one of the suspects he is interrogating, that if he turns state's witness, he will get a shorter sentence than the other two, Sugar Ray Coates and Tyrone Jones. But Leroy knows when he emerges from jail, he may find friends of the other two waiting for him!

But the punishment that comes to mind most naturally in this context arises from the fact that most such games are parts of an ongoing relationship. Cheating may gain one player a short-term advantage, but this can harm the relationship and create a longer-run cost. If this cost is sufficiently large, that can act as a deterrent against cheating in the first place.[17]

Batters in the American League are hit by pitches eleven to seventeen percent more often than their colleagues in the National League. According to Sewanee Professors Doug Drinen and John-Charles Bradbury, most of this difference is explained by the designated hitter rule.[xvi] In the American League, the pitchers don't bat. Thus, an American League pitcher who plunks a batter doesn't have to fear direct retaliation from the opposing team's pitcher. Although pitchers are unlikely to get hit, the chance goes up by a factor of four if they have just plunked someone the previous half inning. The fear of retaliation is clear. As ace pitcher Curt Schilling explained: "Are you seriously going to throw at someone when you are facing Randy Johnson?"[xvii]

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[17] Schelling shared the 2005 Nobel Prize in Economics with Robert Aumann, who was instrumental in the development of the general theory of tacit cooperation in repeated games.

When most people think about one player punishing the other for past cheating, they think of some version of "tit-for-tat." And that was indeed the finding of what is perhaps the most famous experiment on the prisoners' dilemma. Let us recount what happened and what it teaches.

Tit-for-Tat

In the early 1980s, University of Michigan Political Scientist Robert Axelrod invited game theorists from around the world to submit their strategies for playing the prisoners' dilemma in the form of computer programs. The programs were matched against each other in pairs to play a prisoners'-dilemma game repeated 150 times. Contestants were then ranked by the sum of their scores.

The winner was Anatol Rapoport, a mathematics professor at the University of Toronto. His winning strategy was among the simplest: tit-for-tat. Axelrod was surprised by this. He repeated the tournament with an enlarged set of contestants. Once again Rapoport submitted tit-for-tat and beat the competition.

Tit-for-tat is a variation of the "eye for an eye" rule of behavior: do unto others as they have done onto you.[18] More precisely, the strategy cooperates in the first period and from then on mimics the rival's action from the previous period.

Axelrod argues that tit-for-tat embodies four principles that should be evident in any effective strategy for the repeated prisoner's dilemma: clarity, niceness, provocability, and forgivingness. Tit-for-tat is as *clear* and simple as you can get; the opponent does not have to do much thinking or calculation about what you are up to. It is *nice* in that it never initiates cheating. It is *provocable,* that is, it never lets cheating go unpunished. And it is *forgiving,* because it does not hold a grudge for too long and is willing to restore cooperation.

One of the impressive features about tit-for-tat is that it did so well overall even though it did not (nor could it) beat any one of its rivals in a head-on competition. At best, tit-for-tat ties its rival. Hence if Axelrod had scored each competition as a winner-take-all contest, tit-for-tat would have only losses and ties and therefore could not have had the best track record.[19]

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[18] In Exodus (21:22), we are told: "If men who are fighting hit a pregnant woman and she gives birth prematurely but there is no serious injury, the offender must be fined whatever the woman's husband demands. But if there is a serious injury, you are to take life for a life, eye for eye, tooth for tooth, hand for hand, burn for burn, wound for wound, bruise for bruise." The New Testament suggests more cooperative behavior. In Matthew (5:38) we find: "You have heard that it was said, 'Eye for Eye, and Tooth for Tooth.' But I tell you, do not resist an evil person. If someone strikes you on the right cheek, turn to him the other also." We move from "Do unto others as they have done onto you" to the golden rule, "Do unto others as you would have them do unto you" (Luke 6:31). If people were to follow the golden rule, there would be no prisoners' dilemma. And if we think in the larger perspective, although cooperation might lower your payoffs in any particular game, the potential reward in an afterlife may make this a rational strategy even for a selfish individual. You don't think there is an afterlife? Pascal's Wager says that the consequences of acting on that assumption can be quite drastic, so why take the chance.

[19] Since every loser must be paired with a winner, it must be the case that some contestant has more wins than losses, else there will be more losses than wins overall. (The only exception is when every single match is a tie.)

But Axelrod did not score the pairwise plays as winner-take-all: close counted. The big advantage of tit-for-tat is that it always comes close. At worst, tit-for-tat ends up getting beaten by one defection; i.e., it gets taken advantage of once and then ties from then on.

The reason tit-for-tat won the tournament is that it usually managed to encourage cooperation whenever possible while avoiding exploitation. The other entries either were too trusting and open to exploitation, or were too aggressive and knocked one another out.

In spite of all this, we believe that tit-for-tat is a flawed strategy. The slightest possibility of a mistake or a misperception results in a complete breakdown in the success of tit-for-tat. This flaw was not apparent in the artificial setting of a computer tournament, because mistakes and misperceptions did not arise. But when tit-for-tat is applied to real-world problems, mistakes and misperceptions cannot be avoided and the result can be disastrous.

The problem with tit-for-tat is that any mistake "echoes" back and forth. One side punishes the other for a defection, and this sets off a chain reaction. The rival responds to the punishment by hitting back. This response calls for a second punishment. At no point does the strategy accept a punishment without hitting back.

Suppose, for example, that both Alchian and Williams start out playing tit-for-tat. No one initiates a defection, and all goes well for a while. Then, in round 11, say, suppose Alchian chooses Defect by mistake, or Flood and Dresher err, so that when playing round 12, Williams thinks that Alchian had played Defect in round 11. In either case, Williams will play Defect in round 12, but Alchian will play Cooperate because Williams played Cooperate in round 11. In round 13 the roles will be switched. The pattern of one playing Cooperate and the other playing Defect will continue back and forth, until another mistake or misperception restores cooperation, or leads both to defect.

Such cycles or reprisals are often observed in real-life feuds between Israelis and Arabs in the Middle East, or Catholics and Protestants in Northern Ireland, or Hindus and Muslims in India. In the frontier west we had the memorable feud between the Hatfields and the McCoys. And in fiction, Mark Twain's Grangerfords and Shepherdsons offer another vivid examples of how tit-for-tat behavior can end in a cycle of reprisals. When Huck Finn tries to understand the origins of the Grangerfords-Shepherdsons feud, he runs into the chicken-or-egg problem:

> "What was the trouble about, Buck? --- Land?"
> "I reckon maybe --- I don't know."
> "Well, who done the shooting? Was it a Grangerford or a Shepherdson?"
> "Laws, how do I know? It was so long ago."
> "Don't anyone know?"
> "Oh yes, pa knows, I reckon, and some of the other old people, but they don't know now what the row was about in the first place."

What tit-for-tat lacks is a way of saying "Enough is enough." It is too provokable, and not forgiving enough. And indeed, subsequent versions of Axelrod's tournament, which allowed possibilities of mistakes and misperceptions, showed other more generous strategies to be superior to tit-for-tat.[20]

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[20] In 2004, Graham Kendall at Nottingham ran a contest to celebrate the twentieth anniversary of Axelrod's original tournament. It was "won" by a group from England's Southampton University. The Southampton group submitted multiple entries, sixty in all. Their entries all started with an unusual pattern so as to

Here we might even learn something from monkeys. Cotton-top tamarin monkeys were placed in a game where each had the opportunity to pull a lever that would give the other food. But pulling the lever required effort. The ideal for each monkey would be to shirk while his partner pulled the lever. But the monkeys learned to cooperate in order to avoid retaliation. Tamarin cooperation remained stable as long as there were no more than two consecutive defections by one player, a strategy that resembles tit-for-two-tats.[xviii]

More Recent Experiments

Thousands of experiments on the prisoners' dilemma have been performed in classrooms and laboratories. These involve different numbers of players, repetitions, and other treatments. Here are some important findings.[xix]

First and foremost is that cooperation occurs significantly frequently, even when each pair of players meets only once. On the average, almost half of the players choose the cooperative action.

Indeed, the most striking demonstration of this was on the Game Show Network's production of "Friend or Foe." In this show, two-person teams were asked trivia questions. The money earned from correct answers went into a "trust fund," which over the 105 episodes ranged from $200 to $16,400. To divide the trust fund, the two contestants played a one-shot dilemma.

Each privately wrote down "friend" or "foe." When both wrote down friend, the pot was split evenly. If one wrote down foe while the other wrote friend, the person writing foe would get the whole pot. But if both wrote foe, then neither would get anything. Thus, whatever the other side does, you get at least as much and possibly more by writing down foe.

And yet, almost half the contestants wrote down friend. Even as the pot grew larger there was no change in the likelihood of cooperation. People were as likely to cooperate when the fund was below $3,000 as when it was above $5,000. These were some of the findings in a pair of studies by Professors Felix Oberholzer-Gee, Joel Waldfogel, and Matthew White and John List.[xx]

If you are wondering how watching television counts as academic research, it turns out that more than $700,000 was paid out to contestants. This was the best-funded experiment on the prisoners' dilemma, ever.

There was much to learn. It turns out that women were much more likely to cooperate than men, 53.7% versus 47.5% in season 1. The contestants in season 1 didn't have the advantage of seeing the results from the other matches before making their decision. But in season 2, the results of the first 40 episodes had been aired and this pattern became apparent.

In season 2, people learned from the experience of others. When the team consisted of two women, the cooperation rate rose to 55%. But when a woman was paired with a guy, her cooperation rate fell to 34.2%. And they guy's rate fell, too, down to 42.3%. Overall, cooperation dropped by ten points.

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recognize each other. Then the drone program sacrificed itself so that the other would do well. The drone program also refused to cooperate with other programs so as to knock down the opponents' scores. While having an army of drones prepared to sacrifice themselves on your behalf is one way to increase your payoff, it doesn't tell us much about how to play a prisoners' dilemma.

When a group of subjects is assembled and matched pairwise a number of times, with different pairings at different times, the proportion choosing cooperation generally declines over time. However, it does not go to zero, settling instead on a small set of persistent cooperators.

If the same pair plays the basic dilemma game repeatedly, they often build up to a significant sequence of mutual cooperation, until one player defects near the end of the sequence of repetitions. This happened in the very first experiment conducted on the dilemma. Almost immediately after they had thought up the game, Flood and Dresher recruited two of their colleagues to play the dilemma game a hundred times.[xxi] On 60 of these rounds, both players chose Cooperate. A long stretch of mutual cooperation lasted from round 83 to round 98, until one player sneaked in a defection on round 99.

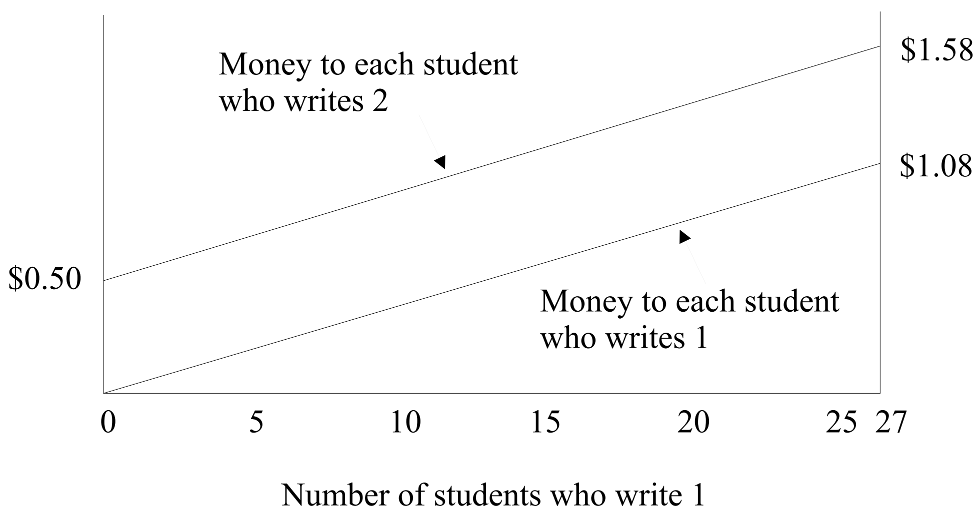
Actually, according to the strict logic of game theory, this should not have happened. When the game is repeated exactly 100 times, it is a sequence of simultaneous move games, and we can apply the logic of backward reasoning to it. Look ahead to what will happen on the 100th play. There are no more games to come, so defection cannot be punished in any future rounds. Dominant strategy calculations dictate that both players should choose Defect on the last round. But once that is a given, the 99th round becomes effectively the last round. Although there is one more round to come, defection on the 99th round is not going to be selectively punished by the other player in the 100th round because his choice in that round is foreordained. Therefore the logic of dominant strategies applies to the 99th round. And one can work back this sequential logic all the way to round 1. But in actual play, both in the laboratory and the real world, players seem to ignore this logic, and achieve the benefits of mutual cooperation. What may seem at first sight to be irrational behavior—departing from one's dominant strategy – turns out to be a good choice, so long as everyone else is similarly "irrational."

Game theorists suggest an explanation for this phenomenon. The world contains some "reciprocators" people who will cooperate so long as the other does likewise. Suppose you are not one of these relatively nice people. If you behaved true to your type in a finitely repeated game of prisoners' dilemma, you would start cheating right away. That would reveal your nature to the other player. To hide the truth (at least for a while) you have to behave nicely. Why would you want to do that? Suppose you started by acting nicely. Then the other player, even if he is not a reciprocator, would think it possible that you are one of the few nice people around. There are real gains to be had by cooperating for a while, and the other player would plan to reciprocate your niceness to achieve these gains. That helps you, too. Of course you are planning to sneak in a defection near the end of the game, just as the other player is. But you two can still have an initial phase of mutually beneficial cooperation. Thus while each side is waiting to take advantage of the other, both are benefiting from this mutual deception.

In some experiments, instead of matching each subject in the group to just one other and playing several two-person dilemmas, the whole group is engaged in just one a multi-person dilemma. We mention a particularly entertaining and instructive instance from the classroom. Professor Raymond Battalio of Texas A&M University had his class of 27 students play the following game.[xxii] Each student owned a hypothetical firm and had to decide (simultaneously and independently, by writing on a slip of paper) whether to produce 1 and help keep the total supply low and the price high, or produce 2 and gain at the expense of others. Depending on the total number of students producing 1, money would be paid to students according to the following table:

Number of students writes 1	Payoff to each student who writes 1	Payoff to each student who writes 2
0		$0.50
1	$0.04	$0.54
2	$0.08	$0.58
3	$0.12	$0.62
…	…	…
25	$1.00	$1.50
26	$1.04	$1.54
27	$1.08	

This is easier to see and more striking in a chart:



The game is "rigged" so that students who write 2 (Defect) always get 50 cents more than those who write 1 (Cooperate), but the more of them that write 2, the less their collective gain. Suppose all 27 start planning to write 1, so each would get $1.08. Now one thinks of sneaking a switch to 2. There would be 26 1's, and each would get $1.04 (4 cents less than in the original plan), but the switcher would get $1.54 (46 cents more). The same is true irrespective of the initial number of students thinking of writing 1 versus 2. Writing 2 is a dominant strategy. Each student who switches from writing 1 to writing 2 increases his own payout by 46 cents, but decreases that of each of his 26 colleagues by 4 cents --- the group as a whole loses 58 cents. By the time everyone acts selfishly, each maximizing his own payoff, they each get 50 cents. If they could have successfully conspired and acted so as to minimize their individual payoff, they would each receive $1.08. How would you play?

