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Trajectory Tracking Control of an Autonomous Vehicle using Model Predictive Control and PID Controller

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Abstract—**Over the years, there has been a substantial increase in the number of vehicular traffic, which has led to vital problems like car crashes and congestion. More than 90 percent of collisions are the result of human error. Technology that allows for autonomous driving has the potential to enhance traffic efficiency and safety. Based on knowledge about the nearby traffic, an autonomous vehicle can create a trajectory and follow it using control algorithms. A significant technology in the study and implementation of autonomous vehicles is trajectory tracking control. Paths are a series of instructions that provide directional directives to get to a specific location, whereas a trajectory includes the schedule of velocity and higher order words, such as acceleration in terms of the body's longitudinal and lateral motion, that are necessary to reach there. In this study, PID controllers and model predictive controllers (MPC) are used to govern the trajectory of an autonomous vehicle. The performance of the autonomous vehicle using both the controllers are then compared. The work is validated using simulations on MATLAB simulink.**

Index Terms—**Autonomous, MPC, PID, Trajectory**

I. Introduction

An autonomous vehicle is one that can sense the environment and move safely with no human aid. Such vehicle uses a variety of sensors to perceive its surroundings, like, radar, lidar, sonar, Global Positioning System and thermographic cameras. Advanced control systems interpret sensory information to determine appropriate navigation paths and to avoid obstacle collision. Self-driving cars are used for commercial purposes such as robotaxis, shuttles, and delivery vehicles. Semi-autonomous vehicles are able to keep in lane and also be able to park themselves, but they are not self-driving.

The use of autonomous cars has the potential to greatly increase the effectiveness of current road transportation systems. It includes reducing pollution and energy use, increasing traffic density in a safe manner, and is currently the subject of intensive development and research [1]. The main area of research for these vehicles is the creation of reliable and computationally practical control frameworks that ensure the trajectory guidance of the vehicles is free from collisions while taking into consideration the limitations of the current road boundaries, other objects, and traffic regulations [2].

The robotics discipline contains the vast majority of work on trajectory planning and guidance. In order to generate collision-free paths under static and dynamic constraints, various algorithms are presented [2]. A trajectory incorporates the specifications of velocity and higher order words necessary to reach the final destination, like acceleration in terms of both the body's longitudinal and lateral motion. A path is a set of instructions that provides directional orders to get somewhere [3].

Three aspects characterize the present state of the art in planning techniques. The first approach is the sampling-based approach, which randomly explores the state space via lattices and effective deterministic searching methods. The second group uses decoupling techniques to plan the route globally and determine the speed necessary to avoid obstacles locally [4]. The third category of planning algorithms includes formulations for mathematically restricted optimization that provide some certainty of the solution's conditional existence and optimality based on the convexity of the problem definition and the accuracy of the initial hypothesis [3].

Model predictive control (MPC), which is a part of the third group of planning strategies, provides a simple and effective technique for replanning that can adapt to dynamic changes in the surroundings of the controlled vehicle through regular updates to the implementation of its receding horizon [4]. Model predictive control is a highly calculated optimization control technique. Model predictive control has been used to follow the trajectory of smart automobiles due to its clear benefits in handling constraints as digital technology and hardware processing capabilities have advanced [2].

The use of MPC for control of vehicle dynamics has been studied extensively. Collision avoidance (CA) for (semi) autonomous systems using a single control input, such as active front steering, is the subject of certain research [5]. Lower level trajectory tracking also uses MPC [6]. A streamlined trajectory planning module and a trajectory tracking mod-

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ule that employs highly accurate nonlinear vehicle dynamics models in the MPC are combined in some works. These models need a lot of vehicle-specific information, including tyre information, vehicle mass, and inertia characteristics. Within the MPC, linearized models are frequently employed to cut down on processing requirements[6]. With such linearization, the execution time can be minimized, but these presumptions may not always be true in dynamic situations or for vehicles requiring integrated lateral and longitudinal control over extended prediction horizons; the results are, at best, inadequate [7].

