Stationarity

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Time Series Econometrics - Økomometri II

Outline

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A General $ARMA(p, q)$ Process

- We can now formulate a more general autoregressive moving average process, ARMA(p, q)

$$Y_t = \mu + \theta_1 Y_{t-1} + ...\theta_p Y_{t-p} + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + ... + \alpha_q \varepsilon_{t-q}$$

$$\varepsilon_t \sim IID(0, \sigma^2).$$

- For nonstationary series, we formulate an $ARIMA\ (p, d, q)$ model.
- For a nonstationary, seasonal time series, we formulate an $ARIMA(p, d, q)\ \times SAR(s)\ \times SMA(s)$ model.

Estimating ARIMA Models

- The goal of ARIMA analysis is a parsimonious representation of the process governing the residual.
- Use only enough AR and MA terms to fit the properties of the residuals correctly.

The Box-Jenkins Methodology

This methodology can be applied to stationary series only. So, you must first deal with unit roots and stochastic seasons.

1. Identification
   1. Address seasonality, s
   2. Determine order of integration, d
   3. Find appropriate values of p and q
2. Estimation
   1. Pure $AR(p)$ models can be estimated (consistently) using OLS, non-linear OLS or maximum likelihood.
   2. Pure $MA(q)$ models can be estimated (consistently) using non-linear OLS or maximum likelihood.
   3. ARIMA models, $AR(p)$ models and $MA(q)$ models can all be estimated (consistently) using non-linear OLS and maximum likelihood.

Outline

The Dickey-Fuller Test

- Model Y_t as an $AR(1)$ process

$$Y_t = \theta Y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim IID(0, \sigma^2).$$

- If $|\theta| < 1$, then the process is stationary.
- If $|\theta| = 1$, then the process exhibits a unit root and the process is nonstationary.
- If $|\theta| > 1$, then the process explodes
- The unit root test examines whether or not $\theta = 1$.
- In practice, we first make the following transformation

$$\begin{aligned} Y_t - Y_{t-1} &= \theta Y_{t-1} - Y_{t-1} + \varepsilon_t \rightarrow \\ \Delta Y_t &= (\theta - 1) Y_{t-1} + \varepsilon_t \\ &= \pi Y_{t-1} + \varepsilon_t \end{aligned}$$

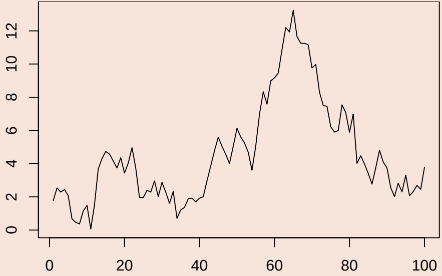
and then test whether or not $\pi = 0$.

The Dickey-Fuller Test

- If $\pi = 0$, then $\theta = 1$ and Y_t has a unit root.
- $H_0 : \pi = 0$
- $H_1 : \pi < 0$.
- Under H_0 the t-value of the estimated coefficient of Y_{t-1} does NOT follow the standard t-distribution. You must compare them with the DF-test statistics like those reported in Table A on p. 488 in Enders (2010) [look at the τ-statistics].

Example: Determining the order of integration - the DF-test

Step 1: Graph the time series, y:



Does this time series look stationary?

Example: Determining the order of integration - the DF-test

Step 2: Implement the DF-test in $\mathbb{R}$ by writing:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(Yt) ~ 0 + L(Yt, 1)))  # 0  means no intercept in the regression


