Trajectory Tracking of Autonomous Driving Vehicles via Output Feedback MPC*

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***Abstract*—This paper aims at addressing a model predictive control technique for trajectory tracking of autonomous driving vehicle under the condition that the system states are difficult to be measured. Firstly, a linearized error model and an dynamic output feedback controller are established, and then an augmented closed-loop model is obtained to predict the dynamic behavior of autonomous driving vehicles. In addition, the dynamic output feedback model predictive controller is derived by minimizing the upper bound of an infinite horizon quadratic objective function which explicitly takes the input constraint into account. Finally, the optimization problem of dynamic output feedback model predictive control (DOFMPC) is solved by linear matrix inequalities (LMIs) technique, and the first one of the optimized control sequence is implemented. It is proved that the output feedback predictive control strategy realizes the goal of accurately tracking the reference trajectory on the simulation platform of CarSim and Simulink, and the effectiveness of the algorithm is verified.**

***Index Terms*—model predictive control, trajectory tracking, autonomous driving vehicle, output feedback**

I. Introduction

Autonomous driving vehicle system is a highly nonlinear and strongly coupled dynamic system, and its high-precision trajectory tracking control problem has been a hot topic for many years. At present, the primary methodologies in solving the trajectory tracking problem include proportional integral and differential (PID) control algorithm, model predictive control (MPC) algorithm, linear quadratic regulator (LQR) control algorithm, pure pursuit algorithm, adaptive control algorithm, etc. Next, we will introduce the application of MPC in trajectory tracking to understand the research status and existing problems, and find a breakthrough for the study of this paper while inheriting the previous excellent algorithms.

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MPC refers to online optimization control algorithm, which is characterized by receding horizon optimization and feedback correction [1]. It can deal effectively with online optimization problems for linear multivariable control systems and handle physical constraints in a systematic way. Throughout the existing MPC methods, in the design of autonomous vehicle trajectory tracking controller, the following two aspects are mainly considered: the state feedback based predictive controller and the output feedback predictive controller [2]. MPC method based on state feedback is widely used in trajectory tracking, such as [3] - [13]. [3] made a detailed introduction to the driverless vehicle driving technology by applying the model predictive control. [4] designed the controller using MPC algorithm and realized the improved method realized the active steering and track tracking of the front wheel. The experimental results shown that the tracking stability and tracking accuracy of the vehicle were greatly improved by using the model predictive controller. [5] - [6] designed a model predictive controller with known road information, and considered the tire model to achieve obstacle avoidance on snow and ice covered roads. [7] provided an nonlinear predictive controller for autonomous driving vehicle system which based on the prediction model involving the dynamic of the vehicle related to the reference trajectory. For lane keeping and obstacle avoidance of autonomous driving vehicles, [8] - [9] proposed two control methods based on MPC. The first method solved a single nonlinear MPC problem. The second approach used a layered scheme. [10] regarded model predictive control as an integrated control algorithm, the tire force distribution based on quadratic programming was studied from the perspective of ensuring the constraints of chassis integrated control. Based on IM-Proved-RRT and linear time-varying MPC algorithm, [11] realized the path planning and trajectory tracking of autonomous driving vehicles. [12] studied vehicle tracking on low-adhesion road surface by adding vehicle dynamics constraints. [13] designed an improved path planning method for driverless vehicles, and build the corresponding software in the loop real-time simulation system to contribute to the simulation work of driverless vehicles. When the above scholars designed the trajectory tracking controller, they all considered that the system state was completely measurable. From the perspective of practical application, in the face of complex motion control problems, most of the actual vehicle systems can not accurately obtain the state information, and can not achieve state feedback. Even if the position information is obtained through the high-precision position detection device, it will greatly increase the cost of the vehicle system, and make it bulky and reduce the reliability of the system. So it is necessary and significant to explore the output feedback MPC algorithm for autonomous driving vehicle system on the basis that the related state information cannot be measured. However, to the best of the author's knowledge, limited words can be found on the topic of output feedback MPC in dealing with the trajectory tracking problem, which motivates our study presented in this paper.