This work does a comparison study about the trajectory tracking control of the vehicle using two different controllers such as PID controller and MPC. Through this work MPC proves to be a better controller than PID from the various simulation results obtained.

The following is how the document is set up. The vehicle's kinematic model is shown in Section 2. The controller design is presented in Section 3, and the simulation results are shown in Section 4. Section 5 presents the work's conclusion.

II. Mathematical Modeling of Vehicle

A. Kinematic Bicycle Model

A linear kinematic bicycle model that takes the longitudinal, lateral, and yaw motions of a vechicle is taken into account. The kinematic bicycle model of the vehicle is depicted schematically in Fig. 1 which is similar to as in [1]. ϕ is

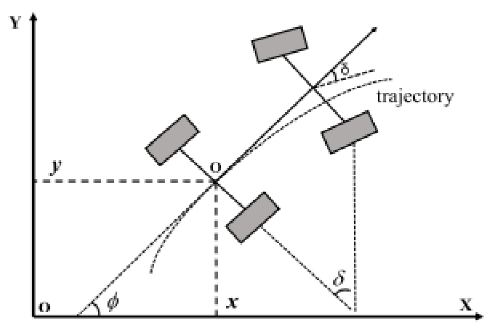

Fig. 1. Vehicle Kinematic Model

the vehicle's orientation, and x and y in Fig. 1 represent the halfway location of the rear axle of the vehicle in inertial coordinates. A system with the equation $\chi = \begin{bmatrix} x & y & \phi \end{bmatrix}^T$ and control variables u = $\begin{bmatrix} v & \delta \end{bmatrix}^T$ are defined according to the three degrees of freedom vehicle dynamics model shown above.

$$\dot{x} = vcos\phi \tag{1}$$

$$\dot{y} = vsin\phi \tag{2}$$

$$\dot{\phi} = \frac{vtan\delta}{l} \tag{3}$$

The differential equations can be written as:

$$\dot{\chi} = f_1(\chi, u) \tag{4}$$

The desired trajectory can be defined as:

$$\dot{\chi}_r = f_1(\chi_r, u_r) \tag{5}$$

where, χ_r is the refernce and u_r is the refernce control output. An approximate value is derived by expanding the Taylor series around the desired point as

$$\dot{\chi} = f_1(\chi_r, u_r) + \frac{\partial(\chi, u)}{\partial \chi}(\chi - \chi_r) + \frac{\partial(\chi, u)}{\partial u}(u - u_r) \tag{6}$$

Combining Eq.(6) and Eq.(7),

$$\dot{\overline{\chi}} = \begin{bmatrix} \dot{x} - \dot{x_r} \\ \dot{y} - \dot{y_r} \\ \dot{\phi} - \dot{\phi_r} \end{bmatrix} + \begin{bmatrix} 0 & 0 & -vsin\phi_r \\ 0 & 0 & vcos\phi_r \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x - x_r \\ y - y_r \\ \phi - \phi_r \end{bmatrix} + \begin{bmatrix} cos\phi_r & 0 \\ sin\phi_r & 0 \\ \frac{tan\phi_r}{l} & \frac{v_r}{lcos^2\delta} \end{bmatrix} \begin{bmatrix} v - v_r \\ \delta - \delta_r \end{bmatrix} \tag{7}$$

where v_r and δ_r are the reference velocity and reference front wheel angle and x_r, y_r, and ϕ_r are the reference x axis position, reference y axis position, and reference heading angle. For a time interval T, the system in Eq. (8) is discretized as

$$\overline{\chi}(k+1) = A_1(k,t)\overline{\chi}(k) + B_1(k,t)\overline{u}(k) \tag{8}$$

with

$$A_{1k,t} = \begin{bmatrix} 1 & 0 & -vsin\phi_r T \\ 0 & 1 & vcos\phi_r T \\ 0 & 0 & 1 \end{bmatrix} \quad B_{1k,t} = \begin{bmatrix} cos\phi_r T & 0 \\ sin\phi_r T & 0 \\ \frac{tan\phi_r T}{l} & \frac{v_r T}{lcos^2\delta} \end{bmatrix}$$

$$\overline{\chi}(k) = \begin{bmatrix} x(k) - x_r(k) \\ y(k) - y_r(k) \\ \phi(k) - \phi_r(k) \end{bmatrix} \quad \overline{u}(k) = \begin{bmatrix} v(k) - v_r(k) \\ \delta(k) - \delta_r(k) \end{bmatrix}$$