##
## Time series regression with "ts" data:
## Start = 2, End = 100
##
## Call:
## dynlm(formula = diff(Yt) ~ 0 + L(Yt, 1))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8821 -0.4968  0.0519  0.7852  2.1766
##
## Coefficients:
##          Estimate Std. Error t value Pr(>|t|)
## L(Yt, 1)  -0.0124     0.0170   -0.73     0.47
##
## Residual standard error: 0.936 on 98 degrees of freedom
## Multiple R-squared:  0.00541,Adjusted R-squared:  -0.00474
## F-statistic: 0.533 on 1 and 98 DF,  p-value: 0.467
```

- The t–statistic associated with the coefficient in front of the lagged value of y is equal to -0.73

Example: Determining the order of integration - the DF-test

- Compare the value of t–statistic (-0.73) to the appropriate (non-standard) critical values from Table A in Enders (2010).
- The 1% and 5% critical values of τ (the t-statistic in a regression with no constant and no deterministic trend) for a sample size $= 100$ are -2.60 and -1.95, respectively.
- Since the t-value from the regression is **greater than** both of these critical values (i.e., closer to zero) $\rightarrow$

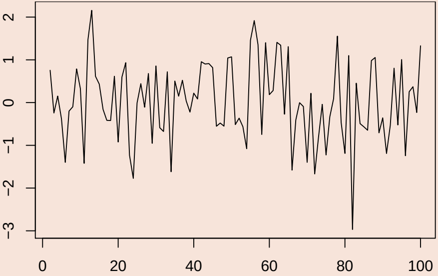
  We can not reject that $\pi = 0$ and that there is a unit root present in this series.
- **Conclusion:** $t > \tau \rightarrow$ do NOT reject $H_0 : \pi = 0$

Example: Determining the order of integration - the DF-test

Step 3: Examine whether y is $I(1)$ or $I(2)$.

Step 4: Graph the time series Δy:

```
Ytd=diff(Yt) # We take the first difference of Yt
```



Does this time series look stationary?

Example: Determining the order of integration - the DF-test

Step 5: Implement the DF-test on the differenced time series in $\mathbb{R}$ by writing:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(Ytd) ~0 + L(Ytd, 1)))


##
## Time series regression with "ts" data:
## Start = 3, End = 100
##
## Call:
## dynlm(formula = diff(Ytd) ~ 0 + L(Ytd, 1))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9510 -0.5573 -0.0463  0.7665  2.1817
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## L(Ytd, 1)   -1.016      0.102   -9.94   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.94 on 97 degrees of freedom
## Multiple R-squared:  0.505,Adjusted R-squared:   0.5
## F-statistic: 98.8 on 1 and 97 DF,  p-value: <2e-16
```

Example: Determining the order of integration - the DF-test

- Examine the regression summary.
- The t–statistic associated with the coefficient in front of the lagged value of y is equal to -9.61.
- Compare this to the appropriate (non-standard) critical values from Table A in Enders (2010). The 1% and 5% critical values of t^*_{nc} (the t -statistic in a regression with no constant and no deterministic trend) for a sample size $= 100$ are -2.60 and -1.95, respectively.
- Since the t-value from the regression is **less than** both of these critical values, we can reject the hypothesis that $\pi = 0$ and that there is a unit root present in this series.

  $t < \tau \rightarrow$ Reject $H_0 : \pi = 0$
- **Conclusion:** Y_t is integrated of order 1, i.e., $I(1)$.

Note: The time series was, in fact, a random walk. It was simulated in $\mathbb{R}$ by writing:

```
set.seed(123123)
Yt <- arima.sim(list(order = c(0, 1, 0)), n = 100)
TT <- 100
Yt <- ts(cumsum(rnorm(TT)))
```

The Dickey-Fuller Test

- Different models have different τ distributions. These values are also reported in Table A in Enders (2010).

$$\Delta Y_t = \delta Y_{t-1} + \varepsilon_t \qquad \text{no intercept, no trend}$$

$$\Delta Y_t = \alpha + \delta Y_{t-1} + \varepsilon_t \qquad \text{intercept, no trend}$$

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \varepsilon_t \qquad \text{intercept, trend}$$

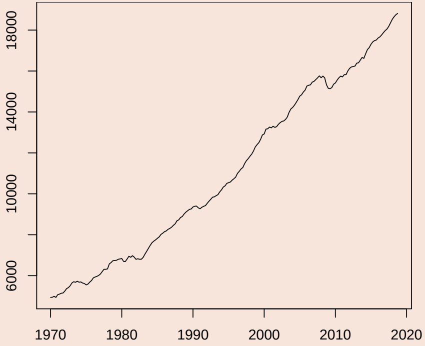
- Note that under H_0 the distribution of the F-test statistic also changes.
- Thes (non-standard) ϕ-test statistics can be found in Table B in Enders (2010).