In some practice plays of this game, first without classroom discussion and then with some discussion to achieve a "conspiracy," the number of cooperative students writing 1 ranged from 3 to a maximum of 14. In a final binding play, the number was 4. The total payout was $15.82, which is $13.34 less than that from totally successful collusion. "I'll never trust anyone again as long as I live," muttered the conspiracy leader. And what was his choice? "Oh, I wrote 2," he replied. Yossarian would have understood.

More recent laboratory experiments of multi-person dilemmas use a format called the contribution game. Each player is given an initial stake, say $10. Each can choose to keep part of this, and contribute a part to a common pool. The experimenter then doubles the accumulated common pool, and divides this equally among all the players, contributors and non-contributors alike.

Suppose there are four players, say A, B, C, and D, in the group. Regardless of what the others are doing, if person A contributes a dollar to the common pool, this increases the common pool by $2 after the doubling. But $1.50 of the increment goes to B, C, and D; A gets only 50 cents. Therefore A loses out by raising his contribution; conversely he would gain by lowering it. And that is true no matter how much, if anything, the others are contributing. In other words, contributing nothing is the dominant strategy for A. The same goes for B, C, and D. This logic says that each should hope to become a "free rider" on the efforts of the others. If all four play their dominant strategy, the common pool is empty and each simply keeps the initial stake of $10. When everyone tries to be a free rider, the bus stays in the garage. If everyone had put all of their initial stakes in the common pool, the pool after doubling would be $80 and the share of each would be $20. But each has the personal incentive to cheat on such an arrangement. This is their dilemma.

The contribution game is not a mere curiosum of the laboratory or theory; it is played in the real world in many social interactions where some communal benefit can be achieved by voluntary contributions from members of the group, but the benefit cannot be withheld from those who did not contribute. Flood control in a village, or conservation of natural resources, are cases in point: it is not possible to build levees or dams so that flood waters will selectively go to the fields of those who did not help in the construction, and it is not practicable to withhold gas or fish in the future from someone who consumed too much in the past. This creates a multi-person dilemma: each player has the temptation to shirk or withhold his contribution, hoping to enjoy the benefits of the others' contributions. When they all think this way, the total of contributions is meager or even zero, and they all suffer. These situations are so ubiquitous and of such large magnitude that all of social theory and policy needs a good understanding of how the dilemmas might be resolved.

In what is perhaps the most interesting variant of the game, players are given an opportunity to punish those who cheat on an implicit social contract of cooperation. However, they must bear a personal cost to do so. After the contribution game is played, the players are informed about the individual contributions of other players. Then a second phase is played, where each player can take an action to lower the payoffs of other players, at a cost to himself of so many cents (typically 33) per dollar reduction chosen. In other words, if player A chooses to reduce B's payoff by three dollars, then this is done and A's payoff is reduced by one dollar. These reductions are not reallocated to anyone else; they simply return to the general funds of the experimenter.

The results of the experiment show that people engage in a significant amount of punishment of "social cheaters," and that the prospect of the punishment increases the contributions in the first phase of the game dramatically. Therefore such punishments seem to be an effective mechanism for achieving cooperation that benefits the whole group. But the fact that individuals carry them out is surprising at first sight. The act of punishing others at a personal cost is itself a contribution for the general benefit and it is a dominated strategy; if it succeeds in eliciting better behavior from the cheater in the future, its benefits will be for the group as a whole, and the punisher will get only his small share of this benefit. Therefore the punishment has to be the result of something other than a selfish calculation. That is indeed the case. Recent experiments on this game have been conducted while the players' brains were being imaged by positron emission tomography (PET scan). [xxiii] These revealed that the act of imposing the penalty activated a brain region called the dorsal striatum, which is involved in experiencing pleasure or satisfaction. In other words, people actually derive a psychological benefit or pleasure from punishing social cheaters. Such an instinct must have deep biological roots, and may have been selected for an evolutionary advantage.[xxiv]

How to Achieve Cooperation

These examples and experiments have suggested several preconditions and strategies for successful cooperation. Let us develop the concepts more systematically, and apply them to some more examples from the real world.

Successful punishment regimes must satisfy several requirements. Let us examine these one by one.

Detection of cheating: Before cheating can be punished, it must be detected. If detection is fast and accurate, the punishment can be immediate and accurate. That reduces the gain from cheating and increases its cost, and increases the prospects for successful cooperation. For example, airlines constantly monitor each other's fares; if American were to lower its fare from New York to Chicago, United can respond in under five minutes. But in other contexts, firms that want to cut their prices can do so in secret deals with the customers, or hide their price cuts in a complicated deal involving many dimensions of delivery time, quality, warranties, and so on. In extreme situations, each firm can only observe its own sales and profits, which can depend on some chance elements as well as on the firms' actions. For example, how much one firm sells can depend on the vagaries of demand, not just on other firms' secret price cuts. Then detection and punishment become not only slow but also inaccurate, raising the temptation to cheat.

Finally, when three or more firms are simultaneously in the market, they must find out not only whether cheating has occurred, but also who has cheated. Otherwise any punishments cannot be targeted to hurt the miscreant, but have to be blunt, for example unleashing a price war that hurts all. A different and striking example of the effect of being able to direct punishment at the miscreant comes from baseball. Batters are 15% more likely to be hit by a pitch in the American League than in the National League. In the latter, pitchers have to bat. Each pitcher knows that if he hits a batter with an "errant" pitch, he will face direct retaliation from the other team's pitchers. In the American League, designated hitters are used. Pitchers never bat, so they don't face any risk of getting a taste of their own medicine. [xxv]

Nature of punishment: Next, there is the choice of punishment. Sometimes the players have available to them actions that hurt others, and these can be invoked after an instance of cheating even in a one-time interaction. As we pointed out in the dilemma in *L.A. Confidential*, the friends of Sugar and Tyrone will punish Leroy when he emerges from jail after his light sentence for turning state's witness. In the Texas A&M classroom experiment, if the students could detect who had reneged on the conspiracy for all of them to write 1, they could inflict social sanctions such as ostracism on the cheaters. Few students would risk that for sake of an extra 50 cents.

Other kinds of punishments arise within the structure of the game. Usually this happens because the game is repeated, and the gain from cheating in one play leads to a loss in future plays. Whether this is enough to deter a player who is contemplating cheating depends on the sizes of the gains and losses, and the importance of the future relative to the present. We will return to this aspect soon.

Clarity: The boundaries of acceptable behavior, and the consequences of cheating, should be clear to a prospective cheater. If these things are complex or confusing, the player may cheat by mistake, or fail to make a rational calculation and play by some hunch. For example, suppose Rainbow's End and B. B. Lean are playing their price-setting game repeatedly, and RE decides that it will infer that BB has cheated if RE's discounted mean of profits from the last seventeen months is 10 percent less than the average real rate of return to industrial capital over the same period. BB does not know this rule directly; it must infer what rule RE is using by observing RE's actions. But the rule stated here is too complicated for BB to figure out. Therefore not a good deterrent against BB's cheating. Something like tit-for-tat is abundantly clear: if BB cheats, it will see RE cutting its price the very next time.

Certainty: Players should have confidence that defection will be punished and cooperation rewarded. This is a major problem in some international agreements like trade liberalization in the World Trade Organization (WTO). When one country complains that another has cheated on the trade agreement, the WTO initiates an administrative process that drags on for months or years. The facts of the case have little bearing on the judgment, which usually depends more on dictates of international politics and diplomacy. Such enforcement procedures are unlikely to be effective.

Size: How harsh should such punishments be? It might seem that there is no limit. If the punishment is strong enough to deter cheating, it need never actually be inflicted. Therefore it may as well be set at a sufficiently high level to ensure deterrence. For example, the World Trade Organization could have a provision to nuke that any nation that breaks its undertakings to keep its protective tariffs at the agreed low levels. Of course you recoil in horror at the suggestion, but that is at least in part because you think it possible that some error may cause this to happen. When errors are possible, as they always are in practice, the size of the punishment should be kept as low as is compatible with successful deterrence in most circumstances. It may even be optimal to forgive occasional defection in extreme situations, for example a firm that is evidently fighting for its survival may be allowed some price cuts without triggering reactions from rivals.

Repetition: Look at the pricing game between Rainbow's End and B. B. Lean. Suppose they are going along merrily from one year to the next holding prices at their joint best, $80. One year the management of RE considers the possibility of cutting the price to $70. They reckon that this will yield them an extra profit of $110,000 – $72,000 = $38,000. But that can lead to a collapse of trust. RE should expect that in future years BB will also choose $70, and each will make only $70,000 each year. If RE had kept to the original arrangement, each would have kept on making $72,000. Thus RE's price-cutting will cost it $72,000 – $70,000 = $2,000 every year in the future. Is a one-time gain of $38,000 worth the loss of $2,000 every year thereafter?

One key variable that determines the balance of present and future considerations is the interest rate. Suppose the interest rate is 10% per year. Then RE can stash away its extra $38,000 and earn $3,800 thousand every year. That comfortably exceeds the loss of $2,000 in each of those years. Therefore the cheating is RE's interest. But if the interest rate is only 5% per year, then the $38,000 earns only $1,900 in each subsequent year, less than the loss of $2,000 due to the collapse of the arrangement; so RE does not cheat. The interest rate at which the two magnitudes just balance is 2/38 = 0.0526, or 5.26% per year.

The key idea here is that when interest rates are low, the future is relatively more valuable. For example, if the interest rate is 100%, then the future has low value relative to the present – a dollar in a year's time is worth only 50 cents right now because you can turn the 50 cents into a dollar in a year by earning another 50 cents as interest during the year. But if the interest rate is zero, then a dollar in a year's time is worth the same as a dollar right away. [21]

In our example, for realistic interest rates a little above 5 percent, the temptation for each firm to cut the price by \$10 below their joint best price of \$80 is quite finely balanced, and collusion in a repeated game may or may not be possible. In Chapter 4 we will see how low the price can fall if there is no shadow of the future and the temptation is irresistible.

Another relevant consideration is the likelihood of continuation of the relationship. If the shirt is a transient fashion item that may not sell at all next year, then the temptation to cheat this year is not offset by any prospect of future losses.

But Rainbow's End and B. B. Lean sell many items besides this shirt. Won't cheating on the shirt price bring about retaliation on all those other items in the future? And isn't the prospect of this huge retaliation enough to deter the defection? Alas, the usefulness of multi-product interactions for sustaining cooperation is not so simple. The prospect of multi-product retaliation goes hand-in-hand with that of immediate gains from simultaneous cheating in all of those dimensions, not just one. If all the products had identical payoff tables, then the gains and losses would both increase by a factor equal to the number of products, and whether the balance is positive or negative would not change. Therefore successful punishments in multi-product dilemmas must depend in a more subtle way on differences among the products.

A third relevant consideration is the expected variation in the size of the business over time. This has two aspects – steady growth or decline, and fluctuations. If the business is expected to grow, then a firm considering defection now will recognize that it stands to lose more in the future due to the collapse of the cooperation, and will be more hesitant to defect. Conversely, if the business is on a path of decline, then firms will be more tempted to defect and take what they can now, knowing that there is less at stake in the future. As for fluctuations, firms will be more tempted to cheat when a temporary boom arrives; cheating will bring them larger immediate profits, whereas the downside from the collapse of the cooperation will hit them in the future, when the volume of business will be only the average, by definition of the average. Therefore we should expect price wars to break out during times of high demand. But this is not always the case. If a period of low demand is caused by a general economic downturn, then the customers will have lower incomes and may become sharper shoppers as a result: their loyalties to one firm or the other may break down and they may respond more quickly to price differences. Then a firm cutting its price can expect to attract more customers away from its rival, and therefore a larger immediate gain from defection.

Finally, the composition of the group of players is important. If this is stable and expected to remain so, that is conducive to the maintenance cooperation. New players who do not have a stake or a history of participation in the cooperative arrangement are less likely to abide by it. And if the current group of players expects new ones to enter and shake up the tacit cooperation in the future, that increases their own incentive to cheat and take some extra benefit right now.

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[21] If you read the financial press, you have often seen the statement that "interest rates and bond prices move in opposite directions." The lower is the interest rate, the higher will be the prices of bonds. And bonds, being promises of future income, reflect the importance of the future. This is another way to remember this role of interest rates.

Solution by Kantian Categorical Imperative?

It is sometimes said that the reason some people cooperate in the prisoners' dilemma is that they are not only making the decision for themselves but also the decision for the other player. That is wrong in point of fact, but the person is acting as if this is the case.

The person truly wants the other side to cooperate and reasons to himself that the other side is going through the same logical decision process that he is. Thus the other side must come to the same logical conclusion that he has. Hence if the player cooperates, he reasons that the other side will do so as well and if he defects, he reasons that will cause the other side to defect. This is similar to the "categorical imperative" of the German philosopher Immanuel Kant: "Take only such actions as you would like to see become a universal law."

Of course, nothing could be further from the truth. The actions of one player have no effect whatsoever on the other player in the game. Still people have this way of thinking that somehow their actions can influence the choice of others, even when their actions are invisible.

The power of this magical type thinking was revealed in an experiment done with Princeton undergraduates by Eldar Shafir and Amos Tversky.[xxvi] In their experiment, they put students in a prisoners' dilemma game. But unlike the usual dilemma, in some treatments they told one side what the other had done.

When students were told that the other side had defected on them, only 3% responded with cooperation. When told that the other side had cooperated, this increased cooperation levels up to 16%. It was still the case that the large majority of students were willing to act selfishly. But many were willing to reciprocate the cooperative behavior exhibited by the other side, even at their own expense.

What do you think would happen when the students were not told anything about the other player's choice at all? Would the percentage of cooperators be between 3 and 16 percent? No; it rose to 37%. At one level, this makes no sense. If you wouldn't cooperate when you learned that the other side had defected and you wouldn't cooperate when you learn that the other side had cooperated, why would you then cooperate when you don't know what the other side had done?

Shafir and Tversky call this "quasi-magical" thinking – the idea that by taking some act, you can influence what the other side will do. People realize they can't change what the other side has done once they've been told what the other side has done. But if it remains open or undisclosed, then they imagine that their actions might have some influence. Or that the other side will somehow be employing the same reasoning chain and thus reach the same outcome they do. Since Cooperate, Cooperate is preferred to Defect, Defect, the person chooses Cooperate.

We want to be clear that such logic is completely illogical. What you do and how you get there has no impact at all on what the other side thinks and acts. They have to make up their mind without reading your mind or seeing your move.

However, the fact remains that if the people in a society engage in such quasi-magical thinking, they will not fall victim to many prisoners' dilemmas and all will reap higher payoffs from their mutual interactions. Could it be that human societies deliberately instill such thinking into their members for just such an ultimate purpose?