By selecting a new state variable vector with the formula $\xi(k|t) = [\overline{\chi}(k|t)]$, an enhanced model is created. The added state variable results in the enhanced model shown below:

$$\xi(k+1|t) = \overline{A_1}_{k,t}\,\xi(k|t) + \overline{B_1}_{k,t}\,\Delta u(k|t) \tag{9}$$

$$\eta(k) = C_{1k,t}\,\xi(k|t) \tag{10}$$

$$\overline{A_1}_{k,t} = \begin{bmatrix} A_{1k,t} & B_{1k,t} \\ O_{m*n} & I_m \end{bmatrix} \tag{11}$$

$$\overline{B_1}_{k,t} = \begin{bmatrix} B_{1k,t} \\ I_m \end{bmatrix} \tag{12}$$

$$C_{1k,t} = [I_n, ..., O_{n*m}] \tag{13}$$

where the dimensions of the state and control variables, respectively, are m and n.

III. Controller Design

A. Proportional-Integral-Derivative Controller

A PID controller is designed initially to guide the car along a desired path so that the vehicle's response may be studied. Assume about the challenge of moving towards the (x_r, y_r) target point. The error in postion of the vehicle is mathematically represented by Eq.(14).The vehicle's speed must be managed proportionally to the distance it travels from the target

$$e = \sqrt{(x - x_r)^2 + (y - y_r)^2} \tag{14}$$

$$v = k_p e + k_i \int e dt + k_d \frac{\mathrm{d}e}{\mathrm{d}t} \tag{15}$$

and to steer toward the goal which is at the vehicle-relative angle

$$\phi_r = tan^{-1} \frac{y_r - y}{x_r - x} \tag{16}$$

using a proportional controller

$$\delta_r = k_h(\phi_r - \phi) \tag{17}$$

which turns the steering wheel toward the target.The control inputs of the vehicle are the longitudinal velocity v and the steering angle δ.The desired outputs are the postion coordinates x,y and ϕ. The PID controller is tuned to suitable gains so as to sucessfully track the reference path. Fig.2 represents

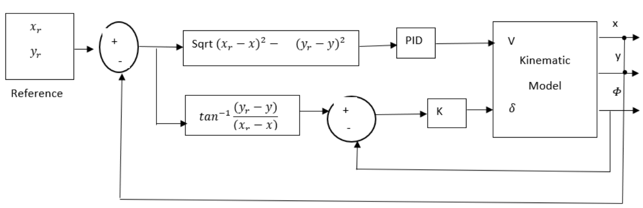

Fig. 2. Control Block diagram

the control block diagram in which a PID controller is used to follow the reference path of the vehicle by reducing the error.The input velocity of the vehicle is obtained by Eq.(15) and the other input,which is the steering angle is obtained by Eq.(17).The controller gains, k_p, k_i, k_d are obtained by tuning is shown in the table below.

The Ziegler-Nichols or Cohen-Coon approaches are suitable for the linearized models. Despite the fact that these techniques are intended for linear models, they can nevertheless serve as a solid foundation for fine-tuning a PID parameters for a nonlinear system.The parameter gains are optimized using these techniques so as to satisfy the output requirement.When system's nonlinearities are considerable and cannot be sufficiently compensated by a PID controller, more sophisticated control approaches, such as adaptive control, fuzzy logic control, or model predictive control are employed.

B. MPC Controller

The system's mathematical model is employed to estimate the future behaviour of the vehicle over a limited period of time. The controller then resolves an optimisation problem to determine the ideal control actions that, given the system and control input constraints, optimize a cost function over the prediction horizon. The system is then subjected to the initial control action, and the procedure is repeated at the following time step using more recent data and projections.