Example: Testing US GDP for a unit root

- Let us test US quarterly GDP for a unit root:

  Step 1: Graph the time series:

```
# install.packages("Quandl")
library(Quandl)
bnp = Quandl("FRED/GDPC1", start_date="1970-01-01", end_date="2018-12-01", type="ts")
bnp=ts(bnp, start=c(1970,1), end=c(2018,4), frequency = 4) # format as a time series object
```



- Does it look stationary?

Example: Testing US GDP for a unit root

Step 2: Implement the DF-test in $\mathbb{R}$ by writing:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(bnp) ~ 0 + L(bnp, 1)))  # 0  means no intercept in the regression


##
## Time series regression with "ts" data:
## Start = 1970(2), End = 2018(4)
##
## Call:
## dynlm(formula = diff(bnp) ~ 0 + L(bnp, 1))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -434.3  -28.8   11.2   48.0  206.1
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## L(bnp, 1) 0.006081   0.000464    13.1   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 76.2 on 194 degrees of freedom
## Multiple R-squared:  0.469,Adjusted R-squared:  0.467
## F-statistic:  172 on 1 and 194 DF,  p-value: <2e-16
```

- The t–statistic associated with the coefficient in front of the lagged value of y is equal to 13.1

Example: Testing US GDP for a unit root

- Compare the value of t–statistic (13.1) to the appropriate (non-standard) critical values from Table A in Enders (2010).
- The 1% and 5% critical values of τ (the t-statistic in a regression with no constant and no deterministic trend) for a sample size $= 100$ are -2.60 and -1.95, respectively.
- Since the t-value from the regression is **greater than** both of these critical values (i.e., closer to zero) $\rightarrow$

  We can not reject that $\pi = 0$ which means there is a unit root present in the US GDP
- **Conclusion:** $t > \tau \rightarrow$ do NOT reject $H_0 : \pi = 0$

Example: Testing US GDP for a unit root

Step 3: Implement the DF-test with a **constant**, but no trend:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(bnp) ~ 1 + L(bnp, 1)))  # 0  means no intercept in the regression


##
## Time series regression with "ts" data:
## Start = 1970(2), End = 2018(4)
##
## Call:
## dynlm(formula = diff(bnp) ~ 1 + L(bnp, 1))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -425.7  -35.4    4.6   43.2  188.9
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.62036   15.22839    2.27    0.024 *
## L(bnp, 1)    0.00333    0.00130    2.57    0.011 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 75.4 on 193 degrees of freedom
## Multiple R-squared:  0.033,Adjusted R-squared:  0.028
## F-statistic: 6.59 on 1 and 193 DF,  p-value: 0.011
```

- The t-statistic on lagged output is 2.53. The critical value, τ_μ, from Enders' Table A is -2.89 (5%, n = 100).
- Since $t > \tau_\mu$, we can not reject $H_0 : \pi = 0$. Means, we cannot reject the presence of a unit root in US GDP

Example: Testing US GDP for a unit root

Step 4: Implement the DF-test with a **constant** and **trend**:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(bnp) ~ 1 + L(bnp, 1) + trend(diff(bnp))))