Dilemmas in Business

Armed with the toolkit of experimental findings and theoretical ideas in the previous sections, let us step outside the laboratory and look at some instances of prisoners' dilemmas in the real world and attempts at resolving them.

Let us begin with the dilemma of rival firms in an industry. Their joint interests are best served by monopolizing or cartelizing the industry and keeping prices high. But each firm can do better for itself by cheating on such an agreement and sneaking in price cuts to steal business from its rivals. What can the firms do? Some factors conducive to successful collusion, such as growing demand, and lack of disruptive entry, may be at least partially outside their control. But they can try to facilitate the detection of cheating, and devise effective punishment strategies.

All these are easier to achieve if the firms meet regularly and communicate. Then they can negotiate and compromise on what are acceptable practices and what constitutes cheating. The process of negotiations and its memory contributes to clarity. If something occurs that looks prima facie like cheating, another meeting can help clarify whether it is something extraneous, or an innocent error by a participant, or deliberate cheating. Therefore unnecessary punishments can be avoided. And the meeting can also help the group implement the appropriate punishment actions.

The problem with all this is that the group's success in resolving their dilemma harms the general public's interest. Consumers must pay higher prices, and the firms withhold some supply from the market to keep the price high. As Adam Smith said, "People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices."[xxvii] Governments that want to protect the general public interest get into the game and enact antitrust laws that make it illegal for firms to collude in this way.[22] In the United States, the Sherman Antitrust Act prohibits conspiracies "in restraint of trade or commerce," of which price-fixing or market-share fixing conspiracies are the prime instance and the ones most frequently attempted. In fact the Supreme Court has ruled that not only are explicit agreements of this kind forbidden, but also any explicit or tacit arrangement among firms that has the effect of price-fixing is a violation of the Sherman Act, regardless of its primary intent. Violation of these laws can lead to jail terms for the firms' executives, not just fines for the corporations that are impersonal entities.

Not that firms don't try to get away with the illegal practices. One of the most recent such conspiracies involved Archer Daniels Midland (ADM), a famous American processor of agricultural products, and their Japanese counterpart, Ajinomoto. They met to decide market sharing and pricing agreements for various products such as lysine (which is produced from corn and used for fattening up chickens and pigs). The aim was to keep the prices high at the expense of their customers. Their philosophy was "The competitors are our friends, and the customers are our enemies." All this came to light because one of the ADM negotiators became an informant for the FBI, and arranged for many of the meetings to be recorded for audio and sometimes also video.[xxviii]

[22] Not all governments care enough about the general interest. Some are beholden to the producers' special interests, and ignore or even facilitate cartels. We won't name any, for fear that they might ban our book in their countries!

An instance famous in antitrust history and business school case studies concerns the large turbines that generate electricity. In the 1950s, the U.S. market for these turbines had three firms: GE was the largest, with a market share of around 60%, Westinghouse was the next with approximately 30%, and Allied-Chalmers about 10%. They kept these shares, and obtained high prices, using a clever coordination device. Electric utilities invite bids for the turbines they intend to buy. If the bid was issued during days 1-17 of a lunar month, Westinghouse and Allied-Chalmers had to put in very high bids that would be sure losers, and GE was the conspiracy's chosen winner by making the lowest bid (but still at a monopolist's price allowing big profits). Similarly, Westinghouse was the designated winner in the conspiracy if the bid was issued during days 18-25, and Allied-Chalmers for days 26-28. Since the utilities do not issue their solicitations for bids according to the lunar calendar, over time each of the three producers gets the agreed market share. Any cheating on the agreement is immediately visible to the rivals. But, so long as the Department of Justice did not think of linking the winners to the lunar cycles, it was proof from detection by the law. Eventually the authorities did figure it out, some executives of the three firms went to jail, and the profitable conspiracy collapsed. Different schemes were tried later. [xxix]

A variant of the turbine scheme appeared later in the bidding at the airwave spectrum auctions in 1996-7. A firm that wanted the right for the licenses in a particular location would signal to the other firms its determination to fight for that right by using the telephone area code for that location as the last three digits of its bid. Then the other firms would let it win. So long as the same set of firms interacts in a large number of such auctions and over time, and so long as the antitrust authorities do not figure it out, the scheme may be sustainable.[xxx]

More commonly, the firms in an industry try to attain and sustain implicit or tacit agreements without explicit communication. This eliminates the risk of criminal antitrust action, although the antitrust authorities can take other measures to break up even the implicit collusion. The downside is some sacrifice of clarity of the tacit arrangement and the accuracy of detection, but firms can devise methods to improve both.

Instead of agreeing on the prices to be charged, the firms can agree on a division of the market, either by geography or by product line or by some similar measure. Cheating is then more visible—your salespeople will quickly come to know if another company has stolen some of your assigned market.

Detection of price cuts, especially in case of retail sales, can be simplified, and retaliation made quick and automatic, by the use of devices like "matching or beating competition" policies and "most favored customer" clauses.

Many companies selling household and electronic goods loudly proclaim that they will beat any competitor's price. Some even guarantee that if you find a better price for the same product within a month after your purchase, they will refund the difference, or in some cases even double the difference. At first sight, these strategies seem to promote competition by guaranteeing low prices. But a little game-theoretic thinking shows that in reality they can have exactly the opposite effect. Suppose Rainbow's End and B. B. Lean had such policies, and their tacit agreement was to price the shirt at \$90. Now each firm knows that if it sneaks a cut to \$85, the rival will find out about it quickly; in fact the strategy is especially clever in that it puts the customers, who have the best natural incentive to locate low prices, in charge of detecting cheating. And the prospective defector also knows that the rival can retaliate instantaneously by cutting its own price; it does not have to wait until next year's catalog is printed. Therefore the cheater is more effectively deterred.

Promises to meet or beat the competition can be clever and indirect. In the competition between Pratt and Whitney (P&W) and Rolls-Royce (RR) for jet aircraft engines to power Boeing 757 and 767 planes, P&W promised all prospective purchasers that its engines would be 8% more fuel-efficient than those of RR, otherwise P&W would pay the difference in fuel costs.[xxxi]

A "most-favored-customer" clause says that the seller will offer to all their customers the best price they offer to the most favored ones. Taken at face value, it seems that the manufacturers are guaranteeing low prices. But let's look deeper. The clause means that the manufacturer cannot compete by offering selective discounts to attract new customers away from his rival, while charging the old higher price to his established clientele. They must make general price cuts, which are more costly, because they reduce the profit margin on all sales. You can see the advantage of this clause to a cartel: the gain from cheating is less, and the cartel is more likely to hold.

A branch of the U.S. antitrust enforcement system, the Federal Trade Commission, considered such a clause that was being used by Du Pont, Ethyl, and other manufacturers of anti-knock additive compounds in gasoline. The Commission ruled that there was an anticompetitive effect, and forbade the companies from using such clauses in their contracts with customers.[23]

Tragedies of the Commons

Among the examples at the start of this chapter, we mentioned problems like overfishing that arise because each person stands to benefit by taking more, while the costs of his action are visited upon numerous others or on future generations. University of California biologist Garrett Harding called this the "tragedy of the commons," using among his examples the overgrazing of commonly-owned land in fifteenth and sixteenth century England.[xxxii] The problem has become well known under this name. Today the problem of global warming is an even more serious example; no one gets enough private benefit from reducing carbon emissions, but all stand to suffer serious consequences when each follows his self-interest.

This is just a multi-person prisoners' dilemma, like the one Yossarian in *Catch-22* faced in his decision about risking his life in wartime. Of course societies recognize the costs of letting such dilemmas go unresolved, and make attempts to achieve better outcomes. What determines whether these attempts succeed?

Indiana University political scientist Elinor Ostrom, and her collaborators and students, have conducted an impressive array of case studies of attempts to resolve dilemmas of the tragedy of the commons, that is, to use and conserve common property resources in their general interest and avoid overexploitation and rapid depletion. They studied some successful and some unsuccessful attempts of this kind, and derived some prerequisites for cooperation.[xxxiii]

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[23] This ruling was not without some controversy. The Commission's chairman, James Miller, dissented. He wrote that the clauses "arguably reduce buyers' search costs and facilitate their ability to find the best price-value among buyers." For more information, see "In the matter of Ethyl Corporation et al." FTC Docket 9128, *FTC Decisions* pp. 425-686.

First, there must be clear rules that identify who is a member of the group of players in the game—those who have the right to use the resource. The criterion is often geography or residence, but can also be based on ethnicity or skills, or sold by auction or for an entry fee.[24]

Second, there must be clear rules defining permissible and forbidden actions. These include restrictions on time of use (open or closed seasons for hunting or fishing, or what kinds of crops can be planted and any requirements to keep the land fallow in certain years), location (a fixed position or a specified rotation for inshore fishing), the technology (size of fishing nets), and finally, the quantity or fraction of the resource (amount of wood from a forest that each person is allowed to gather and take away).

Third, a system of penalties for violation of the above rules must be clear and understood by all parties. This need not be an elaborate written code; shared norms in stable communities can be just as clear and effective. The sanctions used against rule-breakers range from verbal chastisement, social ostracism, fines, the loss of future rights, and in some extreme cases even incarceration. The severity of each type of sanction can also be adjusted. An important principle is graduation. The first instance of suspected cheating is most commonly met simply by a direct approach to the violator and a request to resolve the problem. The fines for a first or second offense are low, and are ratcheted up only if the infractions persist or get more blatant and serious.

Fourth, a good system to detect cheating must be in place. The best method is to make detection automatic in the course of the players' normal routine. For example, a fishery that has good and bad areas may arrange a rotation of the rights to the good areas. Anyone assigned to a good spot will automatically notice if a violator is using it, and has the best incentive to report the violator to others and get the group to invoke the appropriate sanctions. Another example is the commonly found requirement that harvesting from forests or similar common areas must be done in teams; this facilitates mutual monitoring and avoids the need to hire guards.

Sometimes the rules on what is permissible must be designed in the light of feasible methods of detection. For example, the size of a fisherman's catch is often difficult to monitor exactly, and difficult even for a well-intentioned fisherman to control exactly. Therefore rules based on fish quantity quotas are rarely used. Quantity quotas perform better when quantities are more easily and accurately observable, as in the case of water supplied from storage, and harvesting of forest products.

Fifth, when the above categories of rules and enforcement systems are being designed, information that is easily available to the prospective users proves particularly valuable. Although each may have the temptation after the fact to cheat, they all have a common prior interest in

[24] The establishment of property rights is what actually happened in England. In two waves of "enclosures," first by local aristocrats during the Tudor period and later by acts of Parliament in the 18th and 19th centuries, previously common land was given to private owners. When land is private property, the invisible hand will shut the gate to just the right extent. The owner will charge grazing fees to maximize his rental income, and this will cut back the use. This will enhance overall economic efficiency, but alter the distribution of income; the grazing fees will make the owner richer, and the herdsmen poorer. Even absent concern for the distributional consequences, this approach is not always feasible. Property rights over the high seas or SO_2 and CO_2 emissions are hard to define and enforce in the absence of an international government: fish and pollutants move from one ocean to another, SO_2 is carried by the wind across borders, while CO_2 from any country rises to the same atmosphere. For this reason, whaling, acid rain, or global warming must be handled by more direct controls, but securing the necessary international agreements is no easy matter either.

designing a good system. They can make the best use of their knowledge of the resource and of the technologies for exploiting it, the feasibility of detecting various infractions, and the credibility of various kinds of sanctions in their group. Centralized or top-down management has been demonstrated to get many of these things wrong, and therefore perform poorly.

While Ostrom and her collaborators are generally optimistic about finding good solutions to many problems of collective action using local information and systems of norms, she gives a salutary warning against perfection: “The dilemma never fully disappears, even in the best operating systems. … No amount of monitoring or sanctioning reduces the temptation to zero. Instead of thinking of overcoming or conquering tragedies of the commons, effective governance systems cope better than others.”

Nature Red in Tooth and Claw

As you might expect, prisoners’ dilemmas arise in species other than humans. In matters like building shelter, gathering food, and avoiding predators, an animal can act either selfishly in the interest of itself or its immediate kin, or in the interest of a larger group. What circumstances favor good collective outcomes? Evolutionary biologists have studied this question and found some fascinating examples and ideas. Here is a very brief sample.[xxxiv]

The British biologist J.B.S. Haldane was once asked whether he would risk his life to save a fellow human being, and replied: “For more than two brothers, or more than eight cousins, yes.” You share half of your genes with a brother (other than an identical twin), and one-eighth of your genes with a cousin; therefore such action increases the expected number of copies of your genes that propagate to the next generation. Such behavior makes excellent biological sense; the process of evolution would favor it. This purely genetic basis for cooperative behavior among close kin explains the amazing and complex cooperative behavior observed in ant colonies and beehives.

Among animals, altruism without such genetic ties is rare. But reciprocal altruism can arise and persist among members of a group of animals with much less genetic identity, if their interaction is sufficiently stable and long-lasting. Hunting packs of wolves and other animals are examples of this. Here is an even more gruesome but fascinating instance.[xxxv] Vampire bats in Costa Rica live in colonies of a dozen or so, but hunt individually. On any day, some may be lucky and others unlucky. The lucky ones return to the hollow trees where the whole group lives, and can share their luck by regurgitating the blood they have brought from their hunt. A bat that does not get a blood meal for three days is at the risk of death. The colonies develop effective practices of mutual “insurance” against this risk by such sharing.

University of Maryland biologist Gerald Wilkinson explored the basis of this behavior by collecting bats from different locations and putting them together. Then he systematically withheld blood from some of them, and saw whether others shared with them. He found that sharing occurred only when the bat was on the verge of death, and not earlier. Bats seem to be able to distinguish real need from mere temporary bad luck. More interesting, he found that sharing occurred only among bats that already knew each other from their previous group, and that a bat was much more likely to share with someone who had come to its aid in the past! In other words, the bats are able to recognize other individual bats and keep score of their past behavior, to develop an effective system of reciprocal altruism.

Case Study: The Early Bird Kills the Golden Goose

The Galápagos Islands are the home of Darwin's finches. Life on these volcanic islands is difficult and so evolutionary pressures are high. Even a millimeter change in the beak of a finch can make the all difference in the competition for survival.[25]

Each island differs in its food sources and beaks of the finch reflect those differences. On Daphne Major, the primary food source is a cactus. Here the aptly named cactus finch has evolved so that its beak is ideally suited to gather the pollen and nectar of the cactus blossom.

The birds are not consciously playing a game against each other. Yet each adaptation of a bird's beak can be seen as its strategy in life. Strategies that provide an advantage in gathering food will lead to survival, a choice of mating partners, and more offspring. The beak of the finch is a result of this combination of natural and sexual selection.

Even when things seem to be working, genetics throws a few curveballs into the mix. There is the old saying that the early bird gets the worm. On Daphne Major, it was the early finch that got the nectar. Rather than wait until nine in the morning when the cactus blossoms naturally open for business, a dozen finches were trying something new. They were prying open the cactus blossom to get a head start.