This paper mainly solves the trajectory tracking problem of autonomous driving vehicle when the system state is not measurable. Aiming at this problem, the dynamic output feedback model predictive controller [14] for polyhedral uncertain system is presented and an MPC optimization problem is solved by LMIs technology [15]. The addressed technique is verified by the co-simulation of Simulink and Carsim, and the results show that the provided MPC algorithm realizes the high-precision trajectory tracking.

II. Kinematic model of the vehicle

This section mainly introduces the kinematics model of autonomous driving vehicle. Assuming that the vehicle moves in a circle around a certain point at any time and neglecting the effect of suspension, the steering motion model can be obtained. Fig 2.1 is the schematic diagram of vehicle kinematics model in inertial coordinate system.

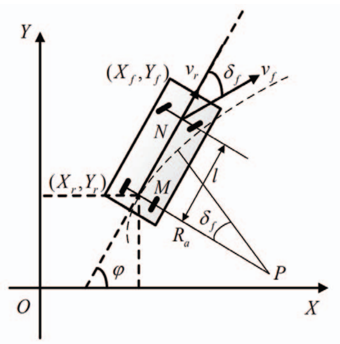

Fig. 1. The kinematic model of autonomous driving vehicle

Among them, (X_r, Y_r) and (X_f, Y_f) are the coordinates of rear axle center and front axle center of the vehicle; the front wheel deflection angle of the vehicle is represented by δ_f; l is the wheelbase of the vehicle, and this data can be modified in the Carsim data interface; the instantaneous center of the vehicle when turning is represented by P; R_a is used to represent the steering radius of the rear wheels of the vehicle; v_f and v_r are velocities of front and rear axle centers; the heading angle is represented by φ. It is assumed that the sideslip angle remains constant when the center of mass rotates.

At coordinate (X_r, Y_r), the vehicle speed is

$$v_r = \dot{X}_r \cos\varphi + \dot{Y}_r \sin\varphi \tag{1}$$

The constraints of front and rear axle of car body are

$$\left\{ \begin{array}{l} \dot{X}_r \sin\varphi - \dot{Y}_r \cos\varphi = 0 \\ \dot{X}_f \sin(\delta_f + \varphi) - \dot{Y}_f \cos(\delta_f + \varphi) = 0 \end{array} \right. \tag{2}$$

Combining (1) and (2), we get the following formula

$$\left\{ \begin{array}{l} \dot{X}_r = v_r \cos\varphi \\ \dot{Y}_r = v_r \sin\varphi \end{array} \right. \tag{3}$$

We can get the following formula according to the geometric relationship between the rear wheel and the front wheel

$$\left\{ \begin{array}{l} X_f = l\cos\varphi + X_r \\ Y_f = l\sin\varphi + Y_r \end{array} \right. \tag{4}$$

Substituting (3) and (4) into (2), the yaw rate ω can be obtained as

$$\omega = \frac{v_r}{l} \tan\delta_r \tag{5}$$

At the same time, steering radius and front wheel deflection angle can be obtained by yaw rate and vehicle speed

$$\left\{ \begin{array}{l} \delta_f = \arctan(l/R) \\ R_a = v_r/\omega \end{array} \right. \tag{6}$$

Combined with the previous formula, the vehicle's kinematics equation is

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ \tan\delta_f/l \end{bmatrix} v_r \tag{7}$$

It can be known from (7), we regard the entire vehicle system as a closed-loop system, its state variable is $\chi(x, y, \varphi)$, and its input variable is $u(v, \delta_f)$, the general form of the system is

$$\dot{\chi} = f(\chi, u) \tag{8}$$

When the reference trajectory is given, since the reference trajectory is limited by the above kinematics equation, the trajectory of the vehicle is

$$\dot{\chi}_r = f(\chi_r, u_r) \tag{9}$$

where r represents the reference quantity, $u_r = [v_r \quad \delta_r]^{\mathrm{T}}$, $\chi_r = [X_r \quad Y_r \quad \varphi_r]^{\mathrm{T}}$.