Let it be assumed that $\overline{A_1}_{k,t} = \overline{A_1}_{t,t}$ and $\overline{B_1}_{k,t} = \overline{B_1}_{t,t}$. On the basis of receding state space model $(\overline{A_1}_{k,t}, \overline{B_1}_{k,t}, \overline{C_1}_{k,t})$,the future control parameters are used to generate the predicted state variables sequentially:

$$Y_1(t) = \varphi_{1t}\xi(t|t) + \Theta_{1t}\Delta U(t) \tag{18}$$

with

$$Y_1(t) = \begin{bmatrix} \eta(t+1|t) \\ \eta(t+2|t) \\ .... \\ \eta(t+N_p|t) \end{bmatrix} \quad \varphi_1(t) = \begin{bmatrix} \overline{C_1}_{t,t}\overline{A_1} - t,t \\ \overline{C_1}_{t,t}\overline{A_1}^2_{t,t} \\ .... \\ \overline{C_1}_{t,t}\overline{A_1}^{N_p}_{t,t} \end{bmatrix}$$

$$\Theta_{1t} = \begin{bmatrix} \overline{C_1}_{t,t}\overline{A_1}_{t,t} & 0 & .. & 0 \\ \overline{C_1}_{t,t}\overline{A_1}_{t,t}\overline{B}_{t,t} & \overline{C_1}_{t,t}\overline{B} & .. & 0 \\ .. & .. & .. & .. \\ \overline{C_1}_{t,t}\overline{A_1}^{N_p-1}_{t,t}\overline{B}_{t,t} & \overline{C_1}_{t,t}\overline{A_1}^{N_p-2}_{t,t}\overline{B}_{t,t} & .. & 0 \end{bmatrix}$$

$$\Delta U(t) = \begin{bmatrix} \Delta u(t|t) \\ \Delta u(t+1|t) \\ .... \\ \Delta u(t+N_c-1|t) \end{bmatrix}$$

The cost function is defined as follows for the path control of the vehicle in control system:[1]

$$J_1(k) = \sum_{i=1}^{N_p} \|\eta(t+i|t) - \eta_{ref}(t+i|t)\|_Q^2 + \sum_{i=1}^{N_c} \|\Delta U(t+i|t)\|_R^2 \tag{19}$$

The first term in Eq. (20) deals with the goal of reducing the difference between the predicted output points and the desired trajectory points, while the second term deals with the size of the $\Delta U(t)$ being taken into consideration. Cost function is minimised as follows to determine the ideal δU:

$$J_1(\xi(t), u(t-1), \Delta U) = \Delta U(t)^T H_{1t} \Delta U(t) + G_{1t} \Delta U(t) \tag{20}$$

$$H_{1t} = \Theta^T Q \Theta + R \tag{21}$$

$$G_{1t} = 2E_1^T{}_t Q \Theta_t \tag{22}$$

$$\eta_{ref}(t+i|t) = [x_p(i), y - p(i), \phi_p(i)]^T \tag{23}$$

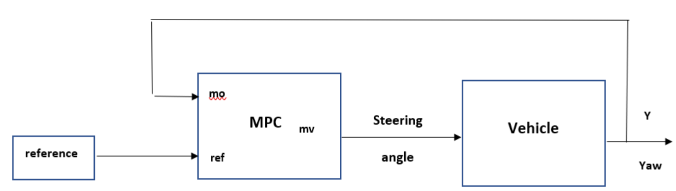

Fig. 3. Block Diagram of MPC

The input and output constraints considered for the designing of the controller are represented as below:

$$-1 \leq X \leq 1 \tag{24}$$

$$-1 \leq y \leq 1 \tag{25}$$

$$-3.14 \leq \phi \leq 3.14 \tag{26}$$

$$-3.14 \leq \delta \leq 3.14 \tag{27}$$

$$0 \leq V \leq 10 \tag{28}$$

Eq.(24) to Eq.(26) are the input constraints and Eq.(27) and Eq.(28) are the output constraints.The weights are applied to the controller for optimization of the cost function.Optimisation techniques like Bryson's rule is used to identify the best weights that minimise the objective function while taking into account the system's constraints and the control inputs . Weights applied are:

$$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 5 \end{bmatrix} \tag{29}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \tag{30}$$