At first glance, this would seem to give these birds an edge over their late-coming rivals. The only problem is that in the process of prying open the blossom, the birds would often snip the stigma. As Weiner explains in *The Beak of the Finch*:

> [The Stigma] is the top of the hollow tube that pokes out like a tall straw from the center of each blossom. When the stigma is cut, the flower is sterilized. The male sex cells in the pollen cannot reach the female sex cells in the flower. The cactus flower withers without bearing fruit.

When the cactus flowers wither, the main source of food disappears for the cactus finch. You can predict the end result of this strategy: No nectar, no pollen, no seeds, no fruit and then no more cactus finch. Does that mean that evolution has led the finches into a prisoners' dilemma where the eventual outcome is extinction?

Case Discussion

Not quite, on two counts. Finches are territorial and so the finches (and their offspring) whose local cactus shut down may end up as losers. Killing next year's neighborhood food supply is not worth today's extra sip of pollen. Therefore these deviant finches would not appear to have a fitness advantage over the others.

But that conclusion changes if this strategy ever becomes too prevalent. The deviant finches will expand their search for food and even those finches that wait will not save their cactus's stigma. Given the famine that is sure to follow, the birds most likely to survive are those who started in the strongest position. The extra sip of nectar could make the difference.

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[25] This example is motivated by Jonathan Weiner's wonderful book, *The Beak of the Finch*. See especially Chapter 20: The Metaphysical Crossbeak.

What we have here is a cancerous adaptation. If it stays small, it can die out. But if it ever grows too large, it will become the fittest strategy on a sinking ship. Once it ever becomes advantageous even on a relative scale, the only way to get rid of it is to eliminate the entire population and start again.

With no finches left on Daphe Major, there will be no one left to snip the stigmas and the cacti will bloom again. When two lucky finches alight on this island, they will have an opportunity to start the process from scratch.

The game we have here is a cousin to the prisoners' dilemma. Instead, it is a life and death case of the "stag hunt" game analyzed by the philosopher Jean-Jacques Rousseau.[26] In the stag hunt, if everyone works together to capture the stag, they succeed and all eat well. A problem arises if some hunters come across a hare along the way. If too many hunters are sidetracked chasing after hares then there won't be enough hunters left to capture the stag. In that case, everyone will do better chasing after rabbits. The best strategy is to go after the stag if *and only if* you can be confident that most everyone is doing the same thing. You have no reason not to chase after the stag, except if you lack confidence in what others will do.

The result is a confidence game. There are two ways it can be played. Everyone works together and life is good. Or everyone looks out for themselves and life is nasty, brutish, and short. This is not the classic prisoners' dilemma in which each person has an incentive to cheat no matter what others do. Here, there is no incentive to cheat, so long as you can trust others to do the same. But can you trust them? And even if you do, can you trust them to trust you? Or can you trust them to trust you to trust them? As FDR famously said (in a different context), we have nothing to fear but fear itself.

For more practice with prisoners' dilemmas, have a look at the following case studies in Chapter 14: The King Lear Problem and What Price a Dollar?

[26] There are other interpretations of Rousseau's stag hunt, which we return to in the history section in the next chapter.

Big Game of Coordination

Fred and Barney are stone-age rabbit-hunters. One evening they chance to meet and carouse, and engage in some "shop talk." As they exchange information and ideas, they realize that by cooperating they could hunt much bigger game, such as stag or bison. One person on his own cannot expect any success hunting either stag or bison. But done jointly, each day's stag or bison hunting is expected to yield six times as much meat as a day's rabbit-hunting by one person. Therefore cooperation promises great advantage: each hunter's share of meat from a big-game hunt is three times what he can get hunting rabbits on his own.

The two agree to go big-game hunting together the following day, and return to their respective caves. Unfortunately they have caroused too well, and have forgotten whether they decided to go after stag or bison. The hunting grounds for the two species are in opposite directions. There were no cell-phones in those days, and this was before the two became neighbors, so one could not quickly visit the other's cave to ascertain where to go. Each would have to make the decision the next morning in isolation.

Therefore the two end up playing a simultaneous-move game of deciding where to go. If we call each hunter's quantity of meat from a day's rabbit-hunting 1, then the share of each from successful coordination in hunting either stag or bison is 3. So the payoff table of the game is as shown here:

Fred's choice	Barney's choice		
	Stag	Bison	Rabbit
Stag	3 3	0 0	1 0
Bison	0 0	3 3	1 0
Rabbit	0 1	0 1	1 1

This game differs from the prisoners' dilemma of the previous chapter in many ways. Let us focus on one crucial difference. Fred's best choice depends on what Barney does and vice versa. For neither player is there a strategy that is best regardless of what the other does; unlike in the prisoners' dilemma, this game has no dominant strategies. So each player has to think about the other's choice, and figure out his own best choice in the light of that.

Fred's thinking goes as follows: "If Barney goes to the grounds where the stags are, then I will get my share of the large catch if I go there too, but nothing if I go to the bison grounds. If Barney goes to the bison grounds, things are the other way around. Rather than take the risk of going to one of these areas and finding that Barney has gone to the other, should I go by myself after rabbits and make sure of my usual, albeit small, quantity of meat? In other words, should I take 1 for sure instead of risking either 3 or nothing? It depends on what I think Barney is likely to do. So let me put myself in his shoes (clogs?) and think what he is thinking. Oh; he is wondering what I am likely to do, and is trying to put himself in my shoes! Is there any end to this circular thinking about thinking?"

A Game of Price Competition

John Nash's beautiful equilibrium was designed as a theoretical way to square just such circles of thinking about thinking about other people's choices in games of strategy.[27] The idea is to look for an outcome where each player in the game chooses the strategy that best serves his or her own interest, in response to the other's strategy. If such a configuration of strategies arises, neither player has any reason to change his choice unilaterally. Therefore this is a potentially stable outcome of the game where the players make individual and simultaneous choices of strategies. We begin by illustrating the idea by means of some examples of it in action. Later in this chapter we discuss how well it predicts outcomes in various games; we find reasons for cautious optimism, and for making the Nash equilibrium a starting point of the analysis of almost all games.

Let us develop the concept by considering a more general version of the pricing game between Rainbow's End and B. B. Lean. In Chapter 3 we allowed each the choice of just two prices for the shirt, namely \$80 and \$70. We also recognized the strength of the temptation for each to cut the price. Let us therefore allow more choices in a lower range, going in \$1 steps from \$42 to \$38.[28] In that example, when both charge \$80, each sells 1,200 shirts. If one of them cuts its price by \$1 while the other holds its price unchanged, then the price-cutter gains 100 customers, 80 of whom shift from the other firm and 20 shift from some other firm that is not a part of this game, or decide to buy a shirt when they would otherwise not have done so. If both firms reduce their price by \$1, existing customers stay put but each gains the 20 new ones. So when both firms charge \$42 instead of \$80, each gains 38 * 20 = 760 customers above the original 1,200. Then each sells 1,960 shirts and makes profit (42 – 20) * 1960 = 43,120 dollars. Doing similar calculations for the other price combinations, we have the game table below.

Rainbow's End's Price	B.B. Lean's Price				
	42	41	40	39	38
42	43120 43120	***43260*** 41360	43200 39600	42940 37840	42480 36080
41	41360 ***43260***	41580 41580	***41600*** 39900	41420 38220	41040 36540
40	39600 43200	39900 ***41600***	***40000*** ***40000***	39900 ***38400***	39600 36800
39	37840 42940	38220 41420	***38400*** 39900	38380 38380	38160 ***36860***

[27] For those readers who have not seen the movie *A Beautiful Mind* starring Russell Crowe as Nash, or read Sylvia Nasar's bestselling book of the same name, we should add that John Nash developed his fundamental concept of equilibrium in games around 1950, and went on to make contributions of equal or greater importance in mathematics. After several decades of severe mental illness, he recovered and was awarded the 1994 Nobel Prize in Economics. This was the first Nobel Prize for game theory.

Nash shared the prize with Reinhard Selten, who developed a much more general version of the backward reasoning method we described in Chapter 2, and with John Harsanyi, who developed methods for solving games in which the players have unequal information about various aspects of the game. Harsanyi's contributions were instrumental to the burgeoning of the field of economics of information, for which George Akerlof, Joseph Stiglitz, and Michael Spence were awarded the Nobel Prize in 2001. We will take up this theme in Chapter 8, and develop its applications in Chapters 10 and 13.

[28] The \$1-increment, and the restricted range of prices, are chosen merely to simplify our entrée into this game by keeping the number of strategies available to each player finite. Later in the chapter we will consider briefly the case where each firm can choose its price from a continuous range of values.

38	36080 42480	36540 41040	36800 39600	***36860*** 38160	36700 36700

The table may seem daunting, but is in fact easy to construct using Microsoft Excel or any other familiar spreadsheet program.

> Trip to the Gym No 2
>
> Try your hand at constructing this table in Excel.

Best Responses

Consider the thinking of RE's executives in charge of setting prices. (From now on, for brevity's sake we will simply say "RE's thinking", and similarly for BB.) If RE believes that BB is choosing $42, then RE's profits from choosing various possible prices are given by the numbers in the south-west corners of the first column of profits in the above table. Of those five numbers, the highest is $43260, corresponding to RE's price $41. Therefore this is RE's "best response" to BB's choice of $42. Similarly, RE's best response is $40 if it believes that BB is choosing $41, $40, or $39, and $39 if it believes BB is choosing $38. We show these best-response profit numbers in bold italics for clarity. We also show BB's best responses to the various possible prices of RE, using bolded and italicized numbers in the north-east corners of the appropriate cells.

Before proceeding, we must make two remarks about "best responses." First, the term itself requires clarification. The two firms' choices are simultaneous. Therefore, unlike the situation in Chapter 2, each firm is not observing the other's choice and then "responding" with its own best choice given the other firm's actual choice. Rather, each firm is formulating a belief (which may be based on thinking or experience or educated guesswork) about what the other firm is choosing, and responding to this belief.

Second, note that it is not always best for one firm to undercut the other's price. If RE believes that BB is choosing $42, RE should choose a lower price, namely $41; but if RE believes that BB is choosing $39, RE's best response is higher, namely $40. In choosing its best price, RE has to balance two opposing considerations: undercutting will increase the quantity it sells, but will leave it a lower profit margin per unit sold. If RE believes that BB is setting a very low price, then the reduction in RE's profit margin from undercutting BB may be too big, and RE's best choice may be to accept a lower sales volume to get a higher profit margin on each shirt. In the extreme case where RE thinks BB is pricing at cost, namely $20, matching this price will yield RE zero profit. RE does better to choose a higher price, keeping some loyal customers and extracting some profit from them.

Nash Equilibrium

Now return to the table and inspect the best responses. One fact immediately stands out: one cell, namely the one where each firm charges $40, has both of its numbers in bold italics, yielding a profit of $40,000 to each firm. If RE believes that BB is choosing the price of $40, then its own best price is $40, and vice versa for BB's beliefs about RE's choice and BB's best response. If the two firms choose to price their shirts at $40 each, the beliefs of each about the other's price are confirmed by experience. Then there would be no reason for one firm to change its price if the truth about the other firm's choice were somehow revealed. Therefore these choices constitute a stable configuration in the game.

Such an outcome in a game, where the action of each player is best for him given his beliefs about the other's action, and the action of each is consistent with the other's beliefs about it, neatly squares the circle of thinking about thinking. Therefore it has a good claim to be called a resting point of the players' thought processes, or an equilibrium of the game. Indeed, this is just a definition of Nash's famous equilibrium.

To highlight the Nash equilibrium, we shade its cell in grey, and will do the same in all the game tables that follow.

The price-setting game in Chapter 2, with just two price choices of $80 and $70, was a prisoners' dilemma. The more general game with several price choices shares this feature. If both firms could make a credible, enforceable agreement to collude, they could both charge prices considerably higher than the Nash equilibrium price of $40, and this would yield larger profits to both. As we saw in Chapter 3, a common price of $80 gives each of them $72,000, as against only $40,000 in the Nash equilibrium. In fact the common price of $80 is best if they can collude to cartelize the industry. This calculation is a trip to the gym even for economics majors, so we ask you to take our word for it. The result should impress upon you how consumers can suffer if an industry is a monopoly or a producers' cartel.

We take you on a different trip to the gym. In the above example, the two firms were very symmetrically situated in all relevant matters of costs, and the quantity sold for each combination of own and rival prices. In general this need not be so, and then in the resulting Nash equilibrium the two firms' prices can be different. For those of you who want to acquire a better grasp of the methods and the concepts, we offer this as an "exercise"; casual readers should feel free to peek at the answer in the workouts.

> Trip to the Gym No. 3
>
> Suppose Rainbow's End locates a cheaper source for its shirts, so its cost per shirt goes down from $20 to $11.60, while B. B. Lean's cost remains at $20. Recalculate the payoff table and find the new Nash equilibrium.

The pricing game has many other features, but they are more complex than the material so far. Therefore we postpone them to a position later in this chapter. To conclude this section, we make a few general remarks about Nash equilibria.

Does every game have a Nash equilibrium? The answer is essentially yes, provided we generalize the concept of actions or strategies to allow mixing of moves. This was Nash's famous theorem. We will develop the idea of mixing moves in the next chapter. Games that have no Nash equilibrium, even when mixing is allowed, are so complex or esoteric that we can safely leave them to very advanced treatments of game theory.

Is Nash equilibrium a good solution for simultaneous-move games? Later in this chapter we will discuss some arguments and evidence bearing on this issue, and our answer will be a guarded "Yes."

Does every game have a unique Nash equilibrium? No. In the rest of this chapter we will encounter some important examples of games with multiple Nash equilibria, and will discuss the new issues of solution they raise.

Which Equilibrium?

Let us try Nash's theory on the hunting game. Finding best responses in the hunting game is easy. Fred should simply make the same choice that he believes Barney is choosing. Here is the result.

Fred's choice	Barney's choice		
	Stag	Bison	Rabbit
Stag	***3*** ***3***	0 0	1 0
Bison	0 0	***3*** ***3***	1 0
Rabbit	0 1	0 1	***1*** ***1***

So the game has three Nash equilibria.[29] Which of these will emerge as the outcome? Or will the two fail to reach any of the equilibria at all? The idea of Nash equilibrium does not by itself give the answers. Some additional and different consideration is needed.

If Fred and Barney had met at the stag party of a mutual friend, that might make the choice of Stag more prominent in their minds. If the ritual in their society is that as the head of the family sets out for the day's hunting he calls out in farewell "Bye, son," the choice of Bison might be prominent. But if the ritual is for the family to call out in farewell "Be safe," the prominence might attach to the safer choice that guarantees some meat regardless of what the other chooses, namely rabbit-hunting.

But what, precisely, constitutes "prominence"? One strategy, say Stag for the sake of definiteness, may be prominent in Fred's mind, but that is not enough for him to make that choice. He must ask himself whether the same strategy is also prominent for Barney. And that in turn involves asking whether Barney will think it prominent to Fred. Selecting among multiple Nash equilibria requires resolution of a similar problem of thinking about thinking as does the concept of Nash equilibrium itself.