For (9), Taylor series are used to deal with the expansion of reference points, while higher-order terms are ignored, we get

$$\dot{\chi} = f(\chi_r, u_r) + \frac{\partial f(\chi, u)}{\partial \chi}|(\chi - \chi_r) + \frac{\partial f(\chi, u)}{\partial u}|(u - u_r) \tag{10}$$

Subtracting (10) and (9), it follows that

$$\dot{\tilde{\chi}} = \begin{bmatrix} \dot{X} - \dot{X}_r \\ \dot{Y} - \dot{Y}_r \\ \dot{\varphi} - \dot{\varphi}_r \end{bmatrix} = \begin{bmatrix} 0 & 0 & -v_r \sin\varphi_r \\ 0 & 0 & v_r \cos\varphi_r \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X - X_r \\ Y - Y_r \\ \varphi - \varphi_r \end{bmatrix} + \begin{bmatrix} \cos\varphi_r & 0 \\ \sin\varphi_r & 0 \\ \frac{\tan\delta_r}{l} & \frac{v_r}{l\cos^2\delta_r} \end{bmatrix} \begin{bmatrix} v - v_r \\ \delta - \delta_r \end{bmatrix} \tag{11}$$

The above equation is the linearized vehicle error model quoted from the [3]. Because the design of the controller requires a discrete vehicle model, the kinematic model of the vehicle after discretization can be represented as

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= C(k)x(k) \end{aligned} \tag{12}$$

with

$$A(k) = \begin{bmatrix} 1 & 0 & -v_r(k)\sin\varphi_r(k)T \\ 0 & 1 & v_r(k)\cos\varphi_r(k)T \\ 0 & 0 & 1 \end{bmatrix}$$

$$B(k) = \begin{bmatrix} \cos\varphi_r(k)T & 0 \\ \sin\varphi_r(k)T & 0 \\ \frac{\tan\delta_r(k)T}{l} & \frac{v_r(k)T}{l\cos^2\delta_r(k)} \end{bmatrix}$$

where T is the sampling period, $x \in \Re^{n_x}$ is unmeasurable state, $u \in \Re^{n_u}$ is input, $y \in \Re^{n_y}$ is output. The input u is constrained by the following inequality

$$-\bar{u} \leqslant u(k) \leqslant \bar{u} \tag{13}$$

where $\bar{u} = [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_{n_u}]^{\mathrm{T}}$, $\bar{u}_s > 0$, $s \in \{1, 2, \ldots, n_u\}$. Assume that

$$[A|B|C](k) \in \Omega,\ \Omega = Co\{[A_1|B_1|C_1], \cdots, [A_N|B_N|C_N]\} \tag{14}$$

where Co means the convex hull, then the non-negative coefficient $\lambda_n(k),\ n \in \{1, \ldots, N\}$ satisfies

$$\sum_{n=1}^{N} \lambda_n(k) = 1, \quad [A|B|C](k) = \sum_{n=1}^{N} \lambda_n(k)[A_n|B_n|C_n].$$

Based on the kinematics model of autonomous driving vehicle, the controller of DOFMPC is designed. The controller can guarantee that the state is always changing in a fixed set, and it can be defined as

$$\begin{aligned} x_c(i+1|k) &= H_c(k)x_c(i|k) + P_c(k)y(i|k) \\ u(i|k) &= R_x(k)x_c(i|k) \end{aligned} \tag{15}$$

where $\{H_c, P_c, R_x\}$ are matrices of output feedback controller; $x_c \in \Re^{n_{x_c}}$ is the controller state. Hence, combining with (12) and (15), the augmented closed-loop system is shown below

$$\tilde{x}(i+1|k) = \Phi(i,k)\tilde{x}(i|k),\ \forall i \geqslant 0, \tilde{x}(k) = \tilde{x}(0|k) \tag{16}$$

where

$$\tilde{x} = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}$$

$$\Phi(i,k) = \begin{bmatrix} A(k+i) & B(k+i)R_x(k) \\ P_c(k)C(k+i) & H_c(k) \end{bmatrix}$$

