lecture 3 presentation

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oktober 17, 2022

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Section 1

Geometric series

Geometriske serier review - Eksempler på serier

Eksempel 1:

$$\sum_{n=1}^{n} x^n = x + x^2 + x^3 + x^4 + ... + x^{n-1}$$

Eksempel 2:

$$x + \sqrt{x} + 1 + \frac{1}{\sqrt{x}} + \frac{1}{x} ...$$

Typer af geometriske serier

Endelig serie

$\sum_{n=1}^{n} ak^n = a * \frac{1-k^n}{1-k},\ k \neq 1$

Uendelig serie

$\sum_{n=1}^{n} ak^n = \frac{a}{1-k},\ |k| < 1$
$\sum_{n=1}^{n} ak^n = na,\ |k| = 1$

Udregning

Endelig serie

$$S_n = \alpha + \alpha * k + \alpha * k^2 + \alpha * k^3 + ... + \alpha * k^{n-1}$$

$$k * S_n = \alpha * k + \alpha * k^2 + \alpha * k^3 + \alpha * k^4 + ... + \alpha * k^n$$

$$S_n - k * S_n = \alpha + (\alpha * k - \alpha * k) + (\alpha * k^2 - \alpha * k^2) + ... + (\alpha * k^{n-1} - \alpha * k^{n-1}) - \alpha * k^n$$

$$S_n - k * S_n = \alpha - \alpha * k^n$$

$$S_n(1 - k) = \alpha(1 - k^n)$$

$$S_n = \alpha * \frac{1 - k^n}{1 - k}$$

Udregning

Uendelig serie

Hvis $|k| < 1$ når $n \to \infty$ vil udtrykket gå mod:

$$S_n = \alpha * \frac{1}{1-k}$$

Dermed kan vi skrive:

$$\sum_{n=1}^{\infty} = \frac{\alpha}{1-k}$$

Section 2

Difference equations

Math to econometrics

Today we will look at 1. order difference equations

In 2. semester math seen like this:

$$x_t = ax_{t-1} + b$$

In time series econometrics you will see it like this, called an AR(1) process:

$$y_t = \mu + \theta y_{t-1} + \varepsilon_t$$

We will look at what differences between these two equations

Lets first see what they got in common:

- Both equations got a constant μ and b
- Both equations got a coefficient θ and a
- Both equations got a variable that changes over time (discrete) y_t and x_t

Math to econometrics

The difference between the two equations is that in econometrics we got the error term ε_t with the diffenition:

$$\varepsilon_t \sim IID(0, \sigma^2)$$

Explain!

Later we take a look at how this changes things!

Differens equations (Math)

Lets look at the difference equation again:

$$x_t = ax_{t-1} + b_t$$

We can start from a given point x_0

$$x_1 = ax_0 + b_1$$
$$x_2 = ax_1 + b_2 = a(ax_0 + b_1) + b_2 = a^2 x_0 + ab_1 + b_2$$
$$x_3 = ax_2 + b_3 = a(a^2 x_0 + ab_1 + b_2) + b_3 = a^3 x_0 + a^2 b_1 + ab_2 + b_3$$

Differens equations (Math)

We can already see the pattern:

$$x_t = a^t x_0 + \sum_{k=1}^{t} a^{t-k} b_k$$

We now assume the case when $b_k = b$ so we now got a constant as in the AR(1) model.

So we can now write the last term as:

$$\sum_{k=1}^{t} a^{t-k} b$$

Which is a geometric series! that we just covered!