To square the circle, the "prominence" must be a multi-level back and forth concept. For the equilibrium to be selected successfully when the two are thinking and acting in isolation, it must be obvious to Fred that it is obvious to Barney that it is obvious to Fred that … that is the choice. If an equilibrium is obvious ad infinitum in this way, that is to say, the players' expectations converge upon it, we call it a *focal point*. The development of this concept was just one of the many pioneering contributions to game theory for which Thomas Schelling of Harvard and Maryland Universities shared the Nobel Prize in Economics in 2005. We will encounter Schelling's other contributions later in this book.

Whether a game has a focal point can depend on many circumstances, including most notably the players' common experiences, which may be historical, cultural, linguistic, or purely accidental. Here are some examples.

[29] If mixing moves is allowed, there are other Nash equilibria as well. But they are somewhat strange and of academic interest. Therefore we leave this additional complication to textbook-level treatments of game theory, for example Avinash Dixit and Susan Skeath, *Games of Strategy,* second edition 2004, New York: W. W. Norton, chapter 8.

Begin with Schelling's classics. Suppose you are told to meet someone in New York City on a specified day, but without being told where or at what time of day. You don't even know who the other person is, so you cannot contact him/her in advance (but you are told how you would identify each other if and when you do meet). You are also told that the other person has been given identical instructions.

Your chances of success might seem slim; New York City is huge and the day is long. But in fact people in this situation succeed surprisingly often. The time is simple: 12 noon is the obvious focal point; expectations converge on it almost instinctively. The location is harder, but there are just a few landmark locations on which expectations can converge. This at least narrows down the choices considerably and improves the chances of a successful meeting.

Schelling conducted experiments in which the subjects were from the Boston or New Haven areas. In those days they traveled to New York by train and arrived at Grand Central Station; for them the clock in that station was focal. Nowadays, many people would think the Empire State Building as a focal point to meet because of the movie *Sleepless in Seattle;* many would think Times Square the obvious "crossroads of the world."

One of us (Nalebuff) performed this experiment in an *ABC Primetime* program, titled *Life: The Game*.[xxxvi] Six pairs of mutual strangers were taken to different parts of New York, and told to find others about whom they had no information except that the other pair would be looking for them under similar conditions. The discussions within each pair followed Schelling's reasoning remarkably well. Each thought about what would be the obvious points to meet, including the next level of the question of what is obvious: each team, say team A, in its thinking recognized the fact that another team, say B, was simultaneously thinking about what was obvious to A. Eventually, three of the pairs went to the Empire State Building, and the other three to Times Square. All chose noon for the time. There remained some further issues to be sorted out: the Empire State Building has observation decks on two different levels, and Times Square is a big place. But with a little ingenuity, including a display of signs, all six pairs were successful in meeting. [30]

What is essential for success is not that the place is obvious to you, or obvious to the other person, but that it is obvious to each that it is obvious to the other that … And, if the Empire State Building has this property, then each team has to go there even though it may be inconvenient for both to get there, because it is the only place both can expect the other person to be. If there were just two teams, one of them might think the Empire State Building the obvious focal point and the other might think Times Square equally obvious; then the two would fail to meet.

Professor David Kreps of Stanford Business School conducted the following experiment in his class. Two students were chosen to play the game, and each had to make his/her choice without any possibility of communication with the other. Their job was to divide up a list of cities between them. One student was assigned Boston, and the other was assigned San Francisco (and

[30] One of the pair sat outside the Empire State Building for almost an hour, waiting for noon. If that had decided to wait inside, they would have done much better. It was also instructive that the team of men went running from one site to another (Port Authority, Penn Station, Time Square, Grand Central, Empire State Building) without any sign that would help them be found by another team. As might be expected, the male teams even crossed paths without recognizing each other. In contrast, the all-women teams made signs and hats. They picked a single spot and waited to be found.

these assignments were public so that each knew the other's city). Each was then given a list of nine other US cities—Atlanta, Chicago, Dallas, Denver, Houston, Los Angeles, New York, Philadelphia, and Seattle—and asked to choose a subset of these cities. If their choices resulted in a complete and non-overlapping division, both got a prize. But if their combined list missed a city or had any duplicates, then they both got nothing.[31]

How many Nash equilibria does this game have? If the student assigned Boston chooses say Atlanta and Chicago, while the student assigned San Francisco chooses the rest (Dallas, Denver, Houston, Los Angeles, New York, Philadelphia, and Seattle) that is a Nash equilibrium: given the choice of one, any change in the choice of the other will create either an omission or an overlap, and would lower the payoff to the deviator. The same argument applies if, say, one chooses Dallas, Los Angeles, and Seattle while the other chooses the other six. In other words, there are as many Nash equilibria as there are ways of dividing up the list of nine numbers into two distinct subsets. There are 2 raised to the power 9, or 512 such ways; therefore the game has a huge number of Nash equilibria. Can the players' expectations converge to create a focal point? When both players were Americans or long-time U.S. residents, over 80% of the time they chose the division geographically; the student assigned Boston chose all the cities east of the Mississippi and the student assigned San Francisco chose those west of the Mississippi.[32] Such coordination was much less likely when one or both students were non-U.S. residents. Thus nationality or culture can help create a focal point. When Kreps' pairs lacked such common experience, choices were sometimes made alphabetically, but even then there was no clear dividing point. If the total number of cities was even, an equal split might be focal, but with nine cities, that is not possible. Thus one should not assume that players will always find a way to select one of multiple Nash equilibria by a convergence of expectations; failure to find a focal point is a distinct possibility.

Next suppose each of two players is asked to choose a positive integer. If both choose the same number, both get a prize. If the two choose different numbers, neither gets anything. The overwhelmingly frequent choice is 1: it is the first among the whole numbers (positive integers), it is the smallest, and so on; therefore it is focal. Here the reason for its salience is basically mathematical.

Schelling gives the example two or more people who have gone to a crowded place together get separated. Where should each go in the expectation of finding the other? If the place, say a department store or a railway station, has a "Lost and Found" window, it has a good claim to be focal. Here the reason for its salience is linguistic. Sometimes meeting-places are deliberately created to guarantee a convergence of expectations; for example, many railway stations in Germany and Switzerland have a well-signposted *Treffpunkt* (meeting point).

What is neat about the game of meeting is not just that the two players find each other, but that the focal point ends up being relevant to so many strategic interactions. Probably the most important is the stock market. John Maynard Keynes, arguably the twentieth century's most

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[31] The game of dividing cities might seem uninteresting or irrelevant, but think of two firms that are trying to divide up the U.S. market between them to allow each to enjoy an uncontested monopoly in its assigned territory. U.S. antitrust laws forbid explicit collusion. To arrive at a tacit understanding requires a convergence of expectations. Kreps' experiment suggests that two American firms may achieve this better than could an American firm and a foreign firm.

[32] Perhaps in a few years' time this will no longer work, if the news stories about the deterioration of geographic knowledge among American school students are true.

famous economist, explained its behavior by analogy with a newspaper contest that was common in his time, where a number of photographs of faces were presented, and readers had to guess which face the majority of other voters would judge the most beautiful. When everyone thinks along these lines, the question becomes which face most people think that most other will thank that most others will think … the most beautiful. The aim becomes not to make any absolute judgment of beauty, but to find a focal point of this process of thinking. How do we agree on that? We look for some distinguishing feature, say a mole (Cindy Crawford) or a gap in the front teeth (Lauren Hutton). In the end, what features become focal is what allows us to converge on who is the most beautiful. Keynes used this as a metaphor for the stock market, where each investor wants to buy the stocks that will rise in price, which means the stocks that investors in general think will appreciate. The hot stock is the one that everyone else thinks … is the hot stock. There can be different reasons why different sectors or stocks become hot at different times – a well-publicized initial public offering, a famous analyst's recommendation, and so on. The focal point concept also explains the attention paid to round numbers: 10,000 for the Dow, or 2,500 for the Nasdaq. These indexes are just values of a specified portfolio of stocks. A number like 10,000 does not have any intrinsic meaning; it serves as a focal point only because expectations can converge more easily on round numbers.

Many mathematical game theorists dislike the dependence of an outcome on historical, cultural, or linguistic aspects of the game or on purely arbitrary devices like round numbers; they would prefer the solution to be determined purely by the abstract mathematical facts about the game – the number of players, the strategies available to each, and the payoffs to each in relation to the strategy choices of all. We disagree. We think it entirely appropriate that the outcome of a game played by humans interacting in a society should depend on the social and psychological aspects of the game.

Next think of bargaining. Here the players' interests seem to be totally conflicting; a larger share for one means a smaller share for the other. But in many negotiations, if the two parties fail to agree, neither will get anything, and both may suffer serious damage, as happens when wage bargaining breaks down and a strike or a lockout ensues. The two parties' interests are aligned to the extent that both want to avoid such disagreement. They can do so if they can find a focal point, with the common expectation that neither will concede anything beyond that point. That is why a 50:50 split is so often observed. It is simple and clear, it has the advantage of appearing fair, and once such considerations get a foothold, it serves for the convergence of expectations.

Next consider the problem of excessive compensation of CEOs. Often a CEO really cares about prestige. Whether the person gets paid \$5 million or \$10 million won't really have a big impact on the person's life. (That's easy for us to say form where we sit, where both numbers are quite abstract.) What's the meeting place that the CEO's care about? It is being better than average. Everyone wants to be in the top half. They all want to meet there. The problem is that this meeting spot only allows in half of the folks. But the way they get around this is via escalating pay. Every firm pays its CEO above last year's average, so everyone can think they have an above average CEO. The end result is wildly escalating CEO salaries. To solve the problem, we need to find some other focal meeting point. For example, historically CEOs got prestige in their community via public service. Competing in that dimension was good all around. The current focal point on pay was created by the Business Week surveys and compensation consultants. Changing the focal point won't be easy.

The issue of fairness is also one of choosing a focal point. The Millennium Development Goals, and Jeff Sachs' book *The End of Poverty*, emphasize that contributing 1% of GDP to development will end poverty by 2025. The key point here is that the focal point of contributions is based on a percent of income, not an absolute amount. Thus rich countries have a bigger obligation to contribute than the less-rich. The apparent fairness of this can contribute to the convergence of expectations. Whether the promised funds actually materialize remains to be seen.

Battles and Chickens

In the hunting game, the two players' interests are perfectly aligned; both prefer one of the big-game equilibria, and the only question is how they can coordinate their beliefs on a focal point. Two other games, which have an element of conflicting interests, also have non-unique Nash equilibria, and each leads to different ideas about strategies. Let us look at an example of each.

Both these games date from the 1950s, and have sexist stories that fit those times. We will illustrate them using variants of the game between our stone-age hunters, Fred and Barney. But we will relate the original sexist stories too, partly because they explain the names that have come to be attached to these games, and partly for the amusement value of looking back on the quaint thoughts and norms of old times.

The first game is generically called "battle of the sexes." The idea is that a husband and wife have different preferences over movies, and the two available choices are very different. The husband likes lots of action and fighting; he wants to see *Independence Day*. The wife likes three-handkerchief weepies; her choice is *A Beautiful Mind*. But both prefer watching either movie in the other's company to watching any movie on their own.

In the hunting version, remove the Rabbit choice, and keep only the Stag and Bison. But suppose Fred prefers stag meat, and rates the outcome of a jointly conducted stag hunt 4 instead of 3, while Barney has the opposite preferences. Therefore game payoff table is as shown below.

Fred's choice	Barney's choice	
	Stag	Bison
Stag	***3*** ***4***	***0*** 0
Bison	0 0	***4*** ***3***

As usual, best responses are shown in bold italics. We see at once that the game has two Nash equilibria, one where both choose Stag, and the other where both choose Bison. Both players prefer to have either equilibrium outcome than to hunt alone in one of the two non-equilibrium outcomes. But they have conflicting preferences over the two equilibria: Fred would rather be in the Stag equilibrium and Barney in the Bison equilibrium.

How might one or the other outcome be sustained? If Fred can somehow convey to Barney that he, Fred, is credibly and unyieldingly determined to choose Stag, then Barney must make the best of the situation by complying. But Fred faces two problems in using such a strategy.

First, it requires some method of communication before the actual choices are made. However, communication is usually a two-way process, so Barney might try the same strategy.

Fred would ideally like to have a device that will let him send messages but not receive them. But that is not without its own problems; how can Fred be sure that Barney has received and understood the message?

Second, and more important, is the problem of credibly conveying an unyielding determination. This can be faked, and Barney might put it to the test by defying Fred and choosing Bison, which leaves Fred with a pair of bad choices: give in and choose Bison, which leads to humiliation and destruction of reputation, or go ahead with the original choice of Stag, which means missing the opportunity of the joint hunt, getting zero meat, and a hungry family.

In Chapter 7 we will examine some ways in which Fred could make his determination credible and achieve his preferred outcome. But we will also examine some ways in which Barney could undermine Fred's commitment.

If the two have two-way communication before the game is played, this is essentially a game of negotiation. The two prefer different outcomes, but both prefer some agreement to complete disagreement. If the game is repeated, they may be able to agree to a compromise, for example alternate between the two grounds on alternate days. Even in a single play, they may be able to achieve a compromise in the sense of a statistical average by tossing a coin and choosing one equilibrium if it comes up heads and the other equilibrium if it comes up tails. We will devote an entire chapter to the important subject of negotiation.

The second classic game is called "chicken." In the standard telling of this story, two teenagers drive toward each other on a straight road, and the first one to swerve to avoid a collision is the loser, or chicken. If both keep straight, however, they crash, and that is the worst outcome for both. To create a game of chicken out of the hunting situation, remove the Stag and Bison choices, but suppose there are two areas for rabbit-hunting. One, located in the South, is large but sparse; both can go there and each will get 1 of meat. The other, located in the North, is plentiful but small. If just one hunter goes there, he can get 2 of meat. If both go there, they will merely interfere and start fighting with each other and get nothing. If one goes North and the other goes South, the one who goes North will enjoy his 2 of meat. The one going South will get his 1. But his and his family's feeling of envy for the other who comes back at the end of the day with 2 will reduce his enjoyment, so we will give him a payoff of only ½ instead of 1. This yields the game payoff table shown below.

Fred's choice	Barney's choice	
	North	South
North	0 0	***1/2*** ***2***
South	***2*** ***1/2***	1 1

As usual, best responses are shown in bold italics. We see at once that the game has two Nash equilibria, with one player going North and the other going South. The latter is then the "chicken"; he has made the best of a bad situation in responding to the other's choice of North.

Both games, the battle of the sexes and chicken, have a mixture of common and conflicting interests: in both, the two players agree in preferring an equilibrium outcome to a non-equilibrium outcome, but they disagree as to which equilibrium is better. This conflict is sharper in chicken, in the sense that if each player tries to achieve his preferred equilibrium, they end up in an outcome that is worst for both.

Methods for selecting one of the equilibria in chicken are similar to those in the battle of the sexes. One of the players, say Fred, may make a commitment to choosing his preferred strategy, namely going North. Once again, it is important to make this commitment credible, and ensure that the other player knows it. In the original version where the two teenagers are driving toward each other, Schelling makes the fanciful suggestion that one driver might disconnect his steering wheel and throw it out of the window; the other driver, on seeing this, recognizes that the first is simply unable to swerve, and accepts that he himself must get out of the way.

Such commitments can be risky. In reality, there are delays in action and observation. Therefore the two drivers may simultaneously see each other's steering wheel in the air, and head helplessly toward a crash!