##
## Time series regression with "ts" data:
## Start = 1970(2), End = 2018(4)
##
## Call:
## dynlm(formula = diff(bnp) ~ 1 + L(bnp, 1) + trend(diff(bnp)))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -416.0  -34.3    5.6   41.0  193.1
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      117.7058    41.8077    2.82   0.0054 **
## L(bnp, 1)         -0.0192     0.0107   -1.80   0.0729 .
## trend(diff(bnp))   6.7196     3.1533    2.13   0.0344 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 74.7 on 192 degrees of freedom
## Multiple R-squared:  0.0553,Adjusted R-squared:  0.0455
## F-statistic: 5.62 on 2 and 192 DF,  p-value: 0.00423
```

- The t-statistic on lagged output is -1.82. The critical value, τ_μ, from Enders' Table A is -2.89 (5%, n $=$ 100).
- Since $t > \tau_\mu$, we can not reject $H_0 : \pi = 0$. Means, we cannot reject the presence of a unit root in US GDP

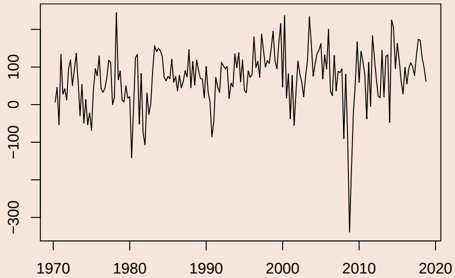
Example: Testing US GDP for a unit root

Conclusion: All three tests tell us that US GDP has at least one unit root.

Step 5: Test if the first difference of GDP has a unit root.

Step 6: Graph the first difference of ΔGDP.

```
Ytd=diff(bnp) # We take the first difference of US GDP
```



Does this time series look stationary?

Example: Testing US GDP for a unit root

Step 7: Implement the DF-test on the differenced data without a const and trend:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(Ytd) ~0 + L(Ytd, 1)))


##
## Time series regression with "ts" data:
## Start = 1970(3), End = 2018(4)
##
## Call:
## dynlm(formula = diff(Ytd) ~ 0 + L(Ytd, 1))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -282.5  -15.4   24.8   73.0  256.2
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## L(Ytd, 1)   -0.337      0.054   -6.24  2.6e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 78.5 on 193 degrees of freedom
## Multiple R-squared:  0.168,Adjusted R-squared:  0.164
## F-statistic:   39 on 1 and 193 DF,  p-value: 2.64e-09
```

- The t-statistic on lagged ΔGDP is -6.26. The critical value, τ_μ, from Enders' Table A is -2.89 (5%, n = 100).
- Since $t < \tau$, we can reject $H_0 : \pi = 0$.
- We can reject the presence of a unit root in ΔGDP.

Example: Testing US GDP for a unit root

Step 8: Implement the DF-test on the differenced data with a const and no trend:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(Ytd) ~1 + L(Ytd, 1)))


##
## Time series regression with "ts" data:
## Start = 1970(3), End = 2018(4)
##
## Call:
## dynlm(formula = diff(Ytd) ~ 1 + L(Ytd, 1))
##
## Residuals:
##    Min      1Q Median      3Q     Max
## -353.1  -35.7    6.3   44.1  196.8
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   45.371      6.999    6.48  7.4e-10 ***
## L(Ytd, 1)     -0.633      0.067   -9.45  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 71.3 on 192 degrees of freedom
## Multiple R-squared:  0.317,Adjusted R-squared:  0.314
## F-statistic: 89.3 on 1 and 192 DF,  p-value: <2e-16
```

- The t-statistic on lagged ΔGDP is -9.47. The critical value, τ_μ, from Enders' Table A is -2.89 (5%, n = 100).
- Since $t < \tau$, we can reject $H_0 : \pi = 0$.
- We reject the presence of a unit root in ΔGDP.

Example: Testing US GDP for a unit root

Step 9: Implement the DF-test on the differenced data with a const and trend:

```
library(dynlm)
summary(dfuller.reg <- dynlm(diff(Ytd) ~1 + L(Ytd, 1) + trend(Ytd)))