III. THE DESIGN OF THE DOFMPC ALGORITHM

The main purpose of this section is to design an appropriate dynamic output feedback algorithm to achieve trajectory tracking. The algorithm should ensure that the performance objective function can obtain the minimum value: $J(k) = \sum_{i=0}^{\infty} (\|y(k+i|k)\|_S^2 + \|u(k+i|k)\|_R^2)$, where S and R are the most appropriate weighting matrices, and both of them are positive definite matrices. Then, let the Lyapunov function be: $V(k+i|k) = \widetilde{x}(k+i|k)^T P \widetilde{x}(k+i|k)$. $P(P>0)$ is a positive-definite and real symmetric matrix to be determined, and $\Omega_P := \left\{x \,\middle|\, x^T P x \leqslant 1\right\}$ represents the ellipsoid related to the matrix P.

Finding an effective solution to the "min-max" optimization problem is the primary goal of DOFMPC approach at each time k:

$$\begin{aligned} &\min_{u(k+i|k)} \max_{[A(k+i)|B(k+i)|C(k+i)] \in \Omega, i \geqslant 0} J(k) \\ &s.t. \quad V(k+i|k) - V(k+i+1||k) \\ &\geqslant \frac{1}{\gamma}(\|y(k+i|k)\|_S^2 + \|u(k+i|k)\|_R^2) \end{aligned} \tag{17}$$

$$x(k) \in \Omega_P \tag{18}$$

$$-\overline{u} \leqslant u(i|k) \leqslant \overline{u} \tag{19}$$

where (17) is a constraint condition for the robustness of the system, (18) is the constraint on initial augmented state, (19) is input constraints. $\gamma(k)$ is appropriate nonnegative variables.

In order to analyze the robustness and optimality of the system, we can assume it satisfies the constraint (17). By summing (17) from $i = 0$ to $i = \infty$, we known $\lim_{i\to\infty} V(k+i|k) = 0$, so we can get $J(k) = \sum_{i=0}^{\infty} (\|y(k+i|k)\|_S^2 + \|u(k+i|k)\|_R^2) \leqslant \gamma(k) V(k)$. In this way, (17) makes $J(k) \leqslant \gamma(k)$ ture. Therefore, the problem of minimizing the upper bound $\gamma(k)$ can replace the optimization problem of minimizing the performance objection function $J(k)$ in the infinite time domain.

Next, the design process of controller is transformed into an optimization problem based on linear matrix inequality constraints. The final result of the derivation is expressed by the following theorem.

Theorem 1: For the vehicle control systems with input constraints (19), if there exist symmetric matrices $\{K, L\}$, matrices $\{\widehat{A}, \widehat{B}, \widehat{C}\}$, and scalars $\{\tau, \gamma\}$, satisfying conditions with $i, l = 1, 2, 3, ..., r$, the following optimization problem of minimizing performance objective function in the infinite time domain can be solved:

$$\min_{\tau,\gamma,\hat{A},\hat{B},\hat{C},K,L} \gamma(k)$$

$$s.t. \quad \begin{bmatrix} \Gamma_1 & * & * & * \\ \Gamma_2 & \Gamma_5 & * & * \\ \Gamma_3 & 0 & \gamma S^{-1} & * \\ \Gamma_4 & 0 & 0 & \gamma R^{-1} \end{bmatrix} \geqslant 0 \tag{20}$$

$$K(k) \leqslant \tau P_e(k) \tag{21}$$

$$\begin{bmatrix} 1-\tau & * & * \\ x_0(k) & L & * \\ 0 & I & K \end{bmatrix} \geqslant 0 \tag{22}$$

$$\begin{bmatrix} \Gamma_5 & * \\ \begin{bmatrix} 0 & \widehat{C} \end{bmatrix} & \bar{u}_{\max}^2 I \end{bmatrix} \geqslant 0 \tag{23}$$

where

$$\Gamma_1 = \Gamma_5 = \begin{bmatrix} K & * \\ I & L \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} A_i & A_i L + B_i \hat{C} \\ K A_i + \hat{B} C_i & \hat{A} \end{bmatrix}$$
$$\Gamma_3 = \begin{bmatrix} 0 & C_i L \end{bmatrix} \qquad \Gamma_4 = \begin{bmatrix} 0 & \hat{C} \end{bmatrix}$$