Prediction horizon=10
Control horizon = 2
Sampling time = 0.01s

The prediction horizon needs to be long enough to include all relevant system dynamics, such as delays and transport lags. To reduce significant prediction mistakes and boost the controller's responsiveness, the prediction horizon should also be as short as is feasible.Choosing the prediction horizon as the minimum of the system's settling time and the time horizon over which the disturbances can be predicted is a common practise.

The control horizon should be brief enough to capture the system's near-term dynamics and permit quick changes to the control inputs. The control horizon needs to be long enough to prevent frequent input switching and enable steady-state operation of the system. Choosing the control horizon as a percentage of the prediction horizon, such as 10% to 20%, is a general rule of thumb.

IV. Simulation Results and analysis

The performance of the two controllers are verified using MATLAB Simulink.Table.1 shows the various values of the controller gains used for the tuning the PID controller to track the reference path.

TABLE I
CONTROLLER GAINS

Controller gains	Values
K_p	5.5
K_I	3
K_d	0.5
K	6

Fig.4 shows the tracking of the reference path using a PID controller. An unit circle is used as the reference path for the vehicle in the simulation.The PID controller sucessfully tracks the unit circular refernce path from the initial position (0,0). Fig.5 shows the tracking errors in the postition of the vehicle. Fig.6 shows the tracking of the reference circular path using

TABLE II
VEHICLE PARAMETERS

Total mass of the vehicle(m)	1575kg
Yaw moment of inertia of vehicle(I_z)	$2875mNs^2$
Longitudinal distance from center of gravity of front tires(I_f)	1.2m
Longitudinal distance from center of gravity of rear tires(I_r)	1.6m
Cornering stiffness of front tire (C_{af})	19000N/rad
Cornering stiffness of rear tire(C_{ar})	33000N/rad

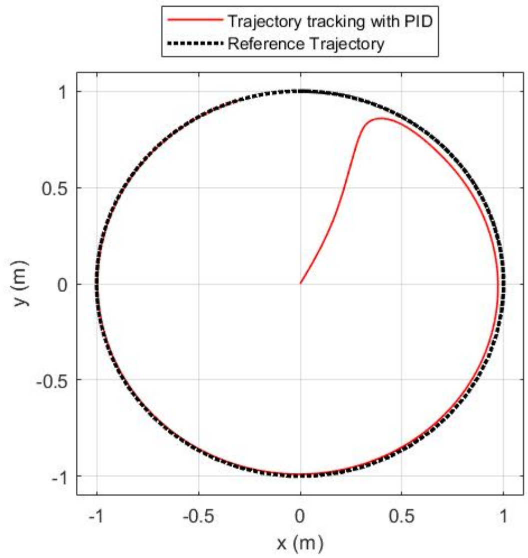

Fig. 4. Trajectory tracking using PID controller

MPC controller. The MPC controller can remove the adverse effects of model simplification and can reduce oscillations. The MPC controller perfectly tracks the reference path with less tracking error as shown in Fig.7.

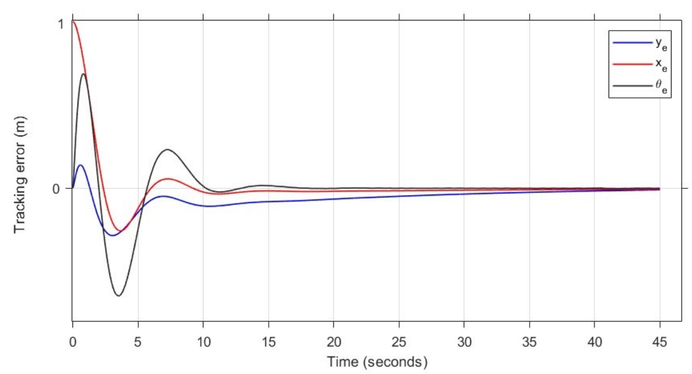

Fig. 5. Trajectory tracking error of PID

Fig.5 and Fig.7 shows that a MPC controller has less tracking errors is compared to a PID controller. Under steady state conditions, the MPC is faster to achieve the set point than the PID controller, and its offsets are also less.