##
## Time series regression with "ts" data:
## Start = 1970(3), End = 2018(4)
##
## Call:
## dynlm(formula = diff(Ytd) ~ 1 + L(Ytd, 1) + trend(Ytd))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -366.5  -35.3    4.2   47.8  201.6
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   30.524     10.728    2.85   0.0049 **
## L(Ytd, 1)     -0.658      0.068   -9.68   <2e-16 ***
## trend(Ytd)     0.675      0.371    1.82   0.0706 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 70.9 on 191 degrees of freedom
## Multiple R-squared:  0.329,Adjusted R-squared:  0.322
## F-statistic: 46.8 on 2 and 191 DF,  p-value: <2e-16
```

- The t-statistic on lagged ΔGDP is -9.69. The critical value, τ_μ, from Enders' Table A is -2.89 (5%, n = 100).
- Since $t < \tau$, we can reject $H_0 : \pi = 0$. We reject the presence of a unit root in ΔGDP.

Example: Testing US GDP for a unit root

Conclusion:

- All three tests reject the presence of a unit root (at the 5% level) in ΔGDP. Thus, we can conclude that annual US GDP is $I(1)$.
- This means we first difference of US GDP is stationary

Note: Most macroeconomic variables are I(1), i.e., they are stationary after taking first difference

Exercise:

- Get data for quarterly GDP for Denmark, and perform a unit root test using DF test.

The Augmented Dickey-Fuller (ADF) Test

- The DF-test assumes that ε_t and ε_{t-k} are uncorrelated for all $k > 0$.
- Check the correlogram (using the **acf** and/or **pacf** functions in $\mathbb{R}$) of the residuals from the DF regression to see if this holds or not.
- If this does not hold, we can augment the DF-test by adding additional lagged differences to the test equation.

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \Sigma_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \varepsilon_t.$$

- How does one choose the appropriate lag length k?
  1. Start with a general model (i.e., start with too many lags) and then reduce it through a series of F- and/or t-tests.
  2. Alternatively, one could start by adding lags until the newly added lag is insignificant. The risk with this method is that significant lags can come afterwards, especially if the data has a periodicity less than one year.
     1. Or, just add lags until their is no autocorrelation!
  3. AIC
  4. BIC

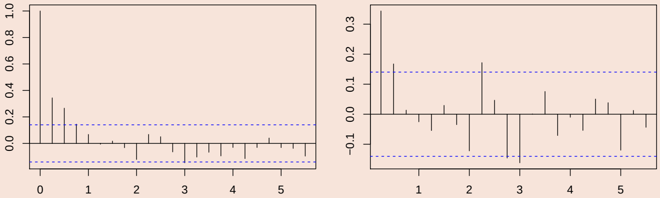
The Augmented Dickey-Fuller (ADF) Test

- My preferred method is to have as few lags as is necessary to eliminate autocorrelation in the residuals. Why? increasing the number of lags lowers the power of the test.
- Keep in mind, however, that with quarterly data (for example) you have to look at 4 lags, 8 lags, etc...not just lags 1, 2, 3, etc. We typically attempt to cover 4 years (12 quarters) for most macro variables (GDP, unemployment, etc.)
- In practice, I always do both. I wouldn't want to publish a paper with results that were overly sensitive to the number of lags included or excluded from this test.

The Augmented Dickey-Fuller (ADF) Test

- Lets check US quarterly GDP whether we our ε_t and ε_{t-k} are uncorrelated
- We can do so by inspecting the **acf** and/or **pacf** of the residuals from our simple DF models.
- Let us just consider a model with const and trend included:

```
library(dynlm)
model<-dynlm(diff(bnp)~1 +  L(bnp,1) + trend(diff(bnp)) )
res=residuals(model)
acf(res);      pacf(res)
```



- Note: the x-axes labels represent quarterly lags, 0 represents time period t, 1 refers to 1 full year (4 quarters, t-4), 2 refers to 2 years (8 quarters, t-8), and so on.
- What do you see? Is ε_t and its lags ε_{t-k} uncorrelated?

The Augmented Dickey-Fuller (ADF) Test

- The correlation between ε_t and its lags ε_{t-k} in time series econometrics goes by the name of "serial correlation".
- Presence of serial correlation does not affect unbiasedness but affects the variance of the model
- So, it is better to use Augmented Dickey-Fuller (ADF) Test to eliminate any correlation ε_t and its lags ε_{t-k}.
- Any evidence of serial correlation implies a systematic movement in the *yt* sequence that is not accounted for by the coefficients included in the model.
- Hence, any of the tentative models yielding nonrandom residuals should be eliminated
- In most econometric models, you will notice that adding lags of the dependent variable to the right hand side of the equation will eliminate serial correlation
- For now, I can proceed to testing GDP for a unit root by running the following model:
$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \Sigma_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \varepsilon_t.$$
- You will notice that Augmented DF test is simply adding additional lags of the dependent variables to the right hand side of the eq.

The Augmented Dickey-Fuller (ADF) Test

Example: Starting from the highest lags

- Since, I have quarterly data, I will typically start with 12th lags