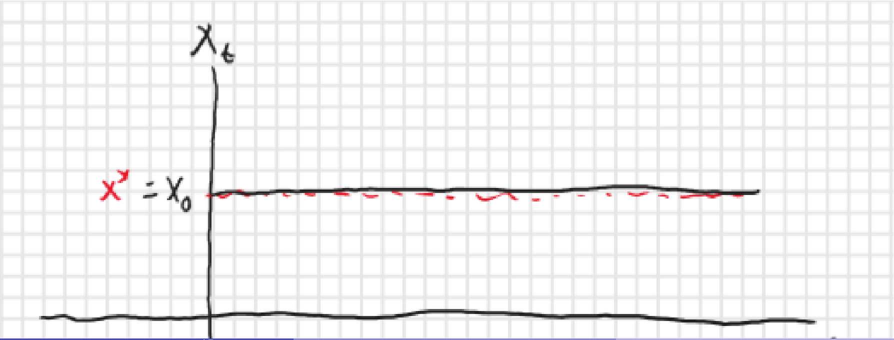
$$\sum_{k=1}^{t} a^{t-k} b = b(a^{t-1} + a^{t-2} + a^{t-3} + \ldots + a + 1) = \frac{(b - ba^t)}{(1-a)}$$

Differens equations (Math)

Therefor we can now write:

$$x_t = a^t(x_0 - \frac{b}{1-a}) + \frac{b}{1-a}$$

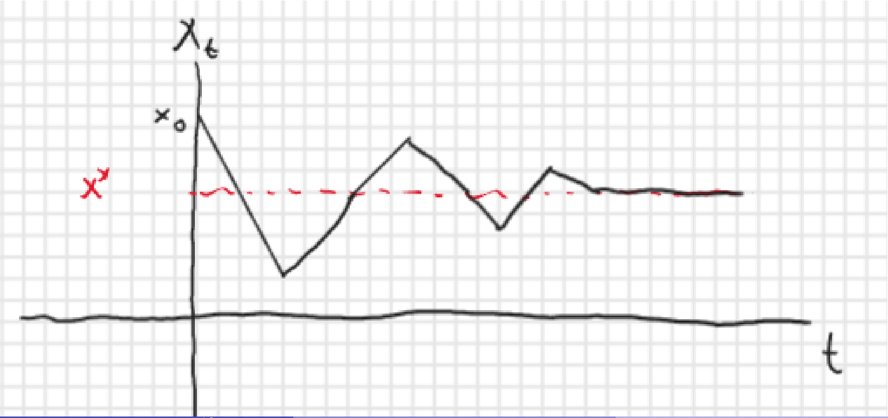
We can see that if $x_0 = \frac{b}{1-a}$ we get that $x_t = \frac{b}{1-a}$ which I have illustrated down below:



Differens equations (Math)

In fact if just x_s at any point hits $\frac{b}{1-a}$ we wont get away from it as:

$$x_{s+1} = a\frac{b}{1-a} + b = \frac{b}{1-a}$$



Difference equations (stability)

Case 1

$$|a| < 1$$

We then see that a^t goes towards 0 as $t \to \infty$ in:

$$x_t = a^t(x_0 - \frac{b}{1-a}) + \frac{b}{1-a}$$

And we will end up with

$$x_t = \frac{b}{1-a}$$

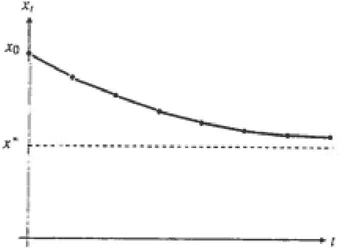
Case 2

$$|a| > 1$$

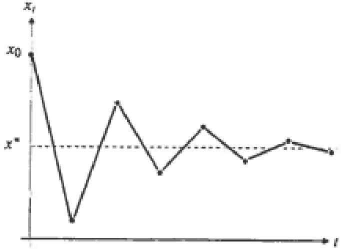
We then see that a^t goes towards ∞ as $t \to \infty$ in and will explode.

But what if x_0 start below and we hit the value it should just stop

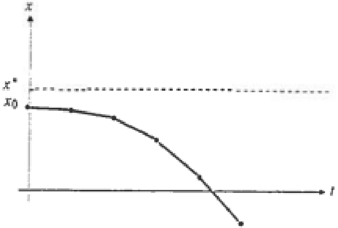
Difference equations (stability)