The MPC controller is designed considering various constraints in velocity and steering angle of the vehicle, while PID does not consider such physical constraints while designing

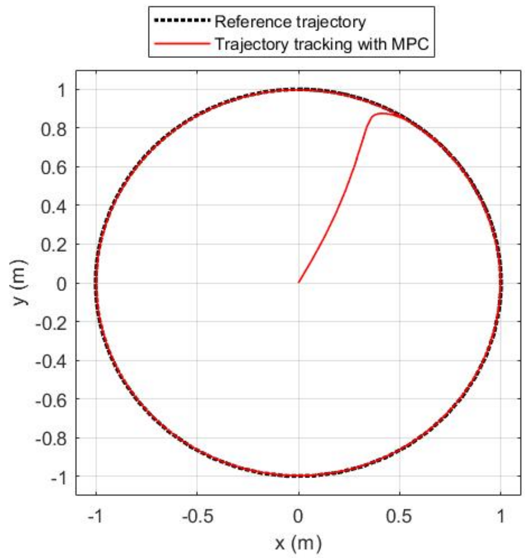

Fig. 6. Trajectory Tracking with MPC

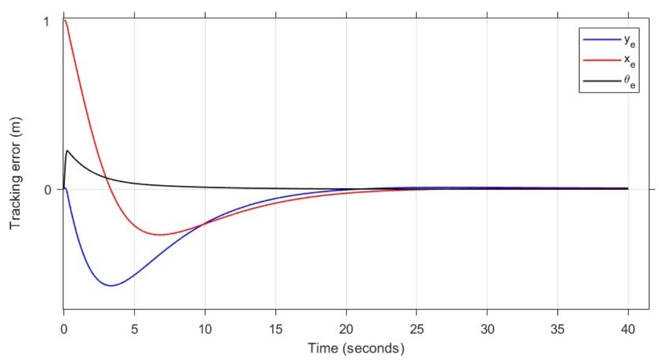

Fig. 7. Trajectory tracking error of MPC

the controller. Fig.8 shows the tracking of the vehicle through the reference defined using way points.On comparing the Fig.9 and Fig.10 tracking error is less for MPC tracking compared to a PID tracking.The tracking error for MPC almost negligible compared to PID.

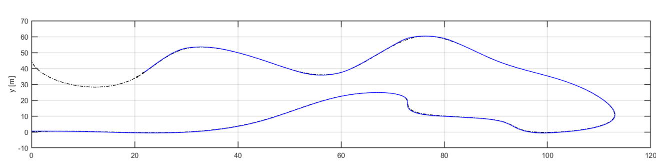

Fig. 8. Vehicle tracking a trajectory

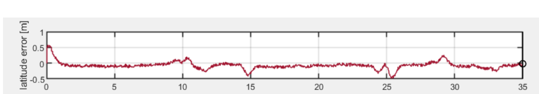

Fig. 9. lattitude error of MPC

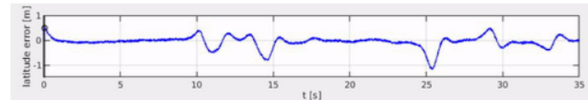

Fig. 10. lattitude error of PID

V. Conclusion

In this work, trajectory tracking of an autonomous vehicle using two different controllers are compared and studied. On validation with the results from simulation, it is known that better response with less error is obtained for MPC rather than PID. The tracking errors are more a PID controller. The monitored planned trajectories using the MPC approach are more precise.The MPC controller considers various constraints in its designing procedure and predicts the behaviour of the system. Considering the dynamic conditions, an MPC controller can track the vehicle more efficiently than the conventional control systems. Collision avoidance and lane keeping assistance are the future scopes of this work.

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