In addition, the parameters of the controller are shown below, and the detailed process of parameters setting will be explained in the following derivation:

$$H_c = M^{-1} \left(\widehat{A} - K A_i L - \widehat{B} C_i L - K B_i \widehat{C} \right) O^{-1}$$
$$P_c = M^{-1} \widehat{B}$$
$$R_x = \widehat{C} O^{-1}$$

Proof of Theorem 1:

(1) The conditions of guaranteeing the robust stability (17). Substituting the expression of the system (16) into (17) to get the following:

$$\widetilde{x}(k+j|k)^T \times$$
$$\left[P - \frac{1}{\gamma} \widetilde{C}(k)^T S \widetilde{C}(k) - \frac{1}{\gamma} \widetilde{C_c}(k)^T S \widetilde{C_c}(k) - \Phi(i|k)^T P \Phi(i|k) \right]$$
$$\times \widetilde{x}(k+j|k) \geqslant 0 \tag{24}$$

where $\widetilde{C}(k) = [C \quad 0], \widetilde{C_c}(k) = [0 \quad C_c]$. Applying the Schur complement, we can get

$$\begin{bmatrix} P & * & * & * \\ P\Phi & P & * & * \\ \widetilde{C} & 0 & \gamma S^{-1} & * \\ \widetilde{C_c} & 0 & 0 & \gamma R^{-1} \end{bmatrix} \geqslant 0 \tag{25}$$

and then, let's define the matrix $D(k)$ and $D(k)^{-1}$ as

$$D(k) = \begin{bmatrix} K(k) & M(k)^T \\ M(k) & V(k) \end{bmatrix}, D(k)^{-1} = \begin{bmatrix} L(k) & O(k)^T \\ O(k) & U(k) \end{bmatrix}$$

where $O(k)$, $M(k)$ are non-singular matrixs, $O(k)^T M(k) + L(k) K(k) = I$ can be obtained by $D(k) D(k)^{-1} = I$. Setting $\Delta_1(k) = \begin{bmatrix} I & L(k) \\ 0 & O(k) \end{bmatrix}$, $\Delta_2(k) = \begin{bmatrix} I & K(k) \\ 0 & M(k) \end{bmatrix}$. By multiplying the left side of (25) by $\mathrm{diag}\{ \Delta_1^T, \Delta_1^T D^{-1}, I, I \}$ and the right side of its transposed matrix, meanwhile setting $\widehat{A} = K A_i L + M P_c C_i L + K B_i R_x O + M H_c O$, $\widehat{B} = M P_c$, $\widehat{C} = R_x O$, after a series of simplifications we can get (19). That is, (19) can ensure the robust stability of the system (16).

(2) The conditions of updating the bound of the state. It is the most appropriate method to replace the unknown state with the bound of the state. Then, there exists $x_o(k), P_e(k)$ and satisfies $e(k) = x(k) - x_o(k) \in \Omega_{P_e(k)}$, where $G = -K^{-1} M^T$, $x_o(k) = G x_c(k)$, and $e(k)$ is the error between the real state and the controller state.

Note that $V = M \left(K - L^{-1} \right)^{-1} M^T$, we get

$$\begin{aligned} \widetilde{x}(k)^T P \widetilde{x}(k) =& x(k)^T K x(k) + x_c(k)^T M x_c(k) + x(k)^T M^T \\ & \times x_c(k) + x_c(k)^T M \left(K - L^{-1} \right)^{-1} M^T x(k) \\ =& e(k)^T K e(k) + x_c(k)^T \times \\ & \left[G^{-1} \left(L - K^{-1} \right) G^{-T} \right]^{-1} x_c(k) \\ =& [x(k) - x_o(k)]^T K [x(k) - x_o(k)] + \\ & x_o(k)^T \left(L - K^{-1} \right)^{-1} x_o(k) \end{aligned} \tag{26}$$

Then there is

$$\begin{aligned} & [x(k) - x_o(k)]^T W [x(k) - x_o(k)] + \\ & x_o(k)^T \left(H - W^{-1} \right)^{-1} x_o(k) \leqslant 1 \end{aligned} \tag{27}$$