We will consider commitments and their credibility more fully in Chapters 6 and 7.

Compromise is also possible in chicken. In a repeated interaction Fred and Barney may agree to alternate between North and South; in a single play, they may use a coin toss or other randomizing method to decide who gets North.

Finally, the chicken game shows a general point about games: even though the players are perfectly symmetric as regards their strategies and payoffs, the Nash equilibria of the game can be asymmetric, that is, the players choose different actions.

A Little History

In the course of developing the example in this chapter and the one before it, we introduce several games that have become classics in the subject. The prisoners' dilemma, of course, everyone knows. But the game of the two stone-age hunters trying to meet is almost equally well known. The eighteenth century French philosopher Jean-Jacques Rousseau introduced it in an almost identical setting; of course he did not have our memorable Flintstones characters to add color to the story.

The hunters' meeting game differs from the prisoners' dilemma because Fred's best response is to take the same action as Barney does (and vice versa), whereas in a prisoners' dilemma game Fred would have a dominant strategy—just one action, say rabbit, would be his best choice regardless of what Barney does—and so would Barney. Another way to express the difference is to say that in the meeting game, Fred would go stag-hunting if he had the assurance, whether by direct communication or because of the existence of a focal point, that Barney would also go stag-hunting, and vice versa. For this reason, the game is often called the *assurance game.*

Rousseau did not put his idea in precise game-theoretic language, and his phrasing leaves his meaning open to different interpretations. In Maurice Cranston's translation,[xxxvii] the large animal is a deer, and the statement of the problem is as follows: "If it was a matter of hunting a deer, everyone well realized that he must remain faithfully at his post; but if a hare happened to pass within the reach of one of them, we cannot doubt that he would have gone off in pursuit of it without scruple and, having caught his own prey, he would have cared very little about having caused his companions to lose theirs." Of course if the others were going for the hare, then there would be no point in any one hunter's attempting the deer. So the statement seems to imply that each hunter's *dominant* strategy is to go after a hare, which makes the game a prisoners' dilemma. However, the game is more commonly interpreted as an assurance game, where each hunter prefers to join the stag hunt if all the others are doing likewise.

The game of chicken is almost as well known as the prisoners' dilemma. The game was supposedly played by American teenagers in the 1950s. The two players drive their cars toward each other on a straight road; the first to swerve to avoid a collision is the chicken, and the one who keeps going straight is the winner. In the version made famous in the movie *Rebel Without a Cause,* the two drive their cars in parallel toward a cliff; the one who first jumps out of his car is the chicken. The metaphor of this game was used for nuclear brinkmanship by Bertrand Russell and others. The game was discussed in detail by Thomas Schelling in his pioneering game-theoretic analysis of strategic moves, and we will pick this up in Chapter 7.

To the best of our knowledge, the battle of the sexes game does not have such roots in philosophy or popular culture. It appears in the book *Games and Decisions* by R. Duncan Luce and Howard Raiffa, early classic on formal game theory.[xxxviii]

Finding Nash Equilibria

How can we find Nash equilibrium for a game? In a table, the worst-case method is cell-by-cell inspection. If both of the payoff entries in a cell are best responses, the strategies and payoffs for that cell constitute a Nash equilibrium. If the table is large, this procedure can get tedious. But god made computers precisely to rescue humans from the tedium of inspection and calculation. Software packages to find Nash equilibria are readily available. We mentioned one, Gambit, in Chapter 2 for drawing and solving trees more complex than ones amenable to paper and pencil; the same program has another module for setting up and solving game tables of similar levels of complexity. [xxxix]

Sometimes there are shortcuts to the process; we now describe one of these that is often useful.

Successive Elimination

Return to the pricing game between Rainbow's End and B. B. Lean. Here is the table of payoffs again:

Rainbow's End's Price	B.B. Lean's Price				
	42	41	40	39	38
42	43120 43120	***43260*** 41360	43200 39600	42940 37840	42480 36080
41	41360 ***43260***	41580 41580	***41600*** 39900	41420 38220	41040 36540
40	39600 43200	39900 ***41600***	***40000*** ***40000***	39900 ***38400***	39600 36800
39	37840 42940	38220 41420	***38400*** 39900	38380 38380	38160 ***36860***
38	36080 42480	36540 41040	36800 39600	***36860*** 38160	36700 36700

RE does not know what price BB is choosing. But it can figure out what price or prices BB is not choosing: BB will never set its price at $42 or $38. There are two reasons, both of which apply in our example, but in other situations only one may apply. [xl]

First, each of these strategies is uniformly worse for BB than another available strategy. No matter what it thinks RE is choosing, the choice of $41 is better for BB than that of $42, and the choice of $39 is better than that of $38. To see this, consider the $41 versus $42 comparison; the other is similar. Look at the five numbers for BB's profits from choosing $41 versus those from $42, corresponding to the five choices for RE. We highlight the former five numbers by shading the background of those cells dark gray, and the latter five numbers by shading the background of those cells a lighter shade of gray. For each of RE's five possible choices, BB's profit from choosing $42 is smaller that from choosing $41 instead:

$$43120 < 43260, \quad 41360 < 41580, \quad 39600 < 39900, \quad 37840 < 38220, \quad 36080 < 36540.$$

So no matter what BB expects RE to do, BB will never choose $42, and RE can confidently expect BB to rule out the $42 strategy.

When one strategy, say A, is uniformly worse for a player than another, say B, we say that A is *dominated* by B. If such is the case, that player will never use A, although whether he uses B remains to be seen. Then the other player can confidently proceed in this thinking on this basis; in particular, he need not consider playing a strategy that is the best response only to A. And when solving the game, we can remove dominated strategies from consideration. This reduces the size of the game table, and simplifies the analysis. [33]

The second avenue for elimination and simplification is to look for strategies that are *never best responses* to anything the other player might be choosing. In this example, $42 is never BB's best response to anything RE might be choosing within the range we are considering: no bolded or italicized entries appear for BB (upper right corner of each cell) in the column corresponding to its price of $42. Therefore RE can confidently think: "No matter what BB is thinking about my choice, it will never choose $42."

Similarly, BB's $38 choice is dominated by $39; and $38 is never BB's best response to anything it might think RE to be choosing (within the range we are considering).

RE's $42 and $38 strategies are similarly eliminated, leaving us with a 3-by-3 game table:

Rainbow's End's Price	41	40	39
41	41580 41580	***41600*** 39900	41420 38220
40	39900 ***41600***	***40000*** ***40000***	39900 ***38400***
39	38220 41420	***38400*** 39900	38380 38380

[33] If A is dominated by B, then conversely, B dominates A. So if A and B were the only two strategies available to that player, B would be a dominant strategy. With more than two strategies available, it is possible that A is dominated by B, but B is not dominant, because it does not dominate some third strategy C. Thus in general, elimination of dominated strategies may be possible even in games that do not have any dominant strategies.

In this simplified game, each firm has a dominant strategy, namely \$40. Therefore our Rule 2 from Chapter 3 indicates that as a solution for the game.

The \$40 strategy was not dominant in the original larger game; for example, if RE thought that BB would charge \$42, its profits from setting its own price at \$41, namely \$43,260, would be more than its profits from choosing \$40, namely \$43,200. The elimination of some strategies can open up the way to eliminate more in a second round. Here just two rounds sufficed to pin down the outcome. In other examples it may take more rounds, and even then the range of outcomes may be narrowed somewhat but not all the way to uniqueness.

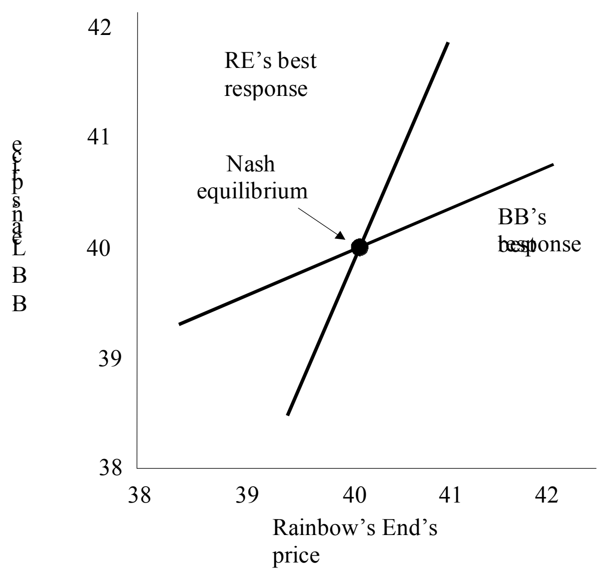
If successive elimination of dominated strategies (or never-best-response strategies) and choice of dominant strategies does lead to a unique outcome, that is a Nash equilibrium. When this works, it is an easier way to find Nash equilibria. Therefore we summarize our discussion of finding Nash equilibria into two rules:

Rule 3: **Eliminate from consideration any dominated strategies and strategies that are never best responses, and go on doing so successively.**

Rule 4: **Having exhausted the simple avenues of looking for dominant strategies or ruling out dominated ones, the next thing to do is to search all the cells of the game table to look for a pair of mutual best responses in the same cell, which is then a Nash equilibrium of the game.**

Games with Infinitely Many Strategies

In each of the versions of the pricing game we discussed so far, we allowed each firm only a small number of price points: only \$80 and \$70 in Chapter 3, and only the between \$42 and \$38 in \$1 steps in this chapter. Our purpose was only to convey the concepts of the prisoners' dilemma and Nash equilibrium in the simplest possible context. But in reality prices can be any number of dollars and cents, and to all intents and purposes that is as if they can be chosen over a continuous range of numbers.



Our theory can cope with this further extension quite easily, using nothing more than basic high-school algebra and geometry. We can show the prices of the two firms in a two-dimensional graph, measuring RE's price along the horizontal or X axis and BB's price along the vertical or Y axis. We can show the best responses in this graph instead of showing bold italic profit numbers in a game table of discrete price points.

We do this for the original example where the cost of each shirt to each store was $20. We omit the details of the mathematics, and merely tell you the result.[xli] The formula for BB's best response in terms of RE's price (or BB's belief about the price RE is setting) is

BB's best response price = 24 + 0.4 * RE's price (or BB's belief about it)

This is shown as the flatter of the two lines in the above graph. We see that for each $1 cut in RE's price, BB's best response should be to cut its own price but by less, namely 40 cents. This is the result of BB's calculation, striking the best balance between losing customers to RE and accepting a lower profit margin.

The steeper of the two curves in the figure is RE's best response to its belief about BB's price. Where the two curves intersect, the best response of each is consistent with the other's beliefs; we have a Nash equilibrium. The figure shows that this occurs when each firm charges $40. Moreover, it shows that this particular game has exactly one Nash equilibrium. Our finding a unique Nash equilibrium in the table where prices had to be multiples of $1 was not an artificial consequence of that restriction.

Such graphs, or tables that allow much more detail than we could in the simple examples, are a standard method for computing Nash equilibria. The calculation or graphing can quickly get too complicated for paper-and-pencil methods, and too boring besides, but computers were made precisely to cope with such mindless number-crunching tasks. The simple examples give us a basic understanding of the concept, and we should reserve our human thinking skills for the higher-level activity of assessing the usefulness of the concept. Indeed, that is our very next topic.

A Beautiful Equilibrium?

John Nash's equilibrium has a lot of conceptual claim to be the solution of a game where each player has the freedom of choice. Perhaps the strongest argument in its favor takes the form of a counterargument to any other proposed solution. A Nash equilibrium is a configuration of strategies where each player's choice is his best response to the other player's choice (or the other players' choices when there are more than two players in the game). Therefore if some outcome is not a Nash equilibrium, at least one player must be choosing an action that is not his best response. Such a player has a clear incentive to deviate from the proposed action; the deviation would destroy the proposed solution.

If there are multiple Nash equilibria, we do need some additional method for figuring out which one will emerge. But that just says we need Nash plus something else; it does not contradict Nash.

So we have a beautiful theory. But does it work in practice? How does one answer this question? One looks for instances where such games are played in the real world, or creates them in a laboratory setting. Then one compares the actual outcomes against the predictions of the theory.

If the agreement is sufficiently good, that supports the theory; if not, the theory should be rejected. Simple, right? In fact the process turns complicated very quickly, both in implementation and in interpretation. The results are mixed, with some reasons for cautious optimism for the theory, but also some ways in which the theory must be augmented or altered.

The two methods—observation and experiment—have different merits and flaws. Laboratory experiments allow proper scientific "control." The experimenters can specify the rules of the game, and the objectives of the participants, quite precisely. For example, in pricing games where the subjects play the roles of the managers of the firms, we can specify the costs of the two firms and the equations for the quantities each would sell in relation to the prices both charge, and give the players the appropriate motivation by paying them in proportion to the profits they achieve for their firm in the game. We can study the effects of the cause on which we want to focus, keeping all other things constant. By contrast, games that occur in real life have too many other things going on that one we not control, and too many things about the players—their true motivations, the firms' costs of production, and so on—that we do not know. That makes it hard to make inferences about the underlying conditions and causes by observing the outcomes.

To set against this, laboratory experiments have an element of artificiality that detracts from their ability to tell us enough about the real world. The subjects are usually students, who have no previous experience of business or similar applications that motivate the games. Many are novices even to the setting of the laboratory where the games are staged. They have to understand the rules of the game and then play it, all in a matter of an hour or two. Think how long it took you to figure out how to play even simple board games or computer games; that will tell you how naïve the play in such settings would be. We already saw some examples of this problem in Chapter 2, and commented on them. Second, although the experimenter can give the students the correct incentives by designing the structure of their monetary payments to fit their performance in the game, the sizes of the payments are usually small, and even college students may not take them sufficiently seriously. By contrast, business games and even professional sports in the real world are played by experienced players for large stakes.

For these reasons, one should not rely on any one form of evidence, whether it supports or rejects a theory, but should use both kinds and learn from each. With all these cautions in mind, let us see how the two types of empirical approaches do.

The field of industrial organization in economics provides the largest body of empirical testing of game-theoretic competition among firms, and industries like auto manufacture have been studied in depth. These empirical investigators start with several handicaps. They do not know the firms' costs and demands from any independent source, and must estimate these things from the same data that they want to use for testing the pricing equilibrium. They do not know precisely how the quantities sold by each firm depend on the prices charged by all. In the examples in this chapter, we simply assumed a linear relationship, but the real world counterparts (demand functions in the jargon of economics) can be nonlinear in quite complicated ways. The investigator must assume some specific form of the nonlinearity. Real-life competition among firms is not just about prices; it has many other dimensions like advertising, investment, and research and development. And real-life managers may not have the pure and simple aims of profit (or shareholder value) maximization that economic theory usually assumes. And competition among firms in real life extends over several years, so an appropriate combination of rollback and Nash equilibrium concepts must be specified. And many other conditions, such as incomes and costs, change from one year to the next, and firms enter or exit the industry; the investigator must think about what all those other things might be and make proper allowance for (control for, in statistical jargon) their effects on quantities and prices. Real world outcomes are also affected by many random factors; therefore uncertainty must be allowed for.[xlii]

A researcher must make a choice in each of these matters, and then derive equations that link the various magnitudes. These equations are then fitted to the data, and statistical tests performed to see how well they do. Then comes an equally difficult problem: What does one conclude from the findings? For example, suppose the data do not fit your equations very well. Something in your specification that led to the equations was not correct, but what was it? It could be the nonlinear form of the equations you chose; it could be the exclusion of some relevant variable like income or of some relevant dimension of competition like advertising; or it could be that the Nash equilibrium concept used in your derivations is invalid. Or it could be a combination of all these things. You cannot conclude that Nash equilibrium is incorrect when something else might be wrong. (But you would be right to raise your level of doubt about the equilibrium concept.)