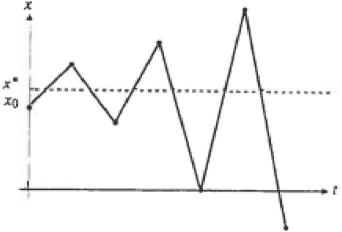


(a) $x_0 > x^* = \frac{b}{1-a}$, $0 < a < 1$



(b) $x_0 > x^* = \frac{b}{1-a}$, $-1 < a < 0$





AR(1) model Econometrics

Lets look at the AR(1) process again:

$$y_t = \mu + \theta y_{t-1} + \varepsilon_t$$

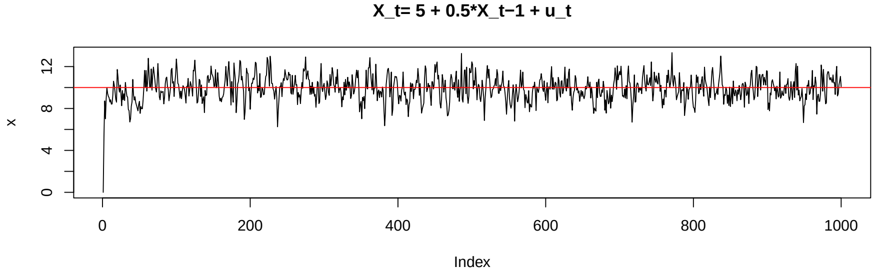
Where the only difference was the error term: ε_t lets see some examples and what the difference is

AR(1) model Exonometrics

To give an example we use the model:

$$y_t = 5 + 0.5y_{t-1} + \varepsilon_t$$

Simulering

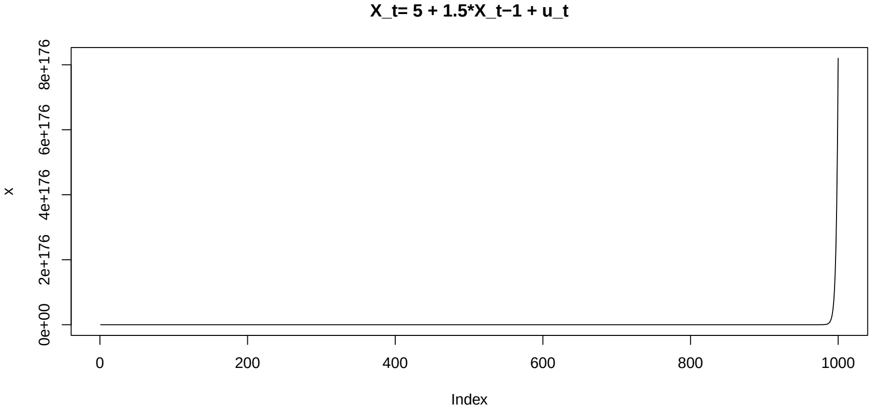


We see the shocks from ε_t does so we never stay in $\frac{b}{1-a}$ as we did with the difference equations before

AR(1) model Exonometrics

$$y_t = 5 + 1.5y_{t-1} + \varepsilon_t$$

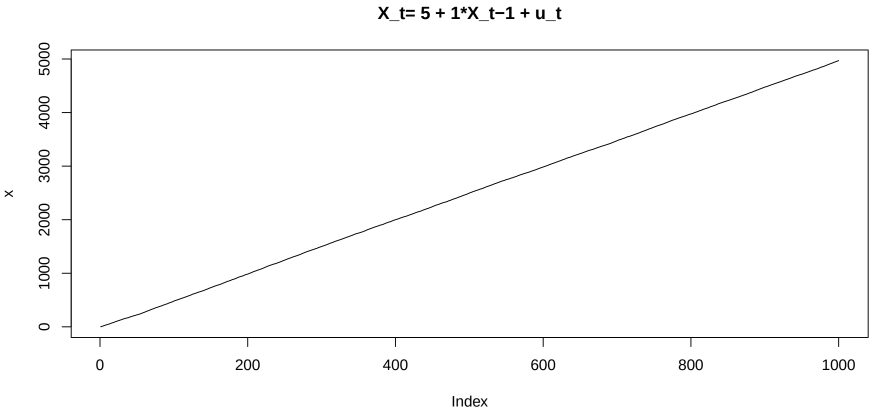
Simulering



AR(1) model Exonometrics

$$y_t = 5 + 1y_{t-1} + \varepsilon_t$$

Simulering



AR(1) model Exonometrics (mean)