We assume that $[x(k) - x_o(k)]^T W [x(k) - x_o(k)] \leqslant \tau$, then get (21) and (22) after derivation. At the sampling point, $x(k+1|k) \in \Omega_P$ holds when the optimization problem has an optimal solution, in turn, we can get

$$\begin{aligned} & e(k+1)^T K^*(k) e(k+1) + x_c(k+1)^T \times \\ & \left[G(k)^{-1} \left(L^*(k) - K^*(k)^{-1} \right) G(k)^{-T} \right]^{-1} x_c(k+1) \leqslant 1 \end{aligned} \tag{28}$$

Let $G(k+1) = G(k)$, $K(k+1) = K^*(k)$, $L(k+1) = L^*(k)$, $x_o(k+1) = G(k+1) x_d(k+1)$, $P_e(0)$ is given, and $P_e(k)$ can be obtained at each sampling time, it is shown that

$$\begin{aligned} & P_e(k+1) = K(k+1) \times \\ & \left[1 - x_o(k+1)^T \left(L(k+1) - K(k+1)^{-1} \right)^{-1} x_o(k+1) \right]^{-1} \end{aligned} \tag{29}$$

(3) The conditions of input constraint. Combined with the augmented closed-loop model (16), we can obtain

$$\begin{aligned} \max_{i \geqslant 0} \| u(i|k) \|_2^2 &= \max_{i \geqslant 0} \left\| \begin{bmatrix} 0 & R_x(k) \end{bmatrix} P^{-\frac{1}{2}} P^{\frac{1}{2}} \widetilde{x}(k) \right\|_2^2 \\ &\leqslant \max_{i \geqslant 0} \left\| \begin{bmatrix} 0 & R_x(k) \end{bmatrix} P^{-\frac{1}{2}} \right\|_2^2 \left\| P^{\frac{1}{2}} \widetilde{x}(k) \right\|_2^2 \\ &\leqslant \max_{i \geqslant 0} \left\| \begin{bmatrix} 0 & R_x(k) \end{bmatrix} P^{-\frac{1}{2}} \right\|_2^2 \end{aligned} \tag{30}$$

By multiplying the left side of (26) by $\mathrm{diag}\{ \Delta_1^T, I \}$ and the right side of its transposed matrix, then combining the Schur complement when simplifying, (23) can be obtained. Simultaneously, (23) can satisfy the condition (20).

IV. Simulation Verification

In this section, regarding the DOFMPC algorithm proposed before, we will verify the performance of the designed dynamic output feedback predictive controller through the Simulink/CarSim co-simulation platform, and compare the tracking effect with the other two algorithms.

The work content of CarSim mainly includes the docking setting with simulink, the setting of simulation working conditions and the setting of various parameters of the vehicle itself.

The work content of Simulink mainly focuses on the writing of S function. Fig 2 is an example of Simulink/CarSim co-simulation based on vehcle kinematics model.

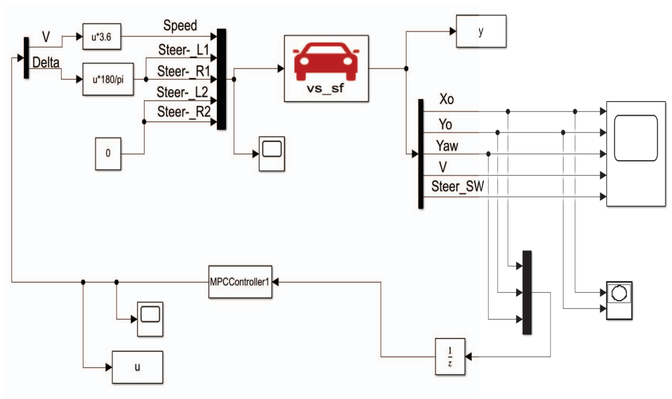

Fig. 2. Co-simulation platform of CarSim and Simulink

The initial position of autonomous driving vehicle is the coordinate origin, and the reference track is a circle with a radius of $25m$. We set the front wheel deflection angle δ_f to $0.1040rad$, the sampling time T to $0.05s$, and the wheelbase l to $2.65m$. The reference speed v_r is set in the specific program and the other coefficients are set in the CarSim interface, then the following matrix can be obtained