```
library(dynlm)
model <- dynlm(diff(bnp) ~ 1 + L(bnp, 1) + L(diff(bnp), 1:12) + trend(diff(bnp)))
summary(model)
```

```
##
## Time series regression with "ts" data:
## Start = 1973(2), End = 2018(4)
##
## Call:
## dynlm(formula = diff(bnp) ~ 1 + L(bnp, 1) + L(diff(bnp), 1:12) +
##     trend(diff(bnp)))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -320.9  -35.1    3.5   40.1  206.9
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           131.0775    41.3158    3.17  0.00180 **
## L(bnp, 1)              -0.0311     0.0117   -2.65  0.00875 **
## L(diff(bnp), 1:12)1     0.2864     0.0741    3.86  0.00016 ***
## L(diff(bnp), 1:12)2     0.2188     0.0770    2.84  0.00505 **
## L(diff(bnp), 1:12)3     0.0237     0.0781    0.30  0.76147
## L(diff(bnp), 1:12)4    -0.0388     0.0769   -0.50  0.61423
## L(diff(bnp), 1:12)5    -0.0260     0.0757   -0.34  0.73169
## L(diff(bnp), 1:12)6     0.0805     0.0755    1.07  0.28836
## L(diff(bnp), 1:12)7    -0.0310     0.0757   -0.41  0.68269
## L(diff(bnp), 1:12)8    -0.1799     0.0756   -2.38  0.01850 *
## L(diff(bnp), 1:12)9     0.2078     0.0760    2.73  0.00694 **
## L(diff(bnp), 1:12)10    0.1324     0.0768    1.72  0.08642 .
## L(diff(bnp), 1:12)11    0.0777     0.0758    1.02  0.30727
```

The Augmented Dickey-Fuller (ADF) Test

- The ADF test can easily be performed in the R urca package:
  - ADF test for model with constant and trend

```
library(urca)
model <- ur.df(bnp, lags = 12, type = "trend")
# summary(model) # Check that the results are similar to our model on
# the previous slide
```

  - ADF test for model with constant and no trend

```
library(urca)
model <- ur.df(bnp, lags = 12, type = "drift")
# summary(model)
```

  - ADF test for model with no constant and no trend

```
library(urca)
model <- ur.df(bnp, lags = 12, type = "none")
# summary(model)
```

- Once you have started with 12 lags, you can start eliminating insignificant lags step by step (preferably removing one lag at a time). You can also confirm this reduction by checking AIC and BIC of the model
- Remember you MUST start by removing lags of the highest order
- Once you have an appropriate model, the critcal values for ADF test are available in R output of the test (these are τ statistics reported for 1, 5, and 10 percent)

The Augmented Dickey-Fuller (ADF) Test

- Performing ADF test in R:

```
library(dynlm)
model <- dynlm(diff(bnp) ~ 1 + L(bnp, 1) + L(diff(bnp), 1:12) + trend(diff(bnp)))
```

Since, 12th lag is significant at 10%, we can start by removing this lag

```
model <- dynlm(diff(bnp) ~ 1 + L(bnp, 1) + L(diff(bnp), 1:11) + trend(diff(bnp)))
```

- You can select the appropriate number of lags using this strategy.
- Our objective is to have a model that is clean of serial correlation with minimum possible lags
- Once you have a model in which ε_t and its lags ε_{t-k} are uncorrelated, you can test your null hypothesis by looking at the t value and comparing it with the τ from Enders (these values are also reported by R output of the test)
- Like before if $t < \tau$, we can reject $H_0 : \pi = 0$. We reject the presence of a unit root
- If you suspect the model should have a trend then you should perform the test by including trend and intercept

The Augmented Dickey-Fuller (ADF) Test

Example: Starting with the less number of lags

- We can also start from less number of lags and keep on adding lags until serial correlation is removed
- To make sure there is no serial correlation in your model, you can visualise your models to select the best model based on Ljnug-Box test[a]
- You need to construct the following function in your R:

```
arpdiag <- function(series, lags) {
    x <- mat.or.vec(lags, 1)
    y <- mat.or.vec(lags, 1)
    y <- y + 0.05
    for (i in 1:lags) {
        x[i] <- Box.test(series, lag = i, type = "Ljung-Box")$p.value
    }
    plot(x, xlab = "Lags", ylab = "p-value H0: no
Autocorrelation", type = "p",
        main = "Ljung-Box Test for
Autocorrelation", ylim = c(0, 1))
    axis(1, 1:lags)
    abline(0.05, 0, lty = 2, col = "blue")
}
```

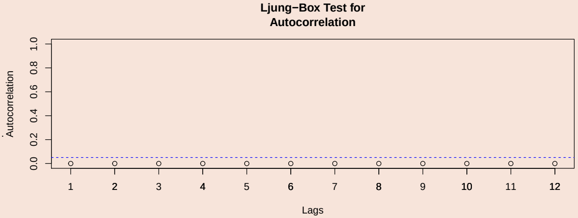
[a] This a test for serial correlation, which we will cover in more detail later.

The Augmented Dickey-Fuller (ADF) Test

Example: Starting with the less number of lags

- Let me start by adding zero lags to the DF test and see if we have any serial correlation in the residuals

```
model1<-ur.df(bnp,lags=0,type='trend')
res<- model1@testreg$residuals # this is  just a way of extracting residuals of your eq. when you use ur.
arpdiag(res, 12)
```



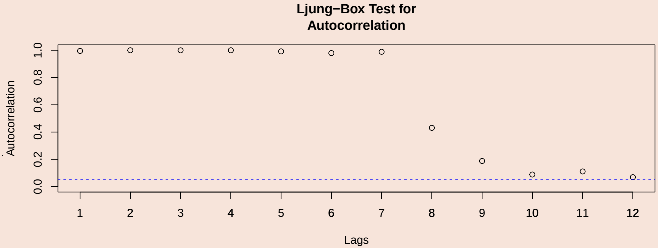
- I can see severe serial correlation in this model. What should I do next?

The Augmented Dickey-Fuller (ADF) Test

Example: Starting with the less number of lags

- If you love working with data, you can start adding one lag at a time
- I am a bit impatient, so I will just add 4 lags and re-test the model:

```
model1<-ur.df(bnp,lags=4,type='trend')
res<- model1@testreg$residuals
arpdiag(res, 12)
```



- I am happy with this model. I can proceed to obtaining the t-statistics from this model and compare it with the critical values from Enders.

The Augmented Dickey-Fuller (ADF) Test

- If you find sufficient evidence to believe that your time series has a unit root, you should take its first difference
- You should then repeat the whole process to re-test for a unit root.
- Let us assume that my series US GDP is stationary after first-differencing, i.e, I(1). Can we say whether it can be considered as a pure random walk or a random walk with drift?

$$Y_t = \alpha_0 + Y_{t-1} + \varepsilon_t$$

Taking first difference of the above eq.

$$\Delta Y_t = \alpha_0 + \varepsilon_t$$

So, if $\alpha = 0$, we have a pure random walk process. If $\alpha \neq 0$ then we have a process that is best described by a random walk with drift

- We can simply test this by running a regression of our stationary series on a constant