We saw before that we never stay the value $\frac{b}{1-a}$ in the AR(1) model, but what if we calculate the mean?

$$\begin{aligned}
E[y_t] &= E[\mu + \theta y_{t-1} + \varepsilon_t] \\
&= \mu + \theta E[y_{t-1}] + E[\varepsilon] \\
&= \mu + \theta E[\mu + \theta y_{t-2} + \varepsilon_{t-1}] \\
&= \mu + \mu\theta + \theta^2 E[y_{t-2}] \\
&= \mu + \mu\theta + \theta^2 E[\mu + \theta y_{t-3} + \varepsilon_{t-2}] \\
&= \mu(1 + \theta + \theta^2 + \theta^3 + \dots + \theta^\infty)
\end{aligned}$$

So back to the geometric series if $|\theta| < 1$ we get $\frac{\mu}{1-\theta}$

AR(1) model Exonometrics (mean)

Lets try calculating the mean using the example from before before

$$y_t = 5 + 0.5y_{t-1} + \varepsilon_t$$
$$E[y_t] = \frac{5}{1 - 0.5} = 10$$

Lets look at the plot again!

Section 3

Statistik opsumering

Expected value

$$E(aX + b) = aE(X) + b$$
$$E(X + Y) = E(X)E(Y)$$

$$E(XY) = E(X)E(Y)$$

- Hvis X og Y er uafhææengige
- Bevis for dette behøves i ik at kende

Variance

$$Var(X) = E[X - E(X)]^2 = E(XX) - E(X)E(X)$$

- Vi kan se hvis $E(X) = 0$ er $Var(X) = E(X^2)$
- Bevis ovenstående

$$Var(aX + b) = a^2 Var(X)$$

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$$

- Bevis ovenstående

$$Var(X + Y) = Var(X) + Var(Y)$$

- Hvis X og Y er uafhængige

Bevis for Var(X)

$$Var(X) = E[X - E(X)]^2 = E(XX) - E(X)E(X)$$

- Løs på tavlen

Bevis for Var(X+Y)

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

Covariance

$$Cov(X,Y) = E[X - E(X)][Y - E(Y)] = E(XY) - E(X)E(Y)$$

- Bevis ovenstående

$$Cov(X,X) = Var(X)$$

$$Cov(aX + b, cY + d) = acCov(X,Y)$$

$$Cov(X + Y, Z) = Cov(X,Z) + Cov(Y,Z)$$

Bevis Cov(X,Y)

$$Cov(X, Y) = E[X - E(X)][Y - E(Y)] = E(XY) - E(X)E(Y)$$

- Løs på tavlen

Section 4

Moving Average process opsumering:

Mean og Varians

$$Y_t = \mu + \varepsilon_t + \alpha\varepsilon_{t-1}$$

$$\varepsilon_t \sim IID(0, \sigma^2)$$

- Calculate mean of Y_t

$$E[Y_t] = E[\mu]E[\varepsilon_t] + E[\alpha\varepsilon_{t-1}]$$

- Vi ved $E[\varepsilon_t] = E[\varepsilon_{t-1}] = 0$

$$E[Y_t] = \mu$$

Mean og varians

- Calculate the variance

$$Var[Y_t] = E[(Y_t - \mu)^2]$$

$$E[(\varepsilon_t + \alpha\varepsilon_{t-1})^2]$$

$$E[\varepsilon_t^2 + 2\alpha\varepsilon_t\varepsilon_{t-1} + \alpha^2\varepsilon_{t-1}^2]$$

$$E[\varepsilon_t^2] + 2\alpha E[\varepsilon_t\varepsilon_{t-1}] + \alpha^2 E[\varepsilon_{t-1}^2]$$

- Hvorfor ved vi $2\alpha E[\varepsilon_t\varepsilon_{t-1}] = 0$?

$$E[\varepsilon_t^2] + \alpha^2 E[\varepsilon_{t-1}^2]$$