$$A_1 = \begin{bmatrix} 1 & 0 & -0.10v_r \\ 0 & 1 & 0.10v_r \\ 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 & 0.10v_r \\ 0 & 1 & 0.10v_r \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1 & 0 & -0.10v_r \\ 0 & 1 & -0.10v_r \\ 0 & 0 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 0 & 0.10v_r \\ 0 & 1 & -0.10v_r \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.10 & 0 \\ 0.10 & 0 \\ 0.004 & 0.04v_r \end{bmatrix}, B_2 = \begin{bmatrix} -0.10 & 0 \\ 0.10 & 0 \\ 0.004 & 0.04v_r \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.10 & 0 \\ -0.10 & 0 \\ 0.004 & 0.04v_r \end{bmatrix}, B_4 = \begin{bmatrix} -0.10 & 0 \\ -0.10 & 0 \\ 0.004 & 0.04v_r \end{bmatrix}.$$

In addition, we need to set the following parameters, $S = 0.1$, $R = 1$, $D_e(0) = diag\{32.3; 10; 10\}$, $G(0) = I$, and $x_c(0) = [5 \quad 10 \quad 5]^{\mathrm{T}}$. Among them, appropriate initial value is crucial in the whole process of simulation verification. Some other parameters are set under reasonable conjecture as follows: $C = diag\{1; 1; 1\}$, $\overline{u}_{\max} = 0.455$.

Now comparing the DOFMPC method proposed in the third chapter with the traditional MPC method and LQR method at different speeds. Fig 3 shows that autonomous driving vehicle using the three methods can track a given trajectory at three different speeds, indicating that the algorithm proposed in this paper has a good control effect. The tracking trajectory at different speeds and the reference trajectory are represented by dashed lines in different colors, respectively. Fig 4 tells us that after using the traditional MPC algorithm and the DOFMPC algorithm, the front wheel steering angle of the vehicle fluctuates greatly at the beginning, but it is always within the limit afterwards, and finally stabilizes. However, when the LQR algorithm is used at 3m/s and 5m/s, we can see that the front wheel rotation angle fluctuates very frequently in the partial enlargement, and even produces huge fluctuation at 8m/s, which is very unfriendly to the vehicle and affects the ride comfort. As shown in Fig 5, we set the initial speed to $0m/s$ in the process of tracking the reference speed, DOFMPC algorithm can track the reference speed faster, the traditional MPC algorithm has large fluctuations in the tracking process, while the LQR algorithm can not complete the tracking task when the speed is slightly faster.