```
# summary(model<-dynlm(diff(bnp)~1))
```

Exercise

- Perform Augmented-DF (ADF) test on Danish GDP
- Make sure there is no serial correlation in your model when you compare your t statistics values with the critical values in Enders
- Make sure you increase the power of the test by including as less number of lags as possible (you can either start with minimum no. of lags and keep adding lags OR you can start with the highest number of lags and start reducing step by step.)
- What is you conclusion regarding unit root?
- Which stochastic process best describes the Danish GDP, random walk with drift or a pure random walk? What is your educated guess before even performing the test?

The Phillips-Perron Test

- In the DF-test we assume that ε_t and ε_{t-k} are uncorrelated for all $k > 0$.
- The ADF-test dealt with this potential problem by added lagged values of the dependent variable to the left hand side of our regression, i.e., we added $\Sigma_{i=1}^{m} \gamma_i \Delta Y_{t-i}$.
- The PP-test deals with this potential problem using nonparametric statistical methods which takes care of serial correlation without added lagged differences.
- The **PP.test** in $\mathbb{R}$ uses the Newey-West estimator of the variance-covariance matrix.
- The **PP.test** in $\mathbb{R}$ estimates the DF style equation with a constant and a time trend.
- The **PP.test** in $\mathbb{R}$ maintains $\mathrm{H}_0 : \theta = 1$, while the alternative is $\mathrm{H}_1 : \theta < 1$.

The Phillips-Perron Test

- Simulate a random walk process and use the PP-test for a unit root:

```
TT <- 200
x = rnorm(TT)
Yt <- ts(cumsum(x))
PP.test(Yt)
```

```
##
##  Phillips-Perron Unit Root Test
##
## data:  Yt
## Dickey-Fuller = -2.6, Truncation lag parameter = 4, p-value =
## 0.3
```

- A p-value greater than 0.10 means we can not reject $H_0 : \theta = 1$.

The Phillips-Perron Test

- Simulate a white noise process and use the PP-test for a unit root:

```
TT <- 200
x = rnorm(TT)
Yt = ts(x)
PP.test(Yt)
```

```
##
##  Phillips-Perron Unit Root Test
##
## data:  Yt
## Dickey-Fuller = -15, Truncation lag parameter = 4, p-value =
## 0.01
```

- Thus, we can reject the unit root hypothesis at the 5% level.

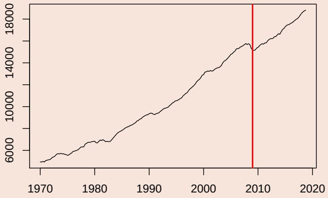
Testing for Unit Roots in the Presence of Structural Breaks

- Structural breaks (in otherwise stationary series) introduce bias in standard unit root towards accepting the unit root hypothesis. Why?
- Pfaff (2008, pp. 110-112) demonstrates the implementation of the Zivot-Andrews unit root test in $\mathbb{R}$. The name of the test is `ur.za()`. It can be found in a package named `urca` along with a large number of other unit root tests.
- Enders (2010, pp. 227-234) discusses the problem of structural change and discusses Perron's (1989) test for unit roots in the presence of structural breaks.
- We will return to the topic of structural breaks in a later lecture.

Testing for Unit Roots in the Presence of Structural Breaks

- You should perform unit root structural break test when you suspect a structural break in the data
- Let us look at US GDP again:

```
plot(bnp)
abline(v=2009, lwd=2, col="red")
```



- I can clearly see there is a structural break in 2008-09 due to financial crisis

Testing for Unit Roots in the Presence of Structural Breaks

- Testing US GDP using unit root in the presence of structural break

```
library(urca)
test <- ur.za(bnp, model = c("both"), lag=12)
# summary(test)
```

- You should select the appropriate lag length following the strategy discussed earlier
- Like ADF test, you can select three types of model (no intercept and no trend, intercept and no trend, both trend and intercept)
- If there is a structural break, it is recommended to perform this test and compare your results with ADF test
- Unfortunately, ZA test only detects one structural break in the data
- It is quite possible that there might be more than one structural break, you can use unit root tests with multiple structural breaks (see, e.g., GLS unit root tests)

Exercise

- Perform PP test on Danish GDP.
- Perform unit root structural break on Danish GDP
  - Is there a break in trend?
  - Is there a break in intercept?
  - Is there a unit root in the data?