Different researchers have made different choices in all these matters, and predictably, have found different results. After a thorough survey of this research, Peter Reiss and Frank Wolak of Stanford University give a mixed verdict: "The bad news is that the underlying economics can make the empirical models extremely complex. The good news is that the attempts so far have begun to define the issues that need to be addressed." In other words, more research is needed.

Another active area for empirical estimation concerns auctions where a small number of strategically aware firms interact in bidding for things like bandwidths in the airwave spectrum. In these auctions, asymmetry of information is a key issue for the bidders and also for the auctioneer. Therefore we postpone the discussion of auctions to Chapter 10, after we have examined the general issues of information in games in Chapter 8. Here we merely mention that empirical estimation of auction games is already having considerable success.[xliii]

What about laboratory experiments? The record of these is also mixed. Among the earliest experiments were the markets set up by Vernon Smith. He found surprisingly good results for game theory as well as for economic theory: small numbers of traders, each with no direct knowledge of the others' costs or values, could achieve equilibrium exchanges very quickly.

Other experiments with different kinds of games yielded outcomes that seemed contradictory to theoretical predictions. For example, in the ultimatum game, where one player makes a take-it-or-leave it offer to the other for dividing a given sum between the two, the offers were surprisingly generous. And in prisoners' dilemmas, good behavior occurred far more frequently than theory might lead people to believe. We discussed some of these findings in Chapters 2 and 3. Our general conclusion was that the participants in these games had different preferences or valuations than the purely selfish ones that used to be the natural assumption in economics. This is an interesting and important finding on its own; however, once the realistic "social" or "other-regarding" preferences are allowed for, the theoretical concepts of equilibrium—rollback in sequential-move games, and Nash in simultaneous-move games—yield generally good explanations of the observed outcomes.

When a game does not have a unique Nash equilibrium, the players have the additional problem of locating a focal point or some other method of selection among the possible equilibria. How well they succeed depends on the context, in just the way that theory suggests. If the players sufficiently common understanding for their expectations to converge, they will succeed in settling on a good outcome; otherwise disequilibrium may persist.

Most experiments work with subjects who have no prior experience of playing the particular game, or even of playing games in the laboratory at all. The behavior of these novices does not conform to equilibrium theory, but often converges to equilibrium as they gain experience. But some uncertainty about what the other player will do persists, and a good concept of equilibrium should allow players to recognize such uncertainty and respond to it. One such extension of the Nash equilibrium concept has become increasingly popular; this is the *quantal response equilibrium,* developed by professors Richard McKelvey and Thomas Palfrey of CalTech. As one might expect of any product of that institution, this is too technical for an elementary semi-popular book like ours, but some readers may be inspired to read and study it.[xliv]

After a detailed review of the relevant work, two of the top researchers in the field of experimental economics, Charles Holt of the University of Virginia and Alvin Roth of Harvard University, offer a guardedly optimistic prognosis: "In the last 20 years, the notion of Nash equilibrium has become a required part of the tool kit for economists and other social and behavioral scientists … There have been modifications, generalizations, and refinements, but the basic equilibrium analysis is the place to begin (and sometimes end) the analysis of strategic interactions."[xlv] We think that to be exactly the right attitude and recommend it to our readers. When studying or playing a game, begin with the Nash equilibrium, and then think of reasons why, and the manner in which, the outcome may differ from the Nash predictions. This dual approach is more likely to give you a good understanding, or success in actual play, than either a totally nihilistic, anything goes, attitude, or a slavishly naïve adherence to the Nash equilibrium and additional assumptions such as selfishness.

Case Study: Half Way

A Nash equilibrium is a combination of two conditions:

> (i) Each player is choosing a best response to what they believe the other players will do in the game.
>
> (ii) Each player's beliefs are correct. The other players are doing just what everyone else thinks they are doing.

It is easier to describe this outcome in a two-player game. Our two players, Abe and Bea each have beliefs about what the other will do. Based on those beliefs, Abe and Bea each choose to take an action that maximizes their payoffs. The beliefs prove right: Abe's best response to what he thinks Bea is doing is just what Bea thought Abe would do and Bea's best response to what she thought Abe would do is indeed just Abe expected her to do.

Let's look at these two conditions separately. The first condition is quite natural. If otherwise, then you'd have to argue that someone is not taking the best action given what he or she believes. If they had something better, why not do it?

Mostly, the rub comes in the second condition—that everyone is correct in what they believe. For Sherlock Holmes and Professor Moriarty this was not a problem: "All I have to say has already crossed your mind." "Then possibly my answer has crossed yours." "You stand firm?" "Absolutely." For the rest of us, correctly anticipating what the other side will do is often a challenge.

The following simple game will help illustrate the interplay between these two conditions and why you might or might not want to accept them.

Abe and Bea are playing a game with the following rules: Each player is to pick a number between 0 and 100, inclusive. There is a $100 prize to the player whose number is closest to half the other person's number.

We'll be Abe and you can play Bea. Any questions?

What if there's a tie?

Okay, in that case we split the prize. Any other questions?

No.

Great, then let's play. We've picked our number. Time for you to pick yours. What is you number? To help keep yourself honest, write it down.

We picked 50. No we didn't. To see what we actually picked, you'll have to read on.

Let's start by taking a step back and use the two-step approach to finding a Nash equilibrium. In step 1, we believe that your strategy had to be an optimal response to something we might have done. Since our number has to be something between 0 and 100, we figure that you couldn't have picked any number bigger than 50. For example, the number 60 is only an optimal response if you thought we would pick 120, something we couldn't do under the rules.[34]

What that tells us is that if your choice was truly a best response to something we might have done, you had to pick a number between 0 and 50.

By the same token, if we picked a number based on something that you might have done, we would have picked something between 0 and 50.

Believe it or not, many folks stop right there. When this game is played among people who haven't read this book, the most common response is 50. Frankly, we think that is a pretty lame answer (with apologies if that's what you picked). Remember that 50 is only the best choice if you think that the other side was going to pick 100. But, in order for the other side to pick 100, they would have to have misunderstood the game. They would have had to pick a number that had (almost) no chance of winning. Any number less than 100 will beat 100.

We will assume that your strategy was a best response to something we might have done and so it is between 0 and 50. That means our best choice should be something between 0 and 25.

Note that at this juncture, we have taken a critical step. It may seem so natural that you didn't even notice. We are no longer relying on our first condition that our strategy is a best response. We have taken the next step and proposed that our strategy should be a best response to something that is a best response from you.

If you are going to do something that is a best response, we should be doing something that is a best response to a best response.

[34] The average of 60 and 180 is 120. Half that average is 60, leading to 60 as a best response.

At this point, we are beginning to form some beliefs about your actions. Instead of imagining that you can do anything allowed by the rules, we are going to assume that you will actually have picked a move that is a best response. Given the quite sensible belief that you are not going to do something that doesn't make sense, it then follows that we should only pick a number between 0 and 25.

Of course, by the same token, you should be realizing that we won't be picking a number bigger than 50. If you think that way, then you won't pick a number bigger than 25.

As you might have guessed, the experimental evidence shows that after 50, 25 is the most common guess in this game. Frankly, 25 is a much better guess than 50. At least it has a chance of winning if the other player was foolish enough to pick 50.

If we take the view that you are only going to pick a number between 0 and 25, then our best response is now limited to numbers between 0 and 12.5. In fact, 12.5 is our guess. We'll win if our guess is closer to half your number than your number is to half ours. That means we win if you picked anything higher than 12.5.

Did we win?

Why did we pick 12.5? We thought you would pick a number between 0 and 25 and that's because we thought you'd think we'd pick a number between 0 and 50. We could of course go on with our reasoning and conclude that you'd figure we'd pick a number between 0 and 25, leading you to choose something between 0 and 12.5. If you had made that thought, then you'd be one step ahead of us and would have won. Our experience suggests that most people don't think more than two or three levels, at least on their first go round.

Now that you've had some practice and better understand the game, you might want a rematch. That's fair. So write down your number again—we promise not to peek.

We are pretty confident that you expect us to pick something less than 12.5. That means you'll pick something less than 6.25. And if we think you'll pick something less than 6.25, we should pick a number less than 3.125.

Now if this were the first go round, we might stop there. But we just explained that most folks stop after two levels of reasoning and this time we expect that you are determined to beat us so you'll engage in at least one more level of thinking ahead. If you expect us to pick 3.125, then you'll pick 1.5625, which leads us to think of 0.78125.

At this point, we are guessing that you can see where this is all heading. If you think we are going to pick a number between 0 and x, then you should pick something between 0 and x/2. And if we think you are going to pick something between 0 and x/2, then we should pick something between 0 and x/4.

The only way that we can both be right is if we both pick 0. That's what we've done. This is a Nash equilibrium. If you pick 0, we want to pick 0; if we pick 0, you want to pick 0. Thus if we both correctly anticipate what the other will do, we both do best picking 0, just what we expected the other to do.

We should have picked 0 the first time round as well. If you pick X and we pick 0, then we win. That is because 0 is closer to X/2 than X is to 0/2=0. We knew this all along, but didn't want to give it away the first time we played.

As it turned out, we didn't actually need to know anything about what you might be doing to pick 0. But that is a highly unusual case and an artifact of our only having two players in the game.

Let's modify the game to add more players. Now the person whose number is closest to half the average number wins. Under these rules, it is no longer the case that 0 always wins.[35] But it is still the case that the best responses converge to zero. In the first round of reasoning, all players will pick something between 0 and 50. (The average number picked can't be above 100, so half the average is bounded by 50.) In the second iteration of logic, if everyone thinks others will play a best response, then in response everyone should pick something between 0 and 25. In the third iteration of logic, they'll all pick something between 0 and 12.5.

How far people are able to go in this reasoning is a judgment call. Again, our experience suggests that most people stop at two or three levels of reasoning. The case of a Nash equilibrium requires that the players follow the logic all the way. Each player picks a best response to what he or she believes that the other players are doing. The logic of Nash equilibrium leads us to the conclusion that all players will pick 0. Everyone picking 0 is the only strategy where each of the players is choosing a best response to what they believe other players are doing and each is right about what they believe the other will be doing.

When people play this game, they rarely pick zero on the first go round. That is convincing evidence against the predictive power of Nash equilibrium. On the other hand, when they play the game even two or three times, they get very close to the Nash result. That is convincing evidence in favor of Nash.

Our view is that both perspectives are correct. To get to a Nash equilibrium, all players have to choose best responses—which is relatively straightforward. They also all have to have correct beliefs about what the other players will be doing in the game. This is much harder. It is theoretically possible to develop a set of internally consistent beliefs without playing the game. But it is often easier by playing the game. To the extent that players learn that their beliefs were wrong by playing the game and then learn how to do a better job predicting what others will do, they will converge to a Nash equilibrium.

While experience is helpful, it is no guarantee of success. One problem arises when there are multiple Nash equilibria. Consider the annoying problem of what to do when a mobile phone call gets dropped. Should you wait for the other person to call you or should you call. Waiting is a best response if you think the other person will call and calling is a best response if you think the other person will wait. The problem here is that there are two equally-attractive Nash equilibria: You call, the other person waits or you wait and the other person calls.

Experience doesn't always help get you there. If you both wait, then you might decide to call, but if you both happen to call at the same time, then you get busy signals (or at least you did in the days before call waiting). To resolve this dilemma, we often turn to social conventions, such as having the person who first made the call do the callback. At least that way you know the person has the number.

[35] Now picking 0 won't always win. If there are three players and the other two have picked 1 and 5, then the average of the three numbers (0, 1, and 5) is 2, half the average is 1, and the person picking 1 will win.

Workouts

Trip to the Gym No. 1:

You win by leaving the other side with 1, which it is forced to take. That means that starting with 2, 3, or 4 is a winning position. Hence a person stuck with 5 loses as whatever he does leaves the other side with 2, 3, or 4. Taking this to the next round of thinking, a person stuck with 9 loses. Carrying on the same reasoning, the player starting with 21 has a losing hand, when the rival player uses the correct strategy always takes the total down in groups of four.

Another way to see this is realize that the person who gets to take the next to last flag is the winner, as that leaves the other side with just one which they are forced to take. Taking the penultimate flag is just like taking the last flag in a game with one fewer flags. In the case with 21 flags, you act as if there are only 20 and try to last the last one from the twenty. Unfortunately, this is a losing position, at least if the other side understands the game.

Incidentally, this again shows that the first mover in a game need not always have the advantage, as we pointed out in the text by considering another variant of the flag game, where the player to take the last flag was the winner but the game started with 20 flags instead of 21.

Trip to the Gym No. 2:

If you want to calculate the numbers in the tables for yourself, the exact formula for the sales of RE is: Quantity sold by RE = 2800 – 100 * Price charged by RE + 80 * Price charged by BB

The formula for the sales of BB is the mirror image of this. Then the profits inferred by recalling that each store's cost is $20, so

RE's profit = (RE's price – 20) * Quantity sold by RE.

The formula for BB's profit is similar.

These formulas can then be embedded into an Excel spreadsheet. In the left column (Column A), enter the prices of RE for which you want to do the calculations in rows 2, 3, … . With five prices in our range, these are rows number 2–6. In the top row (Row 1), enter the corresponding prices of BB in columns B, C, … . With five prices, these are columns B–F. In Cell B2, enter the formula: = MAX(2800 – 100 * $A2 + 80 * B$1 , 0).

Note carefully the dollar signs; in Excel notation they ensure the appropriate "absolute" and "relative" cell references when the formula is copied and pasted to all the other cells with the different price combinations. The formula also ensures that if prices charged by the two firms are too different, the sales of the firm with the higher price do not go negative. This is the table of the quantities sold by RE.

To go from the quantities to the profits of RE, write down in a blank cell somewhere else in the spreadsheet (we used cell J2), the RE's cost, namely 20. On the same page of the spreadsheet, directly below the table of quantities, say in rows 8–12 (leaving row 7 blank for clarity), copy out the prices of RE in a new set of rows in Column A. In cell B8, enter the formula: = B2 * ($A8 – J2).

This yields RE's profit when it charges the first of its set of prices we are considering, and BB charges the first of its corresponding prices. Copy and paste this formula into all the other cells to get the full table of profits for RE.

The formulas for the quantities and profits of BB can be entered in rows 14–18 and 20–24. The formula for quantities is MAX(2800–100*B$1+80*$A14,0). And, entering BB's cost in a spare cell J3, the formula for profits is B14 *(B$1–$J$3).

(Of course, if you want to experiment with different equations for the quantity sold or different costs, you should change the numbers accordingly.)