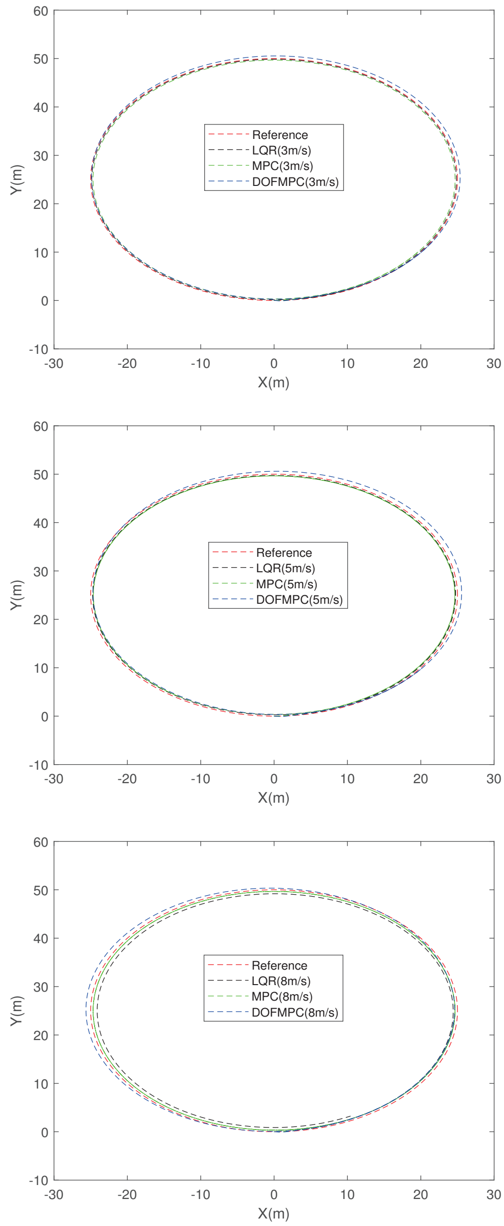

Fig. 3. Trajectory tracking at 3m/s, 5m/s and 8m/s.

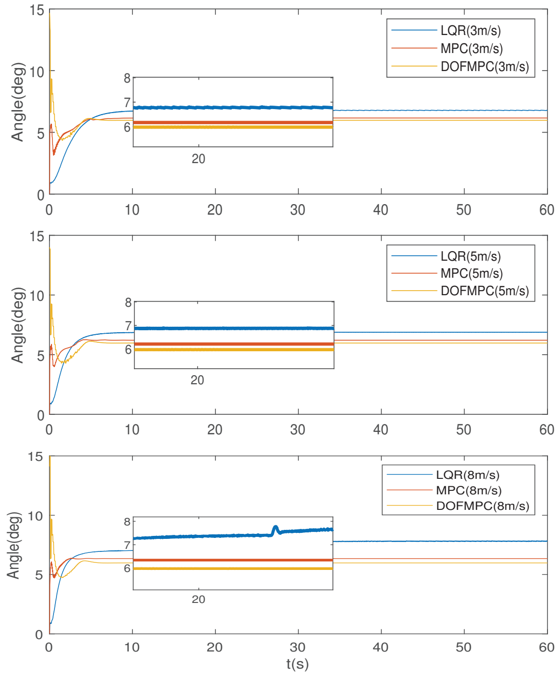

Fig. 4. The front wheel steer angle at 3m/s, 5m/s and 8m/s.

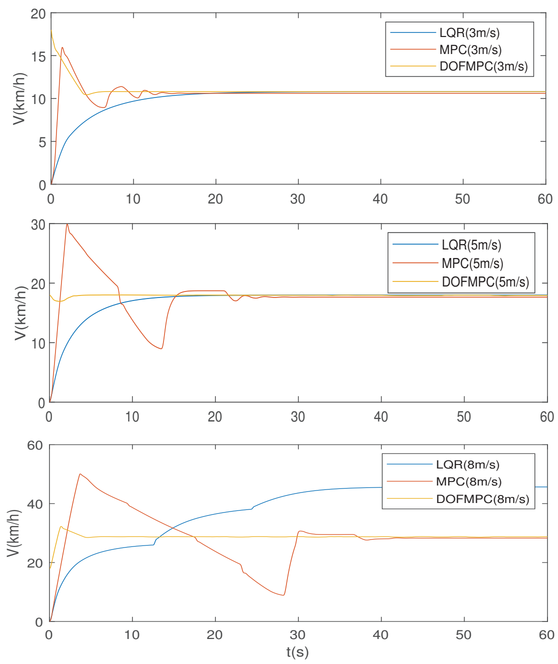

Fig. 5. Speed tracking at 3m/s, 5m/s and 8m/s.

V. Conclusions

This paper studied the application of robust constrained DOFMPC to the trajectory tracking problem of autonomous driving vehicles. The DOFMPC algorithm seeks to solve the constrained optimization problem in the real-time at the sampling point, and finally, the effective solution is regarded as the control sequence. The availability and reliability of the presented algorithm is confirmed by Carsim/Simulink co-simulation platform. This algorithm is capable of tracking a given reference trajectory effectively, and through comparison with the other two algorithms, it is found that it can better complete the trajectory tracking task. Furthermore, we will attempt to apply this method to actual vehicles to do more detailed and in-depth research on other control problems of autonomous driving vehicles.

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