We reproduce below the result of this for our example:

	Col. A	Col. B	Col. C	Col. D	Col. E	Col. F	Col. G	Col.H	Col. I	Col. J
Row 1		42	41	40	39	38			Costs	
Row 2	42	1960	1880	1800	1720	1640			RE	20
Row 3	41	2060	1980	1900	1820	1740	RE's		BB	20
Row 4	40	2160	2080	2000	1920	1840	quantities			
Row 5	39	2260	2180	2100	2020	1940				
Row 6	38	2360	2280	2200	2120	2040				
Row 7										
Row 8	42	43120	41360	39600	37840	36080				
Row 9	41	43260	41580	39900	38220	36540	RE's			
Row 10	40	43200	41600	40000	38400	36800	Profits			
Row 11	39	42940	41420	39900	38380	36860				
Row 12	38	42480	41040	39600	38160	36720				
Row 13										
Row 14	42	1960	2060	2160	2260	2360				
Row 15	41	1880	1980	2080	2180	2280	BB's			
Row 16	40	1800	1900	2000	2100	2200	quantities			
Row 17	39	1720	1820	1920	2020	2120				
Row 18	38	1640	1740	1840	1940	2040				
Row 19										
Row 20	42	43120	43260	43200	42940	42480				
Row 21	41	41360	41580	41600	41420	41040	BB's			
Row 22	40	39600	39900	40000	39900	39600	profits			
Row 23	39	37840	38220	38400	38380	38160				
Row 24	38	36080	36540	36800	36860	36720				

Trip to the Gym No. 3:

The Excel spreadsheet is easily modified by changing RE's cost figure in cell J2 from 20 to 11.60:

	Col. A	Col. B	Col. C	Col. D	Col. E	Col. F	Col. G	Col. H	Col. I	Col. J
Row 1		40	39	38	37	36			Costs	
Row 2	37	2300	2220	2140	2060	1980			RE	11.6
Row 3	36	2400	2320	2240	2160	2080	RE's		BB	20
Row 4	35	2500	2420	2340	2260	2180	quantities			
Row 5	34	2600	2520	2440	2360	2280				
Row 6	33	2700	2620	2540	2460	2380				
Row 7										
Row 8	37	58420	56388	54356	52324	50292				
Row 9	36	58560	56608	54656	52704	50752	RE's			
Row 10	35	58500	56628	54756	52884	51012	Profits			
Row 11	34	58240	56448	54656	52864	51072				
Row 12	33	57780	56068	54356	52644	50932				
Row 13										
Row 14	37	1760	1860	1960	2060	2160				
Row 15	36	1680	1780	1880	1980	2080	BB's			
Row 16	35	1600	1700	1800	1900	2000	quantities			
Row 17	34	1520	1620	1720	1820	1920				
Row 18	33	1440	1540	1640	1740	1840				
Row 19										
Row 20	37	35200	35340	35280	35020	34560				
Row 21	36	33600	33820	33840	33660	33280	BB's			
Row 22	35	32000	32300	32400	32300	32000	profits			
Row 23	34	30400	30780	30960	30940	30720				
Row 24	33	28800	29260	29520	29580	29440				

The profit numbers are then entered into the payoff table for the game:

Rainbow's End's Price	B.B. Lean's Price				
	40	39	38	37	365
38	35200 58420	***35340*** 56388	35280 543560	35020 52324	34560 50292
37	33600 ***58560***	33820 56608	***33840*** 54656	33660 52704	33280 50752
36	32000 58500	32300 ***56628***	***32400*** ***54756***	32300 ***52884***	320080 51012
35	30400 58240	30780 56448	***30960*** 54656	30940 52864	30720 ***51072***
34	28800 57780	29260 56068	29520 54356	***29580*** 52644	29440 50932

Observe that we had to use a range of lower prices to locate best responses. In the new Nash equilibrium, BB charges $38 and RE charges $35. RE benefits twice over, once from its lower cost and further as its price cut shifts some customers to it from BB. As a result, BB's profit goes down by a lot (from $40,000 to $32,400) while RE's profit goes up by a lot (from $40,000 to $54,756). Even though RE's cost advantage is only 42% ($11.60 is 58% of $20), its profit advantage is 69% ($54,756 is 1.69 times $32,400). Now you see why businesses are so keen to eke out seemingly small cost advantages, and why firms quickly move to lower-cost locations and countries.

Endnotes

[i] *Robert Frost's Poems,* ed. Louis Untermeyer (New York: Washington Square Press, 1971).

[ii] A good free and open source package of this kind is Gambit. It can be downloaded from its web site, http://econweb.tamu.edu/gambit/

[iii] The episode is described, and a brief video of the actual game is available, from the show's web site, www.cbs.com/primetime/survivor5/.

[iv] For the mathematically sophisticated readers, a good account of many such games is in the book *Games of No Chance,* ed. Richard J. Nowakowski, Cambridge, UK: Cambridge University Press, 1996.

[v] These experiments are too numerous to cite in full. An excellent survey and discussion can be found in Colin Camerer, *Behavioral Game Theory: Experiments in Strategic Interaction,* Princeton, NJ: Princeton University Press, 2003, pp. 48–83, 467. Camerer also discusses experiments and their findings on other related games, most notably the "trust game" that is like the Charlie-Fredo game (see his pp. 83–90). Once again actual behavior differs from what would be predicted by backward reasoning assuming purely selfish preferences; considerable trusting behavior and its reciprocation are found.

[vi] Alan G. Sanfey, James K. Rilling, Jessica A. Aronson, Leigh E. Nystrom, and Jonathan D. Cohen, "The neural basis of economic decision making in the ultimatum game," *Science,* volume 300, June 2003, pp. 1755–1757.

[vii] Camerer, op. cit., p. 24. Emphasis in the original.

[viii] See Camerer, op. cit., pp. 101–110 for an exposition and discussion of some such theories.

[ix] For a detailed expert discussion of Chess from the game-theoretic perspective, read an article by Herbert A. Simon and Jonathan Schaeffer, "The Game of Chess," in *The Handbook of Game Theory, Volume 1,* eds. Robert J. Aumann and Sergiu Hart, Amsterdam: North-Holland, 1992. Chess-playing computers have improved greatly since the article was written, but its general analysis retains its validity. Simon won the Nobel Prize in Economics in 1978 for his pioneering research into the decision-making process within economic organizations.

[x] From "Brief history of the groundfishing industry of New England," on the U. S. government website http://www.nefsc.noaa.gov/history/stories/groundfish/grndfsh1.html.

[xi] Joseph Heller, Catch-22, New York: Simon and Schuster, 1955. Chapter 42, p. 455 of the Dell paperback edition published in 1961.

[xii] University of California biologist Garrett Harding brought this class of problems to wide attention in his influential article, "The tragedy of the commons," *Science,* volume 162, December 13, 1968, pp. 1243–48.

[xiii] "The Work Of John Nash In Game Theory," Nobel Seminar, December 8, 1994. On the web site http://nobelprize.org/economics/laureates/1994/nash-lecture.pdf.

[xiv] William Poundstone, *Prisoner's Dilemma,* New York: Doubleday, 1992, pp. 8–9; Sylvia Nasar, *A Beautiful Mind,* New York: Simon and Schuster, 1998, pp. 118–9.

[xv] James Andreoni and Hal Varian have developed an experimental game called Zenda, based on this idea. See their "Preplay communication in the prisoners' dilemma," *Proceedings of the National Academy of Sciences,* vol. 96, no. 19, September 14, 1999, pp. 10933–10938. We have tried the game in classrooms, and found it to be very successful in developing cooperation. But its implementation in more realistic setting is harder.

[xvi] This research comes from their working paper "Identifying Moral Hazard: A Natural Experiment in Major League Baseball," available at http://ddrinen.sewanee.edu/Plunk/dhpaper.pdf.

[xvii] At the time, Schilling was pitching for National League's Arizona Diamondbacks and Cy Young winner Randy Johnson was his teammate. Quoted in Ken Rosenthal, "Mets Get Shot with Mighty Clemens at the Bat," *The Sporting News*, June 13, 2002.

[xviii] The results are due to M. Keith Chen and Marc Hauser, "Modeling Reciprocation and Cooperation in Primates: Evidence for a Punishing Strategy," *Journal of Theoretical Biology*, May 2005, pp. 5–12. There is a video of the experiment you can see at www.som.yale.edu/faculty/keith.chen/datafilm.htm.

[xix] See Colin Camerer, *Behavioral Game Theory,* New York: Russell Sage Foundation and Princeton, NJ: Princeton University Press, pp. 46–48.

[xx] See Felix Oberholzer-Gee, Joel Waldfogel and Matthew W. White "Social Learning and Coordination in High-Stakes Games: Evidence from Friend or Foe" (June 2003). NBER Working Paper No. W9805. Available at SSRN: http://ssrn.com/abstract=420319 and John A List, 2006. "Friend or Foe? A Natural Experiment of the Prisoner's Dilemma," The Review of Economics and Statistics, MIT Press, vol. 88(3), pages 463–471.

[xxi] For a detailed account of this experiment, see William Poundstone, *Prisoner's Dilemma,* New York: Doubleday, 1992, pp. 8–9, and Sylvia Nasar, *A Beautiful Mind,* New York: Simon and Schuster, 1998, pp. 118–9.

[xxii] Jerry E. Bishop, "All for One, One for All? Don't Bet On It," *Wall Street Journal,* December 4, 1986.

[xxiii] Reported by Thomas Hayden, "Why we need nosy parkers," *US News and World Report,* June 13, 2005. Details can be found in D. J. de Quervain, U. Fischbacher, V. Treyer, M. Schellhammer, U. Schnyder, and E. Fehr, "The neural basis of altruistic punishment," *Science,* volume 305, number 5688, 27 August 2004, pp. 1254–8.

[xxiv] Cornell University economist Robert Frank discusses in his book *Passions Within Reason* (New York: W. W. Norton, 1988) argues that emotions such as guilt and love evolved, and social values such as trust and honesty were developed and sustained, to counter individuals' short-run temptations to cheat and to secure the long-run advantages of cooperation. And Robin Wright, in *Non-Zero* (New York: Pantheon), develops the sweeping theme that mechanisms that achieve mutually beneficial outcomes in non-zero-sum games explain much of human cultural and social evolution.

[xxv] Source: http://www.eurekalert.org/pub_releases/2004-03/aiop-wab032904.php

[xxvi] Eldar Shafir and Amos Tversky, "Thinking through Uncertainty: Nonconsequential Reasoning and Choice," *Cognitive Psychology*, 24, pp. 449–474, 1992.

[xxvii] *The Wealth of Nations,* volume 1, book 1, chapter 10 (1776).

[xxviii] Kurt Eichenwald gives a brilliant and entertaining account of this case in his book, *The Informant,* New York: Broadway Books, 2000. The "philosophy" quote is on p. 51.

[xxix] David Kreps, *Microeconomics for Managers,* New York: W. W. Norton, 2004, pp. 530–1, gives an account and discussion of the turbine industry.

[xxx] See Paul Klemperer, "What really matters in auction design," Journal of Economic Perspectives, Winter 2002, pp. 169–89 for examples and analysis of collusion in auctions.

[xxxi] Kreps, op. cit., p. 543.

[xxxii] "Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on this commons. … Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit, in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons" Garrett Harding, "The Tragedy of the Commons*," Science,* volume 162, December 13, 1968, pp. 1243–1248.

[xxxiii] Elinor Ostrom, *Governing the Commons,* Cambridge University Press, 1990, and "Coping with the tragedy of the commons," *Annual Review of Political Science,* volume 2, June 1999, pp. 493–535.

[xxxiv] The literature is huge. Two good popular expositions are Matt Ridley, *The Origins of Virtue,* New York: Viking Penguin, 1997, and Lee Dugatkin, *Cheating Monkeys and Citizen Bees,* Cambridge, MA: Harvard University Press, 1999.

[xxxv] Dugatkin, op. cit., pp. 97–99.

[xxxvi] This aired on March 16, 2006. A DVD is available for purchase at http://www.abcnewsstore.com/store/, product code P060316 01.

[xxxvii] Quoted from William Poundstone, *Prisoner's Dilemma,* New York: Knopf, 1993, p. 220.

[xxxviii] Readers who want a little more formal detail on each of these games will find useful articles on the web in the Wikepedia encyclopedia (http://en.wikipedia.org/wiki/Main_Page) or at http://www.gametheory.net/.

[xxxix] A reminder that Gambit is free and open source software, and can be downloaded from its web site, http://econweb.tamu.edu/gambit/

[xl] At a higher level of analysis, the two are seen to be equivalent in two-player games if mixed strategies are allowed for each player; see Avinash Dixit and Susan Skeath, *Games of Strategy,* New York: W. W. Norton, second edition 2004, p. 207.

[xli] For readers with some mathematical background, here are a few steps in the calculation. The formula given in the text for the quantity sold by BB can be written as:

Quantity sold by BB = 2800 – 100 * Price charged by BB + 80 * Price charged by RE

On each unit, BB makes a profit equal to its price minus 20, its cost. Therefore BB's total profit is

BB's profit = (2800 – 100 * BB's price + 80 * RE's price) * (BB's price – 20)

If BB sets its price equal to its cost, namely 20, it makes zero profit. If it sets its price equal to

(2800 + 80 * RE's price) / 100 = 28 + 0.8 * RS's price,

it makes zero sales and therefore zero profit. BB's profit is maximized by choosing a price somewhere between these two extremes, and in fact for a linear demand formula this occurs at a price exactly half way between the extremes. Therefore

BB's best response price = ½ * (20 + 28 + 0.8 * RE's price) = 24 + 0.4 * RE's price.

Similarly, RE's best response price = 24 + 0.4 * BB's price.

When RE's price is $40, BB's best response price is 24 + 0.4 * 40 = 24 + 16 = 40, and vice versa. This confirms that in the Nash equilibrium outcome each firm charges $40. For more details of such calculations, see Avinash Dixit and Susan Skeath, Games of Strategy, New York: W. W. Norton, second edition 2004, pp. 124–128.

[xlii] For readers interested in pursuing this topic, we recommend the survey by Peter C. Reiss and Frank A. Wolak, "Structural Econometric Modeling: Rationales and Examples from Industrial Organization," in *Handbook of Econometrics, Volume 6,* eds. James Heckman and Edward Leamer, Amsterdam: North-Holland, 2006, forthcoming.

[xliii] This research is surveyed by Susan Athey and Philip A. Haile: "Empirical Models of Auctions," in *Advances in Economic Theory and Econometrics, Theory and Applications, Ninth World Congress, Volume II,* eds. Richard Blundell, Whitney K. Newey and Torsten Persson, Cambridge, UK: Cambridge University Press, 2006, pp. 1–45.

[xliv] Richard McKelvey and Thomas Palfrey, "Quantal Response Equilibria for Normal Form Games," *Games and Economic Behavior,* vol. 10, no. 1, July 1995, pp. 6–38.

[xlv] Charles A. Holt and Alvin E. Roth, "The Nash Equilibrium: A Perspective," *Proceedings of the National Academy of Sciences,* vol. 101, no. 12, March 23, 2004, pp. 3999–4002.